The Lighthouse, the Spiral, and the Phase-Locked Ring

A Finite-Stiffness Effective Model of Radial Accommodation, Torque Transfer, and Relativistic Rotation

S. M. H. Emamifar
Independent Research Collaboration on Black Hole and Cosmology Concepts
ORCID: 0009-0007-6257-0163

July 2026

Abstract

The Ehrenfest rotating-disk problem exposes a distinction that is often hidden in elementary discussions of special relativity: relativistic kinematics constrains admissible motions, but it does not by itself select the constitutive response of a real material during spin-up. This paper develops a conditional effective model for one particular response: a rotating ring whose material order parameter carries a conserved winding number and is dynamically attracted toward a preferred proper wavenumber. In the strong-locking limit, preservation of the ring's proper circumference produces the radial map

$$r(\rho, \omega) = \frac{\rho}{\sqrt{1 + \omega^2 \rho^2 / c^2}}.$$

The construction is then extended beyond a hard constraint. A minimal finite-stiffness energy is introduced for the proper circumference and proper wavenumber. The radial map is recovered as the leading term of a stationary solution, while finite stiffness predicts a calculable outward correction. A sufficient local-stability criterion is obtained in terms of a dimensionless stiffness parameter. Treating the rotational angle as a true dynamical coordinate reveals a turning point in the angular momentum of the purely geometric branch. An independent wound "Tick" phase supplies the missing angular-momentum channel, giving $J = J_{\mathrm{orb}} + NQ_{\Theta}$ and a quantitative condition for monotonic torque-driven spin-up. The lighthouse, spiral, and blind-observer pictures are retained as operational intuitions, but they are explicitly separated from the derived results. The model is not presented as a microscopic theory of the electron or as a derivation of special relativity. It is a mathematically explicit, testable effective theory of a phase-locked relativistic ring and a roadmap toward a gauge-invariant finite-width field model.

Keywords: Ehrenfest problem; relativistic rotating ring; phase locking; winding number; finite stiffness; radial accommodation; internal phase current; torque transfer.

Contents

1 Introduction **4**

2 Claim Discipline and Scope **5**

3 The Ehrenfest Problem as a Constitutive Problem **5**

4 The Lighthouse, Spiral, and Blind Observer **6**
4.1 The lighthouse 6
4.2 The spiral 6
4.3 The blind observer 7

5 Two Distinct Quadratic Budgets **7**
5.1 External energy-momentum shares 7
5.2 Internal Tick-Gam lock 8

6 Gauge-Invariant Winding and Independent Phase Sectors **8**
6.1 Material order parameter 8
6.2 Independent Tick phase 9
6.3 Electromagnetic field 9

7 Strong-Locking Ring Kinematics **9**

8 A Minimal Finite-Stiffness Engine **10**
8.1 Collective variables and rest energy 10
8.2 Stationary geometry at prescribed angular velocity 10
8.3 Elimination of the wavenumber 11

9 Strong-Locking Expansion and Radial Correction **11**

10 Local Stability of the Stationary Branch **12**
10.1 Regular collective dynamics 13

11 Worldsheet Derivation of Proper Circumference **13**

12 Torque-Driven Completion **14**
12.1 Turning point of the purely geometric branch 15

13 The Tick Phase as an Angular-Momentum Channel **15**
13.1 Lighthouse phase reservoir 16
13.2 Synchronous branch and minimum Tick inertia 16

14 Adiabatic Spin-Up and Dissipation **17**

15 Predictions Internal to the Effective Model **17**

16 What the Model Does Not Yet Establish **18**
16.1 Derivation of relativistic kinematics 18
16.2 Non-axisymmetric stability 18
16.3 Causality and characteristic speeds 18
16.4 Microscopic confinement 18
16.5 Particle interpretation 18

17 Discussion **18**

18 Conclusion **19**

A Algebraic Details of Wavenumber Elimination **20**

B Derivation of the Strong-Locking Correction **20**

C Notation Summary **21**

1 Introduction

The Ehrenfest problem asks what happens when an extended disk is brought from rest into relativistic rotation. Tangential material elements move with position-dependent speed, while radial directions are instantaneously orthogonal to that motion. A naive combination of Euclidean rigidity and Lorentz contraction produces incompatible circumference and radius assignments. Historically, the problem helped reveal how restrictive relativistic rigidity is and why a real spin-up process must involve stress, deformation, and a specification of the material law [3, 2, 4, 5].

The standard resolution is not that the disk has a unique universal response. Special relativity fixes the causal and kinematic framework; the actual deformation depends on the constitutive properties of the material and on the acceleration protocol. Modern variational treatments of relativistic elasticity make this separation explicit [6].

The present paper asks a narrower constructive question:

> Can a ring endowed with a conserved phase winding and a preferred proper wavenumber possess a stable relativistic branch in which its proper circumference remains approximately fixed while its laboratory radius changes during rotation?

The answer developed below is conditional but nontrivial. A hard phase-locking constraint does generate the map

$$r = \rho/\gamma,$$

and a finite-stiffness completion preserves this map as its leading strong-locking limit. The finite-stiffness theory also shows what the hard constraint hides: a separate circumferential restoring sector is required to prevent a global runaway, and an independent internal phase current is required for an ordinary torque-driven passage through the complete stationary branch.

This paper therefore has three levels:

1. **Kinematic level.** A layered-ring map is derived under an explicit fixed-proper-circumference condition.

2. **Effective dynamical level.** A finite-stiffness collective model is constructed and tested for equilibrium and local stability.

3. **Microscopic programme.** A gauge-invariant order parameter, an electromagnetic sector, and an independent Tick phase are identified as ingredients of a future finite-width field theory.

No claim is made that every rotating disk follows the derived map. No claim is made that a physical electron is literally the thin ring studied here. The capitalised term *Luminous Ring* will denote a proposed ring-like confined excitation used as physical motivation, not an established particle model.

2 Claim Discipline and Scope

To avoid mixing assumptions with results, the logical status of the main statements is fixed from the outset.

Table 1: Logical status of the principal ingredients.

Statement	Status	Meaning
$C_{\text{proper}} = 2\pi\gamma r$	Derived	Proper circumference of a stationary circular material congruence.
$r = \rho/\gamma$	Conditional derivation	Follows if the proper circumference of each ring is fixed at $2\pi\rho$.
Finite-stiffness correction	Derived within model	Follows from the explicit reduced energy introduced below.
$\beta^2 + \eta^2 = 1$	Representation	A normalised rewriting of the standard energy-momentum relation.
Helical speed picture	Interpretation	A geometric visualisation, not an independent proof of relativity.
Phase-locking origin of matter	Hypothesis	Requires a microscopic field solution and is not established here.
Electron identification	Open	The effective ring is not yet a spin-1/2 particle model.

This classification is part of the model rather than a disclaimer added afterward. It specifies exactly what would have to be falsified or extended at each level.

3 The Ehrenfest Problem as a Constitutive Problem

Consider a disk initially labelled by a material radial coordinate ρ. A ring at that label has initial proper circumference

$$C_0(\rho) = 2\pi\rho. \tag{1}$$

After spin-up, let the ring settle at laboratory radius $r(\rho)$ and rotate with angular velocity ω. Its laboratory tangential speed and Lorentz factor are

$$v(\rho) = \omega r(\rho), \qquad \gamma(\rho) = \frac{1}{\sqrt{1 - \omega^2 r(\rho)^2/c^2}}. \tag{2}$$

The laboratory circumference is Euclidean on a laboratory-time slice:

$$C_{\text{lab}}(\rho) = 2\pi r(\rho). \tag{3}$$

The summed local comoving circumferential lengths are larger by the local Lorentz factor:

$$C_{\text{proper}}(\rho) = \gamma(\rho) C_{\text{lab}}(\rho) = 2\pi\gamma(\rho) r(\rho). \tag{4}$$

Equation (4) is kinematic. It does not determine $r(\rho)$. A constitutive or dynamical

condition is still required. The present model selects the branch

$$C_{\text{proper}}(\rho) = C_0(\rho) = 2\pi\rho. \tag{5}$$

This condition is not a radial Lorentz contraction. It is the statement that the integrated comoving circumference of a labelled ring remains at its preferred value while the laboratory radius changes.

Combining (4) and (5) gives

$$\gamma r = \rho. \tag{6}$$

Solving the implicit relation yields

$$\boxed{r(\rho,\omega) = \frac{\rho}{\sqrt{1+\omega^2\rho^2/c^2}}}, \qquad \boxed{\gamma(\rho,\omega) = \sqrt{1+\frac{\omega^2\rho^2}{c^2}}}. \tag{7}$$

The material ordering is preserved because

$$\frac{\partial r}{\partial \rho} = \left(1+\frac{\omega^2\rho^2}{c^2}\right)^{-3/2} > 0. \tag{8}$$

The radial map is therefore exact *on the selected branch*. The remaining problem is to determine whether a physically sensible energy can select this branch without imposing it as an infinitely rigid constraint.

4 The Lighthouse, Spiral, and Blind Observer

The mathematical model was motivated by three linked images.

4.1 The lighthouse

A central or external field acts as a phase reference. The word "lighthouse" does not mean that the field must literally be a narrow beam. It denotes a sector capable of providing a reference phase and, in the dynamical completion, exchanging phase charge with the ring.

4.2 The spiral

A circulating internal pattern combined with directed motion can be represented by a helix. If the total auxiliary phase speed is taken to be c, a forward component v and an orthogonal component $u_\perp^{\text{eff}}$ satisfy

$$v^2 + \left(u_\perp^{\text{eff}}\right)^2 = c^2. \tag{9}$$

This is a useful picture of a normalised quadratic invariant, but it does not derive the invariant speed. The assumption that the total underlying phase propagation is null or lightlike must eventually follow from the field equations.

4.3 The blind observer

The blind observer counts received phase cycles but does not directly see the geometry producing them. This metaphor emphasises that clock rates are operationally inferred from phase counts. It is not, by itself, a complete length-measurement protocol: radar ranging, clock synchronisation, light-travel delay, and Doppler corrections would all be required for that purpose.

The three pictures are retained because they organise the construction, but none is substituted for a variational derivation.

5 Two Distinct Quadratic Budgets

A central clarification is that the internal phase structure of the ring and its external motion relative to a laboratory require different variables.

5.1 External energy-momentum shares

For an excitation of invariant mass m_0, total energy E, momentum p, and speed v, define

$$\boxed{\beta \equiv \frac{pc}{E} = \frac{v}{c}}, \qquad \boxed{\eta \equiv \frac{m_0 c^2}{E} = \frac{1}{\gamma}}. \tag{10}$$

The standard relation

$$E^2 = p^2 c^2 + m_0^2 c^4 \tag{11}$$

becomes

$$\boxed{\beta^2 + \eta^2 = 1}. \tag{12}$$

This is the external Race Protocol in its defensible form. It is a normalised representation of relativistic kinematics, not an independent derivation of it.

In the rest frame,

$$(\beta, \eta) = (0, 1), \tag{13}$$

whereas $\beta = \eta = 1/\sqrt{2}$ occurs at $v = c/\sqrt{2}$, not at rest.

For $m_0 > 0$ and finite E, one has $\eta > 0$, hence

$$\boxed{v < c}. \tag{14}$$

The limit $v \to c$ corresponds to $E \to \infty$ at fixed m_0. The equations above do not imply that the particle dissolves into light; such a transition would require a separate stability calculation and all relevant conservation laws.

5.2 Internal Tick-Gam lock

Let ℓ_{Gam} be a characteristic internal spatial advance and T_{Tick} a characteristic internal cycle period. A dimensionally consistent pair of internal shares is

$$K_{\mathrm{int}} = \frac{\ell_{\mathrm{Gam}}}{\sqrt{\ell_{\mathrm{Gam}}^2 + c^2 T_{\mathrm{Tick}}^2}}, \qquad R_{\mathrm{int}} = \frac{cT_{\mathrm{Tick}}}{\sqrt{\ell_{\mathrm{Gam}}^2 + c^2 T_{\mathrm{Tick}}^2}}. \tag{15}$$

Then

$$K_{\mathrm{int}}^2 + R_{\mathrm{int}}^2 = 1. \tag{16}$$

The proposed internal 45-degree lock is

$$K_{\mathrm{int}} = R_{\mathrm{int}} = \frac{1}{\sqrt{2}}, \qquad \ell_{\mathrm{Gam}} = cT_{\mathrm{Tick}}. \tag{17}$$

At present this is a constitutive hypothesis. It is proposed to participate in selecting an internal rest-energy scale, but the mass scale has not been derived until the full rest energy

$$m_0 c^2 = \int T_{\mathrm{rest}}^{00}\, \mathrm{d}^3 x \tag{18}$$

has been calculated for a stable field solution.

6 Gauge-Invariant Winding and Independent Phase Sectors

A single scalar phase cannot simultaneously be a material pattern frozen into the ring, an internal clock, and the phase of a freely propagating electromagnetic wave. The effective construction therefore distinguishes three sectors.

6.1 Material order parameter

Let

$$\Xi = f \mathrm{e}^{\mathrm{i}\phi_{\mathrm{lock}}} \tag{19}$$

be a complex material order parameter. With electromagnetic potential A_μ, define

$$D_\mu \Xi = \left(\nabla_\mu - \mathrm{i}\frac{e}{\hbar} A_\mu\right) \Xi, \qquad P_\mu = \nabla_\mu \phi_{\mathrm{lock}} - \frac{e}{\hbar} A_\mu. \tag{20}$$

The material pattern may be comoving in the gauge-covariant sense

$$u^\mu D_\mu \Xi = 0. \tag{21}$$

For a closed loop enclosing magnetic flux Φ_B, the physical winding condition is

$$qC_{\mathrm{proper}} = \left| 2\pi N - \frac{e}{\hbar} \Phi_B \right|. \tag{22}$$

Define the effective winding quantity

$$\mathcal{N} \equiv 2\pi N - \frac{e}{\hbar}\Phi_B. \tag{23}$$

In a screened or zero-flux sector, $\mathcal{N} = 2\pi N$.

6.2 Independent Tick phase

The order parameter Ξ is a spatial locking pattern. An internal clock requires another phase, denoted Θ, whose proper-time advance is nonzero:

$$u^\mu \nabla_\mu \Theta = \Omega_{\text{Tick}}. \tag{24}$$

This phase will later carry a conserved or transferable phase charge.

6.3 Electromagnetic field

The electromagnetic sector is described by

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu, \tag{25}$$

and may propagate locally at the invariant speed c. The present paper does not solve the full finite-width system. A schematic microscopic Lagrangian density would contain

$$\begin{aligned}\mathcal{L}_{\text{micro}} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu \Xi|^2 - V(|\Xi|) \\ &- \frac{K_s}{2}\left(h^{\mu\nu}\overline{D_\mu \Xi} D_\nu \Xi - q_0^2|\Xi|^2\right)^2 + \mathcal{L}_\Theta + \mathcal{L}_{\text{coupling}}.\end{aligned} \tag{26}$$

The collective model below is a controlled truncation of the desired structure, not a derivation from (26).

7 Strong-Locking Ring Kinematics

Suppose a material ring carries fixed effective winding $\mathcal{N}$ and a preferred proper wavenumber $q_0 > 0$. The preferred proper circumference is

$$C_0 = \frac{|\mathcal{N}|}{q_0}, \qquad \rho = \frac{C_0}{2\pi}. \tag{27}$$

In the strict strong-locking limit,

$$q = q_0, \qquad qC_{\text{proper}} = |\mathcal{N}|, \tag{28}$$

so

$$C_{\text{proper}} = C_0 = 2\pi\rho. \tag{29}$$

Equation (7) follows immediately.

This establishes a consistent constrained branch. It does not yet show that the branch is energetically preferred, dynamically accessible, or stable under finite perturbations. These questions require finite stiffness.

8 A Minimal Finite-Stiffness Engine

8.1 Collective variables and rest energy

Let

$$C(t) = C_{\mathrm{proper}}(t) \tag{30}$$

be the instantaneous proper circumference and $q(t)$ the gauge-invariant proper wavenumber. Consider the positive rest-energy functional

$$\boxed{U(C,q) = U_0 + \frac{\kappa_C}{2}(C - C_0)^2 + \frac{A}{2}(q - q_0)^2 + \frac{B}{2}(qC - \mathcal{N})^2}, \tag{31}$$

where

$$\kappa_C > 0, \qquad A > 0, \qquad B > 0. \tag{32}$$

The three penalties have distinct roles:

1. $A(q - q_0)^2/2$ selects the preferred proper wavenumber.
2. $B(qC - \mathcal{N})^2/2$ enforces winding compatibility.
3. $\kappa_C(C - C_0)^2/2$ represents finite-width, amplitude, or ordinary circumferential restoring energy.

The last term is essential. Without it, large circumference can be accommodated by decreasing $q \sim \mathcal{N}/C$, allowing the locking sector to approach a finite energy while the ring drifts toward a runaway branch. Winding and wavenumber selection do not, by themselves, provide global confinement.

8.2 Stationary geometry at prescribed angular velocity

For uniform rotation with laboratory radius r,

$$C = 2\pi\gamma r, \qquad \gamma = \frac{1}{\sqrt{1 - \omega^2 r^2/c^2}}. \tag{33}$$

At fixed ω, this can be inverted:

$$\boxed{r(C,\omega) = \frac{C/(2\pi)}{\sqrt{1 + \omega^2 C^2/(4\pi^2 c^2)}}}, \tag{34}$$

$$\gamma(C,\omega) = \sqrt{1 + \frac{\omega^2 C^2}{4\pi^2 c^2}}. \tag{35}$$

Set

$$a \equiv \frac{\omega}{2\pi c}. \tag{36}$$

The stationary reduced functional at prescribed ω is

$$\mathcal{F}_\omega(C, q) = \frac{U(C, q)}{\sqrt{1 + a^2 C^2}}. \tag{37}$$

It is the negative of the stationary Lagrangian and is equivalent to the appropriate fixed-angular-velocity Routhian for this collective truncation.

8.3 Elimination of the wavenumber

Since the Lorentz factor is independent of q, the stationary equation for q is $\partial U/\partial q = 0$, giving

$$\boxed{q_*(C) = \frac{Aq_0 + BC\mathcal{N}}{A + BC^2}}. \tag{38}$$

Substitution produces

$$\boxed{U_{\text{eff}}(C) = U_0 + \frac{\kappa_C}{2}(C - C_0)^2 + \frac{AB(q_0 C - \mathcal{N})^2}{2(A + BC^2)}}. \tag{39}$$

The exact finite-stiffness equilibrium equation is

$$\boxed{U'_{\text{eff}}(C) = U_{\text{eff}}(C)\frac{a^2 C}{1 + a^2 C^2}}. \tag{40}$$

For reference,

$$\begin{aligned} U'_{\text{eff}}(C) = {} & \kappa_C(C - C_0) \\ & + \frac{AB(q_0 C - \mathcal{N})(Aq_0 + BC\mathcal{N})}{(A + BC^2)^2}. \end{aligned} \tag{41}$$

Equation (40), rather than the hard equality $C = C_0$, is the central stationary equation of the finite-stiffness engine.

9 Strong-Locking Expansion and Radial Correction

Near the zero-rotation state, let

$$C = C_0 + \delta C. \tag{42}$$

The phase sector contributes the effective circumferential stiffness

$$\boxed{\kappa_{\text{ph}} = \frac{ABq_0^2}{A + BC_0^2}}, \tag{43}$$

and the total stiffness is

$$\boxed{\kappa_{\rm tot} = \kappa_C + \kappa_{\rm ph}}. \tag{44}$$

Expanding (40) gives

$$\boxed{\delta C = \frac{U_0 a^2 C_0}{\kappa_{\rm tot}(1 + a^2 C_0^2)} + \mathcal{O}(\kappa_{\rm tot}^{-2})}. \tag{45}$$

Thus finite stiffness predicts a small positive proper-circumference expansion under rotation.

The wavenumber shifts by

$$\delta q = -\frac{B q_0 C_0}{A + B C_0^2}\, \delta C + \mathcal{O}(\delta C^2). \tag{46}$$

Define

$$\zeta \equiv a C_0 = \frac{\omega \rho}{c}. \tag{47}$$

Using (34), one obtains

$$\boxed{r(\omega) = \frac{\rho}{\sqrt{1+\zeta^2}} \left[1 + \frac{U_0 a^2}{\kappa_{\rm tot}(1+\zeta^2)^2} + \mathcal{O}(\kappa_{\rm tot}^{-2})\right]}. \tag{48}$$

The hard radial-accommodation map is the leading term, while finite stiffness generates a calculable outward correction.

10 Local Stability of the Stationary Branch

At a stationary point,

$$\frac{\mathrm{d}^2 \mathcal{F}_\omega}{\mathrm{d}C^2} = \frac{1}{\gamma}\left[U''_{\rm eff}(C) - U_{\rm eff}(C)\frac{a^2}{(1 + a^2 C^2)^2}\right]. \tag{49}$$

In the strong-locking approximation, local radial stability requires

$$\boxed{\kappa_{\rm tot} > \frac{U_0 a^2}{(1 + a^2 C_0^2)^2}}. \tag{50}$$

Introduce the dimensionless stiffness

$$\Lambda \equiv \frac{\kappa_{\rm tot} C_0^2}{U_0}. \tag{51}$$

Then

$$\boxed{\Lambda > \frac{\zeta^2}{(1+\zeta^2)^2}}. \tag{52}$$

Since

$$0 \le \frac{\zeta^2}{(1+\zeta^2)^2} \le \frac{1}{4}, \tag{53}$$

a simple sufficient condition is

$$\boxed{\Lambda > \frac{1}{4}}. \tag{54}$$

This criterion guarantees local radial stability of the strong-locking branch for all ζ within the reduced model. It does not establish stability against non-axisymmetric or finite-width field perturbations.

10.1 Regular collective dynamics

Keeping C and q independent avoids the low-velocity Hessian singularity of reductions that eliminate the spatial phase variable before variation. A minimal slow collective dynamics is

$$L_{Cq} = \frac{\mu_C}{2}\dot{C}^2 + \frac{\mu_q}{2}\dot{q}^2 - \mathcal{F}_{\omega(t)}(C, q), \tag{55}$$

with $\mu_C, \mu_q > 0$.

At zero rotation, the static Hessian at (C_0, q_0) is

$$H_0 = \begin{pmatrix} \kappa_C + Bq_0^2 & Bq_0C_0 \\ Bq_0C_0 & A + BC_0^2 \end{pmatrix}. \tag{56}$$

Its determinant is

$$\det H_0 = \kappa_C(A + BC_0^2) + ABq_0^2 > 0, \tag{57}$$

so the static two-variable sector is positive definite.

11 Worldsheet Derivation of Proper Circumference

The geometric relation $C = 2\pi\gamma r$ can be expressed invariantly on the worldsheet of the ring. Let $\xi^a = (t, \sigma)$, with $0 \le \sigma < 2\pi$, and embed the ring in cylindrical Minkowski space:

$$X^0 = ct, \tag{58}$$
$$X^1 = r(t)\cos[\sigma + \Phi(t)], \tag{59}$$
$$X^2 = r(t)\sin[\sigma + \Phi(t)], \tag{60}$$
$$X^3 = 0. \tag{61}$$

The induced metric is

$$h_{tt} = -c^2 + \dot{r}^2 + r^2\dot{\Phi}^2, \tag{62}$$
$$h_{t\sigma} = r^2\dot{\Phi}, \tag{63}$$
$$h_{\sigma\sigma} = r^2, \tag{64}$$

with

$$\det h = -r^2(c^2 - \dot{r}^2). \tag{65}$$

For stationary radius and $\omega = \dot{\Phi}$,

$$h^{\sigma\sigma} = \frac{1}{r^2} - \frac{\omega^2}{c^2} = \frac{1}{\gamma^2 r^2}. \tag{66}$$

Let the material label field be $\psi = \sigma$. Its invariant label density is

$$s^2 = h^{ab}\partial_a\psi\partial_b\psi = \frac{1}{\gamma^2 r^2}. \tag{67}$$

The proper length per material-label interval is

$$\mathrm{d}\ell_{\mathrm{proper}} = \frac{\mathrm{d}\sigma}{s} = \gamma r\,\mathrm{d}\sigma. \tag{68}$$

Therefore

$$\boxed{C_{\mathrm{proper}} = \int_0^{2\pi} \frac{\mathrm{d}\sigma}{s} = 2\pi\gamma r}. \tag{69}$$

12 Torque-Driven Completion

The fixed-ω analysis establishes stationary configurations, but spin-up requires the angular coordinate Φ to remain dynamical. Let the total rest energy of the geometric and locking sectors be $U(C, q)$. The stationary material Lagrangian is

$$L_{\mathrm{el}} = -\frac{U(C,q)}{\gamma}, \qquad C = 2\pi\gamma r. \tag{70}$$

The angular momentum must be calculated at fixed r and q, before eliminating the velocity-dependent circumference:

$$\boxed{J_{\mathrm{el}} = \frac{\partial L_{\mathrm{el}}}{\partial\omega} = \frac{\gamma\omega r^2}{c^2}\left(U - CU_C\right)}. \tag{71}$$

The term CU_C is the contribution of circumferential stress.

The stationary radial equation gives

$$U_C = \frac{U\omega^2 r}{2\pi c^2\gamma}, \tag{72}$$

so

$$CU_C = U\frac{\omega^2 r^2}{c^2} = U\beta^2. \tag{73}$$

On the stationary branch,

$$\boxed{J_{\mathrm{el}} = \frac{U\omega r^2}{c^2\gamma}}. \tag{74}$$

12.1 Turning point of the purely geometric branch

In the strong-locking approximation,

$$C = C_0 = 2\pi\rho, \qquad U \simeq U_0, \tag{75}$$

and hence

$$\boxed{J_{\rm el}(\zeta) = \frac{U_0\rho}{c}\frac{\zeta}{(1+\zeta^2)^{3/2}}}. \tag{76}$$

Differentiation gives

$$\boxed{\frac{{\rm d}J_{\rm el}}{{\rm d}\omega} = \frac{U_0\rho^2}{c^2}\frac{1-2\zeta^2}{(1+\zeta^2)^{5/2}}}. \tag{77}$$

The angular momentum reaches a maximum at

$$\zeta_{\rm turn} = \frac{1}{\sqrt{2}}, \qquad \beta_{\rm turn} = \frac{1}{\sqrt{3}}, \tag{78}$$

with

$$J_{\rm el}^{\rm max} = \frac{2}{3\sqrt{3}}\frac{U_0\rho}{c}. \tag{79}$$

Beyond the turning point, ${\rm d}J_{\rm el}/{\rm d}\omega < 0$. This is not a universal speed limit. It is evidence that the purely geometric locked branch is incomplete as a monotonic torque-driven engine.

13 The Tick Phase as an Angular-Momentum Channel

Introduce an independent wound phase on the worldsheet,

$$\Theta(t,\sigma) = N\sigma - \chi(t). \tag{80}$$

For the stationary worldsheet, the invariant phase gradient contains

$$\dot\chi + N\omega. \tag{81}$$

A quadratic collective approximation is

$$L_\Theta = \frac{I_\Theta}{2}(\dot\chi + N\omega)^2 - V_N(r,q), \qquad I_\Theta > 0. \tag{82}$$

The conjugate phase charge is

$$\boxed{Q_\Theta = \frac{\partial L_\Theta}{\partial\dot\chi} = I_\Theta(\dot\chi + N\omega)}, \tag{83}$$

and the Tick contribution to angular momentum is

$$\boxed{J_\Theta = \frac{\partial L_\Theta}{\partial \omega} = N Q_\Theta}. \tag{84}$$

Thus

$$\boxed{J = J_{\rm el} + N Q_\Theta}. \tag{85}$$

The product of a phase charge and winding number has a close analogue in current-carrying ring and vorton mechanics, where independent charge and winding quantum numbers combine to determine angular momentum [7, 10].

13.1 Lighthouse phase reservoir

Let $\lambda(t)$ be a reference or lighthouse phase and introduce

$$L_{\rm light} = \frac{I_L}{2}\dot\lambda^2 - \epsilon\left[1 - \cos(\chi - \lambda)\right]. \tag{86}$$

Then

$$\dot Q_\Theta = -\epsilon \sin(\chi - \lambda), \qquad I_L \ddot\lambda = +\epsilon \sin(\chi - \lambda). \tag{87}$$

The phases exchange charge. Since $J_\Theta = N Q_\Theta$, they also mediate redistribution between internal and orbital angular-momentum channels.

With external torque $\tau_{\rm ext}$, the angular equation is

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(J_{\rm el} + N Q_\Theta\right) = \tau_{\rm ext}}. \tag{88}$$

13.2 Synchronous branch and minimum Tick inertia

On a synchronous branch,

$$\chi - \lambda = \text{constant}, \qquad \dot\chi = \dot\lambda = \Omega_L, \tag{89}$$

so

$$Q_\Theta = I_\Theta(\Omega_L + N\omega). \tag{90}$$

If I_Θ and Ω_L vary slowly,

$$\frac{\mathrm{d}J}{\mathrm{d}\omega} = \frac{\mathrm{d}J_{\rm el}}{\mathrm{d}\omega} + N^2 I_\Theta. \tag{91}$$

The minimum of the dimensionless geometric term is

$$\min_{\zeta \geq 0} \frac{1 - 2\zeta^2}{(1+\zeta^2)^{5/2}} = -\frac{8\sqrt{10}}{125}, \tag{92}$$

attained at $\zeta^2 = 3/2$. Therefore the sufficient condition

$$\boxed{N^2 I_\Theta > \frac{8\sqrt{10}}{125}\frac{U_0 \rho^2}{c^2}} \tag{93}$$

guarantees $\mathrm{d}J/\mathrm{d}\omega > 0$ for all ζ within the strong-locking, constant-I_Θ approximation. Numerically,

$$N^2 I_\Theta > 0.202386\,\frac{U_0\rho^2}{c^2}. \tag{94}$$

The Tick phase is therefore not decorative. In this reduced model it is the internal channel required to complete monotonic torque-driven spin-up.

14 Adiabatic Spin-Up and Dissipation

Let $\Omega_{\min}$ be the smallest normal-mode frequency of the coupled (C, q, χ, λ) system near a stationary solution. If the driving timescale satisfies

$$\tau_{\mathrm{spin}} \gg \Omega_{\min}^{-1}, \tag{95}$$

the collective variables can follow the instantaneous stationary branch adiabatically. In a strictly conservative model, perturbations oscillate around that branch. Weak radiation, internal damping, or coupling to the driving apparatus is required for settling.

A complete dissipative treatment must preserve the total energy and angular-momentum accounting of the enlarged system, including emitted fields. Friction cannot simply be inserted into the ring equation without specifying the receiving sector.

15 Predictions Internal to the Effective Model

The model becomes scientifically useful only through consequences not identical to the imposed assumptions. Within its domain, it predicts:

1. **Finite-stiffness expansion.** The proper circumference is not exactly constant but increases according to (45).
2. **Outward correction to the ideal map.** The laboratory radius exceeds the hard-lock value by the correction in (48).
3. **A radial-stability threshold.** The strong-locking branch requires (52); $\Lambda > 1/4$ is sufficient.
4. **A geometric angular-momentum turning point.** If no internal phase current participates, the strong-lock branch turns at $\beta = 1/\sqrt{3}$.
5. **A minimum internal rotational susceptibility.** Monotonic torque-driven continuation requires the Tick condition (93) in the simplified synchronous regime.
6. **Coupled collective modes.** The variables C, q, χ, and λ should produce multiple normal-mode frequencies rather than a single radial oscillation.

These predictions are model-specific. Their failure in a numerical field realisation would falsify the present collective completion rather than special relativity.

16 What the Model Does Not Yet Establish

Several decisive tasks remain open.

16.1 Derivation of relativistic kinematics

The external budget $\beta^2 + \eta^2 = 1$ was defined from the standard energy-momentum relation. The present work therefore does not derive the Lorentz factor from phase locking. Such a derivation would require the microscopic action to yield

$$E^2 - p^2c^2 = m_0^2c^4 \tag{96}$$

without assuming it.

16.2 Non-axisymmetric stability

Radial stability is not full stability. Perturbations proportional to

$$\mathrm{e}^{\mathrm{i}m\sigma}, \qquad m \geq 1, \tag{97}$$

can couple extrinsic bending, longitudinal phase, electromagnetic, and amplitude modes. Current-carrying ring models demonstrate that higher modes can possess independent instability criteria [8, 9, 6].

16.3 Causality and characteristic speeds

A valid finite-width theory must have a bounded Hamiltonian and characteristic speeds satisfying

$$0 \leq c_{\mathrm{mode}}^2 \leq c^2. \tag{98}$$

Positive static stiffness alone does not ensure causal propagation.

16.4 Microscopic confinement

The energy (31) is phenomenological. The origin of q_0, κ_C, A, B, I_Θ, and the confinement of electromagnetic energy to a finite ring must be derived from a finite-width field solution.

16.5 Particle interpretation

A complex scalar order parameter describes a bosonic collective field. It does not by itself reproduce fermionic spin, charge quantisation, magnetic moment, or the observed spectrum of elementary particles. Those are separate requirements.

17 Discussion

The main conceptual gain of the construction is not a declaration that one geometry replaces another. It is the identification of the missing constitutive engine behind a particular

rotating branch.

The hard-locking statement

$$\gamma r = \rho \tag{99}$$

is easy to write but hides two questions: what force keeps the proper circumference near its preferred value, and how can the branch be traversed under torque? The finite-stiffness energy answers the first question at collective level. It shows that winding and preferred wavenumber generate part of the required stiffness, but a separate amplitude or circumferential sector is indispensable. The torque analysis answers the second question by revealing a turning point and identifying a wound Tick phase as the missing angular-momentum reservoir.

The resulting division of labour is:

$$\begin{aligned} \text{winding and } q_0 &\longrightarrow \text{preferred proper circumference,} \\ \kappa_C, A, B &\longrightarrow \text{finite restoring dynamics,} \\ Q_\Theta, N &\longrightarrow \text{internal angular momentum,} \\ \lambda &\longrightarrow \text{phase exchange and synchronisation.} \end{aligned} \tag{100}$$

This makes the lighthouse picture more precise. The lighthouse is not a mysterious source of radial contraction. It is a reference or reservoir sector that may exchange phase charge. The radial response comes from the energy landscape of the material order parameter. The spiral is a kinematic representation of mixed internal and external channels. The blind observer reminds us that measured clock rates are phase-count ratios, but the observer metaphor does not replace an operational protocol.

18 Conclusion

A complete universal resolution of the rotating-disk problem cannot be obtained without specifying the material. This paper specifies one conditional material architecture: a ring with a gauge-invariant winding, a preferred proper wavenumber, finite circumferential stiffness, and an independent wound Tick phase.

The principal results are:

1. Fixed proper circumference gives the exact stationary map

$$r(\rho, \omega) = \frac{\rho}{\sqrt{1 + \omega^2 \rho^2 / c^2}}.$$

2. A finite-stiffness energy yields this map as its leading strong-locking limit rather than imposing it as a permanent equality.

3. Finite stiffness predicts a positive proper-circumference shift and an outward correction to the laboratory radius.

4. The dimensionless condition

$$\Lambda > \frac{\zeta^2}{(1 + \zeta^2)^2}$$

controls local radial stability, with $\Lambda > 1/4$ sufficient in the reduced model.

5. The purely geometric branch develops a turning point in angular momentum. A wound Tick phase contributes
$$J_\Theta = NQ_\Theta$$
and can restore monotonic torque-driven continuation when its effective inertia is large enough.

The effective engine is therefore mathematically viable, but the decisive next step is no longer another conceptual appendix. It is the construction and numerical evolution of a finite-width, gauge-invariant field model, followed by a full spectrum of axisymmetric and non-axisymmetric perturbations. The present work provides the equations, thresholds, and failure modes against which that microscopic model can be tested.

A Algebraic Details of Wavenumber Elimination

Starting from the phase part of (31),

$$U_q = \frac{A}{2}(q - q_0)^2 + \frac{B}{2}(qC - \mathcal{N})^2, \tag{101}$$

the stationary condition is

$$A(q - q_0) + BC(qC - \mathcal{N}) = 0. \tag{102}$$

Thus

$$q_*(C) = \frac{Aq_0 + BC\mathcal{N}}{A + BC^2}. \tag{103}$$

The two residuals are

$$q_* - q_0 = -\frac{BC(q_0C - \mathcal{N})}{A + BC^2}, \tag{104}$$

$$q_*C - \mathcal{N} = \frac{A(q_0C - \mathcal{N})}{A + BC^2}. \tag{105}$$

Substitution gives

$$U_q(q_*) = \frac{AB(q_0C - \mathcal{N})^2}{2(A + BC^2)}, \tag{106}$$

which establishes (39).

B Derivation of the Strong-Locking Correction

At $C = C_0$,

$$q_0C_0 - \mathcal{N} = 0, \qquad U'_{\text{eff}}(C_0) = 0. \tag{107}$$

The second derivative of the phase contribution is

$$\frac{\mathrm{d}^2}{\mathrm{d}C^2}\frac{AB(q_0C-\mathcal{N})^2}{2(A+BC^2)}\bigg|_{C=C_0} = \frac{ABq_0^2}{A+BC_0^2}. \tag{108}$$

Hence

$$U'_{\mathrm{eff}}(C_0+\delta C) = \kappa_{\mathrm{tot}}\delta C + \mathcal{O}(\delta C^2). \tag{109}$$

Using $U_{\mathrm{eff}}(C) = U_0 + \mathcal{O}(\delta C^2)$ in (40) gives (45).

Since

$$r(C) = \frac{C/(2\pi)}{\sqrt{1+a^2C^2}}, \tag{110}$$

one has

$$\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}C} = \frac{1}{C(1+a^2C^2)}. \tag{111}$$

Combining this derivative with (45) gives (48).

C Notation Summary

Symbol	Meaning
r	Laboratory radius of the ring.
ρ	Preferred material radius, $C_0/(2\pi)$.
C	Instantaneous proper circumference.
C_0	Preferred zero-rotation proper circumference.
q	Gauge-invariant proper wavenumber.
q_0	Preferred proper wavenumber.
N	Integer winding number.
$\mathcal{N}$	Effective winding including enclosed flux.
ω	Laboratory angular velocity.
γ	Tangential Lorentz factor.
ζ	Dimensionless rotation parameter $\omega\rho/c$.
U_0	Rest energy at the preferred configuration.
κ_C	Direct circumferential stiffness.
A	Preferred-wavenumber stiffness.
B	Winding-compatibility stiffness.
κ_{ph}	Circumferential stiffness induced by the phase sector.
Θ	Independent Tick phase.
Q_Θ	Momentum conjugate to the Tick phase.
I_Θ	Effective Tick-phase inertia.
λ	Lighthouse or reference-reservoir phase.
β	External speed share v/c.
η	External invariant-energy share $1/\gamma$.

Author's Statement and Signature

The author presents this work as an independent theoretical investigation. All distinctions between derived results, model-dependent results, and open hypotheses have been stated explicitly in the manuscript. The effective ring model is offered for critical analysis, mathematical extension, and numerical testing.

Electronically signed

S. M. H. Emamifar
Independent Research Collaboration on Black Hole and Cosmology Concepts
ORCID: 0009-0007-6257-0163
July 2026

References

[1] A. Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik* **17**, 891–921 (1905).

[2] M. Born, "Die Theorie des starren Elektrons in der Kinematik des Relativitätsprinzips," *Annalen der Physik* **30**, 1–56 (1909).

[3] P. Ehrenfest, "Gleichförmige Rotation starrer Körper und Relativitätstheorie," *Physikalische Zeitschrift* **10**, 918 (1909).

[4] G. Herglotz, "Über den vom Standpunkt des Relativitätsprinzips aus als starr zu bezeichnenden Körper," *Annalen der Physik* **336**, 393–415 (1910).

[5] F. Noether, "Zur Kinematik des starren Körpers in der Relativtheorie," *Annalen der Physik* **336**, 919–944 (1910).

[6] P. Mourão, J. Natário, and R. Vicente, "Relativistic elastic membranes: rotating disks and Dyson spheres," arXiv:2409.10602 (2024).

[7] B. Carter, "Mechanics of cosmic rings," arXiv:hep-th/0703023 (2007).

[8] B. Carter and X. Martin, "Dynamic instability criterion for circular (vorton) string loops," arXiv:hep-th/0306111 (2003).

[9] Y. Lemperière and E. P. S. Shellard, "Vorton existence and stability," arXiv:hep-ph/0305156 (2003).

[10] R. A. Battye, S. J. Cotterill, and J. A. Pearson, "Stable cosmic vortons in bosonic field theory," *Physical Review Letters* **127**, 241601 (2021).