From Rotating-Frame Time Holonomy to Dynamical Phase Memory

A Finite-Stiffness Phase-Locked Ring Model with an Operational Clock Readout

S. M. H. Emamifar

Spokesperson

Independent Research Collaboration on

Black Hole and Cosmology Concepts (IRCBHC)

ORCID: 0009-0007-6257-0163

July 2026

Technical Research Note – Version 1.2 (Dynamical Clarifications)

Abstract

A clock transported arbitrarily slowly around a directed closed path on a rotating body retains a finite proper-time difference relative to a co-rotating reference clock. For one ideal eastward equatorial loop on Earth, the exact slow-transport limit is

$$\Delta\tau_{\text{slow,east}} = -\gamma_{\oplus}\frac{2\pi\Omega R^2}{c^2} \simeq -207.386\,\text{ns}.$$

This is a standard relativistic Sagnac/slow-clock-transport holonomy; it is additive in winding number and introduces no new physics. The present work then constructs a logically separate finite-stiffness phase-locked ring model and asks a narrower dynamical question: under what conditions can a reversible loop holonomy be associated with a persistent change in a local phase rate? The model contains a proper circumference, an independent proper wavenumber, a wound Tick phase, and a lighthouse/reference phase. It yields an operational clock count and the exact logarithmic phase-rate equation

$$\frac{\mathrm{d}\ln F}{\mathrm{d}n} = \frac{2\pi}{\omega}\left(\frac{\dot{Q}_\Theta}{Q_\Theta} - \frac{\dot{I}_\Theta}{I_\Theta}\right),$$

where F is the Tick-phase rate, Q_Θ its conjugate charge, I_Θ its effective inertia, and n the completed rotation count. A closed conservative uniform branch reduces

to a phase pendulum and has zero secular average phase pumping when I_Θ is constant. Therefore a multiplicative phase drift is not a consequence of Sagnac physics alone: it requires a changing phase inertia, non-adiabatic drive, an open reservoir, radiation, persistent phase lag, or a structural transition. The ideal loop interval corresponds to approximately 1906.42 cycles of the defining caesium-133 transition; this is a metrological conversion, not an additional model parameter. The article separates standard results, derived effective-model results, conditional regimes, and open tasks, and concludes with explicit energy, angular-momentum, numerical, and falsification requirements. Cosmological applications are intentionally deferred until the phase-memory coefficient is dynamically calculated.

Keywords: Sagnac effect; slow clock transport; rotating frame; time holonomy; finite stiffness; phase locking; operational time; Tick phase; phase memory; angular-momentum transfer.

Claim discipline. Part I is standard relativistic physics. The finite-stiffness ring is an effective collective model. The operational time count is a definition internal to that model. A non-zero secular phase-memory coefficient is an open dynamical result, not an assumption established by the Sagnac calculation. No claim concerning cosmic expansion, the CMB, or dark energy is made in this article.

Contents

1 Introduction **4**

2 Logical Status of the Construction **5**

3 Standard Slow-Clock Holonomy on a Rotating Earth **5**
3.1 Idealised equatorial setup 5
3.2 Leading-order decomposition 6
3.3 Exact slow-transport limit 6
3.4 Westward loop 7

4 Area Form, Winding, and the Sagnac Ledger **7**
4.1 Three distinct terrestrial numbers 8

5 The Boundary Between Holonomy and Phase Memory **8**

6 Finite-Stiffness Phase-Locked Ring **9**
6.1 Collective variables and preferred state 9
6.2 Relativistic geometric embedding 10
6.3 Adiabatic elimination of the proper wavenumber 10
6.4 Large-stiffness branch and radial accommodation 11
6.5 Local radial stability 12

7 Tick Phase, Lighthouse Phase, and the Notation Dictionary **13**
7.1 Closed reference oscillator and open-reservoir extension 14
7.2 Illustrative closures for the Tick-phase inertia 14

8 Operational Time as a Net Phase Count **15**

9 Exact Phase-Rate Evolution **16**

10 Conservative No-Drift Theorem **17**

11 Conditions for Secular Phase Memory **18**

12 Angular Momentum, Torque, and Radial Response **19**

13 Differential–Algebraic Evolution System **21**

14 Energy Accounting **22**

15 Corrected Calibration Ledger and Caesium-Cycle Conversion **22**

16 Decisive Tests and Falsification Criteria **24**

17 Discussion **25**

18 Conclusion **25**

A One Eastward Equatorial Loop Completed in One Year **26**

B Continuous and Discrete Forms of a Constant Logarithmic Drift **26**

C Correction of the Former 542-Cycle Entry **27**

D Textual Signature and Project Attribution **27**

1 Introduction

Relativistic clock comparisons on rotating platforms contain both local rate effects and global path-dependent effects. A clock fixed on the terrestrial equator has the familiar special-relativistic kinematic deficit associated with the equatorial speed $V = \Omega R$. A clock transported once around the equator relative to the ground additionally samples a directed closed path. In the slow-transport limit, the traveller's separate quadratic speed contribution vanishes, while the rotation–transport cross term remains finite. The resulting one-loop value is approximately 207 ns for Earth.

The first objective of this article is to derive this result cleanly and distinguish three frequently conflated quantities: the stationary equatorial kinematic deficit, the one-direction slow-clock holonomy, and the two-direction Sagnac interval.

The second objective is not to reinterpret the standard result as a stationary daily clock drift. Instead, we construct an effective phase-locked rotating system and identify the exact mathematical condition under which its internal phase rate can acquire a secular change. This distinction is essential:

$$\boxed{\text{additive loop holonomy} \quad \neq \quad \text{multiplicative phase-rate evolution.}} \tag{1}$$

The former follows from standard relativity. The latter must emerge, if at all, from a dynamical equation and an identifiable energy and angular-momentum reservoir.

The programme developed here is deliberately modular. The rotating-frame result remains valid even if the phase-memory model fails. Conversely, the phase model is assessed by stability, conservation laws, numerical evolution, and observable dimensionless comparisons rather than by analogy alone.

2 Logical Status of the Construction

Table 1: Logical status of the principal statements.

Statement	Status	Meaning
Slow-clock loop holonomy	Standard	Derived from the proper-time integral in flat spacetime using a rotating platform.
Additivity under repeated winding	Standard	Repeating the same oriented loop multiplies the holonomy by the winding number.
Finite-stiffness energy $U(C, q)$	Effective-model result	Defines a collective model with a preferred proper circumference and proper wavenumber.
Strong-locking radial map	Derived on a selected branch	Recovered as the large-stiffness leading solution, not imposed as a permanent rigid constraint.
Tick charge and Tick angular momentum	Derived in the collective phase sector	The independent wound phase carries a conjugate charge and an angular-momentum contribution.
Operational time count	Definition	Specifies the clock readout used by the model.
Secular phase memory	Open	Requires a non-zero averaged logarithmic phase-rate change.
Constant compound drift	Conditional regime	Follows only when the per-cycle logarithmic rate is approximately constant.
Cosmological interpretation	Deferred	Requires a derived non-zero drift and a complete observable map.

3 Standard Slow-Clock Holonomy on a Rotating Earth

3.1 Idealised equatorial setup

We idealise the Earth as a uniformly rotating body with equatorial radius R and angular velocity Ω. Both clocks remain at the same radius and gravitational potential. The traveller completes one full directed revolution relative to the surface and returns to the reference clock.

Using the WGS 84 equatorial radius and conventional Earth angular velocity [1],

$$R = 6\,378\,137\,\mathrm{m}, \tag{2}$$

$$\Omega = 7.292\,115 \times 10^{-5}\,\mathrm{rad/s}, \tag{3}$$

$$V = \Omega R \simeq 465.10\,\mathrm{m/s}, \tag{4}$$

$$L = 2\pi R \simeq 40\,075\,016.7\,\mathrm{m}. \tag{5}$$

In an Earth-centred approximately inertial frame, the co-rotating reference clock has proper-time rate

$$\frac{\mathrm{d}\tau_{\mathrm{ground}}}{\mathrm{d}t} = \sqrt{1 - \frac{V^2}{c^2}}. \tag{6}$$

For a small eastward coordinate-speed increment $u > 0$, the traveller has speed $V + u$ and

$$\frac{\mathrm{d}\tau_{\mathrm{trav}}}{\mathrm{d}t} = \sqrt{1 - \frac{(V+u)^2}{c^2}}. \tag{7}$$

The coordinate duration required to cover one circumference relative to the surface is

$$T = \frac{L}{u}. \tag{8}$$

The accumulated proper-time difference is therefore

$$\Delta\tau(u) = \frac{L}{u}\left[\sqrt{1 - \frac{(V+u)^2}{c^2}} - \sqrt{1 - \frac{V^2}{c^2}}\right]. \tag{9}$$

3.2 Leading-order decomposition

For $V, u \ll c$,

$$\sqrt{1 - \frac{v^2}{c^2}} = 1 - \frac{v^2}{2c^2} + \mathcal{O}(c^{-4}). \tag{10}$$

Substitution into equation (9) gives

$$\boxed{\Delta\tau_{\mathrm{east}} = -\frac{VL}{c^2} - \frac{uL}{2c^2} + \mathcal{O}(c^{-4}).} \tag{11}$$

The two contributions are algebraically distinct:

$$\Delta\tau_{\mathrm{loop}} = -\frac{VL}{c^2} = -\frac{2\pi\Omega R^2}{c^2}, \tag{12}$$

$$\Delta\tau_{\mathrm{trav}} = -\frac{uL}{2c^2}. \tag{13}$$

As $u \to 0^+$, the traveller's additional quadratic term vanishes, but the directed-loop term remains.

3.3 Exact slow-transport limit

Let the traveller's angular velocity be $\Omega + \delta\Omega$. Completing one revolution relative to Earth requires

$$T = \frac{2\pi}{\delta\Omega}. \tag{14}$$

Define

$$f(\Omega) = \sqrt{1 - \frac{R^2\Omega^2}{c^2}}. \tag{15}$$

Then

$$\Delta\tau(\delta\Omega) = 2\pi \frac{f(\Omega + \delta\Omega) - f(\Omega)}{\delta\Omega}. \tag{16}$$

Taking the one-sided limit,

$$\lim_{\delta\Omega \to 0^+} \Delta\tau = 2\pi f'(\Omega) = -\gamma_\oplus \frac{2\pi\Omega R^2}{c^2}, \tag{17}$$

where

$$\gamma_\oplus = \frac{1}{\sqrt{1 - \Omega^2 R^2/c^2}}. \tag{18}$$

Numerically,

$$\boxed{\Delta\tau_{\text{slow,east}} \simeq -207.386\,\text{ns}.} \tag{19}$$

The correction from $\gamma_\oplus - 1$ is negligible at terrestrial precision.

3.4 Westward loop

For westward transport the leading cross term reverses sign:

$$\Delta\tau_{\text{west}} = +\frac{VL}{c^2} - \frac{uL}{2c^2} + \mathcal{O}(c^{-4}), \tag{20}$$

so

$$\boxed{\lim_{u \to 0^+} \Delta\tau_{\text{west}} \simeq +207.386\,\text{ns}.} \tag{21}$$

4 Area Form, Winding, and the Sagnac Ledger

The rotational velocity field is

$$\boldsymbol{V}(\boldsymbol{r}) = \boldsymbol{\Omega} \times \boldsymbol{r}. \tag{22}$$

At leading order the directed slow-transport contribution along a path element is

$$\mathrm{d}(\Delta\tau_{\text{loop}}) = -\frac{\boldsymbol{V} \cdot \mathrm{d}\boldsymbol{r}}{c^2}. \tag{23}$$

For a closed oriented curve C,

$$\Delta\tau_{\text{loop}}[C] = -\frac{1}{c^2} \oint_C (\boldsymbol{\Omega} \times \boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{r}. \tag{24}$$

Stokes' theorem gives

$$\boxed{\Delta\tau_{\text{loop}}[C] = -\frac{2\boldsymbol{\Omega} \cdot \boldsymbol{A}_C}{c^2} + \mathcal{O}(c^{-4}),} \tag{25}$$

where $\boldsymbol{A}_C$ is the directed area enclosed by the loop.

If the same curve is traversed N times with the same orientation,

$$\boxed{\Delta\tau_{\text{loop}}[C^N] = N\Delta\tau_{\text{loop}}[C].} \tag{26}$$

This is an additive winding law. It is geometric and path dependent, but not a strictly topological invariant because it varies continuously with directed area.

4.1 Three distinct terrestrial numbers

Table 2: Terrestrial rotating-time quantities that must not be conflated.

Quantity	Approximate value	Meaning
Stationary equatorial kinematic deficit	103.7 ns per sidereal day	Ordinary velocity time dilation of a ground clock relative to a hypothetical non-rotating clock at the same potential.
One-direction slow-clock holonomy	207.386 ns per directed equatorial loop	The surviving cross term for one transported clock compared with a co-rotating reference.
Two-direction Sagnac interval	414.8 ns	Difference between oppositely directed light or signal paths around the equatorial loop.

The classic Sagnac interferometric interval is [2, 5]

$$\Delta t_{\text{Sagnac}} = \frac{4\boldsymbol{\Omega}\cdot\boldsymbol{A}}{c^2}, \tag{27}$$

twice the magnitude of the one-direction slow-clock term at leading order. Related corrections are operationally essential in global navigation and time transfer [5].

5 The Boundary Between Holonomy and Phase Memory

The standard result establishes

$$\Delta\tau_N = N\Delta\tau_1. \tag{28}$$

This does not imply

$$F_N = F_0(1+r)^N \tag{29}$$

for a stationary material clock or local phase standard. Additive holonomy describes the result of repeated closed transport comparisons. Multiplicative evolution describes a change of the rate or baseline on which later changes act.

To keep the two notions separate, define the Sagnac-derived dimensionless benchmark

$$\epsilon_{\mathrm{Sag}} \equiv \frac{|\Delta\tau_{\mathrm{loop}}|}{T_\oplus} = \frac{\Omega^2 R^2}{c^2}, \qquad T_\oplus = \frac{2\pi}{\Omega}. \tag{30}$$

This is a benchmark ratio, not yet the phase-memory coefficient of a material system.

The dynamical quantity of interest will instead be

$$\bar{\epsilon}_\phi \equiv \left\langle \frac{\mathrm{d}\ln F}{\mathrm{d}n} \right\rangle. \tag{31}$$

A future identification

$$\bar{\epsilon}_\phi \stackrel{?}{=} \epsilon_{\mathrm{Sag}} \tag{32}$$

would require a derivation from the phase dynamics and the measurement protocol. It is not assumed in this article.

6 Finite-Stiffness Phase-Locked Ring

6.1 Collective variables and preferred state

Let

$$C(t) = C_{\mathrm{proper}}(t) \tag{33}$$

be the instantaneous proper circumference, and let $q(t)$ be a gauge-invariant proper wavenumber. The preferred configuration satisfies

$$q_0 C_0 = \mathcal{N}, \qquad C_0 = 2\pi\rho, \tag{34}$$

where $\mathcal{N}$ is the effective winding quantity and ρ is the preferred proper-radius parameter.

A minimal positive collective rest-energy functional is

$$\boxed{U(C,q) = U_0 + \frac{\kappa_C}{2}(C - C_0)^2 + \frac{A}{2}(q - q_0)^2 + \frac{B}{2}(qC - \mathcal{N})^2,} \tag{35}$$

with

$$\kappa_C > 0, \qquad A > 0, \qquad B > 0. \tag{36}$$

The A sector selects q_0, the B sector enforces winding–circumference compatibility, and κ_C supplies direct circumferential confinement.

6.2 Relativistic geometric embedding

For stationary rotation at angular velocity ω,

$$C = 2\pi\gamma r, \qquad \gamma = \frac{1}{\sqrt{1-\omega^2 r^2/c^2}}. \tag{37}$$

Solving for the laboratory radius gives

$$\boxed{r(C,\omega) = \frac{C/(2\pi)}{\sqrt{1+\omega^2 C^2/(4\pi^2 c^2)}}.} \tag{38}$$

Define

$$a = \frac{\omega}{2\pi c}. \tag{39}$$

At fixed ω, we adopt the reduced worldline action of a rotating collective object whose comoving rest energy is $U(C,q)$,

$$L_{\mathrm{rot}}(C,q;\omega) = -U(C,q)\sqrt{1-\frac{\omega^2 r^2(C,\omega)}{c^2}} = -\frac{U(C,q)}{\gamma(C,\omega)}. \tag{40}$$

Using equation (38),

$$\gamma(C,\omega) = \sqrt{1+a^2C^2}. \tag{41}$$

Consequently, fixed-ω stationary configurations extremize the positive reduced functional

$$\boxed{\mathcal{F}_\omega(C,q) = \frac{U(C,q)}{\sqrt{1+a^2C^2}}.} \tag{42}$$

This form is therefore not inserted merely because it resembles a Lorentz factor. It follows from the chosen relativistic collective action. It is, however, still an effective-model assumption: a microscopic field theory must reproduce the same reduced action before the construction can be regarded as fundamental.

6.3 Adiabatic elimination of the proper wavenumber

The algebraic elimination of q assumes a separation of time scales. The proper-wavenumber mode is taken to relax much faster than the circumference and rotational variables,

$$\tau_q \ll \tau_C, \tau_\omega, \tau_\Theta, \tau_L. \tag{43}$$

Under this slaving approximation, q remains close to the instantaneous minimum selected by C. The stationarity condition $\partial U/\partial q = 0$ gives

$$\boxed{q_*(C) = \frac{Aq_0 + BC\mathcal{N}}{A + BC^2}.} \tag{44}$$

Substitution yields

$$\boxed{U_{\text{eff}}(C) = U_0 + \frac{\kappa_C}{2}(C - C_0)^2 + \frac{AB(q_0 C - \mathcal{N})^2}{2(A + BC^2)}.} \tag{45}$$

The exact fixed-ω equilibrium equation is

$$\boxed{U'_{\text{eff}}(C) = U_{\text{eff}}(C)\frac{a^2 C}{1 + a^2 C^2}.} \tag{46}$$

Thus $C = C_0$ is not imposed as an exact rigid constraint at finite stiffness. If the scale separation in equation (43) fails, q must be retained as an independent dynamical coordinate and stability must be checked using the full two-variable Hessian rather than $U_{\text{eff}}(C)$ alone.

6.4 Large-stiffness branch and radial accommodation

Near $C = C_0$, write

$$C = C_0 + \delta C. \tag{47}$$

The phase sector contributes

$$\kappa_{\text{ph}} = \frac{ABq_0^2}{A + BC_0^2}, \qquad \kappa_{\text{tot}} = \kappa_C + \kappa_{\text{ph}}. \tag{48}$$

To leading order,

$$\boxed{\delta C = \frac{U_0 a^2 C_0}{\kappa_{\text{tot}}(1 + a^2 C_0^2)} + \mathcal{O}(\kappa_{\text{tot}}^{-2}).} \tag{49}$$

The shift is positive: finite stiffness permits a small outward correction in proper circumference.

With

$$\zeta = \frac{\omega\rho}{c}, \tag{50}$$

the laboratory radius becomes

$$\boxed{r(\omega) = \frac{\rho}{\sqrt{1 + \zeta^2}}\left[1 + \frac{U_0 a^2}{\kappa_{\text{tot}}(1 + \zeta^2)^2} + \mathcal{O}(\kappa_{\text{tot}}^{-2})\right].} \tag{51}$$

The strong-locking leading term is

$$\boxed{r_{\rm lock}(\omega) = \frac{\rho}{\sqrt{1+\omega^2\rho^2/c^2}}.} \tag{52}$$

6.5 Local radial stability

Introduce the dimensionless stiffness

$$\Lambda = \frac{\kappa_{\rm tot} C_0^2}{U_0}. \tag{53}$$

Within the reduced axisymmetric branch, local radial stability requires

$$\boxed{\Lambda > \frac{\zeta^2}{(1+\zeta^2)^2}.} \tag{54}$$

Since

$$0 \le \frac{\zeta^2}{(1+\zeta^2)^2} \le \frac{1}{4}, \tag{55}$$

the simple sufficient condition

$$\boxed{\Lambda > \frac{1}{4}} \tag{56}$$

guarantees local stability in this reduced sector.

The sufficient bound $\Lambda > 1/4$ is deliberately uniform in ζ and is generally much stronger than the local requirement. The function

$$g(\zeta) = \frac{\zeta^2}{(1+\zeta^2)^2} \tag{57}$$

reaches its maximum $g(1) = 1/4$ in the intermediate relativistic regime $\zeta = 1$, and decreases as $g(\zeta) \sim \zeta^{-2}$ for $\zeta \gg 1$. For the terrestrial rotational scale used in Part I, taking $\rho = R$ and $\omega = \Omega$ only as an illustrative comparison gives

$$\zeta_\oplus = \frac{\Omega R}{c} \simeq 1.5514 \times 10^{-6}, \qquad g(\zeta_\oplus) \simeq 2.4069 \times 10^{-12}. \tag{58}$$

Thus the Earth-scale value lies extremely deep inside the weak-rotation part of the stability domain. This numerical comparison does not identify the Earth with the microscopic ring; it only displays the scale dependence of the reduced stability criterion.

7 Tick Phase, Lighthouse Phase, and the Notation Dictionary

The construction uses three angular variables:

$$\Phi(t), \qquad \chi(t), \qquad \lambda(t). \tag{59}$$

Their meanings are:

- Φ: laboratory material angle, with $\omega = \dot{\Phi}$;
- χ: collective zero mode of an internal periodic order parameter;
- λ: phase of a second oscillator, reservoir, or external reference.

The names "Tick" and "lighthouse" are mnemonic labels, not additional physical substances. The Tick phase represents the internal periodic degree of freedom whose accumulated cycles define the proposed clock readout. The lighthouse phase represents a distinct comparison channel that can exchange phase charge with the Tick sector. Depending on a later microscopic realisation, it could correspond to a second material oscillator, a field reservoir, or an idealised external reference. The cosine interaction below is the minimal synchronisation coupling between these two phase variables; it does not by itself specify their microscopic origin.

The wound Tick phase on the ring worldsheet is represented schematically as

$$\Theta(t, \sigma) = N\sigma - \chi(t), \tag{60}$$

where $N \in \mathbb{Z}$ is the winding number. The invariant collective phase-rate combination is

$$\dot{\chi} + N\omega. \tag{61}$$

It is useful to define the accumulated Tick phase

$$\boxed{\vartheta(t) = \chi(t) + N\Phi(t)} \tag{62}$$

and its rate

$$\boxed{F(t) = \dot{\vartheta} = \dot{\chi} + N\omega.} \tag{63}$$

The reduced Tick sector is

$$\mathcal{L}_\Theta = \frac{I_\Theta(C, q)}{2}(\dot{\chi} + N\omega)^2 - V_N(C, q), \tag{64}$$

so the conjugate Tick charge is

$$\boxed{Q_\Theta = I_\Theta(C, q)F.} \tag{65}$$

Because the Tick phase is wound, its angular-momentum contribution is

$$\boxed{J_\Theta = NQ_\Theta.} \tag{66}$$

The lighthouse/reference sector is

$$\mathcal{L}_L = \frac{I_L}{2}\dot{\lambda}^2 - \varepsilon[1 - \cos(\chi - \lambda)]. \tag{67}$$

Define the local dynamical lag

$$\boxed{\delta = \chi - \lambda.} \tag{68}$$

The phase equations are

$$\boxed{\dot{Q}_\Theta = -\varepsilon \sin\delta,} \tag{69}$$

$$\boxed{I_L\ddot{\lambda} = +\varepsilon \sin\delta.} \tag{70}$$

Therefore the closed two-phase sector conserves

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(Q_\Theta + I_L\dot{\lambda}\right) = 0.} \tag{71}$$

7.1 Closed reference oscillator and open-reservoir extension

In equation (67), λ is a dynamical phase coordinate and I_L is its effective phase inertia. The coupling coefficient ε has dimensions of energy and sets both the depth of the locking potential and the maximum reversible phase-transfer torque $|\dot{Q}_\Theta| \leq \varepsilon$. In the closed model, λ is therefore a second oscillator, not a prescribed background function.

A genuine reservoir or externally maintained lighthouse requires extra terms and a corresponding energy ledger. A minimal open extension is

$$\boxed{I_L\ddot{\lambda} + \Gamma_L\left(\dot{\lambda} - \Omega_L\right) = \varepsilon \sin(\chi - \lambda) + \xi_L(t),} \tag{72}$$

where $\Gamma_L \geq 0$ is a phase-damping coefficient, Ω_L is the drive rate maintained by the external sector, and ξ_L represents a specified fluctuating or deterministic forcing. Equation (72) is not used in the closed-branch theorem below. It records the minimum structure required if the lighthouse is to supply or remove net phase charge rather than merely exchange it reversibly.

7.2 Illustrative closures for the Tick-phase inertia

A microscopic derivation of $I_\Theta(C, q)$ remains an open task. Nevertheless, the reduced equations can be closed for analytical and numerical tests by introducing an explicitly

provisional constitutive law. A useful two-exponent family is

$$\boxed{I_\Theta(C,q) = I_{\Theta 0}\left(\frac{C}{C_0}\right)^p \left(\frac{q}{q_0}\right)^s,} \tag{73}$$

where p and s parameterise how the participating phase inertia is distributed over proper length and wavenumber. It implies

$$\boxed{\frac{\dot{I}_\Theta}{I_\Theta} = p\frac{\dot{C}}{C} + s\frac{\dot{q}}{q}.} \tag{74}$$

The simplest distributed-line closure is

$$\boxed{I_\Theta(C) = \mu_\Theta C,} \tag{75}$$

with constant phase inertia per unit proper length $\mu_\Theta > 0$, corresponding to $(p,s) = (1,0)$. This is an illustrative closure, not a microscopic result. It makes the reduced system determinate while preserving a decisive constraint: if C, q, and Q_Θ return to their initial values after a closed cycle, then I_Θ and $F = Q_\Theta/I_\Theta$ also return, so no secular memory is generated. The closure can therefore expose a drift produced by the dynamics, but it cannot manufacture one from a periodic conservative trajectory.

8 Operational Time as a Net Phase Count

The dynamical lag δ is not identical to the accumulated operational slip. Define

$$\Psi_{\mathrm{Tick|ref}} = \vartheta - \alpha\lambda, \tag{76}$$

where α is a dimensionless calibration ratio between the Tick and external-reference phases. It is fixed by a stated calibration protocol, not adjusted independently at every observation. If a reference state $(*)$ is chosen in which the mean operational slip is required to vanish, then

$$\boxed{\alpha = \frac{F_*}{\dot{\lambda}_*}.} \tag{77}$$

Operationally, the two phase standards are co-located during a calibration interval, their nominal angular frequencies are compared, and the resulting frequency ratio is retained as α. If λ is already expressed in Tick-normalised phase units, this convention gives $\alpha = 1$.

Definition 1 (Operational time count)**.** *For two events A and B, define*

$$\boxed{\mathcal{N}_T[A,B] = \frac{1}{2\pi}\left[\Delta\Phi[A,B] - \Delta\Psi_{\mathrm{Tick|ref}}[A,B]\right].} \tag{78}$$

Using equation (76),

$$\boxed{\mathcal{N}_T[A,B] = \frac{1}{2\pi}\left[\Delta\Phi - \Delta\vartheta + \alpha\Delta\lambda\right].} \tag{79}$$

Differentiating along the actual history,

$$\boxed{\dot{\mathcal{N}}_T = \frac{1}{2\pi}\left[\omega - F + \alpha\dot{\lambda}\right] = \frac{1}{2\pi}\left[\omega - \frac{Q_\Theta}{I_\Theta(C,q)} + \alpha\dot{\lambda}\right].} \tag{80}$$

This is the direct bridge from the collective ring variables to the model's clock readout. A dimensional duration is introduced only after choosing a reference period T_* or frequency ν_*:

$$\tau_{\mathrm{op}} = T_*\mathcal{N}_T = \frac{\mathcal{N}_T}{\nu_*}. \tag{81}$$

9 Exact Phase-Rate Evolution

From $F = Q_\Theta/I_\Theta$,

$$\boxed{\frac{\dot{F}}{F} = \frac{\dot{Q}_\Theta}{Q_\Theta} - \frac{\dot{I}_\Theta}{I_\Theta}.} \tag{82}$$

Using equation (69),

$$\boxed{\frac{\dot{F}}{F} = -\frac{\varepsilon}{Q_\Theta}\sin\delta - \frac{\mathrm{d}\ln I_\Theta}{\mathrm{d}t}.} \tag{83}$$

If $I_\Theta = I_\Theta(C,q)$, then

$$\boxed{\frac{\dot{F}}{F} = -\frac{\varepsilon}{Q_\Theta}\sin\delta - \frac{\partial\ln I_\Theta}{\partial C}\dot{C} - \frac{\partial\ln I_\Theta}{\partial q}\dot{q}.} \tag{84}$$

Thus the phase rate can change through two conceptually distinct channels:

(i) transfer of phase charge Q_Θ;

(ii) change of phase inertia I_Θ through geometry or internal structure.

For the provisional power-law closure in equation (73), this becomes

$$\boxed{\frac{\dot{F}}{F} = -\frac{\varepsilon}{Q_\Theta}\sin\delta - p\frac{\dot{C}}{C} - s\frac{\dot{q}}{q}.} \tag{85}$$

This equation provides a concrete simulation family while leaving the microscopic values of p, s, and $I_{\Theta 0}$ open to derivation.

Let the completed material-rotation count be

$$n = \frac{\Phi}{2\pi}, \qquad \dot{n} = \frac{\omega}{2\pi}. \tag{86}$$

For monotonic rotation with $\omega \neq 0$, define the instantaneous phase-memory coefficient

$$\boxed{\epsilon_\phi(n) \equiv \frac{\mathrm{d}\ln F}{\mathrm{d}n}.} \tag{87}$$

Then

$$\boxed{\epsilon_\phi(n) = \frac{2\pi}{\omega}\left[-\frac{\varepsilon}{Q_\Theta}\sin\delta - \frac{\partial \ln I_\Theta}{\partial C}\dot{C} - \frac{\partial \ln I_\Theta}{\partial q}\dot{q}\right].} \tag{88}$$

The exact solution is

$$\boxed{F(n) = F(n_0)\exp\left[\int_{n_0}^{n} \epsilon_\phi(n')\,\mathrm{d}n'\right].} \tag{89}$$

If, over a specified interval,

$$\epsilon_\phi(n) \simeq \bar{\epsilon}_\phi = \text{constant}, \tag{90}$$

then

$$F(n) = F_0 e^{\bar{\epsilon}_\phi n}. \tag{91}$$

For one cycle,

$$F_{m+1} = F_m e^{\bar{\epsilon}_\phi} = F_m(1 + r_\phi), \tag{92}$$

where

$$\boxed{r_\phi = e^{\bar{\epsilon}_\phi} - 1.} \tag{93}$$

Therefore the compound law is not an independent assumption once a constant logarithmic phase-rate evolution has been dynamically established. The unresolved issue is whether the complete system produces such a non-zero average.

10 Conservative No-Drift Theorem

Assume temporarily that I_Θ and I_L are constant. Since

$$Q_\Theta = I_\Theta(\dot{\chi} + N\omega), \tag{94}$$

the lag derivative is

$$\dot{\delta} = \frac{Q_\Theta}{I_\Theta} - N\omega - \dot{\lambda}. \tag{95}$$

Define the reduced phase inertia

$$\mu = \frac{I_\Theta I_L}{I_\Theta + I_L}. \tag{96}$$

Using equations (69) and (70), one obtains

$$\boxed{\mu\ddot{\delta} + \varepsilon \sin\delta = -\mu N\dot{\omega}.} \tag{97}$$

For uniform rotation,

$$\dot{\omega} = 0, \tag{98}$$

and the lag obeys the conservative phase-pendulum equation

$$\boxed{\mu\ddot{\delta} + \varepsilon \sin\delta = 0.} \tag{99}$$

Proposition 1 (No secular pumping in the closed uniform branch)**.** *For a bounded or periodic solution of equation (99), the phase charge returns to its initial value after a complete lag period. Hence*

$$\left\langle \dot{Q}_\Theta \right\rangle = 0. \tag{100}$$

If I_Θ is constant, then

$$\boxed{\left\langle \frac{\dot{F}}{F} \right\rangle = 0.} \tag{101}$$

Proof. The conservative phase-pendulum trajectory is closed in the bounded phase space. Since $\dot{Q}_\Theta = -\varepsilon \sin\delta$, integration over one complete period returns Q_Θ to its initial value. With constant I_Θ, $F = Q_\Theta/I_\Theta$ also returns to its initial value, so the cycle average of $\mathrm{d}\ln F/\mathrm{d}t$ vanishes. □

This result is decisive: the cosine coupling alone permits reversible phase exchange, but it does not generate permanent exponential growth of the Tick rate.

11 Conditions for Secular Phase Memory

Over a specified cycle interval, define

$$\boxed{\bar{\epsilon}_\phi = \left\langle \frac{\mathrm{d}\ln F}{\mathrm{d}n} \right\rangle = \left\langle \frac{2\pi}{\omega} \left(\frac{\dot{Q}_\Theta}{Q_\Theta} - \frac{\dot{I}_\Theta}{I_\Theta} \right) \right\rangle.} \tag{102}$$

Then

$$\bar{\epsilon}_\phi > 0 \quad \Longrightarrow \quad \text{increasing Tick phase rate,} \tag{103}$$

$$\bar{\epsilon}_\phi = 0 \quad \Longrightarrow \quad \text{no long-term phase memory,} \tag{104}$$

$$\bar{\epsilon}_\phi < 0 \quad \Longrightarrow \quad \text{decreasing Tick phase rate.} \tag{105}$$

A non-zero secular result requires at least one ingredient absent from the closed uniform constant-inertia branch:

(a) a changing geometric phase inertia $I_\Theta(C, q)$;

(b) non-adiabatic driving, $\dot{\omega} \neq 0$;

(c) an open lighthouse or phase reservoir;

(d) radiation or another explicitly modelled receiving sector;

(e) a persistent non-zero averaged lag;

(f) a structural transition changing N, q_0, $\mathcal{N}$, or another internal parameter.

Every pumping or dissipative term must be accompanied by an identifiable energy and angular-momentum channel.

12 Angular Momentum, Torque, and Radial Response

The total angular momentum of the reduced ring is

$$\boxed{J_{\mathrm{tot}} = J_{\mathrm{el}}^{\mathrm{red}}(C, q, \omega) + NQ_\Theta.} \tag{106}$$

Under an external torque τ_{ext},

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(J_{\mathrm{el}}^{\mathrm{red}} + NQ_\Theta\right) = \tau_{\mathrm{ext}}.} \tag{107}$$

Along a quasi-static branch,

$$\dot{J}_{\mathrm{el}}^{\mathrm{red}} = J_C\dot{C} + J_q\dot{q} + J_\omega\dot{\omega}, \tag{108}$$

where J_C, J_q, and J_ω are partial derivatives. Thus

$$\boxed{\dot{\omega} = \frac{\tau_{\mathrm{ext}} - N\dot{Q}_\Theta - J_C\dot{C} - J_q\dot{q}}{J_\omega}.} \tag{109}$$

The radial rate is

$$\boxed{\dot{r} = r_C\dot{C} + r_\omega\dot{\omega}.} \tag{110}$$

On the ideal strong-locking branch,

$$r(\omega) = \frac{\rho}{\sqrt{1 + \omega^2\rho^2/c^2}}, \tag{111}$$

so for $\omega > 0$,

$$\boxed{r_\omega = -\frac{\omega\rho^3/c^2}{(1 + \omega^2\rho^2/c^2)^{3/2}} < 0.} \tag{112}$$

Externally driven spin-up therefore gives inward radial accommodation when the C correction is subdominant. Internal phase transfer alone, however, does not fix the radial direction; its sign depends on J_ω and the complete coupled response.

On the ideal branch, substituting $r = r_{\rm lock}(\omega)$ into the chosen collective worldline action gives the reduced one-coordinate Lagrangian

$$L_{\rm el}^{\rm red}(\omega) = -\frac{U_0}{\sqrt{1+\zeta^2}}, \qquad \zeta = \frac{\omega\rho}{c}. \tag{113}$$

Its canonical branch-response momentum is

$$\boxed{J_{\rm el}^{\rm red}(\omega) = \frac{\partial L_{\rm el}^{\rm red}}{\partial\omega} = M_0\rho^2\omega(1+\zeta^2)^{-3/2}, \qquad M_0 = \frac{U_0}{c^2}.} \tag{114}$$

The power $-3/2$ is therefore intentional. It refers to the canonical momentum of the already reduced, radially accommodating branch. It must not be confused with the instantaneous bare orbital angular momentum at a fixed radius,

$$J_{\rm orb} = \gamma M_0 r^2\omega = M_0\rho^2\omega(1+\zeta^2)^{-1/2}. \tag{115}$$

A numerical implementation must choose one reduction consistently. The monotonicity analysis below concerns $J_{\rm el}^{\rm red}$.

Differentiation gives

$$\frac{{\rm d}J_{\rm el}^{\rm red}}{{\rm d}\omega} = M_0\rho^2\frac{1-2\zeta^2}{(1+\zeta^2)^{5/2}}, \tag{116}$$

which changes sign at

$$\zeta = \frac{1}{\sqrt{2}}. \tag{117}$$

The independent Tick channel modifies the total rotational susceptibility. In the constant-I_Θ synchronous approximation,

$$\frac{{\rm d}J_{\rm tot}}{{\rm d}\omega} = \frac{{\rm d}J_{\rm el}^{\rm red}}{{\rm d}\omega} + N^2 I_\Theta. \tag{118}$$

To obtain a global sufficient bound, define

$$g(\zeta) = \frac{1-2\zeta^2}{(1+\zeta^2)^{5/2}}. \tag{119}$$

Its minimum occurs at $\zeta^2 = 3/2$ and is

$$\min_{\zeta\geq 0} g(\zeta) = -\frac{8\sqrt{10}}{125}. \tag{120}$$

Therefore the sufficient condition

$$\boxed{N^2 I_\Theta > \frac{8\sqrt{10}}{125}\frac{U_0\rho^2}{c^2}} \tag{121}$$

keeps the reduced total torque response positive for every $\zeta \geq 0$.

13 Differential–Algebraic Evolution System

A minimal numerical implementation evolves

$$C,\ P_C,\ q,\ P_q,\ \Phi,\ \chi,\ Q_\Theta,\ \lambda,\ P_L,\ \mathcal{N}_T. \tag{122}$$

For positive collective inertias μ_C and μ_q,

$$\dot{C} = \frac{P_C}{\mu_C}, \qquad \dot{P}_C = -\frac{\partial \mathcal{F}_\omega}{\partial C}, \tag{123}$$

$$\dot{q} = \frac{P_q}{\mu_q}, \qquad \dot{P}_q = -\frac{\partial \mathcal{F}_\omega}{\partial q}. \tag{124}$$

The phase sector is

$$\dot{Q}_\Theta = -\varepsilon \sin(\chi - \lambda), \qquad \dot{\lambda} = \frac{P_L}{I_L}, \tag{125}$$

$$\dot{P}_L = +\varepsilon \sin(\chi - \lambda), \qquad \dot{\chi} = \frac{Q_\Theta}{I_\Theta(C,q)} - N\omega, \tag{126}$$

$$\dot{\Phi} = \omega, \qquad \dot{\mathcal{N}}_T = \frac{1}{2\pi}\left[\omega - \frac{Q_\Theta}{I_\Theta(C,q)} + \alpha\frac{P_L}{I_L}\right]. \tag{127}$$

The angular velocity is determined at each step from the algebraic constraint

$$\boxed{J_{\rm ext}(t) = J_{\rm el}^{\rm red}(C,q,\omega) + NQ_\Theta,} \tag{128}$$

with

$$\dot{J}_{\rm ext} = \tau_{\rm ext}. \tag{129}$$

The instantaneous memory coefficient recorded by the simulation is

$$\boxed{\epsilon_\phi(t) = \frac{2\pi}{\omega}\left[\frac{\dot{Q}_\Theta}{Q_\Theta} - \frac{\dot{I}_\Theta}{I_\Theta}\right].} \tag{130}$$

A complete numerical audit should output

$$C(t),\ q(t),\ r(t),\ \omega(t),\ Q_\Theta(t),\ F(t), \delta(t), \mathcal{N}_T(t), \epsilon_\phi(t),\ E_{\rm tot}(t),\ J_{\rm tot}(t). \tag{131}$$

14 Energy Accounting

A persistent change in F requires energy. The enlarged system must satisfy a balance law of the form

$$\boxed{\frac{\mathrm{d}E_{\mathrm{tot}}}{\mathrm{d}t} = \tau_{\mathrm{ext}}\omega + P_{\mathrm{in}} - P_{\mathrm{out}}.} \tag{132}$$

Here P_{in} and P_{out} must identify actual driving, lighthouse-reservoir, radiation, environmental, or dissipative channels.

For a closed conservative system,

$$\tau_{\mathrm{ext}} = 0, \qquad P_{\mathrm{in}} = P_{\mathrm{out}} = 0, \tag{133}$$

and

$$E_{\mathrm{tot}} = \text{constant}. \tag{134}$$

Any secular growth of Q_Θ or F must then be compensated by a decrease in another energy sector. Cost-free phase amplification is not physically admissible.

15 Corrected Calibration Ledger and Caesium-Cycle Conversion

The earlier $N_0 = 542$ entry is removed. It was reconstructed from a route-dependent aircraft clock result and is neither a fundamental integer nor a clean calibration of the ideal loop considered here.

The SI defining caesium frequency is exact [7]:

$$\boxed{\Delta\nu_{\mathrm{Cs}} = 9\,192\,631\,770\ \mathrm{Hz}.} \tag{135}$$

The phase-count equivalent of the ideal loop interval is therefore

$$\begin{aligned} N^{\mathrm{eq}}_{\mathrm{Cs,loop}} &= \Delta\nu_{\mathrm{Cs}}\,|\Delta\tau_{\mathrm{loop}}| && (136)\\ &= \left(9\,192\,631\,770\ \mathrm{s}^{-1}\right)\left(207.3861067\times10^{-9}\ \mathrm{s}\right) && (137)\\ &\simeq 1906.424. && (138) \end{aligned}$$

Thus

$$\boxed{N^{\mathrm{eq}}_{\mathrm{Cs,loop}} \simeq 1906.42\ \text{caesium cycles}.} \tag{139}$$

This is not a new empirical parameter. It is the same 207.386 ns interval expressed in caesium phase-count units.

Two different dimensionless normalisations must also be kept separate. As a fraction of

one SI second,

$$\frac{N^{\mathrm{eq}}_{\mathrm{Cs,loop}}}{9\,192\,631\,770} = 2.073861 \times 10^{-7}. \tag{140}$$

As a fraction of one sidereal rotation period,

$$\epsilon_{\mathrm{Sag}} = \frac{|\Delta\tau_{\mathrm{loop}}|}{T_\oplus} = \frac{\Omega^2 R^2}{c^2} \simeq 2.40687 \times 10^{-12}. \tag{141}$$

Table 3: Corrected calibration quantities and their logical status.

Quantity	Value or definition	Present status
Slow-clock loop holonomy	$\lvert\Delta\tau_{\mathrm{loop}}\rvert \simeq 207.386\,\mathrm{ns}$	Standard additive result per completed directed equatorial loop.
Caesium-cycle equivalent	$N^{\mathrm{eq}}_{\mathrm{Cs,loop}} \simeq 1906.42$ cycles	Direct metrological conversion of the same loop interval; not an independent model input.
Sagnac-derived benchmark ratio	$\epsilon_{\mathrm{Sag}} = \lvert\Delta\tau_{\mathrm{loop}}\rvert/T_\oplus$	Dimensionless comparison target; not a derived stationary phase-memory rate.
Dynamical phase-memory coefficient	$\bar{\epsilon}_\phi = \langle \mathrm{d}\ln F/\mathrm{d}n\rangle$	Must be calculated from the complete dynamics.

A future unified model must derive a measurement-protocol relation of the schematic form

$$\boxed{r_\phi = \mathcal{R}(\epsilon_{\mathrm{Sag}}, \alpha, N, I_\Theta, I_L, \varepsilon, C, q, \omega)\,.} \tag{142}$$

The value $N^{\mathrm{eq}}_{\mathrm{Cs,loop}}$ may be used to express the result in atomic-clock phase units, but it carries no information independent of $\Delta\tau_{\mathrm{loop}}$.

Its dynamical role is therefore that of an external comparison target. A solution of the phase equations predicts an accumulated Tick-phase excess relative to the calibrated reference,

$$\Delta N^{\mathrm{model}}_\Theta = \frac{1}{2\pi}\int_{t_1}^{t_2} \left[F(t) - \alpha\dot{\lambda}(t)\right] \mathrm{d}t. \tag{143}$$

Let $\kappa_{\mathrm{Cs}\leftarrow\Theta}$ be the experimentally fixed number of caesium transition cycles represented by one model Tick cycle. The caesium-equivalent model output is then

$$\boxed{\Delta N^{\mathrm{model}}_{\mathrm{Cs}} = \kappa_{\mathrm{Cs}\leftarrow\Theta}\Delta N^{\mathrm{model}}_\Theta.} \tag{144}$$

Only after the path, endpoints, and calibration factor are specified is the comparison

$$\Delta N^{\mathrm{model}}_{\mathrm{Cs}} \stackrel{?}{=} N^{\mathrm{eq}}_{\mathrm{Cs,loop}} \simeq 1906.42 \tag{145}$$

meaningful. Thus 1906.42 is neither a dynamical input nor decorative unit conversion: it is

the metrological benchmark against which a derived phase-history prediction can be tested.

16 Decisive Tests and Falsification Criteria

The remaining programme is finite and explicit.

1. **Derive $I_\Theta(C, q)$.** The provisional closures in equations (73) and (75) permit controlled simulations, but the physical Tick inertia must ultimately arise from a finite-width gauge-invariant field configuration.

2. **Construct a bounded Hamiltonian.** It must reproduce the collective equations without hidden constraints or double counting.

3. **Calculate the full normal-mode spectrum.** Axisymmetric collective modes must be supplemented by non-axisymmetric, amplitude, electromagnetic, and bending modes.

4. **Check causality.** All physical characteristic speeds must satisfy $0 \leq c^2_{\mathrm{mode}} \leq c^2$.

5. **Determine the radial direction dynamically.** The sign of $\dot{r}$ must be obtained from equations (109) and (110), not inferred from phase transfer alone.

6. **Evaluate $\bar{\epsilon}_\phi$.** A zero result falsifies the simple secular-memory branch in the tested regime.

7. **Test the constant-rate approximation.** If $\epsilon_\phi(n)$ is not approximately constant, the compound recurrence must be replaced by the exact integral equation (89).

8. **Derive or reject the benchmark identification.** The dynamics must determine whether $\bar{\epsilon}_\phi$ is related to ϵ_{Sag}. The value 1906.42 is only the caesium-cycle representation of the loop interval and must not be counted as an independent calibration.

9. **Construct dimensionless observables.** Any measurable claim must ultimately be expressed as a ratio of clock frequencies, wavelengths, phases, or propagation standards.

A decisive negative result would be any demonstration that every stable, causal, energy-conserving realisation of the proposed finite-stiffness model satisfies

$$\bar{\epsilon}_\phi = 0 \tag{146}$$

under all physically relevant driving and reservoir conditions. Conversely, a positive theoretical result requires a stable branch with non-zero $\bar{\epsilon}_\phi$ and explicit accounting for its energy and angular momentum.

17 Discussion

The present construction reaches a useful boundary between kinematics and dynamics.

First, the slow-clock result is a clean illustration of global rotating-frame geometry. The transported clock may move arbitrarily slowly relative to the surface, yet the one-loop comparison remains finite because the instantaneous cross term scales as u while the journey duration scales as $1/u$.

Second, repeated winding remains additive. No compound law follows from the Sagnac integral itself.

Third, introducing an independent wound Tick phase gives the ring a separate angular-momentum channel and produces an operational clock readout. The resulting logarithmic rate equation identifies precisely how phase charge and phase inertia can modify the Tick rate. The illustrative constitutive family for $I_\Theta(C, q)$ turns this statement into a closed numerical test without presenting the chosen constitutive exponents as fundamental constants.

Fourth, the conservative no-drift theorem sharply limits the model. Reversible phase exchange is not phase memory. A secular effect requires an open, driven, structurally evolving, or inertia-changing sector. This is a strength of the formulation because it prevents an arbitrary drift from being inserted by hand.

Finally, cosmological or galactic applications should be developed only after the coefficient $\bar{\epsilon}_\phi$ has been computed. Such applications would additionally require a complete relation between local time standards, local length standards, freely propagating radiation, observer motion, and dimensionless observables. Those questions are outside the claim set of the present article.

18 Conclusion

The standard equatorial slow-clock comparison yields

$$\Delta\tau_{\text{slow,east}} = -\gamma_\oplus \frac{2\pi\Omega R^2}{c^2} \simeq -207.386\,\text{ns}, \tag{147}$$

and repeated directed loops add linearly in winding number.

The finite-stiffness phase-locked ring supplies a preferred proper circumference without imposing it as an exact rigid constraint, a stable strong-locking branch, an independent wound Tick phase, a lighthouse/reference phase, and a direct operational clock equation. Its central dynamical result is

$$\boxed{\frac{\mathrm{d}\ln F}{\mathrm{d}n} = \frac{2\pi}{\omega}\left[\frac{\dot{Q}_\Theta}{Q_\Theta} - \frac{\dot{I}_\Theta}{I_\Theta}\right].} \tag{148}$$

For a closed conservative uniform branch with constant phase inertia, the cycle average vanishes. A multiplicative phase-memory law can therefore arise only from additional physical dynamics. The remaining scientific question is no longer whether a qualitative phase-memory narrative can be stated, but whether a stable, causal, energy-conserving realisation produces

$$\boxed{\overline{\epsilon}_\phi = \left\langle \frac{\mathrm{d}\ln F}{\mathrm{d}n} \right\rangle \neq 0.} \tag{149}$$

That quantity is the appropriate target for analytical derivation, numerical simulation, and experimental constraint.

A One Eastward Equatorial Loop Completed in One Year

If one equatorial circumference is completed relative to the ground in one Julian year,

$$u = \frac{L}{365.25 \times 86400\ \mathrm{s}} \simeq 1.270\,\mathrm{m/s}. \tag{150}$$

The loop term remains

$$\Delta\tau_{\mathrm{loop}} \simeq -207.386\,\mathrm{ns}, \tag{151}$$

whereas the traveller's additional quadratic term is

$$\Delta\tau_{\mathrm{trav}} = -\frac{uL}{2c^2} \simeq -0.283\,\mathrm{ns}. \tag{152}$$

Thus

$$\Delta\tau_{\mathrm{total}} \simeq -207.669\,\mathrm{ns}. \tag{153}$$

This illustrates the slow-transport decomposition only; it does not establish a stationary multiplicative drift.

B Continuous and Discrete Forms of a Constant Logarithmic Drift

If

$$\frac{\mathrm{d}\ln F}{\mathrm{d}n} = \overline{\epsilon}_\phi, \tag{154}$$

then

$$F(n) = F_0 e^{\overline{\epsilon}_\phi n}. \tag{155}$$

At integer cycle count m,

$$F_{m+1} = F_m(1 + r_\phi), \qquad r_\phi = e^{\bar{\epsilon}_\phi} - 1. \tag{156}$$

For $|\bar{\epsilon}_\phi| \ll 1$,

$$r_\phi = \bar{\epsilon}_\phi + \mathcal{O}(\bar{\epsilon}_\phi^2). \tag{157}$$

The discrete compound law is therefore the sampled form of continuous exponential evolution, not an independent physical mechanism.

C Correction of the Former 542-Cycle Entry

The former quantity $N_0 = 542$ is not retained in this version. It originated from converting an accumulated aircraft-clock offset into caesium cycles. Because that flight result contains altitude, velocity, latitude, route, and comparison-protocol contributions, it is not a clean representation of the ideal equatorial slow-transport loop.

The self-contained replacement used throughout this article is

$$\boxed{N^{\mathrm{eq}}_{\mathrm{Cs,loop}} = \Delta\nu_{\mathrm{Cs}}|\Delta\tau_{\mathrm{loop}}| \simeq 1906.42.} \tag{158}$$

This number is a phase-count translation of the standard loop interval. It does not establish that a stationary terrestrial caesium clock loses the same number of cycles per Earth rotation. That stronger claim would require a non-zero value of $\bar{\epsilon}_\phi$ derived from the phase dynamics.

D Textual Signature and Project Attribution

This research note is issued under the following authorship and project attribution:

S. M. H. Emamifar
Spokesperson
Independent Research Collaboration on
Black Hole and Cosmology Concepts (IRCBHC)
ORCID: 0009-0007-6257-0163
July 2026

References

[1] National Geospatial-Intelligence Agency, *World Geodetic System 1984 (WGS 84): Its Definition and Relationships with Local Geodetic Systems*, NGA.STND.0036_1.0.0 (2014).

[2] G. Sagnac, "L'ether lumineux demontre par l'effet du vent relatif d'ether dans un interferometre en rotation uniforme," *Comptes Rendus de l'Academie des Sciences* **157**, 708–710 (1913).

[3] J. C. Hafele and R. E. Keating, "Around-the-World Atomic Clocks: Predicted Relativistic Time Gains," *Science* **177**, 166–168 (1972), doi:10.1126/science.177.4044.166.

[4] J. C. Hafele and R. E. Keating, "Around-the-World Atomic Clocks: Observed Relativistic Time Gains," *Science* **177**, 168–170 (1972), doi:10.1126/science.177.4044.168.

[5] N. Ashby, "Relativity in the Global Positioning System," *Living Reviews in Relativity* **6**, 1 (2003), doi:10.12942/lrr-2003-1.

[6] W. Rindler, *Relativity: Special, General, and Cosmological*, 2nd ed. (Oxford University Press, Oxford, 2006).

[7] Bureau International des Poids et Mesures, "SI Base Unit: Second," defining $\Delta\nu_{\mathrm{Cs}} = 9\,192\,631\,770$ Hz exactly, https://www.bipm.org/en/si-base-units/second (accessed July 2026).

[8] S. M. H. Emamifar, "Radial Accommodation in the Ehrenfest Problem: A Conditional Layered-Ring Model with an Effective Covariant Phase-Stiffness Realization," manuscript in preparation (2026).