# The Original Sin

*How Modern Physics Lost the Observer*

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# Reader's Map: Why the Argument Must Begin with the Original Sin

The length and ordering of this reference edition are deliberate. The mathematical architecture developed in Part II cannot perform its conceptual work if the reader continues to assume, without examination, that the local observer, the observer's matter, the clock, and the ruler constitute an unchanging external calibration system. The first part therefore develops the local-observer problem in full: the danger of turning an approximation into ontology, the restricted scope of the Ehrenfest impossibility result, the possibility of dynamical radial accommodation, the dependence of rulers on matter, the hidden-change and readout-inversion problems, and the outward-anchor reconstruction protocol.

Only after that foundation has been established does the second part introduce the constructive response: a separation of dynamics, geometric embedding, chronometric readout, and catalogue provenance; a signed nested phase ledger; a point-dependent phase metric; dimensionless chronometric labels; local observer translation; Sagnac normalisations; and a quality-controlled Galactic phase-space atlas.

The edition is consequently longer than a conventional article. No attempt has been made to shorten the foundational diagnosis merely to make the technical formalism appear sooner. The governing order is

| |
|---|
| diagnose the observer problem $\longrightarrow$ separate mechanism from trajectory |
| construct the phase geometry $\longrightarrow$ define the chronometric readout |

*Part I: The Original Sin and the Local Observer Problem*

**Abstract**

Modern physics was built through a set of exceptionally powerful idealizations: locally flat spacetime, inertial observers, ideal clocks, invariant material rulers, point particles, and perfectly rigid bodies. Within their proper domains these idealizations produce predictions of extraordinary accuracy. Operational success, however, does not by itself establish the ontological finality of the concepts on which an approximation is built.

This paper argues that the central difficulty is not simply a failure to recognize spacetime curvature. General relativity explicitly incorporates curved spacetime. The deeper issue is that matter, clocks, rulers, and local observers are still often described from a locally Minkowskian limit in which the observer's material calibration standards are treated as effectively fixed. Less attention is given to the possibility that the body, the atomic structure of the observer, and the observer's own measuring

standards may dynamically reorganize under rotation, infall, and nested gravitational conditions.

The Ehrenfest paradox provides a historical example. It begins with an impossible ideal object—a perfectly rigid disk—and asks whether it can be accelerated into relativistic rotation while preserving all proper distances. Its valid conclusion is limited: perfect rigidity is incompatible with relativistic rotational acceleration. That result does not establish the impossibility of every extended rotating structure. Atoms, electromagnetic waves, and possible field-like electron configurations are not classical rigid bodies.

Within the framework explored here, the point-like appearance of the electron may be an external readout of a closed, rapidly evolving, self-adjusting field configuration rather than a final explanation of its structure. If matter itself changes scale, the observer's ruler and clock cannot automatically be treated as external to that dynamics. A common change in physical distance and local calibration may remain hidden, and a faster contraction of the ruler may even invert the local readout, causing true approach to be measured as recession.

To break this local calibration degeneracy, an outward-anchor and three-point alignment protocol is developed. A gravitational center, a local observer, and a distant bound anchor are compared at two epochs. Translation, rotation, and common scale change are removed by locking the center and the anchor, while the residual displacement of the observer is retained.

The paper also documents the computational status of this program. In particular, the fixed-radius control known as Code 16 is reinterpreted not as a falsification of the variable-ruler model, but as an internal-consistency test of a model in which radial change and calibration evolution were excluded by construction. It shows that raw alignment residuals are not identical to physical radial displacement. It does not adjudicate between a fixed-ruler framework and a model in which the observer's calibration evolves.

The paper distinguishes methodological criticism, structural hypothesis, numerical reconstruction, and physical inference. It does not claim to have disproved relativity, quantum field theory, or dark energy. It formulates a research program in which matter, measurement standards, observer state, and reconstructed geometry must emerge from a common dynamics.

**Keywords:** local observer, rigid body, Ehrenfest paradox, extended electron, relativistic accommodation, variable ruler, outward anchor, three-point alignment, readout inversion, Zero Theory.

# 1 Introduction: When an Approximation Becomes Reality

The history of physics is filled with idealizations introduced to solve restricted problems and later treated, sometimes implicitly, as if they were direct descriptions of ultimate reality.

Classical mechanics employs point particles, frictionless planes, massless strings, and rigid bodies. None of these objects exists perfectly in nature. Their usefulness lies in separating effects and making a problem analytically tractable.

Relativistic and quantum physics continued the same practice through concepts such as:

- the inertial observer,
- locally flat spacetime,
- the ideal clock,
- the invariant ruler,
- the elementary point particle,
- and the perfectly rigid body.

The use of idealizations is not itself an error. The error begins when their origin, domain of validity, and excluded degrees of freedom are forgotten.

The central question of this paper is therefore:

> Can the extraordinary success of an ideal limit be mistaken for proof that the limit itself is the final ontology of nature?

# 2 The Lesson of the Flat Road

Consider a straight road extending for one hundred kilometres across the surface of Earth.

To a driver, the road is flat. Distance, speed, travel time, acceleration, and friction can all be calculated using nearly Euclidean geometry, and the results will be correct.

The driver may justifiably say:

> For my problem, the flat-road model works extremely well.

But if the driver concludes from this local success that Earth itself is flat, the model has been extended beyond its domain.

An external observer at sufficient altitude sees that the apparently straight road is a small arc on a curved surface. The local calculations were not wrong. The global inference was.

The distinction can be summarized as

> Accuracy within a domain establishes the adequacy of the approximation,
> not the ontological finality of its underlying picture.

A flat geometry can be highly accurate on a curved manifold whenever the scale of observation is small relative to the relevant radius of curvature. In the same way, a theory may be extraordinarily accurate over an intermediate range of scales while remaining incomplete at deeper or larger levels.

# 3 The Original Sin: Extending the Flat Limit

Special relativity was formulated in Minkowski spacetime and is exceptionally successful where gravitational effects are negligible.

General relativity later introduced curved spacetime. It is therefore inaccurate to claim that standard physics simply ignores curvature.

The present criticism is more specific. General relativity places curvature in the spacetime metric, while matter, clocks, rulers, and internal material structures are still often treated through a local Minkowski limit with stable local standards.

The equivalence principle states that, in a sufficiently small freely falling region, the laws of physics can be written approximately in the form of special relativity. This is an extraordinarily powerful local statement.

It does not, without further derivation, prove that:

- the atomic structure of the observer remains absolutely unchanged,
- the observer's ruler and clock are dynamically external to the fall,
- an extended object must preserve its initial geometry during relativistic motion,
- or local material standards can never reorganize with the system in which they are embedded.

The central methodological criticism is therefore:

> Standard physics incorporated curvature into spacetime, but did not generally treat the geometric and scale evolution of matter and measuring instruments as part of the same foundational dynamics.

# 4 Two Different Kinds of Curvature

Two distinct questions must be separated.

## 4.1 Spacetime Curvature

Spacetime curvature is encoded in the metric and curvature tensors. It governs geodesic motion, gravitational redshift, lensing, and the propagation of light and matter.

## 4.2 Geometric Reorganization of the Body

The second question concerns the changing radius, circumference, stress distribution, energy density, proper length, and internal geometry of an actual body or field configuration.

Knowing the exterior metric around a star is not sufficient to determine the shape of an elastic body, a fluid, a plasma, or a field soliton placed within it. Constitutive laws, stress-energy dynamics, boundary conditions, and stability equations must also be solved.

The same distinction is essential for any proposed internal model of the electron and for any theory in which the observer's ruler and clock may participate in gravitational evolution.

# 5 How the Ehrenfest Paradox Was Constructed

The Ehrenfest paradox begins with an idealization: a perfectly rigid disk or ring.

In classical mechanics, a rigid body is one for which the distance between every pair of material points remains fixed:

$$|\mathbf{x}_i(t) - \mathbf{x}_j(t)| = \text{constant}.$$

If such a body were perfectly rigid, a force applied to one region would have to be communicated instantaneously throughout the object so that no internal distance changed.

Relativity forbids instantaneous transmission of information or stress. Therefore a perfectly

rigid body is not a physically realizable object. It is a mathematical limit.

The ideal body is then accelerated into rotation. Points on the rim acquire tangential velocity, and one asks whether all proper distances can remain unchanged throughout the spin-up.

They cannot.

The valid conclusion is therefore limited:

| Perfect rigidity and relativistic rotational acceleration cannot be preserved simultaneously. |
|---|

# 6 An Impossible Assumption Applied to an Impossible Model

The methodological significance lies precisely here.

The Ehrenfest construction places an imaginary rigid body into an ideal relativistic process and demonstrates that two ideal conditions are mutually incompatible.

This is useful because it tells us that real systems must:

- transmit stress at finite speed,
- deform during acceleration,
- redistribute internal energy,
- and generally acquire a new equilibrium geometry.

But the conclusion applies only to the assumptions that generated it.

From

a perfectly rigid disk cannot be Born-rigidly spun up

one cannot infer

no extended rotating structure can exist.

These propositions are not logically equivalent.

If the object is non-rigid from the beginning, is permitted to change radius, redistribute energy, and settle into a new equilibrium, then the principal obstruction in the Ehrenfest setup no longer applies directly.

The relevant question becomes:

> How does a real, non-rigid structure reorganize when its internal motion increases?

## 6.1 From Idealization to Dogma

An idealization becomes dangerous when a restricted impossibility result is silently promoted into a universal prohibition.

The historical chain may be written schematically as

$$\text{perfect rigidity is impossible} \quad \Longrightarrow \quad \text{a rigid extended electron is impossible}$$

and then, without sufficient justification,

$$\text{a rigid extended electron is impossible} \quad \Longrightarrow \quad \text{every extended electron model is impossible.}$$

The second implication does not follow.

It excludes, without analysis, configurations that are not rigid bodies at all: nonlinear fields, closed waves, solitons, topological knots, deformable rings, phase-locked structures, and dynamically self-adjusting energy distributions.

The original paradox can therefore remain correct while its later use as a universal veto becomes logically excessive.

# 7 Do Perfectly Rigid Bodies Exist in Nature?

No known physical body is perfectly rigid.

Stone, metal, planets, stars, molecules, and nuclei all deform. Stress propagates at finite speed. Even the hardest material has a finite elastic modulus.

For any real material,

$$v_{\text{stress}} < c.$$

A real object can stretch, compress, heat, store internal energy, launch stress waves, and settle into a different equilibrium configuration.

Perfect rigidity is therefore a calculational idealization, not an observed material state.

Using the limitations of that idealization to reject a structure that is not rigid is methodologically unjustified.

# 8 Is an Atom a Rigid Body?

No.

The familiar planetary picture of an electron orbiting a nucleus is a pedagogical approximation, not the modern quantum description of an atom.

An atom is a bound state described by quantum fields, amplitudes, and spatial probability distributions. The relation between nucleus and electron is not the relation between two material points joined by a rigid rod.

Atoms can be polarized, excited, ionized, deformed by external fields, and reorganized into different bonding configurations.

Spherical symmetry or spatial closure does not imply classical rigidity.

# 9 Is an Electromagnetic Wave a Rigid Body?

No.

An electromagnetic wave is not a set of material points with permanently fixed separations. It is a dynamical field configuration.

A field may propagate, disperse, interfere, exchange energy, become trapped in a cavity, or form a localized nonlinear structure.

If an electron were a closed electromagnetic or phase-like configuration, the relevant question would not be whether a rigid sphere can rotate.

The correct question would be:

Can the field equations admit a finite-energy, localized, stable, charged, and

dynamically closed configuration?

That is a problem of field dynamics, topology, closure, and stability.

# 10 The Logical Error in Extending Rigidity

The common inferential chain may be written as

a classical rigid sphere is incompatible with relativistic rotation

$$\Downarrow$$

an extended electron cannot rotate

$$\Downarrow$$

the electron must be fundamentally point-like.

The chain is invalid unless one first assumes that every extended electron must be a classical rigid sphere.

Other possibilities include:

- a field configuration,
- a ring or torus of energy,
- a closed wave,
- a soliton,
- a topological knot,
- a phase-locked structure,
- or a deformable multilayer configuration.

The failure of one specific model does not establish the failure of all possible models.

# 11 Point-Likeness of the Electron: Observation or Interpretation?

In the Standard Model, the electron is treated as an elementary point-like excitation, and no internal structure has been resolved at currently tested scales.

Two statements must nevertheless be distinguished.

The experimental statement is

no internal electron radius has been resolved at current experimental scales.

The ontological statement is

the electron has no extended structure in ultimate reality.

The second statement does not follow from the first alone.

Failure to resolve structure may mean:

1. no structure exists,
2. the structure is below present resolution,
3. the probe couples only weakly to the relevant internal degree of freedom,
4. the internal configuration appears point-like in an external readout,
5. or internal motion and self-accommodation reduce the observable scale.

The point-particle model is extraordinarily successful as an effective description. Its success does not by itself prove that no deeper structure is possible.

# 12 Can Internal Motion Hide Structure?

Suppose that the electron is not a rigid classical sphere but a closed configuration of field and energy.

Increasing internal motion could then change several quantities simultaneously:

$$r = r(\omega), \qquad \rho_E = \rho_E(\omega), \qquad T_{ij} = T_{ij}(\omega), \qquad \Phi = \Phi(\omega),$$

where $r$ is an effective radius, $\omega$ an internal angular rate, $\rho_E$ an energy density, $T_{ij}$ an internal stress tensor, and $\Phi$ a phase structure.

The radius would not be an externally fixed input. It would be selected by equilibrium:

$$\frac{\partial E_{\text{total}}}{\partial r} = 0,$$

with local stability requiring

$$\frac{\partial^2 E_{\text{total}}}{\partial r^2} > 0.$$

A rigid-body model imposes

$$r = \text{constant}$$

independently of rotation.

A dynamical model instead permits

$$r = r(\omega).$$

The possibility of contraction or radial accommodation must therefore be derived or ruled out through dynamics, not excluded by invoking the impossibility of perfect rigidity.

# 13 A Conditional Radial-Accommodation Relation

A conditional model may relate a reference proper scale $\rho$ to an observed radius $r$ through

$$r = \frac{\rho}{\gamma},$$

with

$$\gamma = \frac{1}{\sqrt{1 - \omega^2 r^2 / c^2}}.$$

Solving self-consistently gives

$$r(\rho,\omega)=\frac{\rho}{\sqrt{1+\omega^2\rho^2/c^2}},$$

and therefore

$$\gamma(\rho,\omega)=\sqrt{1+\frac{\omega^2\rho^2}{c^2}},$$

while the tangential speed is

$$v=\omega r=\frac{\omega\rho}{\sqrt{1+\omega^2\rho^2/c^2}}.$$

For every finite $\omega$,

$$v<c.$$

This branch shows that an increasing angular rate may coexist with a decreasing observed radius without requiring superluminal tangential motion.

The relation is not yet a complete physical theory. It must emerge from an independent action or energy functional with specified stress, closure, equilibrium, and stability conditions.

# 14 A Re-reading of the Ehrenfest Problem

In the standard kinematic discussion, the circumference of a rotating disk is affected by tangential relativistic motion, while the radius is not aligned with the local tangential velocity and is not assigned the same direct Lorentz contraction.

That conclusion is understandable within a setup in which the radial coordinate is held fixed.

The present framework asks a different question:

> If the circumference, energy, stress, and closure conditions change, why must the equilibrium radius remain independent and fixed?

In a real material or field system, circumference and radius are not externally independent of dynamics. They are linked by constitutive laws, stresses, energy minimization, and boundary conditions.

The correct question is therefore not

why does the radius not Lorentz-contract exactly like the circumference?

but

what is the new equilibrium radius after the system dynamically readjusts?

The Ehrenfest paradox does not answer that question. It only shows that perfect rigidity, relativistic rotation, and unchanged geometry cannot all be maintained simultaneously.

# 15 From the Electron to the Ruler

A ruler is not an abstract geometrical entity. It is made of matter.

Its length ultimately depends on:

- equilibrium interatomic distances,
- electronic structure,
- binding energies,
- electromagnetic interactions,
- particle masses and charges,
- and environmental conditions.

Symbolically,

$$L_{\text{ruler}} = L_{\text{ruler}}\left(r_e, E_{\text{bind}}, \alpha, m_e, m_p, \Phi, \ldots\right).$$

If the effective structure of the electron changes,

$$r_e = r_e(t),$$

then, in principle,

$$L_{\text{ruler}} = L_{\text{ruler}}(t).$$

This remains a hypothesis. It must be derived from a matter model and tested through dimensionless observables. It cannot be inserted merely as a cosmological fitting factor.

# 16 The Local Observer Problem

Let the physical distance from an observer to a center, as described in an external reconstruction, be

$$D_{\text{phys}}(t).$$

Let the local ruler scale be

$$L_{\text{ruler}}(t).$$

The locally reported distance is the number of ruler units spanning the physical interval:

$$D_{\text{meas}}(t) = \frac{D_{\text{phys}}(t)}{L_{\text{ruler}}(t)}.$$

Its logarithmic rate is

$$\frac{\dot{D}_{\text{meas}}}{D_{\text{meas}}} = \frac{\dot{D}_{\text{phys}}}{D_{\text{phys}}} - \frac{\dot{L}_{\text{ruler}}}{L_{\text{ruler}}}.$$

Three regimes follow.

## 16.1 Visible Change

If

$$\dot{L}_{\text{ruler}} = 0,$$

the local readout directly tracks physical change.

## 16.2 Hidden Change

If

$$\frac{\dot{D}_{\text{phys}}}{D_{\text{phys}}} = \frac{\dot{L}_{\text{ruler}}}{L_{\text{ruler}}},$$

then

$$\dot{D}_{\text{meas}} = 0.$$

The physical distance changes, but the observer does not locally register it.

### 16.3 Readout Inversion

If the distance decreases but the ruler decreases faster,

$$\frac{\dot{L}_{\text{ruler}}}{L_{\text{ruler}}} < \frac{\dot{D}_{\text{phys}}}{D_{\text{phys}}} < 0,$$

then

$$\frac{\dot{D}_{\text{meas}}}{D_{\text{meas}}} > 0.$$

The physical distance decreases while the local observer reports an increase.

## 17 Why the Local Observer May Not Detect Its Own Change

Suppose every local length changes by a common scale factor $S(t)$:

$$L_i(t) = S(t)L_{i0}.$$

Then

$$\frac{L_i(t)}{L_j(t)} = \frac{L_{i0}}{L_{j0}}.$$

Purely internal comparisons cannot detect the common factor.

For the hypothesis to have physical content rather than represent a mere change of units, at least one of the following must occur:

1. different standards evolve differently,
2. a dimensionless quantity changes,
3. the evolution is anisotropic,
4. an external anchor follows a different calibration history,
5. or signal delays and independent clock comparisons expose the difference.

A variable-ruler theory must therefore produce a dimensionless or relational prediction.

# 18 The Outward-Anchor Principle

Three points are introduced:

$$P_1 = \text{gravitational center},$$

$$P_2 = \text{local observer or target},$$

$$P_3 = \text{outward anchor}.$$

Two snapshots are taken at times $t_0$ and $t_1$.

The center and the outward anchor are aligned between the two snapshots. The observer is intentionally not used as the reference point. Its residual displacement is then measured.

The operational principle is:

> Lock the center and the anchor; leave the observer free; read the residual.

The outward anchor is not an absolute frame. Its role is to provide a reference with a different position in the nested gravitational and calibration hierarchy.

# 19 The Three-Point Alignment Test

Let

$$\mathbf{x}_i^{(0)} = \mathbf{x}_i(t_0), \qquad \mathbf{x}_i^{(1)} = \mathbf{x}_i(t_1).$$

First translate both snapshots so that the center is at the origin:

$$\mathbf{y}_i^{(a)} = \mathbf{x}_i^{(a)} - \mathbf{x}_1^{(a)}, \qquad a \in \{0, 1\}.$$

Choose a rotation matrix $\mathbf{R}$ such that the center–anchor direction in the later snapshot is mapped onto the baseline direction:

$$\mathbf{R}\frac{\mathbf{y}_3^{(1)}}{\|\mathbf{y}_3^{(1)}\|} = \frac{\mathbf{y}_3^{(0)}}{\|\mathbf{y}_3^{(0)}\|}.$$

Define the common scale factor

$$s = \frac{\|\mathbf{y}_3^{(0)}\|}{\|\mathbf{y}_3^{(1)}\|}.$$

The aligned later configuration is

$$\widetilde{\mathbf{y}}_i^{(1)} = s\mathbf{R}\mathbf{y}_i^{(1)}.$$

The observer residual is

$$\Delta\mathbf{r}_2 = \widetilde{\mathbf{y}}_2^{(1)} - \mathbf{y}_2^{(0)}.$$

Its radial component is

$$\Delta r_{\parallel} = \Delta\mathbf{r}_2 \cdot \widehat{\mathbf{y}}_2^{(0)}.$$

If

$$\Delta r_{\parallel} < 0,$$

the reconstructed residual points toward the center.

This quantity is a *three-point alignment residual*. It is not automatically identical to physical infall.

# 20 Computational Program and Selected Codes

The computational program was developed through a series of Python tests. Only the codes required for the article's logical and numerical structure are retained here.

| Code | Role in the Article |
|---|---|
| `step10_alignment_v2.py` | Baseline Sun–Earth–Neptune two-epoch alignment. Highly phase-sensitive and retained mainly as the initial implementation. |
| `step11_year_averaged_alignment.py` | Year-averaged Sun–Earth–Neptune test designed to reduce dependence on a single calendar date. |
| `step12_analytic_null_alignment.py` | Analytic common-mode expansion control based on solar mass loss. |
| `step13_multi_interval_alignment.py` | Time-stability test over multiple independent 26-year intervals. |
| `step14_anchor_sweep.py` | Robustness test under changes of outward anchor. |
| `step15_all_planets_as_targets.py` | Target-ensemble matrix test across the Solar System. |
| `step16_toy_model_control.py` | Fixed-radius internal-consistency model. It tests alignment geometry under imposed constant radii. |
| `step17_central_fall_reconstruction.py` | Inverse reconstruction toy model. |
| `48.py` / Code 48B | DE421 short-baseline solar reconstruction and baseline selection. |
| `50.py` and `51.py` | Nested-ruler readout and inversion demonstrations. |
| `52.py` | Standard-dynamics Galactic three-point alignment using `galpy`/MWPotential2014. |

# 21 Solar-System Results

The Sun was used as $P_1$, a planet as $P_2$, and an outward planet as $P_3$.

The year-averaged Earth–Neptune alignment produced

$$\Delta r_{\parallel} = -1{,}619{,}677.495 \text{ km}$$

over 26 years, corresponding to

$$-62{,}295.288 \text{ km yr}^{-1}.$$

The multi-interval test examined 25 independent 26-year intervals and reported

$$25 \text{ negative radial residuals}, \qquad 0 \text{ positive radial residuals}.$$

The mean residual was

$$-1{,}586{,}686.218 \text{ km},$$

with a median of

$$-1{,}569{,}898.879 \text{ km}.$$

The anchor sweep showed that Jupiter, Saturn, Uranus, and Neptune all retained the inward-sign residual in the tested configurations, while Mercury was substantially more phase-sensitive.

The all-target/all-anchor matrix reported

$$56 \text{ closer-pattern cases}, \qquad 0 \text{ farther/noise cases}.$$

These results establish reproducibility of the sign within the chosen reconstruction procedure. They do not by themselves establish a unique physical cause.

# 22 The Analytic Common-Mode Null Model

A simple common fractional orbital expansion caused by solar mass loss was tested analytically.

Before alignment, the model produced approximately

$$0.355115 \text{ m}$$

of Earth expansion and

$$10.678321 \text{ m}$$

of Neptune expansion over the selected interval.

After Sun–Neptune locking, the Earth residual was only

$$-3.0517578125 \times 10^{-5}\ \mathrm{m},$$

corresponding to

$$-1.173753 \times 10^{-6}\ \mathrm{m\,yr^{-1}}.$$

Thus, a purely common fractional expansion is almost entirely removed by the three-point scale locking.

This result does not prove the variable-ruler hypothesis. It shows only that the large reconstructed residuals are not reproduced by this particular common-mode null model.

# 23 Code 16: Fixed-Radius Internal Consistency, Not Falsification

Code 16 generated an artificial circular-orbit system with no injected radial fall, no common expansion, and no evolving ruler.

Its defining assumptions were effectively

$$r_i(t) = r_{i0}, \qquad L_{\mathrm{local}}(t) = L_0.$$

After alignment, the code produced

$$55 \text{ closer-pattern cases}, \qquad 1 \text{ farther-pattern case}.$$

This result was initially described as a possible method-bias warning. That interpretation is incomplete.

The code was constructed inside a fixed-radius model. Radial evolution, variable calibration, and nested ruler contraction were excluded by construction. Consequently, any nonzero residual in this model must be interpreted through the geometry that remains: phase differences, angular motion, anchor-locking rotation, and projection.

The result therefore establishes the limited statement

$$\boxed{\Delta r_{\parallel}^{\mathrm{raw}} \neq \Delta r_{\mathrm{physical}}}$$

in general.

It does not establish that all observed inward residuals are nonphysical.

It also cannot falsify a variable-ruler or hidden-infall model, because the defining variables of those models were not present in the simulation.

## 23.1 Code 16 as a Case of Internal Model Self-Confirmation

The structure of Code 16 may be summarized as

assume fixed radii

$$\Downarrow$$

generate artificial data with fixed radii

$$\Downarrow$$

interpret the output within a fixed-radius geometry.

This is not a programming error. A model should reproduce the consequences of its assumptions.

The methodological problem arises only if this internal consistency is used to reject a model whose relevant degrees of freedom were excluded from the test.

A model in which

$$\dot{r}_i = 0 \qquad \text{and} \qquad \dot{L}_{\mathrm{local}} = 0$$

cannot decide whether nature allows

$$\dot{r}_i \neq 0 \qquad \text{or} \qquad \dot{L}_{\mathrm{local}} \neq 0.$$

Code 16 is therefore best classified as

a fixed-radius internal-consistency and phase-sensitivity test,

not as a decisive control against the variable-ruler hypothesis.

# 24 How a Proper Discriminating Control Should Be Built

A meaningful comparison must preserve identical initial conditions, phases, times, and anchor geometry while changing only the physical mechanism under test.

Define a fixed-radius null reconstruction

$$\Delta r_{\mathrm{null}} = \Delta r_{\parallel}\Big|_{\dot{r}=0,\ \dot{L}_{\mathrm{local}}=0}.$$

Then define the excess residual

$$\boxed{\Delta r_{\mathrm{excess}} = \Delta r_{\mathrm{data}} - \Delta r_{\mathrm{null}}.}$$

Alternatively, construct two otherwise identical synthetic systems:

$$\text{Model A:} \qquad r_i(t) = r_{i0},$$

$$\text{Model B:} \qquad r_i(t) = r_{i0} + \delta r_i(t).$$

Then compute

$$\Delta r_{\mathrm{injected}} = \Delta r_B - \Delta r_A.$$

This subtraction removes the phase and alignment geometry common to both models and isolates the response to the injected radial law.

The same strategy must be used for a variable-ruler model:

$$L_{\mathrm{local}}^{(A)}(t) = L_0,$$

$$L_{\text{local}}^{(B)}(t) = L_0 S(t).$$

Only a direct comparison of rival predictions on shared inputs can discriminate between the models.

# 25 Injected Central-Fall Model

Code 17 introduced the toy radial law

$$v_r(r) = -\frac{A}{\sqrt{r}}.$$

After outward-anchor locking, the artificial system produced

$$7 \text{ closer cases}, \qquad 0 \text{ farther cases}.$$

This demonstrates that a central-fall model can produce the expected inward-sign residual.

The result is a prediction-model test, not observational proof. Because a fixed-radius phase geometry can also produce a negative projected residual, the injected model must be compared quantitatively with a matched fixed-radius control rather than only by sign.

# 26 The 75-Year Solar-System Working Baseline

The DE421 short-baseline reconstruction was evaluated over 26, 50, 75, 100, and 120 years.

The reconstructed solar residual remained negative for all tested baselines. The 50–75 year range showed the strongest reported stability, and 75 years was selected as the working comparison interval.

The reported value was

$$A_{\text{Solar}} = -6{,}989{,}071.6 \text{ km},$$

with rate

$$\frac{A_{\text{Solar}}}{75 \text{ yr}} = -93{,}187.6 \text{ km yr}^{-1},$$

and fractional value

$$A_{\text{Solar,fraction}} = -0.0015506063.$$

This number is a reconstruction output. It is not to be presented as a direct measured radial velocity of Earth.

# 27 Nested-Ruler Readout Demonstration

The local readout model was then written as

$$D_{\text{meas}} = \frac{D_{\text{phys}}}{L_{\text{effective}}}.$$

A solar-layer calibration was chosen so that the reconstructed solar approach was exactly cancelled in the local readout.

A second, Galactic calibration layer was then added.

The reported 75-year normalized values were

$$A_{\text{Solar,fraction}} = -0.0015506063,$$

$$A_{\text{Galactic,fraction}} = +4.08 \times 10^{-8}.$$

The solar-only readout gave

$$D_{\text{meas}}/D_0 = 1,$$

while the nested solar-plus-Galactic model gave

$$D_{\text{meas}}/D_0 = 1.0000000408.$$

Thus, within the working model, the physical reconstruction indicated approach while the local readout indicated a very small apparent recession.

The corresponding follow-up calculation reported:

$$\text{reconstructed approach} = 6{,}989{,}071 \text{ km},$$

$$\text{apparent local recession} \approx 6.1 \text{ km}.$$

The required effective ruler scale was

$$L_{\text{new}} = 0.9532809,$$

or a model-implied contraction of approximately

$$4.672\%.$$

This number is not an independently measured ruler contraction. It is the ruler change required by that particular normalized readout construction.

# 28 Galactic Extension

At Galactic scale the three points were defined as:

1. the Galactic Center,
2. the Solar System,
3. an outer Gaia ring proxy.

Because linear backward propagation and full orbit integration can disagree, standard Galactic dynamics was adopted as the operational reference.

The reconstruction used `galpy` with MWPotential2014 over a 75-year baseline.

The reported values were:

$$\text{direct Solar-System approach} = 26{,}271{,}701{,}999 \text{ km},$$

$$\text{scale-induced displacement} = 3{,}906{,}224{,}827 \text{ km},$$

and therefore

$$\text{final three-point aligned approach} = 30{,}177{,}926{,}827 \text{ km}.$$

These values remain model-dependent. They depend on the adopted Galactic potential, the Gaia sample, the construction of the outer ring, and the alignment definition.

They must be reported as standard-dynamics reconstruction outputs, not direct measurements of a radial fall of the Solar System.

# 29 From Model Self-Consistency to Experimental Discrimination

Every mathematical model is expected to reproduce data generated from its own assumptions.

A fixed-radius model should reproduce fixed-radius dynamics.

A variable-ruler model should reproduce the consequences of its own calibration law.

Neither fact is evidence that the corresponding model describes nature.

The scientifically relevant question is whether rival models make different predictions for the same observational input.

The required structure is therefore

$$\text{shared initial data}$$

$$\Downarrow$$

$$\begin{cases} \text{fixed-ruler prediction,} \\ \text{variable-ruler prediction,} \end{cases}$$

$$\Downarrow$$

$$\text{comparison with independent observations.}$$

A true discriminating observable must be insensitive to trivial coordinate redefinitions and should preferably be dimensionless.

Possible targets include:

- differential clock comparisons,
- non-common scaling of independent material standards,
- anisotropic calibration drift,
- anchor-dependent residual laws,
- and cross-system comparisons that cannot be absorbed into a global rescaling.

# 30 Looking Outward: Dark Energy

Dark energy is introduced in standard cosmology to explain the observed expansion history, not merely because some recession rates exceed $c$.

The present framework asks whether part of an inferred distance evolution could arise from evolution of the local measuring standard:

$$D_{\text{meas}} = \frac{D_{\text{phys}}}{L_{\text{ruler}}}.$$

A time-dependent $L_{\text{ruler}}$ could alter the inferred distance history.

This mathematical possibility does not eliminate dark energy.

A viable alternative must reproduce, with one consistent calibration law:

- luminosity distances,
- angular-diameter distances,
- redshift,
- supernova time dilation,
- the cosmic microwave background,
- baryon acoustic oscillations,
- structure growth,
- gravitational lensing,
- and cosmic age constraints.

Until that is done, readout inversion remains a research hypothesis.

# 31 Looking Inward: Point Particles and Divergences

The point-particle framework of quantum field theory is extraordinarily successful.

Raw point-field calculations can contain divergences, and renormalization provides a systematic method for obtaining finite predictions. It is inaccurate to reduce renormalization to a casual subtraction of infinities.

Even so, the success of renormalization does not logically prove that point-likeness is the ultimate ontology.

A deeper extended model would need to reproduce:

- electron mass,
- electric charge,
- spin 1/2,
- Fermi statistics,
- the magnetic moment,
- high-energy scattering limits,
- observed stability,
- and the absence of resolved internal structure.

Without these results, an extended electron remains a conceptual possibility rather than a complete physical theory.

# 32 Connecting the Two Boundaries

The central proposal of the paper is a methodological chain:

$$\text{structure of matter} \longrightarrow \text{ruler and clock} \longrightarrow \text{local observer} \longrightarrow \text{cosmological readout.}$$

If the electron and the atom are not absolutely scale-invariant objects external to dynamics, then the ruler is not automatically absolute.

If the ruler is not absolute, a measured distance records both the evolution of the target and the evolution of the observer's own calibration.

The microscopic and cosmological problems may therefore be linked through the physics of measurement standards.

# 33 The Dynamical Model Still Required

A complete theory must derive at least three functions:

$$r_e(\omega, \Phi, \ldots),$$

$$L_{\text{ruler}}(t),$$

$$T_{\text{clock}}(t).$$

A minimal action or energy should include:

- field energy,
- internal rotational energy,
- phase stiffness,
- closure conditions,
- coupling to gravity,
- and radial dynamics.

The equilibrium condition is

$$\frac{\partial E}{\partial r} = 0,$$

with stability requiring

$$\frac{\partial^2 E}{\partial r^2} > 0.$$

The theory must then derive how a change in the microscopic structure modifies atomic lengths and clock rates.

Without that chain, one cannot infer ruler contraction directly from a three-point residual.

# 34 Falsifiable Tests

The framework must be capable of failure.

## 34.1 Non-Rigid Field Test

A proposed field configuration must show:

- finite total energy,
- subluminal local motion,
- a finite equilibrium radius,
- and stability against perturbations.

## 34.2 Electron-Structure Test

The model must satisfy all experimental limits on electron compositeness and reproduce known electron observables.

## 34.3 Ruler Test

The proposed matter-scale evolution must produce a measurable dimensionless or differential signature.

## 34.4 Three-Point Test

The algorithm must:

- remove rigid translation,
- remove common rotation,
- remove common scale,
- recover injected infall,
- recover injected expansion with the opposite sign,
- and distinguish physical radial change from phase-only residuals.

The last condition replaces the earlier, overly strong demand that every no-infall circular model must yield an exactly zero raw residual.

### 34.5 Cosmological Test

A single calibration law must fit the full cosmological data set without separate ad hoc tuning for each observable.

## 35 What This Paper Does Not Claim

This paper does not claim that:

- general relativity ignores curvature,
- all of standard physics assumes globally flat spacetime,
- the Ehrenfest paradox is mathematically wrong,
- the electron is certainly a ring or closed wave,
- rotation alone proves $r = \rho/\gamma$,
- the three-point residual is automatically physical infall,
- dark energy is definitively nonexistent,
- or all quantum phenomena arise from hidden geometry.

## 36 What This Paper Does Claim

The limited claims are:

1. Local accuracy does not establish ontological finality.
2. Spacetime curvature does not by itself determine the internal geometry of matter.
3. The Ehrenfest paradox excludes perfect relativistic rigidity, not all extended rotating structures.
4. Atoms, waves, and field configurations are not classical rigid bodies.
5. Point-like experimental appearance is not logically identical to proof of no structure.

6. A non-rigid structure may, in principle, adjust radius and energy distribution dynamically.
7. If material structure changes, rulers and clocks may also participate in that change.
8. Co-evolution of distance and ruler can hide physical evolution from the local observer.
9. Faster ruler contraction can invert the local readout.
10. Three-point alignment provides a reconstruction observable after common translation, rotation, and scale are removed.
11. Raw alignment residuals are not automatically identical to physical radial displacement.
12. Code 16 is an internal fixed-radius consistency test, not a falsification of the variable-ruler hypothesis.
13. Physical discrimination requires matched rival models and independent observations.

# 37 Diagnosis Before Reconstruction: Why This Work Is a Methodological Warning Rather Than a Completed Alternative Theory

Every attempt to question an established scientific framework eventually encounters the same objection:

> If the existing description is incomplete, where is the complete theory that will replace it?

The objection appears reasonable, but it joins together two different scientific tasks. One task is to identify a hidden assumption, a methodological limitation, or a conceptual wrong turn. The other is to construct a complete mathematical and experimental framework capable of replacing the existing description.

These tasks may belong to the same research programme, but they are not the same intellectual achievement. A diagnosis is not invalid merely because the cure has not yet been completed.

The purpose of this chapter is therefore to state precisely what the present work has attempted, what it has established within its own argument, and what remains beyond its current scope.

## 37.1 Two Distinct Scientific Acts

Foundational progress in physics frequently involves two logically distinct acts:

1. **Foundational diagnosis:** identifying a point at which an approximation, idealisation, or operational convention has been treated as though it were a necessary property of reality.
2. **Theoretical reconstruction:** constructing a new mathematical framework that removes the identified limitation, recovers the successful predictions of the previous framework, and produces new testable consequences.

These two acts should not be confused. A completed reconstruction ordinarily requires a prior diagnosis. Unless the precise limitation of the former framework has been identified, there is no clear criterion for deciding what the new framework must repair. Conversely, a

diagnosis may be logically meaningful even before the corresponding reconstruction has been completed.

This distinction may be expressed as a methodological principle:

> **Principle of Foundational Diagnosis.** Recognising a foundational error and constructing its complete replacement are logically distinct scientific acts. The validity of the diagnosis must be assessed from the evidence and reasoning supporting the identified error, not solely from the completeness of the theory proposed to replace it.

This principle does not grant unrestricted legitimacy to every criticism of an established theory. A diagnosis must still identify a specific assumption, trace its consequences, and show why the assumption deserves re-examination. What it need not do is arrive already accompanied by the final theory of nature.

## 37.2 The Task Undertaken in This Work

The present work is primarily an exercise in foundational diagnosis.

Its central claim is not that a complete alternative theory of matter, gravitation, cosmology, and measurement has already been constructed. Its claim is that modern physical reasoning may contain a persistent methodological blind spot concerning the status of the observer's own material standards.

The argument developed in the preceding chapters traces a chain of idealisations involving:

- the use of perfectly rigid rulers and bodies in conceptual models;
- the treatment of the local Minkowski limit as though it provided an absolute and dynamically neutral calibration environment;
- the assumption that clocks and rulers may participate in motion but not in the deeper dynamical evolution attributed to the systems they measure;
- the elevation of locally invariant readouts into conclusions about global geometric invariance;
- and the silent identification of operational stability with ontological immutability.

Individually, each idealisation may be useful. The problem arises when their combined effect is forgotten and the resulting description is treated as though it were independent of the observer's material construction.

The question raised by this work is therefore not whether local measurements are inaccurate. Local physics may remain extraordinarily precise and internally consistent. The question is whether local measurement alone is sufficient to determine global physical evolution when the measuring standards may themselves belong to that evolution.

## 37.3 Internal Consistency and Global Completeness

A locally self-consistent system of measurement need not provide a complete account of its own global transformation.

Suppose that a physical distance $D(t)$ and the locally realised unit of length $L(t)$ evolve together. The reported numerical distance is

$$d_{\text{meas}}(t) = \frac{D(t)}{L(t)}.$$

If both quantities acquire the same scale factor $S(t)$,

$$D(t) = D_0 S(t), \qquad L(t) = L_0 S(t),$$

then

$$d_{\text{meas}}(t) = \frac{D_0}{L_0}.$$

No local inconsistency appears. Every locally constructed comparison may remain stable, although both the measured system and the standard used to measure it have changed.

This mathematical degeneracy does not prove that such co-evolution occurs in nature. It establishes a more limited but important point: local invariance by itself does not logically exclude common-mode physical evolution.

Accordingly, the present work distinguishes between two propositions:

1. Local experiments demonstrate the stability of dimensionless relationships within the local measurement system.
2. Local experiments prove that the material standards constituting that system are globally immutable.

The first proposition may be strongly supported. The second does not follow automatically from the first. The methodological warning of this book concerns the unnoticed passage

from one proposition to the other.

## 37.4 Historical Precedents for Diagnosis Preceding Completion

Physics has repeatedly advanced through the recognition of conceptual incompatibilities before the complete successor framework was available.

Einstein's 1905 treatment of special relativity did not include the field equations of general relativity. It addressed a specific conflict among the Galilean transformation, Maxwellian electrodynamics, and inherited notions of simultaneity, length, and time. A complete relativistic theory of gravitation emerged only later.

The absence of general relativity from the 1905 analysis did not make the special theory methodologically incomplete. The earlier work had identified and repaired a definite conceptual inconsistency within its stated domain.

The Ehrenfest paradox provides another relevant example. Its importance did not depend on the immediate availability of a complete constitutive theory for relativistic rotating materials. Its value lay in revealing that perfect rigidity could not be transferred unchanged into relativistic rotational kinematics. The paradox diagnosed a failure of an idealisation. The detailed physics of real rotating matter remained a separate problem.

The present work follows this general pattern. It asks whether the assumption of dynamically external material calibration has been granted a status that it has not independently earned. It then examines the conceptual and observational consequences that would follow if clocks, rulers, and matter standards were themselves participants in scale evolution. It does not claim that this participation has already been derived from a fundamental action or experimentally established.

## 37.5 The Error of Demanding a Complete Map Before Acknowledging a Wrong Turn

Consider a traveller who discovers that a road believed to cross a mountain instead terminates at a cliff.

The traveller must provide sufficient evidence that the road truly ends, that the obstacle is not an illusion, and that the prevailing map has misrepresented the route. She is not, however, required to chart every valley, river, and bridge of the correct route before her warning can be taken seriously.

The warning and the replacement map are different achievements.

Likewise, a foundational criticism of an established physical framework must be judged first by questions appropriate to a diagnosis:

- Has a genuine assumption been identified?
- Has that assumption been distinguished from an empirical result?
- Has the transition from approximation to ontological conclusion been traced explicitly?
- Does the assumption restrict which alternatives may even be formulated or tested?
- Can a conceivable observation distinguish the inherited assumption from its alternative?

These are the relevant criteria for the present work. The demand for a complete successor theory becomes appropriate when the project claims to have replaced the existing framework. This book makes no such claim. It argues that a foundational choice should be reopened and that the tools required to investigate it must not be excluded in advance.

## 37.6 What This Work Has Provided

Within that limited but deliberate purpose, the work has provided several concrete contributions.

### 37.6.1 A reconstruction of the conceptual pathway

The argument has traced the progression from familiar flat-road and rigid-body intuitions to relativistic rotation, the Ehrenfest problem, and the status of the local observer. The purpose of this reconstruction has been to identify where a practical idealisation may have become an implicit statement about the nature of matter and measurement.

### 37.6.2 A formulation of the hidden-change degeneracy

The work has expressed mathematically how common evolution of a physical quantity and its local measuring standard can remain invisible in their ratio. This formulation does not establish a physical ruler law. It establishes the logical structure of the observational degeneracy that such a law would create.

### 37.6.3 A formulation of readout inversion

The nested-layer model shows how a physical contraction may coexist with an apparent local expansion when the ruler responds to more scaling channels than the particular distance under measurement. This is presently a model of logical and mathematical possibility, not a completed derivation from fundamental dynamics.

### 37.6.4 An outward-anchor protocol

The three-point reconstruction introduces an external or more distant reference structure to reduce the common-mode ambiguity of purely local measurements. The protocol removes translation, rotation, and selected scale freedom before constructing a residual.

Its purpose is not to manufacture an absolute ruler. Its purpose is to test whether a geometrical discrepancy survives after the transformations shared by the local system have been removed.

### 37.6.5 Computational demonstrations

The computational experiments described in the main text illustrate the behaviour of the alignment protocol under controlled assumptions, including fixed-radius and injected-displacement scenarios. These demonstrations establish the behaviour of the reconstruction procedure. They do not by themselves prove that the reconstructed quantity corresponds to physical infall.

For that reason, the work explicitly distinguishes

$$\Delta D_{\mathrm{data}}$$

from the candidate physical residual

$$\Delta D_{\mathrm{phys,cand}} = \Delta D_{\mathrm{data}} - \Delta D_{\mathrm{null}}.$$

A matched null model remains necessary before any residual can be given a physical interpretation.

### 37.6.6 A connection to the phase-drift hypothesis

The companion phase-drift construction supplies one possible mechanism through which metrological standards might accumulate scale evolution across nested cyclic environments. That connection is presented as a research hypothesis. It is not treated as an already established consequence of standard relativistic physics.

## 37.7 What This Work Has Not Provided

The boundaries of the present argument must be stated equally clearly. This work has not yet provided:

- a microscopic dynamical model of matter from which the evolution of rulers and clocks follows uniquely;
- a covariant action whose solutions produce the proposed nested scale laws;
- a derivation showing why solar, Galactic, and other hierarchical layers must couple to matter with the particular strengths assumed in the illustrative models;
- a complete cosmological fit replacing dark energy, dark matter, or inflation with one quantitatively consistent mechanism;
- a demonstration that the reconstructed multi-million-kilometre displacement is a physical Earth–Sun infall rather than an artefact of coordinate choice, ephemeris modelling, alignment, anchor uncertainty, or data processing;
- an independent observation of the proposed phase-drift mechanism;
- or a decisive dimensionless observable that has already separated a variable-standard framework from all fixed-standard alternatives.

These omissions are not hidden. They define the next stage of the programme. The present work asks that the foundational issue be examined before the absence of its final dynamical completion is used to declare the issue meaningless.

## 37.8 Why This Work Remains Scientific

The distinction between methodological diagnosis and completed theory does not place the present work outside science. A foundational proposal remains scientific to the extent that it:

1. identifies its assumptions explicitly;
2. distinguishes established results from proposed interpretations;
3. formulates its claims mathematically where possible;
4. exposes its reasoning to criticism;
5. identifies observations capable of constraining or rejecting the proposal;
6. and permits future evidence to show that the proposed diagnosis was mistaken.

The present work does not ask that nested contraction, phase drift, or material-scale evolution be accepted without evidence. It asks that they not be excluded merely because the local observer would fail to detect a perfectly common-mode component.

The project becomes stronger, not weaker, when the possible failure modes are stated openly. For example, the proposed interpretation would lose credibility if:

- residuals failed to persist under independent anchor choices;
- the inferred displacement changed arbitrarily under reasonable alignment conventions;
- the result disappeared when the null model reproduced the same pipeline;
- independent ephemeris, ranging, clock, and astrometric data proved incompatible with the required scale evolution;
- or no dimensionless differential signature could be identified even in principle.

A framework that declares in advance how it may fail remains open to scientific judgment.

## 37.9 The Proper Interpretation of the Present Volume

The present volume should therefore be read neither as a completed replacement for modern physics nor as a purely literary objection to it. It is a structured diagnosis with three aims:

1. to identify a foundational assumption concerning the observer and the observer's material standards;
2. to show that relaxing this assumption produces nontrivial observational degeneracies;
3. and to develop preliminary methods by which those degeneracies may eventually be tested.

The conceptual critique, the geometrical reconstruction, the nested readout model, and the phase-drift hypothesis belong to different levels of completion. They should not be collapsed into one undifferentiated claim.

The diagnosis is the most developed part of the present work. The alternative kinematics and reconstruction procedures are conditional constructions. The material dynamics and cosmological replacement programme remain open.

This hierarchy is not a weakness to be concealed. It is the correct map of the project's current scientific status.

### 37.10 Conclusion of This Chapter

To criticise this work solely because it does not yet contain a finished alternative theory is to judge a diagnosis by the standards of a completed reconstruction.

The relevant first question is not:

> Where is the final theory?

It is:

> Has the work identified a genuine assumption whose consequences have not been adequately examined?

If the answer is no, the diagnosis should be rejected. If the answer is yes, then the incompleteness of the replacement theory is not a reason to close the inquiry. It is a reason to begin the next stage of it.

This work does not claim that the new road has already been mapped. It claims that the old map may have placed the junction incorrectly.

*A warning is not a map. Yet every reliable map begins by recognising where the previous road went wrong.*

## 38 Conclusion

The argument began with a flat road.

The driver's local formulas were accurate. Their success was real. But a conclusion that the entire Earth is flat would confuse local adequacy with global ontology.

The rigid-body problem has the same structure.

A perfectly rigid body is not a physical object. It is a limiting idealization. The Ehrenfest paradox correctly shows that such an object cannot be relativistically spun up while retaining all proper distances.

That conclusion excludes

relativistic rotation of a perfectly rigid, unchanged body

but it does not exclude dynamically adjusting extended field structures.

An atom is not rigid.

An electromagnetic wave is not rigid.

A possible field-like electron is not required to be rigid.

The central physical question is therefore

Can a non-rigid field-and-energy configuration reorganize under rotation,
contract dynamically, and remain stable at finite radius?

If it can, the point-like appearance of the electron may be a readout of a self-accommodating structure rather than proof of no structure.

If the microscopic scale of matter evolves, then the ruler and clock of the observer are not external to the dynamics of the universe.

The local-observer problem then becomes operational:

How can physical motion be separated from evolution of the measuring standard itself?

The outward-anchor and three-point protocols are initial attempts to answer that question.

The computational record already reveals an important methodological lesson. A fixed-radius synthetic model may reproduce a strong inward-sign alignment residual. That result does not refute the alternative framework. It shows that raw residuals contain phase and projection structure and must not be equated directly with radial motion.

The task of the new theory is not to deny the success of the flat road. It is to determine the larger surface on which the road lies, explain why the local traveler cannot see its curvature, and identify an observation that distinguishes motion of the world from evolution of the ruler.

# Transition to the Operational Appendices

The purpose of this paper is not to ask the reader to abandon local measurement. It is to ask whether local measurement alone is sufficient to reconstruct global geometry when the material standards used in measurement may participate in the same evolution as the observer.

If a three-point reconstruction returns a large inward residual while local ranging, timing, and orbital ratios remain stable, two broad possibilities must be distinguished. Either the reconstruction is contaminated by phase, projection, or model-completion error, or the local standards co-evolve with the system and suppress a common-mode change. The first possibility is already represented by the fixed-radius controls discussed in the main text. The second is developed below as an explicitly conditional metrological hypothesis.

The appendices therefore do not convert a residual into a discovery. They state the scale map that would be required to hide a physical displacement, show how a three-point comparison can escape a purely local degeneracy, connect that construction to the separate phase-drift paper, and specify the observations that could reject the proposal.

# A Why Local Measurements Cannot Reveal a Hidden Infall

## A.1 Statement of the apparent paradox

The Solar-System reconstruction discussed in the main text returns, on one model branch, an inward Earth residual of order $7 \times 10^6$ km over a nominal 75-year interval after the centre and an outer anchor are locked. If this number were interpreted as a directly observed reduction of the Earth–Sun distance measured with an invariant material ruler, it would be grossly incompatible with modern planetary ephemerides, radar ranging, lunar laser ranging, spacecraft tracking, and atomic-time orbital dynamics.

That contradiction cannot be removed by rhetoric. It can disappear only if the reconstructed quantity and the locally measured quantity are shown to be different observables.

Let

$$D_{\mathrm{phys}}(t) \equiv \text{physical centre–observer separation}, \tag{1}$$

$$\ell_{\mathrm{local}}(t) \equiv \text{physical size of one realised local length unit}, \tag{2}$$

$$\Theta_{\mathrm{local}}(t) \equiv \text{physical duration of one realised local time unit}. \tag{3}$$

The numerical distance reported by the local observer is then

$$d_{\mathrm{meas}}(t) = \frac{D_{\mathrm{phys}}(t)}{\ell_{\mathrm{local}}(t)}. \tag{4}$$

This equation contains the central degeneracy: a change in the numerator can be hidden by a matching change in the denominator.

## A.2 Common-mode scale evolution

Introduce dimensionless scale factors relative to a reference epoch $t_0$:

$$G_D(t) = \frac{D_{\mathrm{phys}}(t)}{D_{\mathrm{phys}}(t_0)}, \qquad G_L(t) = \frac{\ell_{\mathrm{local}}(t)}{\ell_{\mathrm{local}}(t_0)}, \qquad G_T(t) = \frac{\Theta_{\mathrm{local}}(t)}{\Theta_{\mathrm{local}}(t_0)}. \tag{5}$$

Equation (4) gives

$$\frac{d_{\mathrm{meas}}(t)}{d_{\mathrm{meas}}(t_0)} = \frac{G_D(t)}{G_L(t)}. \tag{6}$$

Therefore, if

$$G_D(t) = G_L(t), \tag{7}$$

the local numerical distance is unchanged even though the physical distance and physical ruler have both evolved.

This is not yet a prediction. It is the exact mathematical condition under which a physical radial change becomes invisible to a local length comparison.

## A.3 Phase-cycle scale hypothesis

The following step is non-standard and must be identified as such. The phase-drift model developed in the companion paper postulates that each completed phase-locked cycle produces a fractional change in a local metrological baseline, and that successive changes act multiplicatively on the already modified baseline.

For a uniformly rotating ring, the one-direction slow-transport holonomy is

$$\Delta\tau_\phi = \frac{2\pi\Omega R^2}{c^2}, \tag{8}$$

which has dimensions of time. Dividing by the cycle period $T_{\mathrm{cyc}} = 2\pi/\Omega$ defines the dimensionless phase-cycle parameter

$$\boxed{\epsilon_\phi \equiv \frac{\Delta\tau_\phi}{T_{\mathrm{cyc}}} = \frac{\Omega^2 R^2}{c^2}.} \tag{9}$$

The distinction between the dimensional holonomy $\Delta\tau_\phi$ and the dimensionless fractional parameter $\epsilon_\phi$ is essential.

A possible operational realisation of the hypothesis is

$$\Theta_{\text{local}}(n) = \Theta_0(1+\epsilon_\phi)^n, \tag{10}$$

$$\ell_{\text{local}}(n) = \ell_0(1+\epsilon_\phi)^n, \tag{11}$$

for $n$ completed cycles. The equal scale factors in Equation (11) are required if the physical propagation speed and the locally assigned numerical value of $c$ are both to remain unchanged. More generally,

$$\frac{G_L}{G_T} = \frac{c_{\text{phys}}(t)}{c_{\text{phys}}(t_0)} \tag{12}$$

under a fixed numerical convention. Thus $G_L = G_T$ follows only on the branch where the physical propagation sector is invariant.

Equations (9)–(11) are a hypothesis imported from the phase-drift programme. They are not a consequence of the standard Sagnac effect and are not established by the three-point reconstruction itself.

## A.4 Why local orbital data may remain insensitive

Let a physical orbital period be $P_{\text{phys}}(t)$. The local numerical period is

$$P_{\text{meas}}(t) = \frac{P_{\text{phys}}(t)}{\Theta_{\text{local}}(t)}. \tag{13}$$

A stable local period therefore requires

$$\frac{G_P(t)}{G_T(t)} \simeq 1, \qquad G_P(t) = \frac{P_{\text{phys}}(t)}{P_{\text{phys}}(t_0)}. \tag{14}$$

Likewise, a locally inferred speed is

$$v_{\text{meas}}(t) = \frac{D_{\text{phys}}(t)/P_{\text{phys}}(t)}{\ell_{\text{local}}(t)/\Theta_{\text{local}}(t)} = v_{\text{meas}}(t_0)\frac{G_D G_T}{G_P G_L}. \tag{15}$$

If $G_D = G_L$ and $G_P = G_T$, the numerical speed also remains unchanged.

It is not sufficient, however, merely to invoke Kepler's third law. A shrinking physical orbit changes the dimensional relation

$$P_{\text{phys}}^2 = \frac{4\pi^2}{\mu_{\text{phys}}} D_{\text{phys}}^3, \tag{16}$$

where $\mu_{\rm phys} = GM$ is itself dimensional. In local units, the measured gravitational parameter is

$$\mu_{\rm meas} = \mu_{\rm phys} \frac{\Theta^2_{\rm local}}{\ell^3_{\rm local}}. \tag{17}$$

Consequently, preservation of the dimensionless local Kepler relation requires a complete scale map for $D$, $P$, $\ell$, $\Theta$, and $\mu$. Co-scaling of ruler and clock alone does not automatically prove orbital invariance; it specifies one necessary part of the cancellation.

The scientifically defensible statement is therefore conditional:

> Local ranging and timing can remain insensitive to a physical infall only if the dynamical orbit and the realised metrological standards evolve according to a mutually consistent scale map that preserves the measured dimensionless ratios.

## A.5 What the reconstructed residual represents

The three-point calculation should not be described as measuring an absolute distance with a hidden fixed ruler. It constructs a differential quantity after translation, rotation, and one declared scale mode have been removed.

Let $\mathbf{C}_i$, $\mathbf{E}_i$, and $\mathbf{A}_i$ denote the centre, Earth, and outer-anchor positions at epochs $t_i$, $i = 0, 1$. Define the baseline vectors

$$\mathbf{b}_i = \mathbf{A}_i - \mathbf{C}_i, \qquad \mathbf{e}_i = \mathbf{E}_i - \mathbf{C}_i. \tag{18}$$

Let $\mathcal{R}$ rotate $\mathbf{b}_1$ onto the direction of $\mathbf{b}_0$, and define the anchor-locking scale

$$s_A = \frac{|\mathbf{b}_0|}{|\mathbf{b}_1|}. \tag{19}$$

The transformed second-epoch Earth vector is

$$\widetilde{\mathbf{e}}_1 = s_A \mathcal{R} \mathbf{e}_1, \tag{20}$$

and the reconstructed displacement is

$$\boxed{\Delta \mathbf{e}_{\rm rec} = \widetilde{\mathbf{e}}_1 - \mathbf{e}_0.} \tag{21}$$

Its radial component is

$$\Delta D_{\rm rec} = \Delta \mathbf{e}_{\rm rec} \cdot \widehat{\mathbf{e}}_0. \tag{22}$$

This is a residual in the anchor-normalised geometry. It becomes a physical infall estimate

only after phase-only, projection-only, ephemeris, and model-completion contributions have been bounded or removed.

### A.6 Resolution of the paradox

The hidden-infall paradox is therefore resolved only at the level of logical compatibility:

1. A large anchor-normalised geometric residual need not equal a directly measured local distance change.
2. A common physical evolution can cancel in local dimensionless ratios.
3. The required cancellation is not automatic; it demands a complete and consistent dynamical scale map.
4. Existing precision ranging data are constraints on that map, not data that may simply be declared irrelevant.

The proposal survives only if one parameter set reproduces both the three-point residual and the observed stability of independent local and non-local dimensionless quantities.

## B Resolution of the Readout-Inversion Puzzle via Nested Contraction Layers

In the main text, a numerical model was presented in which a reconstructed inward displacement of Earth toward the Sun of approximately $7 \times 10^6$ km over 75 years is locally read as a small apparent outward displacement of about 6.1 km. This appendix explains, in elementary terms, why such an inversion is a natural consequence of the nested-contraction hypothesis.

### B.1 The two-layer contraction model

Let the effective length of the local ruler—one realised metre—be

$$L_{\text{eff}}(t) = L_0 \, S_{\text{solar}}(t) \, S_{\text{galactic}}(t), \tag{23}$$

where $S_{\text{solar}}$ is the scale factor associated with Earth's evolution within the solar layer and $S_{\text{galactic}}$ is the scale factor associated with the Solar System's evolution within the Galactic layer.

Within this two-layer construction, the physical Earth–Sun distance is modelled, to leading order, as

$$D_{\mathrm{phys}}(t) \simeq D_0\, S_{\mathrm{solar}}(t), \tag{24}$$

because the Sun and Earth participate almost equally in the Galactic common-mode evolution, whereas their mutual separation is controlled by the local solar layer.

## B.2 The locally measured distance

The locally reported distance is the dimensionless ratio

$$d_{\mathrm{meas}}(t) = \frac{D_{\mathrm{phys}}(t)}{L_{\mathrm{eff}}(t)}. \tag{25}$$

Substitution of Eqs. (23) and (24) gives

$$d_{\mathrm{meas}}(t) \simeq \frac{D_0}{L_0}\,\frac{1}{S_{\mathrm{galactic}}(t)}. \tag{26}$$

The solar factor cancels from the local readout. The reported Earth–Sun distance therefore depends, in this idealised limit, only on the residual Galactic factor. If $S_{\mathrm{galactic}}(t)$ decreases, the realised ruler shrinks and the numerical distance increases even while the physical Earth–Sun distance decreases through the solar factor.

## B.3 The apparent inversion

At the reference epoch, let

$$S_{\mathrm{solar}}(0) = S_{\mathrm{galactic}}(0) = 1. \tag{27}$$

After 75 years, suppose the solar layer contracts the physical Earth–Sun distance by approximately $7 \times 10^6$ km, while contracting the solar component of the ruler by the same fractional amount. In the absence of the Galactic layer, numerator and denominator co-scale and the local readout remains unchanged: the infall is hidden.

A much smaller additional Galactic contraction changes the denominator without producing the same differential change in the Earth–Sun separation. The denominator then shrinks slightly faster than the numerator, so that

$$\Delta D_{\mathrm{phys}} < 0, \qquad \Delta d_{\mathrm{meas}} > 0. \tag{28}$$

A true approach of millions of kilometres can therefore be encoded locally as an apparent recession of only a few kilometres.

For small fractional changes, write

$$S_{\text{solar}} = 1 + \sigma_s, \qquad S_{\text{galactic}} = 1 + \sigma_g, \qquad |\sigma_s|, |\sigma_g| \ll 1. \tag{29}$$

Then

$$\frac{\Delta D_{\text{phys}}}{D_0} \simeq \sigma_s, \tag{30}$$

$$\frac{\Delta d_{\text{meas}}}{d_0} \simeq -\sigma_g. \tag{31}$$

The sign of the physical evolution is set by the solar layer, whereas the sign of the residual local readout is set by the Galactic layer. No contradiction follows from opposite signs because the two quantities are not the same observable.

## B.4 Why the inversion does not violate physical intuition

The apparent paradox arises only if the ruler is assumed to respond solely to the same scale channel that governs the distance under measurement. In a nested environment, the ruler carries the cumulative product of all active layers, while a particular two-body separation may respond primarily to its local layer. The mismatch between those scale histories permits the readout to invert.

The mechanism can be stated generally. If

$$D_{\text{phys}} = D_0 \prod_{a \in \mathcal{D}} S_a, \qquad L_{\text{eff}} = L_0 \prod_{b \in \mathcal{L}} S_b, \tag{32}$$

then

$$d_{\text{meas}} = \frac{D_0}{L_0} \frac{\prod_{a \in \mathcal{D}} S_a}{\prod_{b \in \mathcal{L}} S_b}. \tag{33}$$

Every common factor cancels. The observed residual is controlled only by those channels that act differently on the measured separation and on the realised standard.

## B.5 Relation to the phase-drift model

In the companion phase-drift construction, different dynamical layers carry different cycle periods and therefore different accumulated fractional changes. The Galactic layer evolves over a much longer cycle and contributes a much smaller per-cycle drift than the solar layer. The numerical model in the main text is an explicit realisation of this nested structure: a large common solar contraction is cancelled in local measurement, while a small non-common Galactic contribution survives and determines the sign of the reported

residual.

Thus, readout inversion is not an isolated numerical curiosity. It is the local signature expected whenever the observer's metrological standard contains more independent scale channels than the particular separation being measured.

### B.6 Conclusion

Within the nested-contraction framework, readout inversion is the expected consequence of differential layer participation. The physical separation and the local numerical readout need not share either magnitude or sign. A small apparent expansion can therefore serve as a window into a much larger hidden contraction, provided that the layer structure and its numerical coefficients are independently fixed by the full phase-drift model.

## C Why Three-Point Alignment Can Escape the Local-Observer Trap

### C.1 Why two points are insufficient

With only a centre $C$ and observer $E$, the measured separation at two epochs supplies the ratio

$$\frac{d_{CE,\mathrm{meas}}(t_1)}{d_{CE,\mathrm{meas}}(t_0)} = \frac{G_{CE}}{G_L}. \tag{34}$$

A common scale transformation

$$G_{CE} \mapsto \lambda G_{CE}, \qquad G_L \mapsto \lambda G_L \tag{35}$$

leaves this observable unchanged. The two-point problem therefore possesses a scale gauge degeneracy: physical separation and ruler evolution cannot be identified independently.

### C.2 Addition of an outer anchor

Introduce an outer bound object $A$. The two local ratios are

$$q_E = \frac{D_{CE}}{\ell_{\mathrm{local}}}, \qquad q_A = \frac{D_{CA}}{\ell_{\mathrm{local}}}. \tag{36}$$

Their ratio eliminates the local ruler exactly:

$$\boxed{\Xi_{E/A} \equiv \frac{q_E}{q_A} = \frac{D_{CE}}{D_{CA}}.} \tag{37}$$

The epoch-to-epoch double ratio is

$$\mathcal{Q}_{E/A} = \frac{\Xi_{E/A}(t_1)}{\Xi_{E/A}(t_0)} = \frac{G_{CE}}{G_{CA}}. \tag{38}$$

Thus a three-point comparison can reveal *differential* scale evolution even when a universal local ruler factor cancels.

## C.3 What the anchor must satisfy

The outer point is not automatically a valid fixed reference. A useful anchor must satisfy four requirements:

1. its state must be specified in a declared frame and epoch;
2. its own radial and phase evolution must be modelled or measured;
3. its uncertainty must be propagated through the alignment;
4. its scale history must differ sufficiently from that of the inner observer to make the differential signal identifiable.

If Earth and the anchor share exactly the same scale history, $G_{CE} = G_{CA}$, then $\mathcal{Q}_{E/A} = 1$ and the common evolution remains hidden. The method does not create an absolute ruler; it compares two scale histories.

## C.4 Removal of translation, rotation, and common scale

The alignment transformation may be represented as a similarity map

$$\mathbf{x} \mapsto s_A \mathcal{R}(\mathbf{x} - \mathbf{C}_1) + \mathbf{C}_0, \tag{39}$$

where translation places the centres together, $\mathcal{R}$ aligns the anchor directions, and $s_A$ equalises the anchor radii. The Earth residual after this operation is invariant under the removed common similarity transformation, but it is not invariant under every dynamical or phase model.

This is why the fixed-radius circular control can produce a non-zero raw residual. Alignment removes only the modes explicitly represented in Equation (39); differing orbital phase, eccentricity, precession, inclination, light-time treatment, and catalogue transformation can remain.

### C.5 Matched null construction

A valid claim of hidden infall requires a matched null model. Let $\Delta D_{\text{data}}$ be the observed reconstructed radial residual and $\Delta D_{\text{null}}$ the residual generated by a no-infall model using the same sampling, epochs, frame transformations, phase conventions, and alignment algorithm. Define

$$\boxed{\Delta D_{\text{phys,cand}} = \Delta D_{\text{data}} - \Delta D_{\text{null}}.} \tag{40}$$

Only the null-subtracted residual is a candidate for physical radial change. Its covariance must include both data uncertainty and uncertainty in the null model.

The three-point protocol therefore escapes the local-observer trap only in a restricted sense: it removes a shared local ruler factor and exposes relative evolution. It does not by itself identify the cause of that evolution.

## D Connection with the Phase-Drift Model

The companion paper *From Sagnac Holonomy to a Phase-Drift Model for Cosmic Expansion* separates two logically different sectors.

### D.1 Standard sector

The first sector derives, within standard relativistic kinematics, the one-direction slow-clock-transport holonomy

$$\Delta\tau_{\text{loop}} = -\gamma \frac{2\boldsymbol{\Omega} \cdot \mathbf{A}}{c^2}, \tag{41}$$

which for the terrestrial equatorial ring has magnitude approximately 207.386 ns. This is a path-dependent rotating-frame result. It is not a universal constant, not a stationary-clock daily jump, and not by itself a secular evolution law.

## D.2 Hypothetical sector

The second sector adds an independent phase-locking postulate: completed cycles fractionally rescale a realised local baseline. In discrete form,

$$S_{n+1} = S_n(1 + \epsilon_{\phi,n}), \tag{42}$$

and for constant $\epsilon_\phi$,

$$S_N = S_0(1 + \epsilon_\phi)^N. \tag{43}$$

The hypothesis is multiplicative because each cycle acts on the baseline already produced by previous cycles.

The present appendix uses this hypothetical sector only to formulate a possible common-mode mechanism for the local-observer problem. It does not claim that Equation (41) logically implies Equation (42).

## D.3 Nested histories and differential scaling

For a system with several nested cycles, a general scale factor may be written

$$\ln G_X(t_1, t_0) = \sum_a \int_{N_a(t_0)}^{N_a(t_1)} \eta_{X,a}(N_a)\, \mathrm{d}N_a, \tag{44}$$

where $a$ labels spin, orbital, parent-system, or larger-scale cycles and $\eta_{X,a}$ is the fractional response assigned to body or sector $X$. For constant small responses,

$$G_X \simeq \exp\left(\sum_a \eta_{X,a} \Delta N_a\right). \tag{45}$$

The differential scale history of Earth and anchor is then

$$\ln \frac{G_E}{G_A} = \sum_a \int (\eta_{E,a} - \eta_{A,a})\, \mathrm{d}N_a. \tag{46}$$

This is the conceptual bridge to the three-point protocol: a universal common factor cancels, whereas different phase histories can survive in the Earth-to-anchor ratio.

## D.4 Separation of sectors

A complete observable map must distinguish at least four factors:

$$\frac{\Lambda_{\mathrm{meas}}(t_1)}{\Lambda_{\mathrm{meas}}(t_0)} = G_{\mathrm{prop}}\, G_{\mathrm{Doppler}}\, G_L^{-1}\, G_{\mathrm{instrument}}, \tag{47}$$

where the factors represent external propagation, observer motion, local length-scale evolution, and instrumental calibration. No secular signal can be assigned uniquely to phase drift until these sectors are modelled separately.

For the full derivation, assumptions, objections, and proposed tests, see:

> S. M. H. Emamifar, "From Sagnac Holonomy to a Phase-Drift Model for Cosmic Expansion," Zenodo (2026), doi:10.5281/zenodo.21542498.

# E Observable Predictions and Falsification Tests

A model that can hide every disagreement through universal co-scaling is not testable. The phase-locking and hidden-infall programme becomes scientific only after it specifies observables in which at least two sectors possess different scale histories.

## E.1 Logical levels of the claims

| **Level** | **Status in the present programme** |
|---|---|
| Standard result | Sagnac/slow-transport holonomy and ordinary relativistic proper-time relations. |
| Geometric construction | Similarity alignment and ruler-free three-point ratios. |
| Conditional inference | Interpretation of a null-subtracted radial residual as physical infall. |
| New hypothesis | Multiplicative phase-cycle evolution of realised length and time standards. |
| Open dynamics | Derivation of the scale response from a stable finite-stiffness matter model. |
| Empirical claim | None until matched data fits and falsification tests are completed. |

## E.2 Prediction class I: anchor dependence

If the residual reflects differential phase history rather than a universal coordinate artefact, changing the outer anchor should change the inferred signal according to Equation (46). For two anchors $A$ and $B$, the model must predict

$$\mathcal{Q}_{E/A}, \qquad \mathcal{Q}_{E/B}, \qquad \mathcal{Q}_{A/B} \tag{48}$$

with one parameter set. A signal that changes arbitrarily with anchor choice or disappears under a physically equivalent frame construction is evidence against the interpretation.

## E.3 Prediction class II: baseline dependence

For a constant fractional cycle response, the scale evolution is exponential in accumulated cycle count. Over short baselines it is approximately linear:

$$\ln G_X \simeq \sum_a \eta_{X,a} \Delta N_a. \tag{49}$$

Residuals reconstructed over several independent epoch pairs must therefore follow a coherent baseline law. A one-off residual with no predictable time curvature is insufficient.

## E.4 Prediction class III: common-mode and differential clock tests

A universal scale drift cancels from comparisons between clocks sharing the same response. The model must therefore state whether different atomic transitions, molecular standards, cavity lengths, pulsar periods, and orbital clocks possess identical or different coefficients. For two clocks $i, j$,

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln \frac{\nu_i}{\nu_j} = \dot{g}_i - \dot{g}_j. \tag{50}$$

If the theory predicts $\dot{g}_i \neq \dot{g}_j$, existing and future clock networks directly constrain it. If it predicts exact universality, an external non-co-scaling reference must be identified rather than assumed.

## E.5 Prediction class IV: directional decomposition

Observer motion produces direction-dependent Doppler structure, while a scalar local scale drift is common-mode. Sky data should therefore be decomposed schematically as

$$\delta(\hat{\mathbf{n}}, t) = a_{00}(t) + \mathbf{a}_1(t) \cdot \hat{\mathbf{n}} + \sum_{\ell \geq 2, m} a_{\ell m}(t) Y_{\ell m}(\hat{\mathbf{n}}). \tag{51}$$

A proposed monopole phase drift must not be inferred from a dipolar observer motion term.

## E.6 Prediction class V: ephemeris and ranging closure

The same scale law must be inserted into a global fit of planetary ephemerides, light-time equations, spacecraft tracking, lunar laser ranging, radar ranging, and atomic time. The model is not vindicated merely because a common-mode cancellation is algebraically possible. It must show that the remaining non-common terms lie within observational residuals without object-specific tuning.

## E.7 Prediction class VI: dimensional-constant consistency

Any evolution of length and time standards induces a scale law for every dimensional parameter. The model must specify the transformation of $GM$, masses, charges, field strengths, wavelengths, and energy standards, while confronting dimensionless combinations such as frequency ratios, mass ratios, and the fine-structure constant. A change that is only a redefinition of dimensional units has no physical content.

## E.8 Decisive falsifiers

The relevant branch of the model is rejected if any of the following occurs:

1. The reconstructed signal vanishes after a matched no-infall null model and uncertainty propagation are applied.
2. Different credible anchors yield mutually incompatible inferred Earth histories that cannot be predicted from their phase histories.
3. The scale coefficients required to hide the reconstructed infall violate clock-ratio, ephemeris, ranging, or spacecraft-tracking bounds.
4. No consistent transformation law preserves the observed dimensionless orbital relations while producing the claimed physical contraction.
5. The proposed secular signal has the wrong baseline dependence or the wrong angular multipole structure.
6. A finite-stiffness microscopic model fails to possess a stable branch that generates the assumed metrological scale response.

### E.9 Conditions for positive evidence

Positive evidence would require more than a non-zero residual. At minimum, one parameterised model must simultaneously:

1. recover injected signals in synthetic data and return null on matched controls;
2. predict the sign and magnitude for multiple anchors and epochs;
3. fit local ranging and timing data without separate ad hoc corrections;
4. produce an independently detectable differential signature;
5. and outperform fixed-ruler alternatives under a penalised model comparison.

Until these conditions are met, hidden infall remains a structured and falsifiable hypothesis, not an established astronomical measurement.

## Operational Summary of the Appendices

| |
|---|
| Local readout: $d_{\mathrm{meas}} = D_{\mathrm{phys}}/\ell_{\mathrm{local}}$. |
| Exact hiding condition: $G_D = G_L$. |
| Nested inversion: $D_{\mathrm{phys}} \propto S_{\mathrm{solar}},\ L_{\mathrm{eff}} \propto S_{\mathrm{solar}} S_{\mathrm{galactic}},\ d_{\mathrm{meas}} \propto S_{\mathrm{galactic}}^{-1}$. |
| Ruler-free three-point ratio: $\Xi_{E/A} = D_{CE}/D_{CA}$. |
| Candidate physical residual: $\Delta D_{\mathrm{data}} - \Delta D_{\mathrm{null}}$. |
| Phase-cycle hypothesis: $S_{n+1} = S_n(1 + \epsilon_\phi)$. |

The conceptual possibility is therefore clear, but so is the burden of proof. Co-scaling can explain how a global displacement might be hidden from local standards; only matched dynamical models and differential observations can establish whether nature actually uses that mechanism.

# Practical Summary

1. Local flatness can be accurate without being fundamental.
2. The Ehrenfest paradox constrains perfect rigidity, not all extended fields.
3. A field-like electron is not a classical rigid body.
4. Changing matter structure may change the observer's ruler and clock.
5. Three-point alignment reconstructs a residual, not automatically infall.
6. Code 16 tests a fixed-radius model from within its own assumptions.
7. Only matched rival predictions can discriminate between models.

# Part II

## The Geometry of Motion and the Architecture of Time

From the Rod and the Spinning Top Thought Experiment to Nested Angular Progress and a Galactic Chronometric Atlas

Part II is preserved in full as the constructive mathematical and operational completion of the diagnosis developed in Part I. Its internal title page, abstract, numbering, figures, appendices, executable-code listings, and bibliography are retained so that no prior content is lost. The repeated front matter marks a deliberate boundary between the foundational argument and the technical reference manuscript.

S. M. H. Emamifar

IRCBHC — July 2026

# The Geometry of Motion and the Architecture of Time

## From the Rod and the Spinning Top Thought Experiment to Nested Angular Progress and a Galactic Chronometric Atlas

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**Abstract**

This work joins two constructions that were originally developed separately. The first is the Rod and the Spinning Top Thought Experiment, which asks whether the geometry of an observed trajectory uniquely identifies the mechanism that produced it. A gravitational circular orbit and a rigid rotational constraint can both produce curved motion, although their angular rates, force transmission, energies, and angular momenta obey different laws. The thought experiment therefore establishes the methodological distinction

$$\text{trajectory} \neq \text{mechanism}.$$

The second construction is a gravity-independent kinematic chronometric architecture. A signed, unwrapped ledger records spin, orbital, parent-system, and Galactic phases. These phases are embedded geometrically rather than added as bare scalar angles. If

$$\boldsymbol{x}_P = \boldsymbol{x}_P(q^1, \ldots, q^n), \qquad \mathsf{G}^{(P)}_{ab} = \frac{\partial \boldsymbol{x}_P}{\partial q^a} \cdot \frac{\partial \boldsymbol{x}_P}{\partial q^b},$$

then the phase-generated speed and flat-spacetime kinematic proper time are

$$v_P^2 = \mathsf{G}^{(P)}_{ab} \dot{q}^a \dot{q}^b, \qquad \frac{\mathrm{d}\tau_P}{\mathrm{d}t} = \sqrt{1 - \frac{v_P^2}{c^2}}.$$

The mechanical thought experiment supplies a transparent example of a possible dynamical closure, while the phase metric supplies the independent map from angular progression to velocity and clock rate.

For comparison across scales, the local dimensionless chronometric label is

$$\mathscr{K}_P = \gamma_P \frac{v_P^2}{c^2} = \gamma_P - \frac{1}{\gamma_P}, \qquad \gamma_P = (1 - v_P^2/c^2)^{-1/2}.$$

For a uniform rotating ring it equals the one-winding holonomy divided by the native rotation period. The canonical terrestrial equatorial ring gives

$$\mathscr{K}_\oplus \simeq 2.4068743 \times 10^{-12}, \qquad \mathcal{H}^*_\oplus = \mathscr{K}_\oplus T_{\mathrm{sid},\oplus} \simeq 207.386\,\mathrm{ns}.$$

The value 207.386 ns is therefore a terrestrial one-winding translation, not a universal quantum. The full counter-propagating Sagnac proper-time difference is twice this one-direction quantity.

A second scale-normalised signature follows by dividing the one-winding holonomy by the circumference $C = 2\pi R$:

$$\mathscr{S} \equiv \frac{c|\mathcal{H}^*|}{C} = \gamma \frac{v}{c} = \gamma\beta.$$

Thus $\mathscr{K} = |\mathcal{H}^*|/T = \gamma\beta^2$ compares a ring with its native period, while $\mathscr{S}$ compares the same holonomy with its path length. This distinction formalises the comparison between terrestrial and Galactic rotation scales without confusing a dimensional interval with a frame-declared kinematic label.

The construction is finally extended to a quality-controlled Galactocentric phase-space atlas. Each object receives a frame- and epoch-declared state

$$\mathfrak{A}_X(t_0 \mid \mathcal{F}_\mathrm{G}) = (t_0; \boldsymbol{x}_X, \boldsymbol{v}_X; \boldsymbol{\Sigma}_X, \mathcal{Q}_X),$$

and only a measured or explicitly model-completed velocity may be converted into $\mathscr{K}_X$, $\mathscr{S}_X$, a clock factor, or a local-observer translation. The resulting architecture separates four levels that should not be merged: dynamical closure, geometric embedding, chronometric readout, and catalogue provenance. It is not a replacement for gravitational clock calculations; it is a unified language for the kinematic organisation of nested rotation from a mechanical thought experiment to a Milky Way atlas.

**Keywords:** rod thought experiment; spinning top; angular constraint; nested phase ledger; Sagnac signature; kinematic proper time; dimensionless chronometric label; Galactic phase space; Local Observer Translation Map.

**Epistemic status.** The document deliberately separates four categories. The special-relativistic proper-time relation, vector velocity composition, rotating-ring Sagnac relations, and coordinate transformations are standard kinematics. The rod and spinning-top construction is a controlled mechanical thought experiment. The rod–flywheel capacity and spin-transfer relations are conditional closure models, not measured universal laws. The phase ledger, phase metric, chronometric labels, circumference-normalised Sagnac signature, local-observer translation protocol, and catalogue quality system form the proposed architecture. No gravitational potential or gravitational redshift is used inside the kinematic conversion, so the result is not the complete clock rate of a real gravitating system. The canonical value 207.386 ns is the terrestrial one-winding translation of $\mathscr{K}_\oplus$; the corresponding two-direction Sagnac difference is twice that value.

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# 1 Introduction

Natural clocks are read from recurrent motion, yet an observed path contains less information than is often assumed. From a trajectory $\boldsymbol{x}(t)$ one may calculate velocity and acceleration, but the inverse map

$$\boldsymbol{x}(t) \longrightarrow \text{physical cause}$$

is generally non-unique. A circular trajectory may be maintained by a field, a material constraint, electromagnetic confinement, or another mechanism capable of supplying the same instantaneous inward acceleration.

This observation provides the entrance to the chronometric problem. Motion has at least three logically distinct descriptions:

(1) a *dynamical layer*, which determines why particular rates occur;

(2) a *geometric layer*, which composes nested phases into a world line;

(3) a *chronometric layer*, which converts the resulting velocity into a local kinematic clock factor and comparison label.

A fourth catalogue layer records the reference frame, epoch, uncertainty, and whether the velocity was measured or supplied by a closure model.

The unified chain developed here is

$$\boxed{\begin{array}{c}\text{measured rates or declared dynamical closure} \longrightarrow \text{signed nested phase ledger} \\ \longrightarrow \text{geometric embedding and phase metric} \longrightarrow \text{resultant point velocity} \\ \longrightarrow \begin{cases}\text{kinematic proper time,} \\ \text{dimensionless labels and Sagnac signatures,} \\ \text{frame-declared Galactic atlas record.}\end{cases}\end{array}} \tag{1}$$

The rod and spinning-top experiment is therefore not an ornamental analogy. It is the simplest explicit demonstration of why the first arrow in equation (1) must remain separate from the arrows that follow.

# 2 Mechanical Prelude: The Rod and the Spinning Top

## 2.1 The inverse problem of motion

Let the directly observed quantity be the trajectory

$$\boldsymbol{x}(t). \tag{2}$$

It determines

$$\boldsymbol{v}(t) = \dot{\boldsymbol{x}}(t), \qquad \boldsymbol{a}(t) = \ddot{\boldsymbol{x}}(t), \tag{3}$$

but does not by itself identify the interaction or constraint that generated that acceleration. Symbolically,

$$\boxed{\boldsymbol{x}(t) \not\Rightarrow \text{unique dynamics.}} \tag{4}$$

This is the methodological content of the thought experiment and is closely related to the general inverse-problem caution emphasised in classical discussions of mechanics [19].

## 2.2 Gravitational circular orbit and rigid angular constraint

For a Newtonian circular orbit of a body of mass $m$ around a central mass $M$ [17, 18],

$$\frac{GMm}{r^2} = m\omega^2 r, \tag{5}$$

so

$$\boxed{\omega_{\text{grav}}^2 = \frac{GM}{r^3}, \qquad v_{\text{grav}} = \sqrt{\frac{GM}{r}}.} \tag{6}$$

Therefore $\omega_{\text{grav}} \propto r^{-3/2}$ and $v_{\text{grav}} \propto r^{-1/2}$.

Now replace the field explanation with an idealised rod connected to one rotating axis. The defining constraint is

$$\boxed{\omega_{\text{rod}}(r) = \Omega, \qquad v_{\text{rod}}(r) = \Omega r, \qquad F_{\text{rod}} = m\Omega^2 r.} \tag{7}$$

The circular geometry can look similar, but the radial scalings and mechanism are different:

$$\boxed{\text{similar trajectory} \neq \text{identical dynamics.}} \tag{8}$$

## 2.3 The Earth–Neptune rod experiment

Take two terminal locations at

$$r_N \simeq 30 r_E. \tag{9}$$

If a single idealised rod forces both locations to share Earth's annual angular rate, then

$$\omega_N = \omega_E = \Omega, \qquad \boxed{\frac{v_N}{v_E} = \frac{r_N}{r_E} \simeq 30.} \tag{10}$$

The outer terminal must therefore possess a much larger tangential speed even though the two terminals complete the same number of turns.

A second version of the experiment fixes a radial transport capacity $F_\star$:

$$F_\star = m\Omega^2 r. \tag{11}$$

At fixed $F_\star$ and $\Omega$,

$$mr = \text{constant}. \tag{12}$$

Thus the Neptune terminal requires

$$\boxed{\frac{m_N}{m_E} = \frac{r_E}{r_N} \simeq \frac{1}{30}.} \tag{13}$$

For equal mean density, $m \propto R^3$, and therefore

$$\boxed{\frac{R_N}{R_E} = \left(\frac{1}{30}\right)^{1/3} \simeq 0.322.} \tag{14}$$

This result is conditional on the declared capacity law. It is not a property of the actual Earth–Neptune system.

### 2.4 Mechanical cost and dynamical non-equivalence

For a gravitational circular orbit,

$$E_{\text{grav}} = -\frac{GMm}{2r}, \qquad L_{\text{grav}} = m\sqrt{GMr}, \tag{15}$$

whereas the rigidly constrained terminal has

$$E_{\text{rod}} = \frac{1}{2}m\Omega^2 r^2, \qquad L_{\text{rod}} = m\Omega r^2. \tag{16}$$

Hence

$$E_{\text{grav}} \propto r^{-1}, \quad L_{\text{grav}} \propto r^{1/2}, \qquad E_{\text{rod}} \propto r^2, \quad L_{\text{rod}} \propto r^2. \tag{17}$$

A common path shape therefore does not imply common force transmission, energy architecture, or angular-momentum scaling.

## 3 The Connection: From Angular Constraint to Phase Architecture

The rod experiment imposes angular coherence directly. Every terminal shares the same unwrapped angular progression

$$q(t) = q(t_0) + \Omega(t - t_0), \tag{18}$$

while its tangential speed depends on the embedding radius. This is precisely the distinction required by the phase-ledger construction: the turn count is not yet a clock reading, and the angular rate is not yet a velocity until the geometry is supplied.

The unified state of a point is therefore represented schematically by

$$\boxed{\mathfrak{S}_P = \big(\boldsymbol{q}_P, \dot{\boldsymbol{q}}_P; \boldsymbol{x}_P(\boldsymbol{q}_P), \boldsymbol{v}_P; \mathcal{F}, t_0, \mathcal{Q}_P\big).} \tag{19}$$

The rod or any other dynamical model may supply $\dot{\boldsymbol{q}}_P$. The geometric embedding supplies $\boldsymbol{v}_P$. The chronometric map supplies $\mathrm{d}\tau_P/\mathrm{d}t$, $\mathscr{K}_P$, and the Sagnac-derived signatures. The catalogue layer records how each input was obtained.

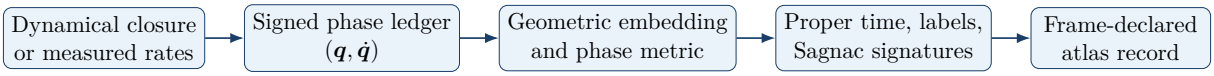


Figure 1: Engineered connection between the mechanical thought experiment and the chronometric atlas. Each layer has a distinct epistemic role.

# 4 Scope and Reference Frames

## 4.1 Chosen reference time

Let $t$ denote the coordinate time of a specified quasi-inertial frame over the domain of the calculation. For Solar-System applications this may be an idealised barycentric inertial frame or a sufficiently accurate local inertial approximation. The speed of a point $P$ in that frame is

$$\boldsymbol{v}_P(t) = \frac{\mathrm{d}\boldsymbol{x}_P}{\mathrm{d}t}. \tag{20}$$

In the flat-spacetime kinematic sector,

$$\boxed{\mathrm{d}\tau_P = \mathrm{d}t\sqrt{1 - \frac{v_P^2}{c^2}}.} \tag{21}$$

Equation (21) is the only clock law assumed in the main derivation.

For Galactic applications, the term "Galactocentric frame" does not denote one universally fixed modern standard. The origin, axis orientation, Solar position, Solar velocity, epoch, and transformation parameters must therefore be stated explicitly. The atlas developed below treats this parameter set as part of the address rather than hiding it in software defaults.

## 4.2 Meaning of "gravity-independent

No term of the form $U/c^2$, $GM/(rc^2)$, or any metric-specific gravitational potential is introduced. The word "gravity-independent" therefore means that the construction is a self-contained kinematic sector. It does not mean that gravitational effects are physically absent from actual astronomical clocks.

When two real clocks occupy different gravitational potentials, their total observed rate difference generally contains both kinematic and gravitational components. The present paper isolates the former so that the rotational and orbital geometry can be studied without reversing, reinterpreting, or absorbing it into a gravitational term.

# 5 The Nested Angular-Progress Ledger

## 5.1 Immediate-parent chain

Let $P$ be a material point, surface marker, atmospheric parcel, or ideal clock associated with a natural celestial body. Introduce an immediate-parent chain

$$P \longrightarrow B_0 \longrightarrow B_1 \longrightarrow \cdots \longrightarrow B_m \longrightarrow \mathcal{F}_{\mathrm{ref}}, \tag{22}$$

where $B_0$ is the host body and $\mathcal{F}_{\text{ref}}$ is an operational orientation frame, such as the ICRF when appropriate [3, 4].

Each link has an oriented phase $q^a$ and an unwrapped turn count $N_a$:

$$q^a(t) = q^a(t_0) + 2\pi N_a(t), \qquad N_a(t) \in \mathbb{R}. \tag{23}$$

The sign of $\dot{N}_a$ records prograde or retrograde progression.

**Definition 1** (Nested angular-progress ledger)**.** *The ledger of point $P$ is*

$$\boxed{\boldsymbol{N}_P(t) = \big(N_1(t), N_2(t), \ldots, N_n(t)\big).} \tag{24}$$

*Its rate vector is*

$$\boxed{\dot{\boldsymbol{N}}_P(t) = \big(\nu_1(t), \ldots, \nu_n(t)\big), \qquad \nu_a = \dot{N}_a = \frac{\omega_a}{2\pi}.} \tag{25}$$

The ledger is an address in nested cycle space. It is not a clock reading and its entries need not be commensurate.




Figure 2: Immediate-parent phase chain. The phase at each level is defined relative to the next parent rather than being referred ambiguously to one common origin.

## 5.2 Signed recurrence relations

In a planar common-axis approximation, unwrapped angles add and subtract exactly. If a body has signed sidereal spin rate $\omega_{\text{sid}}$ and orbital phase rate $\omega_{\text{orb}}$, the apparent solar rate is

$$\omega_{\text{sol}} = \omega_{\text{sid}} - \omega_{\text{orb}}. \tag{26}$$

Using signed periods $P^*$,

$$\boxed{\frac{1}{P^*_{\text{sol}}} = \frac{1}{P^*_{\text{sid}}} - \frac{1}{P_{\text{orb}}}.} \tag{27}$$

Synodic recurrence and spin–orbit locking are analogous differences between selected ledger components. In three dimensions, however, finite rotations must be composed as ordered products in SO(3) rather than as scalar angle sums.

# 6 Geometric Embedding of the Ledger

## 6.1 Position as a function of angular phases

Let the spatial position of point $P$ be a differentiable function of the angular coordinates:

$$\boxed{\boldsymbol{x}_P = \boldsymbol{x}_P(q^1, q^2, \ldots, q^n; t).} \tag{28}$$

An explicit $t$ dependence is allowed for translations, slowly changing axes, deformation, precession parameters, or other prescribed non-angular inputs.

Differentiating gives

$$\boldsymbol{v}_P = \frac{\partial \boldsymbol{x}_P}{\partial t} + \sum_{a=1}^{n} \frac{\partial \boldsymbol{x}_P}{\partial q^a} \dot{q}^a. \tag{29}$$

Define

$$\boldsymbol{v}_0^{(P)} \equiv \frac{\partial \boldsymbol{x}_P}{\partial t}, \qquad \boldsymbol{e}_a^{(P)} \equiv \frac{\partial \boldsymbol{x}_P}{\partial q^a}. \tag{30}$$

Then

$$\boldsymbol{v}_P = \boldsymbol{v}_0^{(P)} + \boldsymbol{e}_a^{(P)} \dot{q}^a, \tag{31}$$

where repeated phase indices are summed.

## 6.2 The phase metric

**Definition 2** (Point-dependent phase metric)**.** *The Gram matrix of angular displacement vectors is*

$$\boxed{\mathsf{G}_{ab}^{(P)} \equiv \boldsymbol{e}_a^{(P)} \cdot \boldsymbol{e}_b^{(P)} = \frac{\partial \boldsymbol{x}_P}{\partial q^a} \cdot \frac{\partial \boldsymbol{x}_P}{\partial q^b}.} \tag{32}$$

The diagonal entries encode the squared effective radii associated with each phase. The off-diagonal entries encode the instantaneous geometric alignment between different phase-generated velocities.

**Proposition 1** (Angular composition of speed)**.** *If $\partial \boldsymbol{x}_P / \partial t = 0$ in the chosen embedding, then*

$$\boxed{v_P^2 = \mathsf{G}_{ab}^{(P)} \dot{q}^a \dot{q}^b.} \tag{33}$$

*For a general embedding,*

$$\boxed{v_P^2 = |\boldsymbol{v}_0^{(P)}|^2 + 2\boldsymbol{v}_0^{(P)} \cdot \boldsymbol{e}_a^{(P)} \dot{q}^a + \mathsf{G}_{ab}^{(P)} \dot{q}^a \dot{q}^b.} \tag{34}$$

*Proof.* Take the Euclidean inner product of equation (31) with itself and use the definition (32). □

This is the precise meaning of "adding angular progressions." The correct composition is not

$$\omega_1 + \omega_2 + \cdots, \tag{35}$$

but a point-dependent quadratic form whose cross terms retain directional information.

# 7 Local Kinematic Chronometry

## 7.1 Angular-progress norm

Define the dimensionless angular-progress norm

$$\boxed{\mathcal{A}_P(t) \equiv \frac{v_P^2(t)}{c^2}.} \tag{36}$$

When the position is generated entirely by the phase coordinates,

$$\mathcal{A}_P = \frac{1}{c^2}\mathsf{G}^{(P)}_{ab}\dot{q}^a\dot{q}^b = \frac{4\pi^2}{c^2}\mathsf{G}^{(P)}_{ab}\nu_a\nu_b. \tag{37}$$

**Definition 3** (Local kinematic clock factor)**.**

$$\boxed{\alpha_P(t) \equiv \frac{\mathrm{d}\tau_P}{\mathrm{d}t} = \sqrt{1-\mathcal{A}_P(t)}.} \tag{38}$$

The accumulated kinematic proper time between $t_0$ and $t_1$ is

$$\boxed{\tau_P(t_1) - \tau_P(t_0) = \int_{t_0}^{t_1}\sqrt{1-\mathcal{A}_P(t)}\,\mathrm{d}t.} \tag{39}$$

**Master relation.** Given the nested angular phases, their rates, and the geometric embedding of a point, the gravity-independent kinematic time of that point is

$$\boxed{\mathrm{d}\tau_P = \mathrm{d}t\sqrt{1-\frac{1}{c^2}\mathsf{G}^{(P)}_{ab}\dot{q}^a\dot{q}^b}} \tag{40}$$

when the embedding has no additional explicit translation, and is obtained from equations (21) and (34) in the general case.

## 7.2 Weak-speed approximation

For $v_P \ll c$,

$$\alpha_P = 1 - \frac{\mathcal{A}_P}{2} + \mathcal{O}(\mathcal{A}_P^2), \tag{41}$$

so that

$$\Delta\tau_P \simeq \Delta t - \frac{1}{2c^2}\int v_P^2(t)\,\mathrm{d}t. \tag{42}$$

The clock deficit is therefore governed by the time integral of the squared resultant velocity, not by the arithmetic total of traversed angle.

# 8 Dimensionless Chronometric Labels and Local Observer Translation

## 8.1 The local dimensionless label

The angular-progress norm $\mathcal{A}_P = v_P^2/c^2$ already supplies a dimensionless measure of motion. For translation and comparison it is useful to introduce a second, exactly invertible label that is also the period-normalised rotating-ring holonomy.

**Definition 4** (Dimensionless chronometric label)**.** *For a point $P$ with $0 \le \mathcal{A}_P < 1$, define*

$$\boxed{\mathscr{K}_P \equiv \gamma_P\mathcal{A}_P = \gamma_P\frac{v_P^2}{c^2}, \qquad \gamma_P = (1-\mathcal{A}_P)^{-1/2}.} \tag{43}$$

Using $\alpha_P = 1/\gamma_P$, the same label has the exact identities

$$\boxed{\mathscr{K}_P = \gamma_P - \frac{1}{\gamma_P} = \frac{1}{\alpha_P} - \alpha_P.} \tag{44}$$

**Proposition 2** (Exact label–rate correspondence)**.** *For $\mathscr{K}_P \geq 0$, the local kinematic clock factor is uniquely recovered by*

$$\boxed{\alpha_P = \frac{\sqrt{\mathscr{K}_P^2 + 4} - \mathscr{K}_P}{2} = \frac{2}{\sqrt{\mathscr{K}_P^2 + 4} + \mathscr{K}_P}.} \tag{45}$$

*Proof.* Equation (44) gives $\alpha_P^2 + \mathscr{K}_P \alpha_P - 1 = 0$. Only the positive root can represent a future-directed clock-rate factor. The second expression in equation (45) is algebraically equivalent and is numerically stable for small labels. □

In the weak-speed regime,

$$\mathscr{K}_P = \mathcal{A}_P + \mathcal{O}(\mathcal{A}_P^2), \qquad \alpha_P = 1 - \frac{\mathscr{K}_P}{2} + \mathcal{O}(\mathscr{K}_P^2). \tag{46}$$

## 8.2 Kinematic equivalence classes

Two specified points $A$ and $B$, evaluated in the same reference frame and under the same convention, are *instantaneously kinematically chronometric-equivalent* when

$$\boxed{\mathscr{K}_A(t) = \mathscr{K}_B(t).} \tag{47}$$

Then $\alpha_A(t) = \alpha_B(t)$ follows exactly from equation (45). Equality of one averaged label is not, by itself, sufficient to guarantee equality of accumulated proper time when the labels vary, because the inverse map is nonlinear. Equality of the complete functions $\mathscr{K}_A(t)$ and $\mathscr{K}_B(t)$ over a common interval does guarantee equal accumulated kinematic proper time over that interval.

The label is not an intrinsic name detached from measurement convention. A reproducible label must be accompanied by the selected reference frame, point or canonical ring, latitude, altitude, epoch, and any centre-motion contribution. These metadata define what is being compared.

## 8.3 Uniform rotating ring and one-winding translation

For a uniformly rotating ring,

$$\beta = \frac{|\Omega| R}{c}, \qquad \mathscr{K}_{\mathrm{ring}} = \gamma \beta^2. \tag{48}$$

The one-direction slow-transport holonomy satisfies

$$\boxed{|\mathcal{H}^*| = \mathscr{K}_{\mathrm{ring}} T_{\mathrm{rot}}, \qquad T_{\mathrm{rot}} = \frac{2\pi}{|\Omega|}.} \tag{49}$$

Thus $\mathscr{K}$ is the unit-independent dimensionless content of this particular ring observable, while multiplication by a selected period translates it into a time interval.

## 8.4 Canonical Earth calibration

Adopt the WGS 84 equatorial radius and the terrestrial sidereal rotation period as the canonical Earth ring. Its dimensionless label is

$$\boxed{\mathscr{K}_\oplus \equiv \gamma_\oplus \frac{\Omega_\oplus^2 R_\oplus^2}{c^2} \simeq 2.4068743 \times 10^{-12}.} \tag{50}$$

Define the Earth-cycle translation constant

$$\boxed{\Theta_\oplus \equiv \mathscr{K}_\oplus T_{\mathrm{sid},\oplus} \simeq 207.386\,\mathrm{ns}.} \tag{51}$$

The number $\Theta_\oplus$ is therefore the terrestrial time-language rendering of $\mathscr{K}_\oplus$ under the canonical equatorial sidereal-cycle convention. It is not the fundamental label itself.

## 8.5 Earth-normalised comparison and translation

For any point or canonical body state $X$, define the Earth-normalised ratio

$$\boxed{\mathscr{R}_{X/\oplus} \equiv \frac{\mathscr{K}_X}{\mathscr{K}_\oplus}.} \tag{52}$$

Its Earth-language translated interval is

$$\boxed{\Theta_X^{(\oplus)} \equiv \mathscr{K}_X T_{\mathrm{sid},\oplus} = \mathscr{R}_{X/\oplus}\Theta_\oplus.} \tag{53}$$

When expressed in terrestrial nanoseconds,

$$\boxed{\Theta_X^{(\oplus)}[\mathrm{ns}_\oplus] = \frac{\mathscr{K}_X}{\mathscr{K}_\oplus} \times 207.386\ldots} \tag{54}$$

This is a comparison coordinate in the Earth calibration language. It must not be silently identified with the physical one-winding holonomy of $X$. That identification is valid only when $X$ is itself represented by a uniform ring and its own native period is used.

For a uniform ring $X$, define the native-cycle translation

$$\boxed{\Theta_X^{(X)} \equiv \mathscr{K}_X T_X = |\mathcal{H}_X^*|.} \tag{55}$$

The coefficient that converts the native-cycle representation to the Earth-cycle representation is

$$\boxed{C_{X\to\oplus} \equiv \frac{T_{\mathrm{sid},\oplus}}{T_X}, \qquad \Theta_X^{(\oplus)} = C_{X\to\oplus}\Theta_X^{(X)}.} \tag{56}$$

**Remark 1.** *The translation map changes the numerical language, not the underlying kinematic label. Equal Local Observer Translation Maps imply equal labels because the same*

*positive calibration period is used. Equal native-cycle intervals do not necessarily imply equal labels, since the native periods may differ.*

## 8.6 Scale-normalised Sagnac signatures

For a uniform ring of circumference $C = 2\pi R$, native period $T = C/v$, and $\beta = v/c$, the one-winding proper-time holonomy used in this work is

$$\boxed{|\mathcal{H}^*| = \gamma \frac{2\pi\Omega R^2}{c^2} = \gamma \frac{Cv}{c^2}.} \tag{57}$$

It produces two complementary dimensionless normalisations:

$$\boxed{\frac{|\mathcal{H}^*|}{T} = \gamma\beta^2 = \mathscr{K}, \qquad \mathscr{S} \equiv \frac{c|\mathcal{H}^*|}{C} = \gamma\beta.} \tag{58}$$

The label $\mathscr{K}$ is period normalised. The signature $\mathscr{S}$ is circumference normalised. They encode different questions and should not be interchanged.

For two counter-propagating light signals received by the co-rotating detector, the proper-time Sagnac difference is

$$\boxed{\Delta\tau_{\leftrightarrow} = 2|\mathcal{H}^*|, \qquad \frac{c\Delta\tau_{\leftrightarrow}}{C} = 2\mathscr{S} = 2\gamma\beta.} \tag{59}$$

Consequently the canonical terrestrial value 207.386 ns is the one-direction quantity $|\mathcal{H}^*_\oplus|$. The associated counter-propagating proper-time difference is approximately 414.772 ns. Stating which convention is used removes the otherwise persistent factor-of-two ambiguity.

## 8.7 Illustrative terrestrial–Galactic ring comparison

A galaxy is not a rigid wheel, so no single global angular velocity is assigned here. Instead, consider a declared canonical Galactic ring at radius $R_G$ with local azimuthal speed $v_{\phi,G}$. Its ring labels are

$$\boxed{\mathscr{K}_G = \gamma_G \left(\frac{v_{\phi,G}}{c}\right)^2, \qquad \mathscr{S}_G = \gamma_G \frac{v_{\phi,G}}{c}, \qquad |\mathcal{H}^*_G| = \frac{C_G}{c}\mathscr{S}_G.} \tag{60}$$

For the explicitly illustrative values

$$R_G = 50{,}000\,\mathrm{ly}, \qquad v_{\phi,G} = 220\,\mathrm{km/s}, \tag{61}$$

one obtains approximately

$$\boxed{|\mathcal{H}^*_G| \simeq 230.5\,\mathrm{yr}, \qquad \Delta\tau_{\leftrightarrow,G} \simeq 461.1\,\mathrm{yr}.} \tag{62}$$

These large dimensional intervals arise mainly from the enormous circumference. Dividing

by circumference isolates the speed-controlled comparison:

$$\boxed{\frac{\mathscr{S}_G}{\mathscr{S}_\oplus} \simeq \frac{v_{\phi,G}}{v_{\mathrm{eq},\oplus}} \simeq 4.73 \times 10^2} \tag{63}$$

in the weak-speed regime. By contrast, the period-normalised labels obey

$$\frac{\mathscr{K}_G}{\mathscr{K}_\oplus} \simeq 2.24 \times 10^5. \tag{64}$$

The two ratios differ because $\mathscr{S} \propto \beta$ while $\mathscr{K} \propto \beta^2$ at low speed.

This comparison formalises the useful intuition that a dimensional Sagnac interval should not be compared across radically different path sizes without also reporting its native period or circumference normalisation.

# 9 Galactic Phase-Space Addressing and the Chronometric Atlas

## 9.1 Declared Galactocentric frame

A Galactic atlas requires a declared reference construction. Let

$$\boxed{\mathcal{F}_{\mathrm{G}} = \left(O_{\mathrm{G}}, \hat{\boldsymbol{e}}_x, \hat{\boldsymbol{e}}_y, \hat{\boldsymbol{e}}_z, t_{\mathrm{G}}; \boldsymbol{\Pi}_{\mathrm{G}}\right)} \tag{65}$$

contain the adopted Galactic centre, oriented axes, coordinate time, and the parameter set $\boldsymbol{\Pi}_{\mathrm{G}}$ used to transform observations into that frame. The latter may include the Solar distance from the Galactic centre, the Solar height above the adopted midplane, and the Solar velocity vector. There is no single modern standard Galactocentric frame with universally fixed numerical parameters; consequently, reproducible science code must record them explicitly rather than relying on a changing default [15].

This Galactocentric frame is distinct from the conventional *Galactic* sky-coordinate system. The latter has its three-dimensional origin at the Solar-System barycentre, whereas the former is constructed with the Galactic centre as origin. Confusing these origins changes both the position and the velocity attached to an object.

## 9.2 Six-dimensional Galactic chronometric address

**Definition 5** (Galactic phase-space address)**.** *For an object $X$ at epoch $t_0$, define*

$$\boxed{\mathfrak{A}_X(t_0 \mid \mathcal{F}_{\mathrm{G}}) = (t_0; \boldsymbol{x}_X, \boldsymbol{v}_X; \boldsymbol{\Sigma}_X, \mathcal{Q}_X)\,,} \tag{66}$$

*where $\boldsymbol{x}_X$ and $\boldsymbol{v}_X$ are its Galactocentric position and velocity, $\boldsymbol{\Sigma}_X$ is the adopted covariance or uncertainty description, and $\mathcal{Q}_X$ records whether the state is measured, transformed, incomplete, or model-completed.*

In Cartesian coordinates,

$$\boldsymbol{x}_X = (x_X, y_X, z_X), \qquad \boldsymbol{v}_X = (v_{x,X}, v_{y,X}, v_{z,X}). \tag{67}$$

The corresponding cylindrical quantities are

$$R_X = \sqrt{x_X^2 + y_X^2}, \qquad \phi_X = \operatorname{atan2}(y_X, x_X), \tag{68}$$

$$v_{R,X} = \frac{x_X v_{x,X} + y_X v_{y,X}}{R_X}, \qquad v_{\phi,X} = \frac{x_X v_{y,X} - y_X v_{x,X}}{R_X}, \tag{69}$$

for $R_X > 0$. Therefore

$$\boxed{v_X^2 = v_{x,X}^2 + v_{y,X}^2 + v_{z,X}^2 = v_{R,X}^2 + v_{\phi,X}^2 + v_{z,X}^2.} \tag{70}$$

Equivalently, when the path is differentiable,

$$\boxed{v_X^2 = \dot{R}_X^2 + R_X^2 \dot{\phi}_X^2 + \dot{z}_X^2.} \tag{71}$$

The Galactic azimuthal phase $\phi_X$ is thus one ledger component, with rate $\dot{\phi}_X = v_{\phi,X}/R_X$, but the radial and vertical components must also be retained.

## 9.3 Why position alone does not determine speed

A single position $\boldsymbol{x}_X(t_0)$ does not determine $\boldsymbol{v}_X(t_0)$. Distinct world lines can intersect the same event in configuration space with different tangent vectors. Hence no unique inference

$$\boldsymbol{x}_X \longmapsto \boldsymbol{v}_X \tag{72}$$

exists without additional information.

There are only two legitimate routes from position to speed in this framework:

(1) observe the position at multiple epochs or provide the full measured phase-space state;

(2) declare a conditional velocity-field closure

$$\boxed{\boldsymbol{v}_X = \boldsymbol{\mathcal{V}}_{\mathrm{MW}}(\boldsymbol{x}_X, t; \boldsymbol{\eta}),} \tag{73}$$

where $\boldsymbol{\eta}$ records all model parameters.

Examples of conditional closures include a constant tangential-speed field, a solid-rotation field, a tabulated rotation curve, or a fitted non-axisymmetric velocity map. A velocity supplied this way is not labelled "measured" in the atlas.

## 9.4 Nested local refinement inside a Galactic address

Let $X$ denote the centre of a star, planet, moon, cluster, or other host object, and let $P$ be a point attached to or moving within it. Write

$$\boldsymbol{x}_P^{\mathrm{G}} = \boldsymbol{x}_X^{\mathrm{G}} + \mathsf{R}_X(t)\boldsymbol{r}_P, \tag{74}$$

where $\mathsf{R}_X(t) \in \mathrm{SO}(3)$ embeds the local coordinates into $\mathcal{F}_{\mathrm{G}}$. Differentiation gives

$$\boldsymbol{v}_P^{\mathrm{G}} = \boldsymbol{v}_X^{\mathrm{G}} + \dot{\mathsf{R}}_X \boldsymbol{r}_P + \mathsf{R}_X \dot{\boldsymbol{r}}_P. \tag{75}$$

For a fixed point co-rotating rigidly with the host,

$$\boxed{\boldsymbol{v}_P^{\mathrm{G}} = \boldsymbol{v}_X^{\mathrm{G}} + \boldsymbol{\Omega}_X^{\mathrm{G}} \times \boldsymbol{r}_P^{\mathrm{G}}.} \tag{76}$$

Additional moon–planet, planet–star, binary, cluster, or stream layers are inserted through the same ordered phase embedding used in the earlier sections. The centre address is therefore not a replacement for the nested ledger; it is its outer Galactic layer.

## 9.5 Atlas label, clock factor, and Earth-language coordinate

For every complete phase-space address, define

$$\beta_X^{\mathrm{G}} = \frac{|\boldsymbol{v}_X^{\mathrm{G}}|}{c}, \qquad \gamma_X^{\mathrm{G}} = \left(1 - (\beta_X^{\mathrm{G}})^2\right)^{-1/2}, \tag{77}$$

$$\boxed{\mathscr{K}_X^{\mathrm{G}} = \gamma_X^{\mathrm{G}} (\beta_X^{\mathrm{G}})^2, \qquad \alpha_X^{\mathrm{G}} = \frac{1}{\gamma_X^{\mathrm{G}}}.} \tag{78}$$

The Earth-language atlas coordinate is

$$\boxed{\Theta_{X,\mathrm{G}}^{(\oplus)} = \mathscr{K}_X^{\mathrm{G}} T_{\mathrm{sid},\oplus} = \frac{\mathscr{K}_X^{\mathrm{G}}}{\mathscr{K}_\oplus} \Theta_\oplus.} \tag{79}$$

The full catalogue record is

$$\boxed{\mathfrak{C}_X = \left(\mathrm{ID}_X, t_0, \mathcal{F}_{\mathrm{G}}, \boldsymbol{x}_X, \boldsymbol{v}_X, \mathscr{K}_X, \alpha_X, \Theta_X^{(\oplus)}, \boldsymbol{\Sigma}_X, \mathcal{Q}_X\right).} \tag{80}$$

The translated interval is a coordinate in the declared terrestrial calibration language. It is not a claim that the object physically performs an Earth-length cycle or possesses a closed-loop holonomy equal to that interval.

## 9.6 Catalogue quality classes

This classification prevents a position-only map from being silently promoted into a measured time map. The catalogue may contain all classes, but only complete M6, T6, or explicitly conditional C6 rows receive $\mathscr{K}$.

## 9.7 Observational input and Gaia-compatible ingestion

A complete stellar input can be formed from sky position, distance, two proper-motion components, radial velocity, epoch, and their covariance. Gaia DR3 provides astrometry for a very large source set and combined radial velocities for more than $3.3 \times 10^7$ stars, making a large six-dimensional implementation feasible for the measured subset [12, 13]. The

Table 1: Minimum provenance classes for a Galactic chronometric atlas.

| Code | State | Interpretation |
|---|---|---|
| M6 | Measured six-dimensional | Position and all three velocity components are observational inputs in a declared frame. |
| T6 | Transformed six-dimensional | A complete observed sky state has been transformed into the declared Galactocentric frame. |
| P5 | Position plus transverse motion | Radial velocity is absent; no unique total-speed label is issued. |
| P3 | Position only | The atlas stores the address but leaves the chronometric label undefined. |
| C6 | Conditional six-dimensional | Missing velocity components are supplied by a named model closure and are never presented as measurements. |

Gaia archive documents the fields `ra`, `dec`, `parallax`, `pmra`, `pmdec`, and `radial_velocity`, including their reference-frame and uncertainty conventions [14].

For a source with distance $d$ in kiloparsecs and total proper motion $\mu$ in milliarcseconds per year, the transverse-speed magnitude is

$$\boxed{v_\perp[\mathrm{km\,s^{-1}}] = 4.74047\,\mu[\mathrm{mas\,yr^{-1}}]\,d[\mathrm{kpc}].} \tag{81}$$

The complete vector transformation must nevertheless preserve the directions of both proper-motion components and add the line-of-sight velocity before transforming to $\mathcal{F}_{\mathrm{G}}$. The accompanying code offers an optional Astropy path for this transformation, with the Galactocentric parameters supplied explicitly rather than inherited silently from software defaults [15].

## 9.8 Uncertainty propagation

Let $\boldsymbol{y}$ denote the measured astrometric and velocity variables with covariance $\boldsymbol{\Sigma}_y$. To first order,

$$\boxed{\sigma_{\mathscr{K}}^2 \simeq \nabla_{\boldsymbol{y}}\mathscr{K}^{\mathsf{T}}\boldsymbol{\Sigma}_y\nabla_{\boldsymbol{y}}\mathscr{K}\,.} \tag{82}$$

For a directly measured velocity vector, with $v = |\boldsymbol{v}|$,

$$\frac{\mathrm{d}\mathscr{K}}{\mathrm{d}v} = \frac{1}{c}\left(2\beta\gamma + \beta^3\gamma^3\right), \qquad \nabla_{\boldsymbol{v}}\mathscr{K} = \frac{\mathrm{d}\mathscr{K}}{\mathrm{d}v}\frac{\boldsymbol{v}}{v}. \tag{83}$$

The Earth-language uncertainty is simply

$$\boxed{\sigma_{\Theta^{(\oplus)}} = T_{\mathrm{sid},\oplus}\sigma_{\mathscr{K}}.} \tag{84}$$

For nonlinear distance transformations, asymmetric distance posteriors, or correlated astrometric errors, Monte Carlo propagation from the full input covariance is preferable to a diagonal first-order approximation.

## 9.9 Demonstration of address and velocity non-uniqueness

Table 2 uses an explicit illustrative Galactocentric convention with a Solar-frame velocity vector $(12.9, 245.6, 7.78)\,\mathrm{km/s}$. These values demonstrate the pipeline; they do not define a unique Galactic standard. The second row occupies the same position as the first but carries an additional velocity component, whereas the third is placed elsewhere with the same velocity magnitude as the first.

Table 2: Illustrative Galactic atlas records. $\Theta^{(\oplus)}$ is the Earth-language comparison coordinate.

| **Record** | $v$ (km/s) | $\mathscr{K}$ | $\mathscr{K}/\mathscr{K}_\oplus$ | $\Theta^{(\oplus)}$ (ms) |
|---|---|---|---|---|
| Illustrative Solar-frame state | 246.062 | $6.73669 \times 10^{-7}$ | $2.79894 \times 10^{5}$ | 58.046 |
| Same position, extra component | 270.419 | $8.13640 \times 10^{-7}$ | $3.38049 \times 10^{5}$ | 70.107 |
| Different position, equal speed | 246.062 | $6.73669 \times 10^{-7}$ | $2.79894 \times 10^{5}$ | 58.046 |
| Position only | – | – | – | – |

The first and third rows belong to the same instantaneous kinematic chronometric equivalence class despite different positions. The first and second do not, despite identical positions. This is the operational reason that the atlas is fundamentally a phase-space atlas rather than a position-only map.

## 9.10 Atlas construction algorithm

A catalogue implementation follows this sequence:

**A1:** Freeze the Galactocentric frame parameters, axis convention, and epoch.

**A2:** Ingest either Cartesian phase-space rows or complete sky-coordinate rows.

**A3:** Transform every complete state to the same $\mathcal{F}_{\mathrm{G}}$; retain the full covariance when available.

**A4:** Assign the provenance class M6, T6, P5, P3, or C6.

**A5:** Compute cylindrical coordinates and verify $v^2 = v_R^2 + v_\phi^2 + v_z^2$.

**A6:** Calculate $\mathscr{K}$, $\alpha$, $\mathscr{K}/\mathscr{K}_\oplus$, and $\Theta^{(\oplus)}$ only for complete states.

**A7:** Propagate uncertainties and attach quality flags rather than dropping them from the map.

**A8:** Store the result as a versioned table and visualise position separately from colour- or contour-coded chronometric labels.

**A9:** Never mix labels calculated in different frames, epochs, or Solar-parameter conventions without an explicit transformation.

# 10 A Planetary Specialisation

## 10.1 Centre motion plus body rotation

Let $C$ be the centre of a rotating planet. Let $\boldsymbol{V}_C(t)$ be its velocity in the chosen quasi-inertial frame, and let $\boldsymbol{r}_P(t)$ be the position of point $P$ relative to the centre. For rigid rotation with angular velocity $\boldsymbol{\Omega}(t)$,

$$\frac{\mathrm{d}\boldsymbol{r}_P}{\mathrm{d}t} = \boldsymbol{\Omega} \times \boldsymbol{r}_P. \tag{85}$$

Therefore

$$\boxed{\boldsymbol{v}_P = \boldsymbol{V}_C + \boldsymbol{\Omega} \times \boldsymbol{r}_P.} \tag{86}$$

The exact local kinematic clock factor is

$$\boxed{\alpha_P = \sqrt{1 - \frac{|\boldsymbol{V}_C + \boldsymbol{\Omega} \times \boldsymbol{r}_P|^2}{c^2}}.} \tag{87}$$

Expanding,

$$\boxed{|\boldsymbol{v}_P|^2 = V_C^2 + |\boldsymbol{\Omega} \times \boldsymbol{r}_P|^2 + 2\boldsymbol{V}_C \cdot (\boldsymbol{\Omega} \times \boldsymbol{r}_P).} \tag{88}$$

The three terms have distinct meanings:

(1) motion of the planetary centre through the selected frame;

(2) local spin motion around the planetary axis;

(3) instantaneous orbital–spin alignment or opposition.

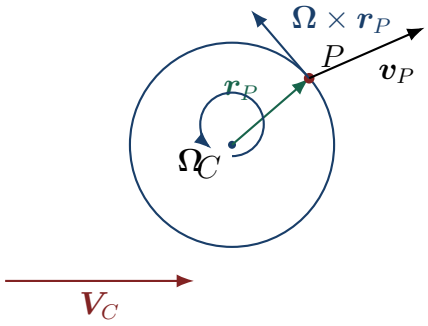


Figure 3: Velocity composition for a point on a rotating planet. The total velocity is the vector sum of centre motion and local spin motion.

## 10.2 Planar circular approximation

For a circular planetary orbit of radius $a$, orbital angular rate $\omega_{\mathrm{orb}}$, and local cylindrical spin radius $\rho$, let

$$V_{\mathrm{orb}} = a|\omega_{\mathrm{orb}}|, \qquad v_{\mathrm{spin}} = \rho|\Omega_{\mathrm{spin}}|. \tag{89}$$

If $\psi(t)$ is the instantaneous angle between the orbital and spin velocity vectors,

$$\boxed{v_P^2 = V_{\mathrm{orb}}^2 + v_{\mathrm{spin}}^2 + 2V_{\mathrm{orb}}v_{\mathrm{spin}}\cos\psi(t).} \tag{90}$$

Thus

$$\boxed{\alpha_P(t) = \sqrt{1 - \frac{V_{\text{orb}}^2 + v_{\text{spin}}^2 + 2V_{\text{orb}}v_{\text{spin}}\cos\psi(t)}{c^2}}.} \tag{91}$$

The cross term is required for instantaneous chronometry. It cannot be recovered from the magnitudes of the two angular rates alone; their geometric orientation must also be known.

# 11 Any Point on or Above a Rotating Planet

## 11.1 Latitude and altitude

For a spherical reference body of radius $R$, a point at altitude $h$ and latitude $\lambda$ has cylindrical distance from the spin axis

$$\boxed{\rho(h,\lambda) = (R+h)\cos\lambda.} \tag{92}$$

The local spin speed is

$$\boxed{v_{\text{spin}}(h,\lambda) = |\Omega_{\text{spin}}|(R+h)\cos\lambda.} \tag{93}$$

Substitution into equation (91) gives a direct kinematic time field over the body and its co-rotating atmosphere:

$$\alpha(h,\lambda,t) = \Bigg[1 - \frac{1}{c^2}\Big(V_{\text{orb}}^2 + \Omega_{\text{spin}}^2(R+h)^2\cos^2\lambda + 2V_{\text{orb}}|\Omega_{\text{spin}}|(R+h)\cos\lambda\cos\psi(t)\Big)\Bigg]^{1/2}. \tag{94}$$

This expression contains no gravitational variable. Its inputs are the selected reference-frame motion of the centre, the spin rate, the point's position relative to the axis, and the instantaneous orientation between the nested motions.

## 11.2 Full-rotation average

If $\boldsymbol{V}_C$ changes negligibly over one body rotation and the spin velocity explores a complete symmetric cycle, then

$$\langle \boldsymbol{V}_C \cdot (\boldsymbol{\Omega} \times \boldsymbol{r}_P)\rangle_{\text{spin}} = 0. \tag{95}$$

Hence

$$\boxed{\left\langle v_P^2 \right\rangle_{\text{spin}} = V_C^2 + \Omega_{\text{spin}}^2\rho^2.} \tag{96}$$

In the weak-speed limit,

$$\boxed{\langle \alpha_P \rangle_{\text{spin}} \simeq 1 - \frac{V_C^2}{2c^2} - \frac{\Omega_{\text{spin}}^2\rho^2}{2c^2}.} \tag{97}$$

The first deficit is common to points sharing the same centre motion. The second distinguishes latitude and altitude.

### 11.3 Difference between two altitudes

At the same latitude, compare $h_1$ and $h_2$. The spin-only difference in squared speed is

$$\Delta v_{\text{spin}}^2 = \Omega_{\text{spin}}^2 \left[(R+h_2)^2 - (R+h_1)^2\right] \cos^2\lambda. \tag{98}$$

For $|h_2 - h_1| \ll R$,

$$\Delta v_{\text{spin}}^2 \simeq 2\Omega_{\text{spin}}^2 R \Delta h \cos^2\lambda. \tag{99}$$

The full-spin-cycle averaged kinematic clock-rate difference is therefore

$$\boxed{\Delta \left\langle \frac{\mathrm{d}\tau}{\mathrm{d}t} \right\rangle_{\text{kin}} \simeq -\frac{\Omega_{\text{spin}}^2 R \Delta h \cos^2\lambda}{c^2}.} \tag{100}$$

Within this gravity-independent sector, the higher co-rotating point is farther from the axis, has greater spin speed, and accumulates slightly less proper time over the same coordinate interval.

## 12 Multiple Nested Motions

### 12.1 General two-phase form

For two angular phases $q^1$ and $q^2$,

$$\boxed{v_P^2 = \mathsf{G}_{11}(\dot{q}^1)^2 + \mathsf{G}_{22}(\dot{q}^2)^2 + 2\mathsf{G}_{12}\dot{q}^1\dot{q}^2.} \tag{101}$$

Writing

$$\mathsf{G}_{11} = r_1^2, \qquad \mathsf{G}_{22} = r_2^2, \qquad \mathsf{G}_{12} = r_1 r_2 \cos\psi, \tag{102}$$

recovers

$$\boxed{v_P^2 = r_1^2\omega_1^2 + r_2^2\omega_2^2 + 2r_1 r_2 \omega_1\omega_2 \cos\psi.} \tag{103}$$

The sign of each $\omega_a$ and the geometry contained in $\cos\psi$ jointly determine whether two phase-generated velocities reinforce or oppose each other.

### 12.2 Moon, planet, star, and galactic layers

For a lunar surface point, the position may contain at least the following motions:

(1) lunar spin about the lunar axis;

(2) lunar orbital motion about the Earth–Moon barycentre;

(3) barycentric motion of the Earth–Moon system about the Solar-System barycentre;

(4) optional higher motion relative to a Galactic or extragalactic reference construction.

The ledger stores these phase rates separately. The embedding $\boldsymbol{x}_P(\boldsymbol{q})$ determines their exact vector composition. Adding more layers changes the dimension of $\mathsf{G}_{ab}^{(P)}$ but not the chronometric law (40).

### 12.3 Non-commuting axes

Finite orientations in three dimensions satisfy

$$R_{P/\mathrm{ref}}(t) = R_n(q^n)R_{n-1}(q^{n-1})\cdots R_1(q^1), \qquad R_a \in \mathrm{SO}(3). \tag{104}$$

The order is physical. The position embedding is built from the ordered product, after which the phase metric is computed by differentiation. The quadratic chronometric relation remains valid even though the finite angles themselves do not commute.

## 13 Conditional Rod–Flywheel Closure

### 13.1 Role of the closure

The mechanical prelude and the chronometric construction answer different questions. The closure layer asks

> What determines the angular rates and how are stresses, torque, or phase locks transmitted?

The chronometric layer asks

> Given the rates and geometry, what velocity, clock factor, Sagnac signature, and catalogue label follow?

Keeping these questions separate prevents a kinematic identity from being mistaken for a dynamical explanation.

### 13.2 Radial-capacity relation

Suppose an idealised rod–flywheel channel assigns a radial transport capacity $F_\star$ to a terminal body of inertial mass $m$ at lever length $a$:

$$\boxed{F_\star = m\omega_{\mathrm{orb}}^2 a.} \tag{105}$$

Then

$$\boxed{\omega_{\mathrm{orb}} = \sqrt{\frac{F_\star}{ma}}.} \tag{106}$$

For uniform density $\varrho$ and body radius $R$,

$$m = \frac{4\pi}{3}\varrho R^3, \tag{107}$$

so

$$\boxed{\omega_{\mathrm{orb}} = \sqrt{\frac{3F_\star}{4\pi\varrho R^3 a}}.} \tag{108}$$

At fixed $F_\star$, $\varrho$, and $\omega_{\mathrm{orb}}$,

$$\boxed{R \propto a^{-1/3}.} \tag{109}$$

Equation (14) is the corresponding Earth–Neptune example.

### 13.3 Spin-transfer law

Orbital angular progression does not by itself determine the terminal body's spin. Introduce an independent dimensionless transfer function

$$\boxed{\Omega_{\mathrm{spin}} = \omega_{\mathrm{orb}}\,\Phi\!\left(\frac{a}{R};\boldsymbol{\eta}\right),} \tag{110}$$

where $\boldsymbol{\eta}$ may contain moment-of-inertia coefficients, coupling fractions, density ratios, compliance parameters, or phase-lock indices. The function $\Phi$ must be derived from a mechanical model or inferred from data; it is not supplied by the phase ledger.

### 13.4 Finite propagation and the limit of the rigid-rod picture

An exactly rigid rod is an idealisation. In any physical completion, stress and torque propagate at finite speed and the outer terminal cannot be updated instantaneously. The useful content of the thought experiment is therefore not literal rigidity but a declared rule that correlates angular phases across scales. A finite-stiffness completion would replace the algebraic constraint by dynamical fields or coupled phase variables and would test equilibrium, stability, and spin-up.

### 13.5 Insertion into the chronometric map

Once a closure or observation supplies $\omega_{\mathrm{orb}}$ and $\Omega_{\mathrm{spin}}$, the ledger stores them separately. The embedding then constructs the resultant velocity with all cross terms, after which $\mathscr{K}$, $\mathscr{S}$, and the local clock factor follow. No rod force, mass, or body size is inserted directly into the special-relativistic clock law; those quantities affect chronometry only through the motion generated by the declared closure.

## 14 Earth as a Numerical Demonstration

### 14.1 Reference values

For illustration, adopt the WGS 84 equatorial radius

$$R_{\oplus} = 6\,378\,137\,\mathrm{m}, \tag{111}$$

and the mean sidereal day

$$T_{\mathrm{sid}} = 86\,164.090\,54\,\mathrm{s}, \tag{112}$$

so that [9, 6]

$$\Omega_{\oplus} = \frac{2\pi}{T_{\mathrm{sid}}} \simeq 7.292\,115 \times 10^{-5}\,\mathrm{rad/s}. \tag{113}$$

The equatorial spin speed is

$$\boxed{v_{\mathrm{eq}} = \Omega_{\oplus} R_{\oplus} \simeq 465.101\,\mathrm{m/s}.} \tag{114}$$

The spin-only dimensionless progress norm at the equator is

$$\mathcal{A}_{\mathrm{spin,eq}} = \frac{v_{\mathrm{eq}}^2}{c^2} \simeq 2.4068743 \times 10^{-12}. \tag{115}$$

The exact canonical Earth label is

$$\boxed{\mathscr{K}_\oplus = \gamma_\oplus \mathcal{A}_{\mathrm{spin,eq}} \simeq 2.4068743 \times 10^{-12}.} \tag{116}$$

The difference between $\mathscr{K}_\oplus$ and $\mathcal{A}_{\mathrm{spin,eq}}$ begins only beyond the displayed precision. Multiplication by the sidereal period gives the Earth-language calibration

$$\boxed{\Theta_\oplus = \mathscr{K}_\oplus T_{\mathrm{sid}} \simeq 207.386\,\mathrm{ns}.} \tag{117}$$

The corresponding proper-time deficit of the co-rotating equatorial clock over one sidereal rotation is a different quantity,

$$\delta\tau_{\mathrm{spin,eq}} = T_{\mathrm{sid}} \left(1 - \frac{1}{\gamma_\oplus}\right) \simeq 103.7\,\mathrm{ns}. \tag{118}$$

At weak speed, this deficit is approximately one half of the one-direction holonomy translation $\Theta_\oplus$.

## 14.2 Orbital centre motion

Using the exact astronomical unit and the sidereal year listed by JPL [6], a circular mean-speed estimate is

$$V_{\mathrm{orb},\oplus} \simeq \frac{2\pi\,\mathrm{au}}{T_{\mathrm{year}}} \simeq 29.785\,\mathrm{km/s}. \tag{119}$$

This common centre-motion contribution produces a weak-speed kinematic deficit of approximately

$$\frac{V_{\mathrm{orb},\oplus}^2}{2c^2} \simeq 4.94 \times 10^{-9}, \tag{120}$$

or approximately 425 µs over one sidereal day in the selected barycentric flat-spacetime approximation. It is common to nearby terrestrial points and therefore largely cancels in comparisons between them.

The instantaneous cross term in equation (88) can be much larger than the spin-squared term, reaching an equatorial magnitude of order

$$\frac{V_{\mathrm{orb},\oplus} v_{\mathrm{eq}}}{c^2} \simeq 1.54 \times 10^{-10}, \tag{121}$$

but it changes sign through the daily cycle and averages to zero under the symmetric approximation (95).

## 14.3 Sea level and a higher co-rotating point

At the equator, compare sea level with a point 1000 m higher. Equation (100) gives

$$\Delta \left\langle \frac{\mathrm{d}\tau}{\mathrm{d}t} \right\rangle_{\mathrm{kin}} \simeq -3.77 \times 10^{-16}. \tag{122}$$

Over one sidereal day,

$$\boxed{\Delta\tau_{\mathrm{kin}} \simeq -32.5\,\mathrm{ps}.} \tag{123}$$

The sign is negative because the higher co-rotating point has the greater tangential spin speed. This number is intentionally only the rotational kinematic contribution. It is not the complete observed clock difference between the two altitudes.

## 14.4 Latitude dependence

At latitude $\lambda$, the spin-only contribution scales as

$$\mathcal{A}_{\mathrm{spin}}(\lambda, h) = \frac{\Omega_\oplus^2 (R_\oplus + h)^2 \cos^2 \lambda}{c^2}. \tag{124}$$

It is maximal at the equator and vanishes on the rotation axis. The orbital centre-motion term does not vanish at the poles; only the local spin component does.

## 14.5 Illustrative rescaled-Earth translation

Consider the conditional scaling discussed in the rod–flywheel model,

$$a' = 27 a_\oplus, \qquad R' = \frac{R_\oplus}{3}, \qquad \Omega' = 9\Omega_\oplus. \tag{125}$$

At leading weak-speed order, the native one-winding interval remains equal to the terrestrial value because

$$\Omega' R'^2 = 9\Omega_\oplus \frac{R_\oplus^2}{9} = \Omega_\oplus R_\oplus^2. \tag{126}$$

However, the equatorial speed becomes $v' = 3v_\oplus$, so the dimensionless label becomes, to leading order,

$$\mathscr{K}' \simeq 9\mathscr{K}_\oplus. \tag{127}$$

Its Local Observer Translation Map is therefore

$$\Theta^{(\oplus)}_{\mathrm{scaled}} \simeq 9\Theta_\oplus \simeq 1866.48\,\mathrm{ns}_\oplus, \tag{128}$$

while its native-cycle translation remains approximately 207.386 ns because its native period is $T' = T_\oplus/9$. This example demonstrates why a native interval and an Earth-language comparison value must be kept distinct.

# 15 Natural Celestial Examples in Ledger Form

## 15.1 Mars

For a point on Mars, the same equations apply after replacing the terrestrial inputs by the Martian centre trajectory, rotation axis, angular rate, reference radius, latitude, altitude, and local phase. The mean Martian solar day follows from the signed phase difference

$$\omega_{\mathrm{sol,Mars}} = \omega_{\mathrm{sid,Mars}} - \omega_{\mathrm{orb,Mars}}. \tag{129}$$

The local kinematic time field is not tied to the solar day definition; it is computed from the resultant velocity in the chosen reference frame.

## 15.2 Venus

Venus has a retrograde signed sidereal spin period. In the ledger, $\Omega_{\mathrm{spin}} < 0$. The local spin velocity $\boldsymbol{\Omega} \times \boldsymbol{r}_P$ therefore reverses direction, changing the instantaneous cross term with orbital motion. The proper-time factor remains positive because it depends on the squared magnitude of the resultant velocity.

## 15.3 The Moon

For a lunar surface point, synchronous rotation is expressed by near equality between the lunar spin rate and the Moon's orbital phase rate about Earth. These two rates are distinct ledger entries even when their difference is nearly zero. The local velocity still contains the Moon-centre motion plus the surface spin term; phase locking does not erase either physical motion.

## 15.4 Differentially rotating bodies

For gas giants and stars, $\Omega_{\mathrm{spin}}$ may depend on latitude, depth, or pattern. Then

$$\Omega_{\mathrm{spin}} = \Omega_{\mathrm{spin}}(\lambda, h, t), \tag{130}$$

and equation (94) remains valid pointwise after the appropriate local angular rate is supplied. A unique body-level spin number requires an explicit convention, whereas the local chronometric field does not.

# 16 Operational Calculation Protocol

A reproducible calculation for one point, one object, or an atlas catalogue proceeds as follows:

**Step 1:** Choose the quasi-inertial or Galactocentric reference frame, freeze all frame parameters, and state the coordinate time and epoch.

**Step 2:** Construct the immediate-parent phase chain and define signed unwrapped phases.

**Step 3:** For an atlas row, store the position, velocity, covariance, and provenance class separately; do not infer a unique velocity from position alone.

**Step 4:** Specify the position embedding $\boldsymbol{x}_P(\boldsymbol{q}; t)$, including all relevant radii, axes, inclinations, translations, and ordered rotations.

**Step 5:** Differentiate the embedding to obtain $\boldsymbol{v}_P$, or compute the phase metric $\mathsf{G}^{(P)}_{ab}$.

**Step 6:** Evaluate the angular-progress norm $\mathcal{A}_P = v_P^2/c^2$.

**Step 7:** Compute the dimensionless chronometric label $\mathscr{K}_P = \gamma_P \mathcal{A}_P$ and recover the local clock factor from equation (45).

**Step 8:** If an Earth-language comparison is required, compute $\mathscr{R}_{P/\oplus}$ and $\Theta_P^{(\oplus)} = \mathscr{K}_P T_{\mathrm{sid},\oplus}$.

**Step 9:** Integrate $\alpha_P$ over the desired coordinate-time interval when accumulated proper time, rather than an instantaneous label, is required.

**Step 10:** For comparisons of altitude, latitude, or nested motions, retain instantaneous vector cross terms unless a complete symmetric-cycle average is explicitly intended.

**Step 11:** If velocity or angular rates are not measured inputs, name and parameterise the separate closure law and mark the output as conditional.

**Step 12:** Propagate uncertainty from the input covariance or by Monte Carlo sampling.

**Step 13:** Record the frame, epoch, point convention, latitude, altitude, closure status, and whether the label is spin-only, centre-motion-only, or fully nested.

**Step 14:** Do not interpret the result as a complete gravitational clock prediction unless a metric-specific gravitational sector is added separately.

# 17 What Is Established and What Remains Conditional

## 17.1 Established within the stated kinematic framework

The following statements follow from the geometry and standard relativistic kinematics used here:

(1) Nested celestial cycles can be represented by a signed, unwrapped phase ledger.

(2) A point's resultant velocity is obtained from the geometric embedding of those phases.

(3) The correct combination of angular rates is a point-dependent quadratic form with cross terms.

(4) The local flat-spacetime kinematic proper time follows exactly from the resultant speed.

(5) Latitude, altitude, longitude, epoch, spin direction, and parent-system motion enter through explicit geometric variables.

(6) A complete spin-cycle average removes the centre–spin cross term under the stated symmetry assumptions.

(7) No gravitational potential is required to calculate this isolated kinematic sector.

(8) The dimensionless label $\mathscr{K}_P$ uniquely reconstructs the local kinematic clock factor in the selected frame.

(9) For a uniform rotating ring, $\mathscr{K} = |\mathcal{H}^*|/T_{\mathrm{rot}}$ exactly.

(10) The Earth-language map is an algebraically reproducible calibration once the canonical terrestrial ring is fixed.

(11) A complete Galactocentric six-dimensional state uniquely determines the isolated kinematic label in the declared frame.

(12) Cartesian and cylindrical representations give the same speed and therefore the same label when transformed consistently.

(13) A versioned catalogue can retain measured, incomplete, and conditionally completed rows without conflating their provenance.

## 17.2 Not established by the chronometric construction

The present framework does not establish that

(1) a scalar arithmetic sum of angles is identical to physical time;

(2) planetary spin and orbital rates are determined by distance and diameter alone;

(3) the rod–flywheel capacity law is an observed universal law of planetary systems;

(4) one universal function $\Phi(a/R)$ explains every planetary day;

(5) gravitational clock effects are absent from real measurements;

(6) strong-field compact objects can be treated by the flat-spacetime formula without metric-specific corrections;

(7) the selected quasi-inertial frame is an absolute state of rest;

(8) 207.386 ns is a universal time quantum or an observer-independent number;

(9) the Local Observer Translation Map of a general point is automatically its physical one-winding holonomy;

(10) equality of native-cycle intervals alone guarantees equality of dimensionless labels or clock rates;

(11) one instantaneous Galactic position uniquely determines an object's velocity;

(12) a conditional Galactic rotation field is observationally measured merely because it supplies a velocity for a position-only row;

(13) a Galactocentric kinematic label is frame-independent or a complete replacement for Galactic gravitational dynamics.

# 18 Discussion

The main gain of the phase-metric formulation is conceptual economy. Spin, orbit, parent motion, and higher cycles are not forced into one arbitrary scalar. They remain separate in the ledger, while their physical effect on a selected point is combined by the geometry of the point's actual path.

This resolves three recurrent ambiguities.

First, equal angular velocities do not imply equal linear velocities. The local contribution of a phase is weighted by the appropriate effective radius contained in $\mathsf{G}_{aa}$.

Second, the relative direction of two motions matters. The off-diagonal entries $\mathsf{G}_{ab}$ retain reinforcement and opposition through the cross terms.

Third, a body-level period does not by itself define the time rate at every location. The local radius from the rotation axis, latitude, altitude, and phase orientation must be supplied.

The framework is therefore naturally local while remaining nested. It can describe a surface clock, a mountain clock, a co-rotating atmospheric clock, a lunar marker, or a point in a multi-body chain with the same master equation.

The optional rod–flywheel closure addresses a different problem: how angular rates might be related to mass, size, and lever length under a specified transport capacity. Keeping this closure separate prevents a tentative mechanical hypothesis from being confused with the exact kinematic map from motion to proper time.

The dimensionless label introduces a second kind of economy. Instead of attaching a dimensional interval directly to every body, the framework first records $\mathscr{K}_X$. The same label can then be displayed in its native-cycle language, in the canonical Earth-cycle language, or in another declared calibration. The translated number changes with the chosen calibration period; the underlying label does not. In this precise sense, 207.386 ns is a terrestrial translation of the canonical Earth label rather than the label itself.

This translation architecture also defines equivalence classes. Bodies or points with the same $\mathscr{K}$, evaluated under the same frame and point convention, have the same kinematic clock factor even when their radii, angular rates, and native periods differ. Conversely, equal one-cycle dimensional intervals can conceal different clock factors if the native cycle durations are different.

The Galactic extension changes the scale but not the logic. A catalogue row is first a phase-space address, not a scalar time tag. Only after a velocity has been measured or conditionally supplied does the row acquire $\mathscr{K}$. This prevents a colour map of position from being mistaken for a measured map of time rates. The resulting product is best understood as a *kinematic chronometric atlas*: a versioned map of frame-dependent phase-space labels, Local Observer Translation Maps, and uncertainties.

The atlas also provides a practical test of the framework. Objects at different positions but equal resultant speeds must occupy the same instantaneous $\mathscr{K}$ class in the selected frame. Objects at the same position but different velocities must not. Large catalogues such as Gaia DR3 allow these classifications to be generated for measured six-dimensional subsets, while incomplete rows remain visibly incomplete or are processed under explicitly named model closures.

# 19 Conclusion

The combined construction begins with a simple methodological fact:

$$\boxed{\text{trajectory} \neq \text{mechanism.}} \tag{131}$$

The rod and spinning-top thought experiment demonstrates this by producing circular, phase-coherent motion through a mechanical constraint whose force, energy, and angular-momentum scalings differ from those of a gravitational orbit. It also supplies a transparent conditional closure in which mass, lever length, density, and a transfer law determine angular rates.

The chronometric architecture begins only after those rates are measured or supplied. A signed nested phase ledger and its geometric embedding produce

$$\mathsf{G}^{(P)}_{ab} = \frac{\partial \boldsymbol{x}_P}{\partial q^a} \cdot \frac{\partial \boldsymbol{x}_P}{\partial q^b}, \qquad v_P^2 = \mathsf{G}^{(P)}_{ab} \dot{q}^a \dot{q}^b, \tag{132}$$

and therefore

$$\boxed{\mathrm{d}\tau_P = \mathrm{d}t \sqrt{1 - \frac{v_P^2}{c^2}}.} \tag{133}$$

For a point on or above a rotating body,

$$\boxed{\boldsymbol{v}_P = \boldsymbol{V}_C + \boldsymbol{\Omega} \times \boldsymbol{r}_P.} \tag{134}$$

This expression preserves the instantaneous vector cross term rather than adding orbital and spin angles as scalars.

For a uniform rotating ring, the architecture supplies two complementary scale-independent observables:

$$\boxed{\mathscr{K} = \frac{|\mathcal{H}^*|}{T} = \gamma\beta^2, \qquad \mathscr{S} = \frac{c|\mathcal{H}^*|}{C} = \gamma\beta.} \tag{135}$$

The canonical terrestrial one-winding translation is 207.386 ns, while the complete counter-propagating Sagnac proper-time difference is twice that value. At Galactic scale, a canonical ring must be declared by its radius, azimuthal speed, frame, and epoch; the Milky Way is not treated as a rigid wheel. Dimensional holonomies, period-normalised labels, and circumference-normalised signatures are stored separately so that scale and speed are not conflated.

The final atlas record is

$$\boxed{\mathfrak{A}_X(t_0 \mid \mathcal{F}_{\mathrm{G}}) = (t_0; \boldsymbol{x}_X, \boldsymbol{v}_X; \boldsymbol{\Sigma}_X, \mathcal{Q}_X)\,.} \tag{136}$$

Position alone does not determine a speed or chronometric label. A velocity must be measured, transformed, or supplied under an explicitly named closure. The complete architecture is therefore modular:

$$\boxed{\begin{array}{c}\text{dynamics or observation} \to \text{phase ledger} \to \text{geometry} \\ \to \text{chronometry} \to \text{catalogue provenance.}\end{array}} \tag{137}$$

The rod gives a mechanism for asking how angular coherence might be imposed. The phase metric tells how nested angular coherence becomes motion. The chronometric labels tell how that motion is read as a local kinematic clock state. The Galactic atlas tells how those states can be compared without hiding the reference frame, scale, uncertainty, or origin of the velocity.

# A Derivation in Turn-Count Coordinates

Since $q^a = 2\pi N_a$,

$$\frac{\partial \boldsymbol{x}_P}{\partial N_a} = 2\pi \frac{\partial \boldsymbol{x}_P}{\partial q^a}. \tag{138}$$

Define the turn-count metric

$$\widetilde{\mathsf{G}}^{(P)}_{ab} \equiv \frac{\partial \boldsymbol{x}_P}{\partial N_a} \cdot \frac{\partial \boldsymbol{x}_P}{\partial N_b} = 4\pi^2 \mathsf{G}^{(P)}_{ab}. \tag{139}$$

Then

$$\boxed{v_P^2 = \widetilde{\mathsf{G}}^{(P)}_{ab} \nu_a \nu_b, \qquad \nu_a = \dot{N}_a.} \tag{140}$$

The proper-time law may be written directly in ledger rates:

$$\boxed{\mathrm{d}\tau_P = \mathrm{d}t \sqrt{1 - \frac{\widetilde{\mathsf{G}}^{(P)}_{ab} \nu_a \nu_b}{c^2}}.} \tag{141}$$

# B Exact Two-Phase Example

Let

$$\boldsymbol{x}(q^1, q^2) = r_1 \begin{pmatrix} \cos q^1 \\ \sin q^1 \end{pmatrix} + r_2 \begin{pmatrix} \cos q^2 \\ \sin q^2 \end{pmatrix}. \tag{142}$$

Then

$$\frac{\partial \boldsymbol{x}}{\partial q^1} = r_1 \begin{pmatrix} -\sin q^1 \\ \cos q^1 \end{pmatrix}, \tag{143}$$

$$\frac{\partial \boldsymbol{x}}{\partial q^2} = r_2 \begin{pmatrix} -\sin q^2 \\ \cos q^2 \end{pmatrix}. \tag{144}$$

Therefore

$$\mathsf{G} = \begin{pmatrix} r_1^2 & r_1 r_2 \cos(q^1 - q^2) \\ r_1 r_2 \cos(q^1 - q^2) & r_2^2 \end{pmatrix}, \tag{145}$$

and

$$v^2 = r_1^2\omega_1^2 + r_2^2\omega_2^2 + 2r_1 r_2 \omega_1 \omega_2 \cos(q^1 - q^2). \tag{146}$$

This explicit example shows that the cross term is generated by the geometry of the phase embedding and not added by hand.

# C Relation to a Rotating-Ring Holonomy

For one uniformly rotating ring of radius $R$ and angular velocity $\Omega$, the local speed is $v = |\Omega|R$ and

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \sqrt{1 - \frac{\Omega^2 R^2}{c^2}}. \tag{147}$$

A slow directed transport around the same ring yields the standard one-winding time holonomy

$$\mathcal{H}^* = -\gamma \frac{2\pi\Omega R^2}{c^2}, \qquad \gamma = \left(1 - \frac{\Omega^2 R^2}{c^2}\right)^{-1/2}. \tag{148}$$

Since $T_{\mathrm{rot}} = 2\pi/|\Omega|$, the magnitude obeys

$$\boxed{\frac{|\mathcal{H}^*|}{T_{\mathrm{rot}}} = \gamma \frac{\Omega^2 R^2}{c^2} = \mathscr{K}_{\mathrm{ring}}.} \tag{149}$$

Thus the dimensionless chronometric label is exactly the period-normalised one-direction holonomy for a uniform ring. The phase-metric framework is more general because it applies to open intervals, arbitrary points, multiple nested phases, and non-closed trajectories. For those more general cases, $\mathscr{K}_P$ remains a local rate label, whereas multiplying it by a calibration period is a declared translation rather than an automatically realised loop observable.

# D Stable Numerical Formulae

For a known speed $v$ with $\beta = v/c \ll 1$,

$$1 - \sqrt{1 - \beta^2} = \frac{\beta^2}{1 + \sqrt{1 - \beta^2}}, \tag{150}$$

which avoids subtractive loss of precision. For a time series sampled at $t_i$, the accumulated proper time may be approximated by

$$\tau_P \simeq \sum_i \Delta t_i \sqrt{1 - \frac{v_P^2(t_i)}{c^2}}, \tag{151}$$

or by a higher-order quadrature when ephemeris accuracy requires it.

For a label $\mathscr{K}$, the stable inverse clock factor is

$$\alpha = \frac{2}{\sqrt{\mathscr{K}^2 + 4} + \mathscr{K}}. \tag{152}$$

The Local Observer Translation Map in nanoseconds is

$$\Theta_X^{(\oplus)}[\mathrm{ns}_\oplus] = 10^9\, T_{\mathrm{sid},\oplus} \mathscr{K}_X. \tag{153}$$

# E Reference Python Implementations

The accompanying source package includes three executable files. The first, `chronometric_label_calculator.py`, computes canonical spin-only and local nested labels. The second, `rod_phase_sagnac_calculator.py`, implements the engineered bridge from a declared rod-capacity closure or a uniform-ring state to $\mathscr{K}$, $\mathscr{S}$, the one-winding holonomy, and the full counter-propagating Sagnac proper-time difference. The third, `galactic_chronometric_atlas.py`, ingests single phase-space states or catalogue CSV files, converts Cartesian addresses to cylindrical components, distinguishes measured from conditionally completed rows, computes $\mathscr{K}$, and writes local-observer atlas tables. Its optional ICRS mode uses Astropy with explicit Galactocentric parameters. The package also includes Cartesian and Gaia-style CSV templates plus example measured and conditional outputs. None of the implementations includes a gravitational clock sector.

Listing 1: Local dimensionless chronometric-label and Local Observer Translation Map calculator.

```
#!/usr/bin/env python3
"""
Dimensionless chronometric-label and Earth-language translation calculator.

The calculator implements the gravity-independent kinematic sector developed in
"Nested Angular Progress and Local Kinematic Time".

Core definitions
----------------
    beta = v / c
    gamma = 1 / sqrt(1 - beta**2)
    K = gamma * beta**2 = gamma - 1/gamma
    alpha = d_tau/dt = 1/gamma

For Earth's canonical equatorial rotating ring,
    K_E = gamma_E * (Omega_E R_E / c)**2
and the Earth-language calibration interval is
    Theta_E = K_E * T_E ~= 207.386 ns.

For another object or point X, its Earth-language translation is
    Theta_X^(E) = K_X * T_E
                = (K_X / K_E) * Theta_E.

This translated interval is a comparison convention. It is not automatically
the physical one-winding holonomy of X. For a uniformly rotating canonical
ring, the native one-winding holonomy magnitude is K_X * T_X.

No gravitational potential or gravitational redshift is included.
"""

from __future__ import annotations

```

```python
import argparse
import math
from dataclasses import dataclass, asdict
from typing import Any

C = 299_792_458.0  # m/s, exact SI value

# Canonical Earth calibration adopted in the article.
EARTH_RADIUS_M = 6_378_137.0
EARTH_SIDEREAL_PERIOD_S = 86_164.09054


@dataclass(frozen=True)
class ChronometricRecord:
    """Computed dimensionless label and translation quantities."""

    name: str
    speed_m_s: float
    beta: float
    gamma: float
    clock_factor_alpha: float
    label_K: float
    earth_normalized_ratio: float
    earth_language_seconds: float
    earth_language_nanoseconds: float
    native_period_s: float | None
    native_cycle_equivalent_seconds: float | None
    native_cycle_equivalent_nanoseconds: float | None
    native_to_earth_conversion: float | None


def angular_velocity(rotation_period_s: float) -> float:
    """Return signed angular velocity in rad/s."""
    if rotation_period_s == 0.0:
        raise ValueError("rotation_period_s cannot be zero")
    return 2.0 * math.pi / rotation_period_s


def cylindrical_radius(
    body_radius_m: float,
    altitude_m: float = 0.0,
    latitude_deg: float = 0.0,
) -> float:
    """Distance from the rotation axis: rho = (R + h) cos(latitude)."""
    if body_radius_m <= 0.0:
        raise ValueError("body_radius_m must be positive")
    if body_radius_m + altitude_m <= 0.0:
        raise ValueError("body_radius_m + altitude_m must be positive")
    if not -90.0 <= latitude_deg <= 90.0:
        raise ValueError("latitude_deg must lie between -90 and +90")
    return (body_radius_m + altitude_m) * math.cos(math.radians(latitude_deg))


def spin_speed(
    body_radius_m: float,
    rotation_period_s: float,
    altitude_m: float = 0.0,
    latitude_deg: float = 0.0,
) -> float:
    """Return signed local spin speed Omega*rho in m/s."""
    omega = angular_velocity(rotation_period_s)
    rho = cylindrical_radius(body_radius_m, altitude_m, latitude_deg)
    return omega * rho


def resultant_speed(
    center_speed_m_s: float,
    local_spin_speed_m_s: float,
    alignment_angle_deg: float,
```

```
) -> float:
    """
    Magnitude of V_center + v_spin using their included angle psi.

    v^2 = V^2 + v_spin^2 + 2 V v_spin cos(psi)

    The sign of local_spin_speed_m_s is retained, so retrograde rotation can be
    represented either by a negative rotation period or by the chosen angle.
    """
    if center_speed_m_s < 0.0:
        raise ValueError("center_speed_m_s must be non-negative")
    psi = math.radians(alignment_angle_deg)
    v2 = (
        center_speed_m_s**2
        + local_spin_speed_m_s**2
        + 2.0
        * center_speed_m_s
        * local_spin_speed_m_s
        * math.cos(psi)
    )
    # Protect against tiny negative roundoff in an exact cancellation case.
    return math.sqrt(max(v2, 0.0))


def dimensionless_label_from_speed(speed_m_s: float) -> tuple[float, float, float, float]:
    """Return beta, gamma, alpha, and K for a speed magnitude."""
    if speed_m_s < 0.0:
        raise ValueError("speed_m_s must be non-negative")
    beta = speed_m_s / C
    if beta >= 1.0:
        raise ValueError("speed must be smaller than c")
    gamma = 1.0 / math.sqrt(1.0 - beta * beta)
    alpha = 1.0 / gamma
    label_k = gamma * beta * beta
    return beta, gamma, alpha, label_k


def clock_factor_from_label(label_k: float) -> float:
    """Exact inverse map alpha(K) for K >= 0."""
    if label_k < 0.0:
        raise ValueError("label_k must be non-negative")
    # Stable form of (sqrt(K^2 + 4) - K)/2.
    return 2.0 / (math.sqrt(label_k * label_k + 4.0) + label_k)


def earth_calibration() -> dict[str, float]:
    """Return Earth's canonical equatorial calibration constants."""
    v_earth = abs(
        spin_speed(
            body_radius_m=EARTH_RADIUS_M,
            rotation_period_s=EARTH_SIDEREAL_PERIOD_S,
        )
    )
    beta, gamma, alpha, label_k = dimensionless_label_from_speed(v_earth)
    theta_s = label_k * EARTH_SIDEREAL_PERIOD_S
    return {
        "speed_m_s": v_earth,
        "beta": beta,
        "gamma": gamma,
        "alpha": alpha,
        "label_K": label_k,
        "translation_seconds": theta_s,
        "translation_nanoseconds": theta_s * 1.0e9,
    }


EARTH = earth_calibration()
EARTH_LABEL_K = EARTH["label_K"]
EARTH_TRANSLATION_S = EARTH["translation_seconds"]
```

```python
EARTH_TRANSLATION_NS = EARTH["translation_nanoseconds"]


def make_record(
    *,
    name: str,
    speed_m_s: float,
    native_period_s: float | None = None,
) -> ChronometricRecord:
    """Create a complete label and Earth-translation record from speed."""
    beta, gamma, alpha, label_k = dimensionless_label_from_speed(speed_m_s)
    ratio = label_k / EARTH_LABEL_K
    earth_s = label_k * EARTH_SIDEREAL_PERIOD_S

    if native_period_s is None:
        native_s = None
        native_ns = None
        conversion = None
    else:
        period = abs(native_period_s)
        if period == 0.0:
            raise ValueError("native_period_s cannot be zero")
        native_s = label_k * period
        native_ns = native_s * 1.0e9
        conversion = EARTH_SIDEREAL_PERIOD_S / period

    return ChronometricRecord(
        name=name,
        speed_m_s=speed_m_s,
        beta=beta,
        gamma=gamma,
        clock_factor_alpha=alpha,
        label_K=label_k,
        earth_normalized_ratio=ratio,
        earth_language_seconds=earth_s,
        earth_language_nanoseconds=earth_s * 1.0e9,
        native_period_s=abs(native_period_s) if native_period_s is not None else None,
        native_cycle_equivalent_seconds=native_s,
        native_cycle_equivalent_nanoseconds=native_ns,
        native_to_earth_conversion=conversion,
    )


def canonical_rotating_body_record(
    *,
    name: str,
    radius_m: float,
    rotation_period_s: float,
    latitude_deg: float = 0.0,
    altitude_m: float = 0.0,
) -> ChronometricRecord:
    """
    Canonical spin-only record for a point fixed to a rotating body.

    For the standard body label, use latitude_deg=0 and altitude_m=0.
    """
    v_spin = abs(
        spin_speed(
            body_radius_m=radius_m,
            rotation_period_s=rotation_period_s,
            altitude_m=altitude_m,
            latitude_deg=latitude_deg,
        )
    )
    return make_record(
        name=name,
        speed_m_s=v_spin,
        native_period_s=rotation_period_s,
    )
```

```python


def local_resultant_record(
    *,
    name: str,
    radius_m: float,
    rotation_period_s: float,
    center_speed_m_s: float,
    alignment_angle_deg: float,
    latitude_deg: float = 0.0,
    altitude_m: float = 0.0,
) -> ChronometricRecord:
    """Instantaneous local label including centre motion and local spin."""
    v_spin = spin_speed(
        body_radius_m=radius_m,
        rotation_period_s=rotation_period_s,
        altitude_m=altitude_m,
        latitude_deg=latitude_deg,
    )
    v_total = resultant_speed(
        center_speed_m_s=center_speed_m_s,
        local_spin_speed_m_s=v_spin,
        alignment_angle_deg=alignment_angle_deg,
    )
    # The native-cycle field here is only a translation convention. If the
    # alignment changes during a cycle, it is not an accumulated physical loop
    # observable; integrate alpha(t) for that calculation.
    return make_record(
        name=name,
        speed_m_s=v_total,
        native_period_s=rotation_period_s,
    )


def print_record(record: ChronometricRecord) -> None:
    """Print one record in a compact scientific format."""
    print("=" * 72)
    print(record.name)
    print("=" * 72)
    print(f"speed                          = {record.speed_m_s:.12g} m/s")
    print(f"beta                           = {record.beta:.12e}")
    print(f"gamma                          = {record.gamma:.16g}")
    print(f"clock factor alpha             = {record.clock_factor_alpha:.16g}")
    print(f"dimensionless label K          = {record.label_K:.12e}")
    print(f"Earth-normalized ratio         = {record.earth_normalized_ratio:.12e}")
    print(
        "Earth-language translation     = "
        f"{record.earth_language_nanoseconds:.12g} ns_E"
    )
    if record.native_period_s is not None:
        print(f"native period                  = {record.native_period_s:.12g} s")
        print(
            "native-cycle equivalent       = "
            f"{record.native_cycle_equivalent_nanoseconds:.12g} ns"
        )
        print(
            "native -> Earth coefficient   = "
            f"{record.native_to_earth_conversion:.12e}"
        )
    print()


def record_as_dict(record: ChronometricRecord) -> dict[str, Any]:
    """Return a serialization-friendly dictionary."""
    return asdict(record)


def build_argument_parser() -> argparse.ArgumentParser:
    parser = argparse.ArgumentParser(
```

```
        description=(
            "Calculate a dimensionless kinematic chronometric label and its "
            "Earth-language translation."
        )
    )
    parser.add_argument("--name", default="Custom body")
    parser.add_argument("--radius-km", type=float, required=True)
    parser.add_argument(
        "--rotation-period-hours",
        type=float,
        required=True,
        help="Use a negative value for retrograde rotation.",
    )
    parser.add_argument("--latitude-deg", type=float, default=0.0)
    parser.add_argument("--altitude-km", type=float, default=0.0)
    parser.add_argument(
        "--center-speed-km-s",
        type=float,
        default=0.0,
        help="Optional centre-of-body speed in the selected reference frame.",
    )
    parser.add_argument(
        "--alignment-angle-deg",
        type=float,
        default=90.0,
        help="Angle between centre velocity and signed local spin velocity.",
    )
    return parser


def main() -> None:
    parser = build_argument_parser()
    args = parser.parse_args()

    radius_m = args.radius_km * 1_000.0
    period_s = args.rotation_period_hours * 3_600.0
    altitude_m = args.altitude_km * 1_000.0
    center_speed = args.center_speed_km_s * 1_000.0

    print("Earth calibration")
    print(f"K_E       = {EARTH_LABEL_K:.12e}")
    print(f"Theta_E   = {EARTH_TRANSLATION_NS:.12f} ns_E")
    print()

    if center_speed == 0.0:
        record = canonical_rotating_body_record(
            name=args.name,
            radius_m=radius_m,
            rotation_period_s=period_s,
            latitude_deg=args.latitude_deg,
            altitude_m=altitude_m,
        )
    else:
        record = local_resultant_record(
            name=args.name,
            radius_m=radius_m,
            rotation_period_s=period_s,
            center_speed_m_s=center_speed,
            alignment_angle_deg=args.alignment_angle_deg,
            latitude_deg=args.latitude_deg,
            altitude_m=altitude_m,
        )

    print_record(record)


if __name__ == "__main__":
    main()
```

Listing 2: Integrated rod–phase–Sagnac calculator.

```
#!/usr/bin/env python3
"""
Integrated rod--phase--Sagnac calculator.

This utility implements the executable bridge used in the combined manuscript:

1. A declared mechanical closure may provide an angular rate.
2. The angular rate and radius determine the tangential speed.
3. The speed determines the dimensionless chronometric label K.
4. A uniform ring also has a one-winding holonomy H*, a full
   counter-propagating Sagnac proper-time difference, and a circumference-
   normalised signature S.

Definitions
-----------
    beta = v / c
    gamma = 1 / sqrt(1 - beta**2)
    K = gamma * beta**2
    S = gamma * beta
    |H*| = K * T = S * C_ring / c
    Delta_tau_counter = 2 * |H*|

For the conditional rod-capacity closure:
    F_star = m * omega**2 * lever
    omega = sqrt(F_star / (m * lever))

The code does not include gravitational redshift or a universal dynamical law.
"""

from __future__ import annotations

import argparse
import math
from dataclasses import asdict, dataclass
from typing import Any

C = 299_792_458.0
EARTH_RADIUS_M = 6_378_137.0
EARTH_SIDEREAL_PERIOD_S = 86_164.09054


@dataclass(frozen=True)
class RingSignature:
    name: str
    radius_m: float
    circumference_m: float
    angular_velocity_rad_s: float
    period_s: float
    speed_m_s: float
    beta: float
    gamma: float
    clock_factor_alpha: float
    label_K: float
    circumference_signature_S: float
    one_winding_holonomy_s: float
    counter_propagating_sagnac_s: float
    earth_normalized_K: float
    earth_language_translation_s: float


def _validate_radius(radius_m: float) -> None:
    if radius_m <= 0.0:
        raise ValueError("radius_m must be positive")


def _validate_speed(speed_m_s: float) -> None:
    if not 0.0 <= speed_m_s < C:
```

```
        raise ValueError("speed magnitude must satisfy 0 <= v < c")


def earth_label() -> float:
    speed = 2.0 * math.pi * EARTH_RADIUS_M / EARTH_SIDEREAL_PERIOD_S
    beta = speed / C
    gamma = 1.0 / math.sqrt(1.0 - beta * beta)
    return gamma * beta * beta


EARTH_LABEL_K = earth_label()


def ring_from_speed(*, name: str, radius_m: float, speed_m_s: float) -> RingSignature:
    """Construct all ring signatures from radius and tangential speed."""
    _validate_radius(radius_m)
    speed = abs(speed_m_s)
    _validate_speed(speed)
    if speed == 0.0:
        raise ValueError("speed must be non-zero for a finite native period")

    circumference = 2.0 * math.pi * radius_m
    omega = speed / radius_m
    period = circumference / speed
    beta = speed / C
    gamma = 1.0 / math.sqrt(1.0 - beta * beta)
    alpha = 1.0 / gamma
    label_k = gamma * beta * beta
    signature_s = gamma * beta
    one_winding = label_k * period
    two_way = 2.0 * one_winding
    earth_translation = label_k * EARTH_SIDEREAL_PERIOD_S

    return RingSignature(
        name=name,
        radius_m=radius_m,
        circumference_m=circumference,
        angular_velocity_rad_s=omega,
        period_s=period,
        speed_m_s=speed,
        beta=beta,
        gamma=gamma,
        clock_factor_alpha=alpha,
        label_K=label_k,
        circumference_signature_S=signature_s,
        one_winding_holonomy_s=one_winding,
        counter_propagating_sagnac_s=two_way,
        earth_normalized_K=label_k / EARTH_LABEL_K,
        earth_language_translation_s=earth_translation,
    )


def ring_from_period(*, name: str, radius_m: float, period_s: float) -> RingSignature:
    """Construct all ring signatures from radius and native rotation period."""
    _validate_radius(radius_m)
    period = abs(period_s)
    if period == 0.0:
        raise ValueError("period_s must be non-zero")
    speed = 2.0 * math.pi * radius_m / period
    return ring_from_speed(name=name, radius_m=radius_m, speed_m_s=speed)


def rod_capacity_ring(
    *,
    name: str,
    force_capacity_n: float,
    terminal_mass_kg: float,
    lever_m: float,
) -> RingSignature:
```

```
    """Apply F_star = m omega^2 a and convert the result to ring signatures."""
    if force_capacity_n <= 0.0:
        raise ValueError("force_capacity_n must be positive")
    if terminal_mass_kg <= 0.0:
        raise ValueError("terminal_mass_kg must be positive")
    _validate_radius(lever_m)

    omega = math.sqrt(force_capacity_n / (terminal_mass_kg * lever_m))
    speed = omega * lever_m
    return ring_from_speed(name=name, radius_m=lever_m, speed_m_s=speed)


def equal_density_radius_ratio(distance_ratio: float) -> float:
    """At fixed capacity and angular rate, return R2/R1 = distance_ratio^(-1/3)."""
    if distance_ratio <= 0.0:
        raise ValueError("distance_ratio must be positive")
    return distance_ratio ** (-1.0 / 3.0)


def record_as_dict(record: RingSignature) -> dict[str, Any]:
    return asdict(record)


def print_record(record: RingSignature) -> None:
    print("=" * 78)
    print(record.name)
    print("=" * 78)
    print(f"radius                          = {record.radius_m:.12g} m")
    print(f"circumference                   = {record.circumference_m:.12g} m")
    print(f"speed                           = {record.speed_m_s:.12g} m/s")
    print(f"angular velocity                = {record.angular_velocity_rad_s:.12e} rad/s")
    print(f"native period                   = {record.period_s:.12g} s")
    print(f"beta                            = {record.beta:.12e}")
    print(f"gamma                           = {record.gamma:.16g}")
    print(f"clock factor alpha              = {record.clock_factor_alpha:.16g}")
    print(f"period-normalised label K       = {record.label_K:.12e}")
    print(f"circumference signature S       = {record.circumference_signature_S:.12e}")
    print(f"one-winding holonomy |H*|       = {record.one_winding_holonomy_s:.12e} s")
    print(f"counter-propagating Delta_tau   = {record.counter_propagating_sagnac_s:.12e} s")
    print(f"Earth-normalised K ratio        = {record.earth_normalized_K:.12e}")
    print(f"Earth-language translation      = {record.earth_language_translation_s:.12e} s_E")


def build_parser() -> argparse.ArgumentParser:
    parser = argparse.ArgumentParser(description=__doc__)
    sub = parser.add_subparsers(dest="mode", required=True)

    ring = sub.add_parser("ring", help="Calculate signatures for a uniform ring")
    ring.add_argument("--name", default="Uniform ring")
    ring.add_argument("--radius-m", type=float, required=True)
    source = ring.add_mutually_exclusive_group(required=True)
    source.add_argument("--speed-m-s", type=float)
    source.add_argument("--period-s", type=float)

    rod = sub.add_parser("rod", help="Apply the conditional rod-capacity closure")
    rod.add_argument("--name", default="Rod terminal")
    rod.add_argument("--force-n", type=float, required=True)
    rod.add_argument("--mass-kg", type=float, required=True)
    rod.add_argument("--lever-m", type=float, required=True)
    rod.add_argument(
        "--distance-ratio",
        type=float,
        help="Also report the equal-density terminal radius ratio r^(-1/3).",
    )
    return parser


def main() -> None:
    args = build_parser().parse_args()
```

```
206
207     if args.mode == "ring":
208         if args.speed_m_s is not None:
209             record = ring_from_speed(
210                 name=args.name,
211                 radius_m=args.radius_m,
212                 speed_m_s=args.speed_m_s,
213             )
214         else:
215             record = ring_from_period(
216                 name=args.name,
217                 radius_m=args.radius_m,
218                 period_s=args.period_s,
219             )
220     else:
221         record = rod_capacity_ring(
222             name=args.name,
223             force_capacity_n=args.force_n,
224             terminal_mass_kg=args.mass_kg,
225             lever_m=args.lever_m,
226         )
227
228     print_record(record)
229
230     if args.mode == "rod" and args.distance_ratio is not None:
231         ratio = equal_density_radius_ratio(args.distance_ratio)
232         print(f"equal-density terminal radius ratio = {ratio:.12g}")
233
234
235 if __name__ == "__main__":
236     main()
```

Listing 3: Galactic phase-space chronometric atlas generator.

```
1  #!/usr/bin/env python3
2  """
3  Galactic phase-space chronometric atlas generator.
4
5  This script implements the gravity-independent kinematic sector developed in
6  "Nested Angular Progress, Galactic Phase-Space Addressing, and Earth-Language
7  Chronometric Labels".
8
9  The fundamental catalogue input is a declared Galactocentric phase-space state
10 at one epoch:
11
12     (x, y, z, vx, vy, vz)
13
14 Position alone does not determine velocity.  Rows without measured velocities
15 are therefore either marked INCOMPLETE or completed by an explicitly selected
16 conditional velocity-field closure.
17
18 For each complete row the script calculates:
19
20     R, phi, z, v_R, v_phi, v_z, |v|,
21     beta, gamma, alpha=d_tau/dt,
22     K=gamma*beta^2,
23     K/K_Earth,
24     Theta_Earth=K*T_sidereal_Earth.
25
26 The Earth-language interval is a comparison coordinate.  It is not claimed to
27 be a native physical loop holonomy unless a separate closed-loop construction
28 applies to the object being represented.
29
30 No gravitational potential or gravitational redshift is included.
31
32 Optional dependency:
33     astropy -- only required for ICRS/Gaia-style sky-coordinate transformation.
34
```

```
Author: S. M. H. Emamifar
Affiliation: IRCBHC
"""

from __future__ import annotations

import argparse
import csv
import json
import math
import random
import statistics
from dataclasses import asdict, dataclass
from pathlib import Path
from typing import Any, Iterable

C_M_S = 299_792_458.0
KPC_M = 3.085_677_581_491_367e19
KM_S_TO_M_S = 1_000.0

EARTH_RADIUS_M = 6_378_137.0
EARTH_SIDEREAL_PERIOD_S = 86_164.09054
EARTH_OMEGA_RAD_S = 2.0 * math.pi / EARTH_SIDEREAL_PERIOD_S
EARTH_EQUATORIAL_SPEED_M_S = EARTH_OMEGA_RAD_S * EARTH_RADIUS_M

# Explicit illustrative Galactocentric frame parameters.  They match the
# Astropy v4.0 example convention and are not asserted to be a unique standard.
DEFAULT_GALCEN_DISTANCE_KPC = 8.122
DEFAULT_Z_SUN_PC = 20.8
DEFAULT_V_SUN_KM_S = (12.9, 245.6, 7.78)


@dataclass(frozen=True)
class Vector3:
    x: float
    y: float
    z: float

    def norm(self) -> float:
        return math.sqrt(self.x * self.x + self.y * self.y + self.z * self.z)

    def __add__(self, other: "Vector3") -> "Vector3":
        return Vector3(self.x + other.x, self.y + other.y, self.z + other.z)

    def scale(self, factor: float) -> "Vector3":
        return Vector3(self.x * factor, self.y * factor, self.z * factor)


@dataclass(frozen=True)
class LabelState:
    speed_m_s: float
    beta: float
    gamma: float
    alpha: float
    label_K: float
    earth_ratio: float
    earth_translation_s: float
    earth_translation_ns: float


@dataclass(frozen=True)
class GalacticChronometricRecord:
    name: str
    epoch: str
    frame_name: str
    quality_class: str
    closure_name: str
    x_kpc: float
    y_kpc: float
```

```
    z_kpc: float
    R_kpc: float
    phi_deg: float
    vx_km_s: float | None
    vy_km_s: float | None
    vz_km_s: float | None
    vR_km_s: float | None
    vphi_km_s: float | None
    speed_km_s: float | None
    beta: float | None
    gamma: float | None
    alpha: float | None
    label_K: float | None
    earth_ratio: float | None
    earth_translation_s: float | None
    earth_translation_ms: float | None
    label_K_sigma: float | None
    earth_translation_ms_sigma: float | None
    notes: str


def label_from_speed_m_s(speed_m_s: float) -> LabelState:
    """Calculate the exact dimensionless label and Earth translation."""
    if speed_m_s < 0.0:
        raise ValueError("speed_m_s must be non-negative")
    beta = speed_m_s / C_M_S
    if beta >= 1.0:
        raise ValueError("speed must be smaller than c")
    gamma = 1.0 / math.sqrt(1.0 - beta * beta)
    alpha = 1.0 / gamma
    label_k = gamma * beta * beta
    return LabelState(
        speed_m_s=speed_m_s,
        beta=beta,
        gamma=gamma,
        alpha=alpha,
        label_K=label_k,
        earth_ratio=label_k / EARTH_LABEL.label_K,
        earth_translation_s=label_k * EARTH_SIDEREAL_PERIOD_S,
        earth_translation_ns=label_k * EARTH_SIDEREAL_PERIOD_S * 1.0e9,
    )


# Bootstrap Earth after the function exists.  The temporary calculation is
# written explicitly to avoid recursive access to EARTH_LABEL.
def _earth_label() -> LabelState:
    beta = EARTH_EQUATORIAL_SPEED_M_S / C_M_S
    gamma = 1.0 / math.sqrt(1.0 - beta * beta)
    alpha = 1.0 / gamma
    label_k = gamma * beta * beta
    return LabelState(
        speed_m_s=EARTH_EQUATORIAL_SPEED_M_S,
        beta=beta,
        gamma=gamma,
        alpha=alpha,
        label_K=label_k,
        earth_ratio=1.0,
        earth_translation_s=label_k * EARTH_SIDEREAL_PERIOD_S,
        earth_translation_ns=label_k * EARTH_SIDEREAL_PERIOD_S * 1.0e9,
    )


EARTH_LABEL = _earth_label()


def cartesian_to_cylindrical(
    position_kpc: Vector3,
    velocity_km_s: Vector3 | None,
) -> tuple[float, float, float | None, float | None]:
```

```
    """Return R, phi(deg), v_R and v_phi in a right-handed frame."""
    x, y = position_kpc.x, position_kpc.y
    radius = math.hypot(x, y)
    phi_deg = math.degrees(math.atan2(y, x))
    if velocity_km_s is None:
        return radius, phi_deg, None, None
    if radius == 0.0:
        return radius, phi_deg, None, None
    v_r = (x * velocity_km_s.x + y * velocity_km_s.y) / radius
    v_phi = (x * velocity_km_s.y - y * velocity_km_s.x) / radius
    return radius, phi_deg, v_r, v_phi


def flat_rotation_closure(position_kpc: Vector3, speed_km_s: float) -> Vector3:
    """Conditional circular field with constant signed tangential speed."""
    radius = math.hypot(position_kpc.x, position_kpc.y)
    if radius == 0.0:
        raise ValueError("flat rotation closure is undefined at R=0")
    return Vector3(
        -speed_km_s * position_kpc.y / radius,
        speed_km_s * position_kpc.x / radius,
        0.0,
    )


def solid_rotation_closure(
    position_kpc: Vector3,
    omega_km_s_kpc: float,
) -> Vector3:
    """Conditional solid-rotation field v_phi=Omega*R."""
    return Vector3(
        -omega_km_s_kpc * position_kpc.y,
        omega_km_s_kpc * position_kpc.x,
        0.0,
    )


def apply_velocity_closure(
    position_kpc: Vector3,
    closure: str,
    flat_speed_km_s: float,
    solid_omega_km_s_kpc: float,
) -> tuple[Vector3 | None, str, str]:
    if closure == "none":
        return None, "P3_POSITION_ONLY", "none"
    if closure == "flat":
        return (
            flat_rotation_closure(position_kpc, flat_speed_km_s),
            "C6_MODEL_CLOSED",
            f"flat_rotation({flat_speed_km_s:g} km/s)",
        )
    if closure == "solid":
        return (
            solid_rotation_closure(position_kpc, solid_omega_km_s_kpc),
            "C6_MODEL_CLOSED",
            f"solid_rotation({solid_omega_km_s_kpc:g} km/s/kpc)",
        )
    raise ValueError(f"unsupported closure: {closure}")


def monte_carlo_label_sigma(
    velocity_km_s: Vector3,
    sigma_km_s: Vector3,
    samples: int,
    seed: int,
) -> tuple[float | None, float | None]:
    """Diagonal-error Monte Carlo. Full covariance should be used when available."""
    if samples <= 1:
        return None, None
```

```
    if sigma_km_s.x < 0 or sigma_km_s.y < 0 or sigma_km_s.z < 0:
        raise ValueError("velocity uncertainties must be non-negative")
    if sigma_km_s.norm() == 0.0:
        return 0.0, 0.0
    rng = random.Random(seed)
    labels: list[float] = []
    translations_ms: list[float] = []
    for _ in range(samples):
        draw = Vector3(
            rng.gauss(velocity_km_s.x, sigma_km_s.x),
            rng.gauss(velocity_km_s.y, sigma_km_s.y),
            rng.gauss(velocity_km_s.z, sigma_km_s.z),
        )
        state = label_from_speed_m_s(draw.norm() * KM_S_TO_M_S)
        labels.append(state.label_K)
        translations_ms.append(state.earth_translation_s * 1.0e3)
    return statistics.stdev(labels), statistics.stdev(translations_ms)


def build_record(
    *,
    name: str,
    epoch: str,
    frame_name: str,
    position_kpc: Vector3,
    velocity_km_s: Vector3 | None,
    quality_class: str,
    closure_name: str,
    velocity_sigma_km_s: Vector3 | None = None,
    mc_samples: int = 0,
    seed: int = 20260722,
    notes: str = "",
) -> GalacticChronometricRecord:
    R_kpc, phi_deg, v_r, v_phi = cartesian_to_cylindrical(
        position_kpc, velocity_km_s
    )

    if velocity_km_s is None:
        return GalacticChronometricRecord(
            name=name,
            epoch=epoch,
            frame_name=frame_name,
            quality_class=quality_class,
            closure_name=closure_name,
            x_kpc=position_kpc.x,
            y_kpc=position_kpc.y,
            z_kpc=position_kpc.z,
            R_kpc=R_kpc,
            phi_deg=phi_deg,
            vx_km_s=None,
            vy_km_s=None,
            vz_km_s=None,
            vR_km_s=None,
            vphi_km_s=None,
            speed_km_s=None,
            beta=None,
            gamma=None,
            alpha=None,
            label_K=None,
            earth_ratio=None,
            earth_translation_s=None,
            earth_translation_ms=None,
            label_K_sigma=None,
            earth_translation_ms_sigma=None,
            notes=notes,
        )

    speed_km_s = velocity_km_s.norm()
    state = label_from_speed_m_s(speed_km_s * KM_S_TO_M_S)
```

```
    if velocity_sigma_km_s is None:
        k_sigma, theta_sigma = None, None
    else:
        k_sigma, theta_sigma = monte_carlo_label_sigma(
            velocity_km_s,
            velocity_sigma_km_s,
            mc_samples,
            seed,
        )

    return GalacticChronometricRecord(
        name=name,
        epoch=epoch,
        frame_name=frame_name,
        quality_class=quality_class,
        closure_name=closure_name,
        x_kpc=position_kpc.x,
        y_kpc=position_kpc.y,
        z_kpc=position_kpc.z,
        R_kpc=R_kpc,
        phi_deg=phi_deg,
        vx_km_s=velocity_km_s.x,
        vy_km_s=velocity_km_s.y,
        vz_km_s=velocity_km_s.z,
        vR_km_s=v_r,
        vphi_km_s=v_phi,
        speed_km_s=speed_km_s,
        beta=state.beta,
        gamma=state.gamma,
        alpha=state.alpha,
        label_K=state.label_K,
        earth_ratio=state.earth_ratio,
        earth_translation_s=state.earth_translation_s,
        earth_translation_ms=state.earth_translation_s * 1.0e3,
        label_K_sigma=k_sigma,
        earth_translation_ms_sigma=theta_sigma,
        notes=notes,
    )


def parse_optional_float(row: dict[str, str], key: str) -> float | None:
    value = (row.get(key) or "").strip()
    if value == "":
        return None
    return float(value)


def records_from_cartesian_csv(
    input_path: Path,
    *,
    closure: str,
    flat_speed_km_s: float,
    solid_omega_km_s_kpc: float,
    frame_name: str,
    default_epoch: str,
    mc_samples: int,
) -> list[GalacticChronometricRecord]:
    records: list[GalacticChronometricRecord] = []
    with input_path.open("r", encoding="utf-8-sig", newline="") as handle:
        reader = csv.DictReader(handle)
        required = {"name", "x_kpc", "y_kpc", "z_kpc"}
        missing = required.difference(reader.fieldnames or [])
        if missing:
            raise ValueError(f"missing CSV columns: {sorted(missing)}")

        for index, row in enumerate(reader, start=1):
            position = Vector3(
                float(row["x_kpc"]),
                float(row["y_kpc"]),
```

```
                float(row["z_kpc"]),
            )
            vx = parse_optional_float(row, "vx_km_s")
            vy = parse_optional_float(row, "vy_km_s")
            vz = parse_optional_float(row, "vz_km_s")
            velocity_values = (vx, vy, vz)

            if all(value is not None for value in velocity_values):
                velocity = Vector3(float(vx), float(vy), float(vz))
                quality = "M6_MEASURED"
                closure_name = "none"
            elif all(value is None for value in velocity_values):
                velocity, quality, closure_name = apply_velocity_closure(
                    position,
                    closure,
                    flat_speed_km_s,
                    solid_omega_km_s_kpc,
                )
            else:
                velocity = None
                quality = "P5_PARTIAL_VELOCITY"
                closure_name = "none"

            svx = parse_optional_float(row, "vx_error_km_s") or 0.0
            svy = parse_optional_float(row, "vy_error_km_s") or 0.0
            svz = parse_optional_float(row, "vz_error_km_s") or 0.0
            sigma = Vector3(svx, svy, svz) if velocity is not None else None

            records.append(
                build_record(
                    name=(row.get("name") or f"row-{index}").strip(),
                    epoch=(row.get("epoch") or default_epoch).strip(),
                    frame_name=(row.get("frame_name") or frame_name).strip(),
                    position_kpc=position,
                    velocity_km_s=velocity,
                    quality_class=quality,
                    closure_name=closure_name,
                    velocity_sigma_km_s=sigma,
                    mc_samples=mc_samples,
                    seed=20260722 + index,
                    notes=(row.get("notes") or "").strip(),
                )
            )
    return records


def icrs_to_galactocentric(
    *,
    ra_deg: float,
    dec_deg: float,
    distance_pc: float,
    pmra_mas_yr: float,
    pmdec_mas_yr: float,
    radial_velocity_km_s: float,
    galcen_distance_kpc: float = DEFAULT_GALCEN_DISTANCE_KPC,
    z_sun_pc: float = DEFAULT_Z_SUN_PC,
    v_sun_km_s: tuple[float, float, float] = DEFAULT_V_SUN_KM_S,
) -> tuple[Vector3, Vector3]:
    """Transform one full ICRS phase-space state using explicit Astropy settings."""
    try:
        import astropy.coordinates as coord
        import astropy.units as u
    except ImportError as exc:
        raise RuntimeError(
            "Astropy is required for ICRS transformation. Install with: "
            "pip install astropy"
        ) from exc

    sky = coord.SkyCoord(
```

```
        ra=ra_deg * u.deg,
        dec=dec_deg * u.deg,
        distance=distance_pc * u.pc,
        pm_ra_cosdec=pmra_mas_yr * u.mas / u.yr,
        pm_dec=pmdec_mas_yr * u.mas / u.yr,
        radial_velocity=radial_velocity_km_s * u.km / u.s,
        frame="icrs",
    )
    velocity_components = tuple(
        component * u.km / u.s for component in v_sun_km_s
    )
    # Astropy versions before 8 use CartesianDifferential here, whereas
    # Astropy 8+ accepts CartesianRepresentation.  Keep both paths explicit.
    try:
        solar_velocity = coord.CartesianRepresentation(*velocity_components)
        frame = coord.Galactocentric(
            galcen_distance=galcen_distance_kpc * u.kpc,
            z_sun=z_sun_pc * u.pc,
            galcen_v_sun=solar_velocity,
        )
    except (TypeError, ValueError):
        solar_velocity = coord.CartesianDifferential(*velocity_components)
        frame = coord.Galactocentric(
            galcen_distance=galcen_distance_kpc * u.kpc,
            z_sun=z_sun_pc * u.pc,
            galcen_v_sun=solar_velocity,
        )
    gc = sky.transform_to(frame)
    position = Vector3(
        gc.x.to_value(u.kpc),
        gc.y.to_value(u.kpc),
        gc.z.to_value(u.kpc),
    )
    velocity = Vector3(
        gc.v_x.to_value(u.km / u.s),
        gc.v_y.to_value(u.km / u.s),
        gc.v_z.to_value(u.km / u.s),
    )
    return position, velocity


def records_from_icrs_csv(
    input_path: Path,
    *,
    frame_name: str,
    default_epoch: str,
) -> list[GalacticChronometricRecord]:
    records: list[GalacticChronometricRecord] = []
    with input_path.open("r", encoding="utf-8-sig", newline="") as handle:
        reader = csv.DictReader(handle)
        required = {
            "name",
            "ra_deg",
            "dec_deg",
            "distance_pc",
            "pmra_mas_yr",
            "pmdec_mas_yr",
            "radial_velocity_km_s",
        }
        missing = required.difference(reader.fieldnames or [])
        if missing:
            raise ValueError(f"missing ICRS CSV columns: {sorted(missing)}")
        for row in reader:
            position, velocity = icrs_to_galactocentric(
                ra_deg=float(row["ra_deg"]),
                dec_deg=float(row["dec_deg"]),
                distance_pc=float(row["distance_pc"]),
                pmra_mas_yr=float(row["pmra_mas_yr"]),
                pmdec_mas_yr=float(row["pmdec_mas_yr"]),
```

```
                radial_velocity_km_s=float(row["radial_velocity_km_s"]),
            )
            records.append(
                build_record(
                    name=row["name"].strip(),
                    epoch=(row.get("epoch") or default_epoch).strip(),
                    frame_name=frame_name,
                    position_kpc=position,
                    velocity_km_s=velocity,
                    quality_class="T6_TRANSFORMED",
                    closure_name="none",
                    notes=(row.get("notes") or "").strip(),
                )
            )
    return records


def write_records_csv(records: Iterable[GalacticChronometricRecord], path: Path) -> None:
    rows = [asdict(record) for record in records]
    if not rows:
        raise ValueError("no records to write")
    with path.open("w", encoding="utf-8", newline="") as handle:
        writer = csv.DictWriter(handle, fieldnames=list(rows[0].keys()))
        writer.writeheader()
        writer.writerows(rows)


def write_records_json(records: Iterable[GalacticChronometricRecord], path: Path) -> None:
    with path.open("w", encoding="utf-8") as handle:
        json.dump([asdict(record) for record in records], handle, indent=2)


def print_summary(records: list[GalacticChronometricRecord]) -> None:
    print("Earth canonical calibration")
    print(f"  K_Earth    = {EARTH_LABEL.label_K:.12e}")
    print(f"  Theta_Earth= {EARTH_LABEL.earth_translation_ns:.12f} ns_E")
    print()
    header = (
        f"{'name':24s} {'quality':25s} {'speed km/s':>12s} "
        f"{'K':>13s} {'K/K_E':>12s} {'Theta_E ms':>12s}"
    )
    print(header)
    print("-" * len(header))
    for record in records:
        if record.speed_km_s is None:
            speed = k_value = ratio = theta = "--"
        else:
            speed = f"{record.speed_km_s:.6g}"
            k_value = f"{record.label_K:.6e}"
            ratio = f"{record.earth_ratio:.6e}"
            theta = f"{record.earth_translation_ms:.6g}"
        print(
            f"{record.name[:24]:24s} {record.quality_class[:25]:25s} "
            f"{speed:>12s} {k_value:>13s} {ratio:>12s} {theta:>12s}"
        )


def build_parser() -> argparse.ArgumentParser:
    parser = argparse.ArgumentParser(
        description="Build a gravity-independent Galactic chronometric atlas."
    )
    subparsers = parser.add_subparsers(dest="command", required=True)

    cart = subparsers.add_parser("cartesian-csv", help="Process Galactocentric Cartesian CSV")
    cart.add_argument("input_csv", type=Path)
    cart.add_argument("--output-csv", type=Path, default=Path("atlas_output.csv"))
    cart.add_argument("--output-json", type=Path)
    cart.add_argument("--frame-name", default="Declared Galactocentric frame")
    cart.add_argument("--epoch", default="J2016.0")
```

```
    cart.add_argument("--closure", choices=("none", "flat", "solid"), default="none")
    cart.add_argument("--flat-speed-km-s", type=float, default=220.0)
    cart.add_argument("--solid-omega-km-s-kpc", type=float, default=27.0)
    cart.add_argument("--mc-samples", type=int, default=0)

    icrs = subparsers.add_parser("icrs-csv", help="Transform full ICRS CSV with Astropy")
    icrs.add_argument("input_csv", type=Path)
    icrs.add_argument("--output-csv", type=Path, default=Path("atlas_output.csv"))
    icrs.add_argument("--output-json", type=Path)
    icrs.add_argument("--frame-name", default="Astropy explicit Galactocentric frame")
    icrs.add_argument("--epoch", default="J2016.0")

    single = subparsers.add_parser("single", help="Calculate one Cartesian phase-space state")
    single.add_argument("--name", default="Custom object")
    single.add_argument("--epoch", default="J2016.0")
    single.add_argument("--frame-name", default="Declared Galactocentric frame")
    single.add_argument("--x-kpc", type=float, required=True)
    single.add_argument("--y-kpc", type=float, required=True)
    single.add_argument("--z-kpc", type=float, required=True)
    single.add_argument("--vx-km-s", type=float, required=True)
    single.add_argument("--vy-km-s", type=float, required=True)
    single.add_argument("--vz-km-s", type=float, required=True)
    return parser


def main() -> None:
    args = build_parser().parse_args()
    if args.command == "cartesian-csv":
        records = records_from_cartesian_csv(
            args.input_csv,
            closure=args.closure,
            flat_speed_km_s=args.flat_speed_km_s,
            solid_omega_km_s_kpc=args.solid_omega_km_s_kpc,
            frame_name=args.frame_name,
            default_epoch=args.epoch,
            mc_samples=args.mc_samples,
        )
        write_records_csv(records, args.output_csv)
        if args.output_json:
            write_records_json(records, args.output_json)
        print_summary(records)
        return

    if args.command == "icrs-csv":
        records = records_from_icrs_csv(
            args.input_csv,
            frame_name=args.frame_name,
            default_epoch=args.epoch,
        )
        write_records_csv(records, args.output_csv)
        if args.output_json:
            write_records_json(records, args.output_json)
        print_summary(records)
        return

    if args.command == "single":
        record = build_record(
            name=args.name,
            epoch=args.epoch,
            frame_name=args.frame_name,
            position_kpc=Vector3(args.x_kpc, args.y_kpc, args.z_kpc),
            velocity_km_s=Vector3(args.vx_km_s, args.vy_km_s, args.vz_km_s),
            quality_class="M6_MEASURED",
            closure_name="none",
        )
        print_summary([record])
        print(json.dumps(asdict(record), indent=2))
        return

```

```
    raise RuntimeError("unreachable command")


if __name__ == "__main__":
    main()
```

# F On the Novelty, Architecture, and Applications of the Framework

This appendix clarifies the nature of the contribution and the intended scope of the framework. It does not modify any mathematical result or physical claim derived in the main text. It provides a transparent description of the architectural organisation of standard kinematics, phase bookkeeping, and chronometric labelling used in this work.

## F.1 Standard building blocks, novel architecture

The individual physical ingredients employed here are established: special-relativistic proper-time relations, vector composition of velocities generated by nested rotations and translations, and standard Galactic coordinate descriptions.

The contribution of this work is not the introduction of a new gravitational law or a replacement for General Relativity. Rather, it is the systematic organisation of these ingredients into a single operational framework consisting of:

1. a signed, unwrapped nested phase ledger separating spin, orbital, parent-system, and Galactic phases;

2. a point-dependent phase metric encoding the embedding geometry and the resulting velocity norm within the stated construction;

3. a dimensionless chronometric label

$$\mathscr{K}_X = \gamma_X \frac{v_X^2}{c^2},$$

   together with its inverse mapping to the local kinematic clock factor;

4. a Local Observer Translation Map convention

$$\Theta_X^{(\oplus)} = \mathscr{K}_X T_{\mathrm{sid},\oplus},$$

   which is a comparison coordinate and not a universal unit of time;

5. a quality-controlled Galactic phase-space atlas that distinguishes measured states, incomplete states, and model-completed states.

The framework therefore represents a new organisation of known kinematical territory rather than a claim of new fundamental dynamics.

## F.2 What the framework is not

The following limitations are explicit:

- The framework is not a replacement for General Relativity or any metric theory of gravity. Gravitational redshift and strong-field corrections must be added separately when total observed clock rates are required.
- It does not determine orbital or spin rates from position and size alone. Angular rates entering the chronometric equations must be supplied by observation or by an explicitly declared dynamical closure.
- The quantity $\mathscr{K}_X$ is not an absolute scalar independent of convention. Reference frame, epoch, and point definition must always be specified.
- The local-observer quantity $\Theta^{(O)}$ is a calibration for comparison with terrestrial units, not a universal temporal quantum.

## F.3 Intended applications

The architecture is intended for:

1. comparison of kinematic chronometric states of planets, stars, and catalogued Galactic sources;
2. separation of kinematic and gravitational contributions in precision timing problems;
3. pedagogical visualisation of the chain

$$\text{phase ledger} \to \text{phase geometry} \to \text{velocity} \to \mathscr{K} \to \text{local translation};$$

4. future construction of empirical chronometric atlases when reliable relations between measured positions and velocity fields are available;
5. enrichment of astronomical catalogues with dimensionless kinematic labels and terrestrial comparison coordinates.

## F.4 Epistemic hierarchy of quantities

The framework separates quantities according to their origin:

| Level | Quantity | Status |
|---|---|---|
| Observation | position, velocity, period | measured input |
| Derivation | $\mathscr{K}_X$, $\alpha_X$ | mathematical output |
| Translation | $\Theta_X^{(\oplus)}$ | comparison convention |
| Closure model | position–velocity relation | explicit hypothesis |

This hierarchy prevents confusion between observed data, mathematical consequences, and future modelling assumptions.

# G Galactic Reference Chronometry and Local Observer Translation

The chronometric framework does not assign a privileged status to the Earth. The Earth is one possible local observer among many. The primary comparison structure is an operational Galactic reference frame $\mathcal{F}_{\mathrm{MW}}$, defined by a declared origin, orientation, and reference epoch.

This choice does not introduce an absolute cosmic time. It provides a common kinematic language for comparing celestial systems.

## G.1 Galactic reference state

For an object $X$, define the complete state

$$\boxed{\mathcal{A}_X(t_0) = (\mathbf{x}_X, \mathbf{v}_X, \boldsymbol{N}_X, \mathcal{F}_{\mathrm{MW}}, t_0)} \tag{154}$$

where position, velocity, nested phase ledger, reference frame, and epoch are recorded together.

Position alone does not determine a chronometric label:

$$\mathbf{x}_X \not\Rightarrow \mathscr{K}_X. \tag{155}$$

A complete phase-space state is required:

$$(\mathbf{x}_X, \mathbf{v}_X) \Rightarrow \mathscr{K}_X. \tag{156}$$

## G.2 Frame-declared dimensionless kinematic label

The dimensionless kinematic label is defined by

$$\boxed{\mathscr{K}_X = \gamma_X \beta_X^2} \tag{157}$$

with

$$\beta_X = \frac{|\mathbf{v}_X|}{c}, \qquad \gamma_X = (1 - \beta_X^2)^{-1/2}. \tag{158}$$

The associated kinematic clock factor is

$$\boxed{\alpha_X = \frac{\sqrt{\mathscr{K}_X^2 + 4} - \mathscr{K}_X}{2}} \tag{159}$$

## G.3 Local observer translation

The dimensionless label is primary. A local observer $O$ may translate it into the observer's own units:

$$\boxed{\Theta_X^{(O)} = \mathscr{K}_X T_O} \tag{160}$$

The Earth expression is only the special case $O = \oplus$:

$$\Theta_X^{(\oplus)} = \mathscr{K}_X T_\oplus. \tag{161}$$

Therefore terrestrial units are a calibration language, not the fundamental definition of the framework.

# Author Declaration

This manuscript unifies a mechanical thought experiment with a phase-geometric chronometric and Galactic catalogue architecture. The rod and spinning-top model is presented as a controlled conceptual and conditional mechanical closure, not as a literal description of the Solar System or the Milky Way. The kinematic proper-time, velocity-composition, and rotating-ring Sagnac relations are used within their declared reference-frame assumptions. The values $\mathscr{K}$, $\mathscr{S}$, and local-observer translations are not absolute names detached from frame, epoch, path, or point convention. The value 207.386 ns denotes the canonical terrestrial one-winding translation; it is not a universal time quantum. Any Galactic position-to-velocity law remains an explicitly declared closure and must not be confused with measured six-dimensional phase-space data.

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