Lorentz Contraction as a Dynamical Inversion

A Strong-Locking Phase-Field Model for Radial Contraction in the Ehrenfest Ring

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Abstract

The Ehrenfest problem exposes a tension between relativistic rigidity, rotation, and the material response of an extended body. A freely rotating elastic ring develops tensile stress and tends to expand, whereas the Lorentz factor conventionally associated with tangential motion points toward a reduced laboratory circumference. This paper proposes that the two tendencies need not be identified with the same dynamical mechanism. It develops an effective phase-field model in which a closed material ring carries an integer phase winding and a preferred proper wavenumber. In the strong-locking limit, the phase pattern is comoving and its proper spatial gradient is fixed.

For a stationary ring of laboratory radius r, angular velocity ω, winding number N, and preferred proper wavenumber q_0, the covariant locking constraints give

$$Q_s = \frac{|N|}{\gamma r} = q_0, \qquad \gamma = \frac{1}{\sqrt{1-\omega^2 r^2/c^2}}.$$

Defining the reference radius $\rho = |N|/q_0$ yields the derived constraint $\gamma r = \rho$ and therefore

$$\boxed{r(\rho,\omega) = \frac{\rho}{\sqrt{1+\omega^2\rho^2/c^2}}}.$$

Thus $r < \rho$ for every nonzero ω: within the proposed strong-locking model, the laboratory radius undergoes a real inward accommodation rather than a merely apparent contraction. The unconstrained elastic radius r_{free} remains a separate counterfactual quantity and is expected to satisfy $r_{\mathrm{free}} > \rho$ for an ordinary tensile ring. The ordering

$$r_{\mathrm{free}} > \rho > r_{\mathrm{lock}}$$

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defines the paper's central notion of *dynamical inversion*: the phase constraint reverses the outward response that the material would exhibit in the absence of locking.

The result is conditional rather than universal. The radial map is derived from the proposed action, but the existence, strength, and material universality of the phase-locking sector remain empirical questions. The paper therefore separates established relativistic kinematics, model assumptions, derived consequences, and falsifiability conditions. It does not claim to replace special relativity or to derive the Lorentz contraction of an arbitrary uniformly translating rod.

Keywords: Ehrenfest paradox; rotating ring; Lorentz contraction; Born rigidity; phase field; winding number; relativistic elasticity.

Before the Equations: Why This Paper Exists

There are two ways to read this paper.

The first is to read it as a technical proposal: a rotating ring, a covariant scalar phase field, a winding number, a strong-locking constraint, and a radial map derived from those ingredients. Read in this way, the paper must stand or fall by its equations. Its assumptions must be stated openly, its derivations must be reproducible, and its predictions must remain vulnerable to experiment.

But there is another way to read it.

One may ask why this paper needed to be written at all.

Why return, in 2026, to a problem introduced more than a century ago? Why reopen the Ehrenfest disk when relativistic kinematics already provides a mathematically successful description? Why stand before the immense authority of established physics and say:

> Let us look once more.

The answer is a single word:

Need.

The need to understand.

Not merely to calculate what an observer records, but to ask what the material system itself is doing. Not merely to accept a transformation law, but to search for the physical mechanism that may lie beneath it. Not merely to repeat that a result is kinematical, but to ask whether a dynamical structure could produce that same result.

Modern physics has achieved a predictive power without precedent in human history. Nothing in this paper denies that achievement. Yet prediction and explanation are not identical. An equation may tell us what will be measured while leaving unanswered the deeper question:

> What physical process makes the measured result possible?

The present work began from a very simple observation:

> The absence of a direct Lorentz contraction along the radial direction does not imply that the radius of a rotating material system cannot change dynamically.

These two statements are not equivalent.

The first concerns the direction of a kinematical contraction. The second concerns the actual response of matter, fields, stresses, and internal constraints. A radius may change even when no direct Lorentz contraction acts along it. It may expand under centrifugal stress. It may contract under an inward constraint. It may settle at a radius determined by the competition between the two.

Once that distinction is made, a new question appears:

> Could the relativistic contraction of a rotating ring be the final result of a dynamical inversion—an inward phase-locking response that overcomes the ring's ordinary outward centrifugal tendency?

This paper proposes that it could.

The argument developed below does not begin by assuming the final radial contraction. It begins with a closed phase field, an integer winding number, and a preferred proper phase gradient. In the strong-locking limit, these ingredients impose the constraint

$$\gamma r = \rho,$$

from which the laboratory radius follows:

$$r(\rho, \omega) = \frac{\rho}{\sqrt{1 + \omega^2 \rho^2 / c^2}}.$$

For every nonzero angular velocity,

$$r < \rho.$$

Within the proposed model, the contraction is therefore not merely an optical appearance, nor an error made by a distant observer. It is a real radial accommodation of the phase-locked system. The unconstrained material tendency points outward; the locked solution points inward. The response has been dynamically inverted.

Whether nature actually contains such a phase-locking mechanism remains an open question. A mathematical derivation inside a proposed model is not the same as an experimental proof of that model. This distinction will be maintained throughout the paper.

But an unproven question is not an illegitimate question.

Science is not weakened when established interpretations are examined. It is weakened only when successful formulas are treated as reasons to stop asking what they mean. We stand in a long queue formed by generations of extraordinary thinkers. Their work should nourish the questions that follow, not leave us holding the stale bread of "it is simply so" and mistaking surrender for understanding.

This paper is therefore both a technical proposal and a declaration of intent.

It does not ask the reader to reject relativity.

It asks the reader to distinguish a successful kinematical description from a possible dynamical explanation.

It does not claim to have delivered the final answer.

It claims only that the question is still alive.

The equations now begin.

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1. Introduction

The rotating-disk problem entered relativistic physics immediately after Born proposed a covariant definition of rigidity in 1909 [1]. Ehrenfest observed that a body cannot in general be brought from rest into uniform rotation while maintaining Born rigidity throughout the acceleration process [2]. Herglotz and Noether subsequently established the severe restrictions on rotational Born-rigid motions now summarized by the Herglotz–Noether theorem [3, 4, 10]. Modern treatments continue to emphasize that the final state of a real rotating ring depends on the spin-up protocol, its constraints, and its constitutive behavior [7, 8, 12, 14].

A central source of confusion is the silent replacement of one question by another. The statement that radial line elements are locally perpendicular to the tangential velocity does not imply that the material radius of a spun-up disk must remain unchanged. A real disk is not an ideal Born-rigid continuum during spin-up. Its radius may change through elastic strain, stress redistribution, fracture, thermal effects, or externally applied constraints. Recent analyses explicitly demonstrate that different acceleration protocols lead to different material outcomes [14].

The present work asks a narrower but more radical question:

> Can a closed phase constraint produce a genuine inward radial accommodation that reverses the outward centrifugal tendency of an otherwise free rotating ring?

The proposed answer is conditional. A phase field ϕ is assigned to a thin ring. The field has an integer winding number and a preferred proper wavenumber. In a strong-locking limit, the winding pattern is frozen into the material and its proper spatial gradient is fixed. These two requirements are sufficient to derive the relation

$$\gamma r = \rho,$$

where ρ is the radius selected by the winding number and preferred wavenumber in the nonrotating state. The corresponding laboratory radius is

$$r = \frac{\rho}{\sqrt{1+\omega^2\rho^2/c^2}} < \rho.$$

The conceptual claim is therefore not that Lorentz transformations are algebraically wrong. Nor is standard Lorentz contraction treated as a literal crushing force in orthodox special relativity. Rather, the paper proposes a new dynamical interpretation for a specific rotating, phase-locked system: the same Lorentz factor that relates proper and laboratory circumferences emerges from a covariant phase constraint, and the resulting real inward displacement reverses the ring's unconstrained centrifugal response.

This distinction motivates the phrase *dynamical inversion*. Let r_{free} denote the radius the ring would adopt under its mechanical constitutive law without the phase-locking sector, and let r_{lock} denote the actual radius in the strong-locking model. The inversion claim is

$$\boxed{r_{\text{free}} > \rho > r_{\text{lock}}.} \tag{1}$$

The first inequality belongs to the mechanical model; the second is derived from the phase constraint.

The paper is organized as follows. Section 2 summarizes the standard relativistic status of the Ehrenfest problem. Section 3 separates assumptions from derived results. Section 4 introduces the effective phase-field action. Sections 5 and 6 derive the radial map and its geometric consequences. Section 7 analyzes the unconstrained centrifugal response and formulates the dynamical inversion. Section 8 discusses constrained equilibrium and finite-stiffness stability. Section 9 clarifies the relation to Lorentz contraction. Sections 10 and 11 give falsifiability conditions and limitations.

2. Standard Relativistic Background

2.1. Born rigidity and spin-up

Born rigidity preserves the infinitesimal orthogonal distance between neighboring material worldlines [1]. It is a kinematic restriction on a congruence of worldlines, not a general constitutive theory of matter. The Herglotz–Noether theorem shows that rotational Born-rigid motions in Minkowski spacetime are generated by Killing motions and therefore possess far fewer degrees of freedom than a Newtonian rigid body [3, 4, 10]. In particular, a disk cannot be taken from rest to rotation by a globally Born-rigid spin-up.

Consequently, a real disk's final state cannot be inferred from Lorentz contraction alone. One must specify how forces are applied, whether laboratory or proper separations are controlled, and what stress–strain law governs the material. A ring may remain intact, expand, or tear depending on the protocol [14]. This point is essential: the Ehrenfest problem is not resolved by simply declaring either the radius or circumference to be immutable.

2.2. Laboratory and proper circumference

For a stationary circular congruence rotating at angular velocity ω, the tangential speed at laboratory radius r is

$$v = \omega r, \qquad \beta = \frac{\omega r}{c}, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{2}$$

The laboratory circumference of the circular coordinate locus is

$$C_{\mathrm{lab}} = 2\pi r. \tag{3}$$

The circumference obtained by summing local comoving tangential rods is

$$C_{\mathrm{proper}} = \gamma C_{\mathrm{lab}} = 2\pi\gamma r. \tag{4}$$

This relation underlies the non-Euclidean spatial geometry assigned to the rotating observers' proper space in standard analyses [7, 12, 9].

Equation (4) does not by itself determine r. Standard relativity supplies the relation between the two circumferences for a specified rotating state, but a constitutive or constraint equation is still required to determine which state the material actually occupies.

2.3. What standard Lorentz contraction does and does not mean

For an inertial rod moving uniformly along its own length, Lorentz contraction is a frame-dependent comparison of endpoint positions that are simultaneous in the measuring frame. It is not, by itself, a material pressure or a claim that the rod is crushed in its own rest frame. The present model does not dispute this statement.

The new claim concerns a different object: a rotating ring endowed with an additional phase-locking sector. In this model, the Lorentz factor appears in a dynamical constraint that selects a smaller physical laboratory radius. That result is not part of standard special relativity; it is a prediction of the proposed effective theory.

3. Logical Status of the Model

To avoid confusing a model postulate with a derived result, the logical status of each ingredient is stated explicitly.

Table 1: Logical status of the main statements.

Statement	Meaning	Status
Minkowski cylindrical metric	Standard background spacetime used for the calculation	Standard
$N \in \mathbb{Z}$	Closed phase winding of the ring	Model assumption / topology
$u^\mu \nabla_\mu \phi = 0$	The locking phase pattern is comoving	Strong-locking assumption
$Q_s = q_0$	The proper spatial phase gradient is fixed	Strong-locking assumption
$\gamma r = \rho$	Constraint relating laboratory radius to reference radius	Derived
$r = \rho/\sqrt{1+\omega^2\rho^2/c^2}$	Radial map	Derived
$r < \rho$ for $\omega \neq 0$	Real inward radial accommodation in the model	Derived
$r_{\text{free}} > \rho$	Unconstrained tensile ring expands	Mechanical result, constitutive-law dependent
$r_{\text{free}} > \rho > r_{\text{lock}}$	Dynamical inversion	Conditional synthesis
Universality for ordinary matter	All bulk matter obeys strong phase locking	Open empirical claim
General inertial Lorentz contraction	Same mechanism applies to every translating rod	Not derived here

The phrase “derived” will therefore always mean “derived from the stated action and locking assumptions,” not “established as a universal law of nature.”

4. Effective Phase-Field Action

4.1. Geometric setting

Work in flat spacetime with cylindrical coordinates $x^\mu = (t, r, \theta, z)$ and line element

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + \mathrm{d}z^2. \tag{5}$$

A thin ring rotates uniformly at fixed radius r and angular velocity ω. Its four-velocity is

$$u^\mu = \gamma(1, 0, \omega, 0), \qquad \gamma = \frac{1}{\sqrt{1 - \omega^2 r^2/c^2}}. \tag{6}$$

The spatial projector into the instantaneous local rest space of the material is

$$h^{\mu\nu} = g^{\mu\nu} + \frac{u^\mu u^\nu}{c^2}. \tag{7}$$

4.2. Locking field

The field ϕ used in the radial derivation is a *spatial locking field*. It should not be identified automatically with a physical clock field. The distinction avoids the inconsistency of demanding both a frozen comoving pattern and a nonzero comoving ticking rate from the same scalar.

For a closed ring, use the ansatz

$$\phi(t, \theta) = N\theta + \chi(t), \qquad N \in \mathbb{Z}. \tag{8}$$

Single-valuedness modulo 2π implies

$$\phi(t, \theta + 2\pi) - \phi(t, \theta) = 2\pi N. \tag{9}$$

The integer N is the phase winding number.

4.3. Candidate action

A minimal effective action that penalizes departure from comoving temporal locking and a preferred proper spatial wavenumber is

$$S = \int \sqrt{-g}\, \mathcal{L}\, \mathrm{d}^4x, \tag{10}$$

with

$$\begin{aligned} \mathcal{L} = & -\mathcal{E}_{\mathrm{mat}} - \frac{K_t}{2}\left(u^\mu \nabla_\mu \phi\right)^2 \\ & - \frac{K_s}{4}\left(h^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - q_0^2\right)^2. \end{aligned} \tag{11}$$

Here $\mathcal{E}_{\mathrm{mat}}$ denotes the material energy density, $K_t > 0$ and $K_s > 0$ are effective locking stiffnesses, and $q_0 > 0$ is a preferred proper wavenumber. The action is phenomenological.

It is chosen because its strong-coupling limit imposes two transparent covariant constraints; it is not derived here from a more microscopic field theory.

In the formal limit $K_t, K_s \to \infty$, finite-action configurations satisfy

$$u^\mu \nabla_\mu \phi = 0, \tag{12}$$
$$h^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi = q_0^2. \tag{13}$$

Equation (12) says that the winding pattern is carried with the material. Equation (13) fixes its proper spatial gradient.

5. Derivation of the Strong-Locking Constraint

5.1. Comoving phase condition

For the ansatz (8),

$$\partial_\mu \phi = (\dot{\chi}, 0, N, 0). \tag{14}$$

The comoving derivative is

$$u^\mu \nabla_\mu \phi = \gamma(\dot{\chi} + \omega N). \tag{15}$$

The temporal locking condition (12) therefore gives

$$\dot{\chi} = -\omega N. \tag{16}$$

The time dependence of χ is thus the coordinate compensation required to keep the spatial winding pattern fixed in the material frame.

5.2. Proper spatial gradient

Define

$$Q_s^2 \equiv h^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi. \tag{17}$$

Using the projector identity,

$$Q_s^2 = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{(u^\mu \partial_\mu \phi)^2}{c^2}. \tag{18}$$

The second term vanishes by (12). With the inverse metric associated with (5),

$$Q_s^2 = -\frac{\dot{\chi}^2}{c^2} + \frac{N^2}{r^2}. \tag{19}$$

Substituting (16),

$$Q_s^2 = N^2 \left(\frac{1}{r^2} - \frac{\omega^2}{c^2} \right) \tag{20}$$
$$= \frac{N^2}{r^2} \left(1 - \frac{\omega^2 r^2}{c^2} \right) \tag{21}$$
$$= \frac{N^2}{\gamma^2 r^2}. \tag{22}$$

Hence

$$\boxed{Q_s = \frac{|N|}{\gamma r}.} \tag{23}$$

Proposition 1 (Strong-locking radius constraint) *For a stationary uniformly rotating ring satisfying the ansatz (8) and the strong-locking constraints (12)–(13), define*

$$\rho \equiv \frac{|N|}{q_0}. \tag{24}$$

Then the physical laboratory radius satisfies

$$\boxed{\gamma r = \rho.} \tag{25}$$

Proof. Equation (13) requires $Q_s = q_0$. Using (23),

$$\frac{|N|}{\gamma r} = q_0. \tag{26}$$

Rearranging and applying (24) gives (25). □

This is the central mathematical step. The relation $r = \rho/\gamma$ is not inserted as a separate length postulate; it follows from the covariant spatial locking condition once the comoving winding constraint is imposed.

6. Proper Circumference and the Radial Map

6.1. Conservation of proper phase circumference

The local-rest-space metric is $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu/c^2$. For the rotating congruence,

$$h_{\theta\theta} = \gamma^2 r^2. \tag{27}$$

Therefore the proper circumference obtained by integrating local comoving lengths around the ring is

$$C_{\text{proper}} = \int_0^{2\pi} \sqrt{h_{\theta\theta}}\, \mathrm{d}\theta = 2\pi\gamma r. \tag{28}$$

Using (25),

$$\boxed{C_{\text{proper}} = 2\pi\rho = \frac{2\pi|N|}{q_0}.} \tag{29}$$

Thus the proper circumference is fixed because both the integer winding number and the preferred proper wavenumber are fixed. Winding conservation alone would not be sufficient.

6.2. Explicit radial map

Starting from

$$r = \frac{\rho}{\gamma} = \rho\sqrt{1 - \frac{\omega^2 r^2}{c^2}}, \tag{30}$$

squaring gives

$$r^2 = \rho^2 - \frac{\omega^2\rho^2 r^2}{c^2}. \tag{31}$$

Therefore

$$r^2\left(1 + \frac{\omega^2\rho^2}{c^2}\right) = \rho^2, \tag{32}$$

and the positive-radius solution is

$$\boxed{r_{\text{lock}}(\rho,\omega) = \frac{\rho}{\sqrt{1 + \omega^2\rho^2/c^2}}.} \tag{33}$$

Corollary 1 (Inward accommodation) *For every $\rho > 0$ and every $\omega \neq 0$,*

$$0 < r_{\text{lock}} < \rho. \tag{34}$$

Proof. The denominator in (33) is strictly greater than unity for nonzero ω. □

The radius in (33) is the actual coordinate radius of the constrained ring in the laboratory frame. It is not introduced as a visual or optical radius.

6.3. Derived Lorentz quantities

Define

$$x \equiv \frac{\omega\rho}{c}. \tag{35}$$

Then

$$\frac{r_{\text{lock}}}{\rho} = \frac{1}{\sqrt{1 + x^2}}, \tag{36}$$

$$\beta = \frac{\omega r_{\text{lock}}}{c} = \frac{x}{\sqrt{1 + x^2}}, \tag{37}$$

$$\gamma = \sqrt{1 + x^2}. \tag{38}$$

Equations (36) and (38) confirm

$$\frac{r_{\text{lock}}}{\rho} = \frac{1}{\gamma}. \tag{39}$$

The local tangential speed remains subluminal for every finite x and approaches c asymptotically as $x \to \infty$.

6.4. Low-speed expansion

For $x \ll 1$,

$$r_{\text{lock}} = \rho\left(1 - \frac{x^2}{2} + \frac{3x^4}{8} + \mathcal{O}(x^6)\right). \tag{40}$$

The leading inward displacement is

$$\boxed{\Delta r_{\text{phase}} \equiv r_{\text{lock}} - \rho \simeq -\frac{\omega^2\rho^3}{2c^2}.} \tag{41}$$

This is the phase-locking contraction predicted by the model at low tangential speed.

6.5. Continuum disk map

If each reference ring of a disk is assigned a label ρ, the map (33) is monotonic:

$$\frac{\mathrm{d}r}{\mathrm{d}\rho} = \left(1 + \frac{\omega^2\rho^2}{c^2}\right)^{-3/2} > 0. \tag{42}$$

Hence rings do not cross. The planar radial-area Jacobian is

$$J_A = \frac{r\,\mathrm{d}r}{\rho\,\mathrm{d}\rho} = \left(1 + \frac{\omega^2\rho^2}{c^2}\right)^{-2}. \tag{43}$$

Any inference about density or stress from (43) requires an explicit material model and a specification of thickness deformation; the phase map alone does not determine them.

7. The Unconstrained Mechanical Response

7.1. Why a free elastic ring tends to expand

A rotating ring requires inward centripetal acceleration. In an ordinary ring, the required inward force is supplied by circumferential tension. That tension produces positive circumferential strain, so the mechanically free equilibrium radius generally exceeds the unstressed radius. This is standard elasticity rather than a relativistic novelty [15, 16].

A simple low-speed estimate illustrates the sign. Consider a thin ring with cross-sectional area A_c, mass per unit length λ, volumetric mass density $\varrho_m = \lambda/A_c$, and Young modulus E. Force balance on a small angular element gives, to leading order,

$$T \simeq \lambda\omega^2 r^2, \tag{44}$$

where T is the circumferential tension. The corresponding stress is

$$\sigma_\theta = \frac{T}{A_c} \simeq \varrho_m\omega^2 r^2. \tag{45}$$

In linear elasticity,

$$\varepsilon_\theta \simeq \frac{\sigma_\theta}{E} > 0. \tag{46}$$

Since $\varepsilon_\theta \simeq (r_{\text{free}} - \rho)/\rho$ for a thin ring,

$$\boxed{\frac{r_{\text{free}} - \rho}{\rho} \simeq \frac{\varrho_m \omega^2 \rho^2}{E} > 0} \tag{47}$$

to lowest order, after replacing r by ρ on the right-hand side.

Equation (47) is not a universal relativistic constitutive law. It is a transparent demonstration that the unconstrained mechanical response has the outward sign for a conventional tensile ring.

7.2. Definition of dynamical inversion

The phase-locked and mechanically free radii answer different counterfactual questions:

$$r_{\text{free}} : \quad \text{equilibrium without the phase-locking constraint,} \tag{48}$$

$$r_{\text{lock}} : \quad \text{equilibrium in the strong-locking phase model.} \tag{49}$$

For a tensile material satisfying (47), and a phase-locked ring satisfying (33),

$$\boxed{r_{\text{free}} > \rho > r_{\text{lock}}.} \tag{50}$$

This ordering is the precise meaning of dynamical inversion in this paper. It does not assert that the same unconstrained material configuration is simultaneously larger and smaller than ρ. It compares two different dynamical models of the same reference ring.

7.3. Scale comparison

The leading fractional phase contraction is

$$\left|\frac{\Delta r_{\text{phase}}}{\rho}\right| \simeq \frac{\omega^2 \rho^2}{2c^2}, \tag{51}$$

whereas the leading free elastic expansion is

$$\frac{\Delta r_{\text{free}}}{\rho} \simeq \frac{\varrho_m \omega^2 \rho^2}{E}. \tag{52}$$

Their ratio is approximately

$$\frac{\Delta r_{\text{free}}}{|\Delta r_{\text{phase}}|} \simeq \frac{2\varrho_m c^2}{E}. \tag{53}$$

For ordinary condensed matter, $E \ll \varrho_m c^2$, so the unconstrained elastic expansion is generally much larger than the relativistic-scale inward displacement. A universal claim that ordinary macroscopic disks must follow the strong-locking map would therefore require an exceptionally stiff additional phase sector. This is a substantive empirical burden, not a detail that can be omitted.

The model is consequently most plausible, at present, as an effective description of a genuinely phase-coherent closed structure rather than an established constitutive law for arbitrary engineering materials.

8. Constrained Equilibrium and Stability

8.1. Constraint reaction

The strong-locking limit may be represented as a constrained variational problem. Let $E_{\mathrm{mat}}(r,\omega)$ be the mechanical energy of the ring and define

$$F(r;\rho,\omega) = \gamma(r,\omega)r - \rho. \tag{54}$$

The constrained equilibrium extremizes

$$\mathcal{I}(r,\Lambda) = E_{\mathrm{mat}}(r,\omega) + \Lambda F(r;\rho,\omega), \tag{55}$$

so that

$$\frac{\partial E_{\mathrm{mat}}}{\partial r} + \Lambda \frac{\partial F}{\partial r} = 0, \tag{56}$$

$$F(r;\rho,\omega) = 0. \tag{57}$$

The multiplier Λ represents the reaction supplied by the locking sector. If the unconstrained mechanical energy decreases as r moves outward from the locked value, the reaction required by (56) is inward. The phase constraint can therefore reverse the free centrifugal tendency, but its required stress must be supplied by the underlying field theory.

8.2. Finite locking stiffness

For finite K_s, the ring need not satisfy $Q_s = q_0$ exactly. A reduced effective radial energy has the schematic form

$$E_{\mathrm{eff}}(r) = E_{\mathrm{mat}}(r,\omega) + \frac{\kappa_s}{4}\left[N^2\left(\frac{1}{r^2} - \frac{\omega^2}{c^2}\right) - q_0^2\right]^2, \tag{58}$$

where $\kappa_s > 0$ is an integrated stiffness. The stationary radius satisfies

$$\frac{\mathrm{d}E_{\mathrm{eff}}}{\mathrm{d}r} = 0. \tag{59}$$

In the limit $\kappa_s \to \infty$, the solution approaches (33), provided the material energy remains finite. For finite κ_s, the mechanical term shifts the radius away from exact locking.

The square penalty is locally restoring in the direction normal to the constraint surface, but full dynamical stability additionally requires positive kinetic energy, a causal constitutive law, and

$$\left.\frac{\mathrm{d}^2 E_{\mathrm{eff}}}{\mathrm{d}r^2}\right|_{r=r_*} > 0. \tag{60}$$

Those conditions are not established by the stationary derivation alone.

8.3. Energy accounting

A real contraction against centrifugal loading requires work and stored field energy. The strong-locking constraint cannot be interpreted as a cost-free geometric instruction. A complete theory must derive the stress-energy tensor of the locking sector,

$$T_{\mu\nu}^{(\phi)} = -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}}, \tag{61}$$

and show that the total energy is bounded below and that the required inward stress is physically admissible. This calculation is a necessary next step.

9. Relation to the Lorentz Factor and to Length Contraction

9.1. Where the Lorentz factor enters

The factor $1/\gamma$ is not inserted into the radial map by hand. It enters through the projection of the phase gradient into the local material rest space. Specifically,

$$Q_s = \frac{|N|}{\gamma r} \tag{62}$$

combines the comoving condition with the Minkowski metric. Fixing $Q_s = q_0$ then produces $r/\rho = 1/\gamma$.

It is therefore legitimate to say:

> Within the proposed strong-locking model, a Lorentz-equivalent radial contraction is derived from a covariant phase-winding constraint.

It would be too strong to say:

> All Lorentz contraction in nature has been proved to arise from this phase field.

The latter statement requires a derivation for arbitrary inertial motion and independent experimental evidence for the locking field.

9.2. Phase-share notation

If one wishes to retain a phase-budget interpretation, the derived geometric ratio may be defined as

$$\mathcal{R}_\phi \equiv \frac{r_{\text{lock}}}{\rho}. \tag{63}$$

The radial derivation gives

$$\mathcal{R}_\phi = \frac{1}{\gamma} = \sqrt{1-\beta^2}. \tag{64}$$

Consequently,

$$\boxed{\mathcal{R}_\phi^2 + \beta^2 = 1.} \tag{65}$$

In this restricted rotating-ring setting, the quadratic phase-budget relation is a rewriting of the derived radial constraint rather than an independent premise.

Care is still required: $\mathcal{R}_\phi$ is a geometric locking ratio. Identifying it with the ticking rate of a physical atomic clock would require a separate clock field or a microscopic coupling not supplied by the present action.

9.3. Inversion of interpretation

The proposed inversion is now precise. Standard kinematics relates local proper circumference and laboratory circumference by $C_{\text{proper}} = \gamma C_{\text{lab}}$. The phase model adds the dynamical law $C_{\text{proper}} = 2\pi\rho$. Together they imply

$$C_{\text{lab}} = \frac{2\pi\rho}{\gamma}, \tag{66}$$

and hence a smaller real laboratory radius. The contraction is not a mistaken visual inference; it is the physical state selected by the phase constraint.

What is inverted is the sign of the unconstrained mechanical response:

$$\text{centrifugal elasticity: outward}, \qquad \text{strong phase locking: inward}. \tag{67}$$

The Lorentz factor is thereby reinterpreted as part of the constraint that reverses the free expansion, not as a standalone mechanical compression law.

10. Falsifiability and Experimental Consequences

The model is meaningful only if it can fail. The following statements provide concrete failure conditions.

10.1. Radial law

A phase-coherent ring in the strong-locking regime should obey

$$r(\omega) = \frac{\rho}{\sqrt{1 + \omega^2\rho^2/c^2}}. \tag{68}$$

At low speed, the residual after subtracting conventional elastic, thermal, electromagnetic, and instrumental effects should contain

$$\Delta r_{\text{phase}} = -\frac{\omega^2\rho^3}{2c^2} + \mathcal{O}(\omega^4). \tag{69}$$

Failure to detect this correction at the sensitivity required for a specified locking stiffness would constrain or exclude the model.

10.2. Winding and wavenumber dependence

The reference radius is predicted to satisfy

$$\rho = \frac{|N|}{q_0}. \tag{70}$$

If systems with independently measurable winding N and proper wavenumber q_0 do not exhibit this relation, the proposed locking mechanism is false in those systems.

10.3. Finite-stiffness departures

At finite locking stiffness, the radial displacement should interpolate between the mechanically free radius and the strong-locking radius. A useful experimental parameter would be an effective stiffness inferred by fitting (58). If no consistent positive stiffness can account for the observed radius across angular velocities, the effective action is inadequate.

10.4. Required stress-energy

The phase sector must supply a measurable inward stress. Any complete version of the model must predict this stress and its stored energy. If the stress required to enforce (33) violates energy positivity, causality, or measured material limits, the strong-locking interpretation is excluded.

10.5. Scope of suitable experiments

Ordinary macroscopic rotors are dominated by conventional elastic expansion, as indicated by (53). More promising tests would involve closed phase-coherent systems in which winding is physically meaningful and long-lived. Candidate platforms might include superconducting loops, superfluid rings, coherent condensates, or engineered waveguide resonators, but applying the present scalar action to any such system requires a platform-specific derivation. No claim is made here that these systems already realize the proposed q_0 locking.

11. Limitations

The present work has several explicit limitations.

1. **Phenomenological action.** The action (11) is proposed, not derived from a microscopic theory of elementary matter.
2. **Stationary thin-ring approximation.** The derivation assumes a thin ring, uniform angular velocity, fixed stationary radius, and flat background spacetime.
3. **Strong-locking idealization.** Exact conservation of the preferred proper wavenumber is an infinite-stiffness limit. Real systems would have finite deviations.

4. **Incomplete material coupling.** A full relativistic constitutive model, including thickness, Poisson response, fracture, and thermal effects, has not been coupled to the phase field.
5. **No general rod derivation.** The paper does not derive the Lorentz contraction of an arbitrary rod in uniform rectilinear motion.
6. **No clock-rate derivation.** The locking scalar ϕ is a comoving spatial pattern. A physical clock requires a separate dynamical phase sector.
7. **No microscopic identification of q_0.** The preferred proper wavenumber is a model parameter whose origin and universality remain open.
8. **Macroscopic empirical burden.** Conventional rotors expand elastically. If strong phase locking is claimed for ordinary matter, the model must quantitatively explain why that expansion does not dominate.

These limitations do not invalidate the conditional derivation. They define the work needed to turn the effective model into a physical theory.

12. Discussion

The main mathematical result can be stated independently of the broader interpretation. A closed scalar phase winding that is (i) comoving and (ii) locked to a fixed proper spatial gradient imposes a fixed proper circumference. In a uniformly rotating ring, the proper circumference is $2\pi\gamma r$. Hence the laboratory radius must decrease according to (33).

The conceptual value of the model lies in making explicit what standard rotating-disk kinematics leaves undetermined. Standard relativity relates measurements between local and laboratory frames, but it does not choose a material constitutive law. The phase action supplies an additional constitutive constraint. Once supplied, the radius is no longer free.

This also clarifies the status of the Ehrenfest problem. The familiar contradiction results from combining a fixed radius with a modified circumference without specifying a physically realizable spin-up or material response. In the present model, the radius is not fixed. It changes inward so that the laboratory geometry remains Euclidean,

$$\frac{C_{\mathrm{lab}}}{r} = 2\pi, \tag{71}$$

while the proper phase circumference remains fixed,

$$C_{\mathrm{proper}} = 2\pi\rho. \tag{72}$$

There is therefore no algebraic contradiction between the two circumferences.

Nevertheless, calling this a complete resolution of the century-old problem would be premature. It is one conditional dynamical completion among many possible material models. Its distinctiveness lies in predicting an inward radius where conventional free elasticity predicts an outward one. That difference makes the proposal testable and potentially falsifiable.

13. Conclusion

A covariant strong-locking phase-field model for a rotating ring has been constructed and analyzed. The model assumes an integer winding number, a comoving phase pattern, and a fixed preferred proper wavenumber. From these assumptions, the proper spatial phase gradient is

$$Q_s = \frac{|N|}{\gamma r}, \tag{73}$$

and the locking condition $Q_s = q_0$ yields

$$\gamma r = \rho, \qquad \rho = \frac{|N|}{q_0}. \tag{74}$$

The resulting physical laboratory radius is

$$\boxed{r_{\text{lock}} = \frac{\rho}{\sqrt{1+\omega^2\rho^2/c^2}} < \rho \quad (\omega \neq 0).} \tag{75}$$

The central intuitive proposal is thereby converted into a precise conditional result. The contraction is real within the model; it is not merely an observational misreading. The "inversion" consists in the reversal of the unconstrained centrifugal response: an ordinary tensile ring tends toward $r_{\text{free}} > \rho$, whereas the strong phase constraint selects $r_{\text{lock}} < \rho$.

What has been proved is the radial map from the proposed locking action. What has not been proved is that ordinary matter possesses this universal locking sector, that the required stress-energy is physically realized, or that the same mechanism accounts for all inertial Lorentz contraction. Those questions are experimentally and theoretically open. The next decisive tasks are to derive the phase stress-energy tensor, couple it to a causal relativistic elastic law, analyze finite-stiffness dynamics, and identify a phase-coherent system in which N, q_0, and the predicted radial correction can be measured independently.

A. Direct Projector Calculation

For completeness, the nonzero contravariant components of the four-velocity are

$$u^t = \gamma, \qquad u^\theta = \gamma\omega. \tag{76}$$

The relevant inverse metric components are

$$g^{tt} = -\frac{1}{c^2}, \qquad g^{\theta\theta} = \frac{1}{r^2}. \tag{77}$$

The projector components needed for the phase ansatz are

$$h^{tt} = -\frac{1}{c^2} + \frac{\gamma^2}{c^2} = \frac{\gamma^2\beta^2}{c^2}, \tag{78}$$

$$h^{t\theta} = \frac{\gamma^2\omega}{c^2}, \tag{79}$$

$$h^{\theta\theta} = \frac{1}{r^2} + \frac{\gamma^2\omega^2}{c^2} = \frac{\gamma^2}{r^2}. \tag{80}$$

Therefore

$$Q_s^2 = h^{tt}\dot{\chi}^2 + 2h^{t\theta}\dot{\chi}N + h^{\theta\theta}N^2. \tag{81}$$

Using $\dot{\chi} = -\omega N$ gives

$$Q_s^2 = \frac{N^2}{\gamma^2 r^2}, \tag{82}$$

in agreement with (22).

B. Alternative Derivation from Proper Circumference

If the fixed proper wavenumber is assumed directly, the total winding is

$$2\pi|N| = \oint Q_s \, \mathrm{d}\ell_{\mathrm{proper}} = q_0 C_{\mathrm{proper}}. \tag{83}$$

Thus

$$C_{\mathrm{proper}} = \frac{2\pi|N|}{q_0} = 2\pi\rho. \tag{84}$$

For a rotating ring, $C_{\mathrm{proper}} = 2\pi\gamma r$, so

$$\gamma r = \rho. \tag{85}$$

This short derivation is equivalent to the local projector calculation but makes clear that both winding conservation and proper-wavenumber locking are required.

C. Low-Speed Competition Between Elasticity and Phase Locking

Combining (41) and (47), the two leading radial tendencies are

$$\Delta r_{\mathrm{phase}} \simeq -\frac{\omega^2\rho^3}{2c^2}, \tag{86}$$

$$\Delta r_{\mathrm{free}} \simeq +\frac{\varrho_m\omega^2\rho^3}{E}. \tag{87}$$

If the two sectors were treated as additive weak perturbations rather than an exact constraint, the net displacement would have the schematic form

$$\Delta r_{\mathrm{net}} \simeq \omega^2\rho^3\left(\frac{\varrho_m}{E} - \frac{\eta}{2c^2}\right), \tag{88}$$

where η is an effective dimensionless coupling of the phase sector to the material radius. Net contraction would require

$$\eta > \frac{2\varrho_m c^2}{E}. \tag{89}$$

For ordinary solids this threshold is extremely large. The exact strong-locking model corresponds not to a small additive correction but to the limit in which the phase constraint dominates the allowed configuration space.

Symbol	Meaning
ρ	Reference radius selected by $\lvert N\rvert/q_0$
r_{lock}	Actual laboratory radius in the strong-locking model
r_{free}	Counterfactual free elastic radius without phase locking
N	Integer phase winding number
q_0	Preferred proper spatial wavenumber
Q_s	Proper spatial magnitude of the phase gradient
ω	Laboratory angular velocity
β	Tangential speed ratio $\omega r/c$
γ	Lorentz factor $(1-\beta^2)^{-1/2}$
K_t, K_s	Temporal and spatial locking stiffnesses
$\mathcal{R}_\phi$	Derived ratio $r_{\mathrm{lock}}/\rho = 1/\gamma$

D. Notation Summary

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