

Mathematical Audit and Operator Interfaces of Lo Shu Structures

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Abstract

This paper organizes research on Lo Shu as a reproducible mathematical audit and a collection of operator interfaces. Its positive thesis is that the classification of numerical labels and the action of geometric operators must be defined separately and connected through explicit readouts, compressions, and error operators. The classification of order-three magic squares is established mathematically; we give self-contained proofs. Under specified integer, zero-sum, distinctness, and minimum-amplitude conditions, normal Lo Shu squares form one D_4 orbit. Within the two-ternary-component integer-readout model, the smallest distinct range requires coefficient magnitudes 3 and 1.

On fixed real grids with eight lines in two dimensions and 49 in three, we compare the kernels of line sums, midpoint differences, scalar-potential gradients, and edge-flow divergence. The real three-dimensional all-line zero-sum kernel is trivial, whereas the midpoint kernel consists of four-dimensional affine fields; modulo three, the two equations coincide and both kernels are four-dimensional. Given Lo Shu and a matching integer readout, a finite-field quotient reproduces its nine cells by a three-to-one compression of 27 states, but does not select Lo Shu. Coordinate replication satisfies $T^T L_3 T = 5L_2$. The quotient closure defect has rank seven and Euclidean operator norm $\sqrt{(175 + \sqrt{29977})/27} \approx 3.590825$; we also give a conditional hidden-channel error bound. All 432 affine bijections and 512 stacks of normal layers are enumerated.

Trigram, Five-Phase, and stem-branch configurations are traditional documentary inputs (H), rather than dictionaries derived from magic-square classification. Fourier, vibration, and appendix tight-binding results depend on explicit model choices (M); matched non-Lo-Shu controls distinguish specificity. No observational data are fitted, but model selection and target-based construction remain present. Independent selection of Lo Shu, controlled elimination of hidden channels, and physical actions compatible with traditional topology remain open (O). Mathematical consistency of finite operators is not identified with evidence for a natural or elementary-particle correspondence.

Keywords: Lo Shu; order-three magic squares; trigrams; stems and branches; discrete projection; constraint kernels; closure error; Fourier transform; dynamics.

1 Problem, scope, and research discipline

Lo Shu has a completely solvable finite combinatorial core and also appears in traditional symbolic configurations and modern models. Connecting these settings requires distinguishing five objects: grid positions, numerical labels, trigram or stem-branch labels, state variables, and operators acting on states. A bijection does not establish an isomorphism of operations, and static constraints do not specify time evolution.

We use the following evidence levels: T, mathematical results with proofs; E, exhaustive finite enumerations or numerical checks on specified domains; H, traditional configurations supported by sources; M, explicitly selected models; O, unresolved or undetermined questions; and N, candidate claims that fail under specified assumptions. Floating-point checks under E do not replace proofs on infinite domains. H is not physical evidence, and M does not imply uniqueness. In mixed labels, H or M supplies the

premise and T identifies what is proved conditional on it. T neither upgrades premises to natural laws nor guarantees an error-free manuscript. Abstracts, tables, captions, and isolated quotations must retain the assumptions essential to their conclusions.

The scope covers integer and real order-three magic squares, minimum ternary prototypes, binary trigram encodings, counters of lengths ten and twelve, three-level three-dimensional grids, real linear and finite-field quotient projections, and finite operator models. It does not exhaust all divination schools, higher-order magic squares, crystals, or fundamental interactions. Completeness refers to the stated solution spaces, enumeration domains, and projection information, not all conceivable connections.

Order-three construction and enumeration already have a literature [1, 2, 3, 4]. The weak-magic convention in [4] differs from our requirement of nine distinct entries. Our self-contained rederivations do not constitute priority claims. We revise the earlier argument [12] and compare interfaces with the public unit-cell-framework record [13] and the author's same-title working manuscript; recalculation within an author's own program is not independent third-party validation. Traditional sources include the *Shuogua*, *Xici*, and later symbolic literature [5, 6, 7, 8]. Similar diagram names do not establish historical causation.

The main deliverables are the constraint-space table in Section 6.2a, exact compression and defect spectra with conditional error control in Sections 7.1b and 7.3, and the finite enumerations in Propositions 2 and 8. Whether a structure can constrain labels is examined separately from whether it can serve as a dynamical operator. An obstruction to static equal sums does not prevent a model from using the collinearity-graph Laplacian. No unverified claim of novelty is made for this organization.

1.1 Types of ternary arithmetic and cross-domain readout

The real-coordinate ternary set $E = \{-1, 0, 1\}$ lies in real space and can be translated to $\{0, 1, 2\}$; it uses ordinary addition, real lines, and Euclidean distance. The field $\mathbb{F}_3 = \mathbb{Z}/3\mathbb{Z}$ uses arithmetic modulo three, with either set as representatives. Similar symbols do not identify algebraic rules: $1 + 2 = 3$ over the reals but $1 + 2 = 0$ in $\mathbb{F}_3$. Translating coordinates does not turn real geometry into finite-field geometry.

Section 7.1a uses three operations in sequence: the quotient map and digit components are defined over $\mathbb{F}_3$; after choosing representatives 0, 1, 2, the readout $N = 1 + 3u + v$ uses ordinary integers; and $P_\pi = C/3$ averages real fields from $\mathbb{R}^{27}$ to $\mathbb{R}^9$. The last operation cannot be performed in $\mathbb{F}_3$, where three has no multiplicative inverse. Ranks, integer readouts, and geometric lines are interpreted in their stated domains throughout.

2 The complete linear structure of Lo Shu

2.1 Definition and the center theorem

Definition 1. A real magic square of order three is a 3×3 matrix whose three row sums, three column sums, and two principal diagonal sums all equal S . Integer squares have integer entries; normal squares contain each integer from one to nine once. A zero-sum matrix has all eight sums equal to zero.

Theorem 1 (center and complementarity; T). If c is the center of an order-three magic square, then $S = 3c$, and any pair of cells opposite about the center has sum $2c$.

Proof. The four lines through the center cover each other cell once and the center four times. Thus $4S = 3S + 3c$, giving $S = 3c$. The endpoints of each such line sum to $S - c = 2c$. $\square$

An integer magic sum is therefore divisible by three. Subtracting the center gives a zero-sum matrix. This additive complementarity is not a priori trigram complementation, matter–antimatter pairing, or charge conjugation.

2.2 General solution, lattice, and rank

Theorem 2 (complete parameterization; T). Every real order-three magic square is $X = cJ + M(a, b)$, where J is the all-ones matrix and

$$M(a, b) = \begin{pmatrix} a & b & -a-b \\ -2a-b & 0 & 2a+b \\ a+b & -b & -a \end{pmatrix}. \quad (1)$$

Integer squares correspond exactly to integer a, b, c . The 8×9 zero-sum constraint matrix A_2 has rank seven over $\mathbb{Q}$ or $\mathbb{R}$.

Proof. Centering sets the center to zero. The first two entries a, b determine the third as $-a - b$. Opposite-cell relations give the last row $a + b, -b, -a$. The first-column sum fixes the middle-left entry at $-2a - b$, and the middle-right entry is $2a + b$. Substitution verifies all eight constraints. The parameters can be read from the matrix, so the representation is unique. Integrality is equivalent in both directions; the integer kernel is a free module on two integer matrices. Its dimension is two, hence the rank is $9 - 2 = 7$. $\square$

The full integer magic-square lattice has basis $J, M(1, 0), M(0, 1)$. Fixing the magic sum leaves a two-dimensional affine lattice. This differs from treating four signed symbols as four independent free generators.

2.3 Distinctness and the exact feasible region

Proposition 1 (distinctness criterion; T). The entries of M are zero and four opposite pairs with signed amplitudes $a, b, a + b, 2a + b$. All nine are distinct precisely when those amplitudes are nonzero and their absolute values are pairwise distinct, equivalently when

$$a, b, a + b, 2a + b, a - b, 3a + b, a + 2b, 3a + 2b \quad (2)$$

are all nonzero.

Proof. A zero amplitude repeats the center; equal or opposite amplitudes repeat an entry. Listing pairwise sums and differences and removing proportional equations gives exactly these eight excluded lines. $\square$

Set $R = \max(|a|, |b|, |a + b|, |2a + b|)$. The extreme entries are $c - R$ and $c + R$. Nonnegative integer squares require $c \geq R$, and positive integer squares require $c \geq R + 1$. Combining these inequalities with the eight excluded lines describes lattice points in polyhedral cones with coincidence hyperplanes removed. This is an interface with enumeration literature [1, 2, 3, 4], not a new physical axiom.

2.4 Minimality and normal uniqueness

Theorem 3 (smallest order under the specified zero-sum constraints; T). Require all rows, columns, and both principal diagonals to sum to zero. Over the integers, or any nontrivial abelian group without two-torsion, orders one and two have only the zero solution, whereas order three has a nonzero solution. Absence of two-torsion means $2g = 0$ implies $g = 0$.

Proof. Order one is immediate. Row and column constraints in order two give $\begin{pmatrix} t & -t \\ -t & t \end{pmatrix}$; diagonals require $2t = 0$. For any nonzero group element g , $M(g, -g)$ is a nonzero order-three solution. Take $g = 1$ over the integers. $\square$ The conclusion changes in characteristic two, making the coefficient domain essential.

Theorem 4 (minimum amplitude and one orbit; T/E). A nine-distinct-entry integer zero-sum matrix has $R \geq 4$. Exactly eight solutions attain $R = 4$, forming one D_4 orbit. Nine distinct positive integer entries have maximum at least nine, with equality precisely for the eight normal Lo Shu squares.

Proof. There are only $2R + 1$ integers in $[-R, R]$, so $R \geq 4$. At equality the entries are exactly $-4, \dots, 4$ and $|a|, |b| \leq 4$. Exhaustive filtering of 81 parameter pairs yields

$$(-3, 2), (-3, 4), (-1, -2), (-1, 4), (1, -4), (1, 2), (3, -4), (3, -2). \quad (3)$$

Adding five gives normal squares. For an enumeration-free classification, the center is five. If nine is in a corner, rotate it to the upper left and denote the upper-right entry by u . The general solution gives middle-left entry $u - 4$ and bottom-middle entry $u + 4$. Both must lie between one and nine, forcing $u = 5$, which repeats the center. Thus nine is at an edge midpoint.

Rotate nine to the top-middle position. The general solution becomes

$$\begin{pmatrix} u & 9 & 6 - u \\ 11 - 2u & 5 & 2u - 1 \\ 4 + u & 1 & 10 - u \end{pmatrix}.$$

The range gives $1 \leq u \leq 5$. Values one, three, and five cause repetitions; only two and four work, and they are reflections. Four possible positions for nine give eight squares in one orbit. Independent enumeration of all $9! = 362880$ permutations agrees. Nine distinct positive integers have maximum at least nine, and equality forces the set $1, \dots, 9$. $\square$

The sum 15 follows from total 45 divided among three rows, and the center five from Theorem 1. The relevant optimization variable is the centered amplitude R . The earlier alternative objective is withdrawn; see Section 11.

2.5 Minimum ternary prototypes and integer readout under specified assumptions

“Discrete” here means integer states; “conservation” means the stated eight zero sums; and “unique” requires a comparison criterion and symmetry equivalence. Theorems 3 and 4 establish minimum nonzero order three and one D_4 orbit of distinct minimum-amplitude integer solutions. They retain the valid conditional core of the earlier argument, without proving that arbitrary local flow conservation produces Lo Shu. Fixing order three and zero sums alone leaves infinitely many integer solutions; minimum order, minimum amplitude, and distinctness are separate requirements.

Proposition 1a (minimum ternary prototypes and two-component readout; T/E). The smallest amplitude of a nonzero integer zero-sum matrix is one. Exactly four matrices attain it, in one D_4 orbit. For any two linearly independent such matrices A, B , the pairs (A_{ij}, B_{ij}) run through $\{-1, 0, 1\}^2$ once, and $5J + 3A + B$ is a normal Lo Shu square. Exactly eight ordered independent pairs produce all eight normal squares.

Proof. Nonzero integer amplitude is at least one. The simultaneous bounds on $a, b, a + b, 2a + b$ leave precisely $(0, 1), (0, -1), (1, -1), (-1, 1)$. Rotations connect them. Set $q = M(0, 1)$ and $r = M(-1, 1)$; the four matrices are $\pm q, \pm r$. Direct inspection shows that (q, r) takes all nine ordered ternary values. Independent pairs are sign changes and exchanges, giving $4 \times 2 = 8$. The readout preserves zero sums and takes values one to nine after adding five. Theorem 4 gives completeness. $\square$

In this two-ternary-component model, requiring integer coefficients α, β , nine distinct readouts, and minimum amplitude four gives $R = |\alpha| + |\beta| = 4$. Both coefficients must be nonzero, and magnitudes $(2, 2)$ cause repetition, leaving $(3, 1)$ or $(1, 3)$. The model layer chooses two-component linear readout, integer coefficients, distinctness, and minimum range; the theorem layer derives coefficient magnitudes three and one, up to signs and exchange. This is neither an arbitrary fitted ratio nor a universal constant outside those assumptions. No observational fitting is used. Each cell now carries two ternary components; this does not restore the earlier single-state readout assumption.

The precise conditional statement is: the smallest nonzero integer square under the specified line constraints has order three; minimum-amplitude ternary prototypes form one orbit; and the minimum distinct integer readout of two independent minimum prototypes gives the unique normal orbit. Square

geometry, two diagonals, integer units, distinctness, and the maximum-absolute-value criterion are inputs not yet derived from a general continuity equation or action. Without amplitude minimization, $M(1, 3)$ is another distinct zero-sum matrix of radius five. Over real states nonzero solutions can be scaled arbitrarily small, with no positive minimum amplitude.

3 Symmetry, combinations, encoding, and matrix spectra

3.1 The D_4 orbit versus spatial symmetry

Clockwise rotation acts by $r(a, b) = (a + b, -2a - b)$, and left-right reflection by $s(a, b) = (-a - b, b)$. They satisfy $r^4 = s^2 = 1$ and $srs = r^{-1}$. Distinct labels prevent any nonidentity grid symmetry from fixing a square, so each normal square has trivial stabilizer and orbit size eight. The eight solutions form one orbit.

An individual numerically labeled Lo Shu has no nontrivial spatial symmetry in the square's D_4 . Eight symmetry-related arrays do not give a single weighted cell a D_4 point group. A half-turn changes M to $-M$, so $X \mapsto 2cJ - X$ already belongs to the orbit action.

3.2 Sum-15 triples and the ternary decomposition of a given Lo Shu

The distinct unordered triples from one to nine summing to 15 are exactly 159, 168, 249, 258, 267, 348, 357, 456. They correspond to the eight Lo Shu lines and to the winning-line structure of tic-tac-toe. The enumeration domain consists of $\binom{9}{3} = 84$ triples; no physical quantity is involved.

Choose the standard orientation $L = \begin{pmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{pmatrix}$. Writing $L - J = 3Q + R$ entrywise gives

$$Q = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}. \quad (4)$$

Every row and column of each contains zero, one, and two, and all nine ordered pairs differ: these are orthogonal Latin squares. Both diagonal sums are also three in this construction, so each is a ternary magic square with repetitions allowed. Set $q = Q - J$, $r = R - J$; both have eight zero sums and $L = 5J + 3q + r$. Nine different labels come from joint readout, not one ternary variable. For $i, j \in \mathbb{F}_3$, $Q = j - i + 1$ and $R = 2(i + j)$, taking representatives zero to two. The coefficient determinant is nonzero modulo three, ensuring unique digit pairs. Diagonal sums still require checking; arbitrary superposition of orthogonal Latin squares is not automatically magic.

Proposition 2 (complete affine-readout census; E). Fix the $\mathbb{F}_3^2$ position grid and integer readout $N = 1 + 3u + v$, with digit representatives zero, one, two. Among all 432 invertible affine maps $(u, v) = A(i, j) + b$, exactly eight give normal Lo Shu squares; the other 424 fail the eight integer line-sum conditions.

Enumeration domain and verification. The first column of a matrix in $\text{GL}(2, 3)$ has eight nonzero choices and the second has six choices outside its span, giving 48 matrices. Nine translations give 432 maps. Check determinant, digit representatives, distinctness, and eight sums of 15 for every map. The eight accepted arrays coincide individually with all normal arrays of Theorem 4. Code and acceptance records accompany the paper. This finite census assigns neither physical probabilities nor a selection mechanism.

3.3 Matrix spectra and quadratic invariants

Let $u = (1, 0, -1)/\sqrt{2}$ and $v = (1, -2, 1)/\sqrt{6}$. Then $Mu = \sqrt{3}(2a + b)v$ and $Mv = -\sqrt{3}bu$. The constant vector is a zero eigenvector of M ; J acts as three on it and zero on its orthogonal complement.

Hence

$$\begin{aligned}\det(\lambda I - X) &= (\lambda - 3c)(\lambda^2 + 3b(2a + b)), \\ \det X &= 9cb(2a + b).\end{aligned}\tag{5}$$

The singular-value multiset is $3|c|, \sqrt{3}|b|, \sqrt{3}|2a + b|$, and

$$\|X\|_F^2 = 9c^2 + 12a^2 + 12ab + 6b^2.\tag{6}$$

For standard L , eigenvalues are $15, \pm 2i\sqrt{6}$, determinant 360, and squared Frobenius norm 285. Grid rotations and reflections are generally not matrix similarities, so eigenvalues need not be constant along the D_4 orbit. These are not energy spectra of an unspecified physical Hamiltonian.

If both a real order-three matrix and its entrywise square are magic, the latter has center c^2 and sum $3c^2$. A line $c - d, c, c + d$ instead has squared sum $3c^2 + 2d^2$, forcing $d = 0$. Thus no nonconstant real bimagic square of order three exists. This is a different question from whether nine distinct perfect squares can form an ordinary order-three magic square.

4 Trigrams: separate encoding, orientation, and complementation

4.1 Traditional trigrams (H) and the chosen three-bit code

The *Xici* describes the hierarchy from two modes to four images and eight trigrams; the *Shuogua* discusses pairings and directions; later symbolic works connect nine cells and trigram numbers [5, 6]. These establish traditional correspondences, not modern dynamical laws of nature.

Encode from bottom line to top, yang as one and yin as zero: Qian 111, Kun 000, Zhen 100, Xun 011, Kan 010, Li 101, Gen 001, Dui 110. The code set is $\mathbb{F}_2^3$. Bitwise complement is $b \mapsto b + (1, 1, 1)$, pairing Qian–Kun, Zhen–Xun, Kan–Li, and Gen–Dui.

4.2 Orientations and pairings in the adopted Later-Heaven configuration (H)

The diagram convention puts south above, north below, east left, and west right. Trigram positions are traditional input H; bottom-up binary coding is a separate explicit convention. The correspondences are not a historical origin derived from magic-square equations. Only after fixing these inputs do we test complementation against opposite positions. Zhen, Xun, Li, Qian, Kan, and Gen have explicit directions

Direction	Lo Shu number	Trigram (H)	Bottom-up code
North	1	Kan (坎)	010
Northeast	8	Gen (艮)	001
East	3	Zhen (震)	100
Southeast	4	Xun (巽)	011
South	9	Li (离)	101
Southwest	2	Kun (坤)	000
West	7	Dui (兑)	110
Northwest	6	Qian (乾)	111
Center	5	No exclusive trigram	Not applicable

in the cited *Shuogua* passage [5]; Dui in the west appears in its commentary. Kun in the southwest is completed here using the sequence and the convention of occupying eight directions, not a verbatim assertion of that passage. Numbers follow standard Lo Shu and may be compared with the later nine-cell mnemonic [6]. This formalizes the adopted configuration, without deriving trigram names or proving the entire historical lineage.

Proposition 3 (Later-Heaven opposites are not generally bitwise complements; T/H). The opposite number pairs $(1, 9), (8, 2), (3, 7), (4, 6)$ correspond to Kan–Li, Gen–Kun, Zhen–Dui, and Xun–Qian.

Only Kan–Li are bitwise complements. The assertion that opposite sums of ten mean complete yin–yang reversal in Later-Heaven trigrams therefore fails.

The Heaven–Earth, Mountain–Lake, Thunder–Wind, and Water–Fire pairings in *Shuogua* can connect with the common Earlier-Heaven complement convention [5], but cannot silently replace Later-Heaven orientation. Retaining only four opposite pairs, there are $4!2^4 = 384$ bijections onto four complementary trigram pairs. Pairing alone does not determine one number–trigram assignment.

4.3 Algebraic and geometric obstructions

Eight cyclic positions form C_8 , whereas trigrams under exclusive-or form C_2^3 . The former has an element of order eight; every nonzero element in the latter has order two. They are not isomorphic groups. The Hamming cube has degree three and the surrounding cycle degree two, so those graphs are not isomorphic either. Enumeration does not remove these distinctions. Eight directions also do not establish eightfold rotation of a three-dimensional periodic crystal, whose rotation orders satisfy the crystallographic restriction [11]; this statement does not concern quasicrystals or arbitrary finite patterns.

Proposition 4 (no direct linear projection of eight cube vertices onto the eight outer cells; T). No linear map sends $\{\pm 1\}^3$ bijectively onto $\{-1, 0, 1\}^2 \setminus \{(0, 0)\}$.

Proof. Central symmetry forces an affine bijection’s translation to vanish. The first target coordinate has multiplicities $(3, 2, 3)$ for $-1, 0, 1$. If a linear form takes only these values on cube vertices and reaches both extremes, its coefficient magnitudes sum to one. Flipping one nonzero coefficient from the maximizing sign pattern forces each magnitude to be either one or one-half. Thus the coordinate is one signed coordinate, or the half-sum/half-difference of two signed coordinates, with multiplicities $(4, 0, 4)$ or $(2, 4, 2)$. Neither matches the target. $\square$

This does not forbid nonlinear encodings or the full 27-point grid. It excludes the specific direct geometric projection of the eight three-bit cube vertices onto the eight outer Lo Shu cells.

5 Stems, branches, and Five Phases: computable cycles without unique dynamics

5.1 The exact synchronization model of the sexagenary cycle

Stem and branch orders and their 60-pair table can be checked against Hong Kong Observatory [7]. For integer steps t , define

$$g_t = t \pmod{10}, \quad z_t = t \pmod{12}. \quad (7)$$

Theorem 5 (stem–branch period; T/H). The synchronous state visits exactly the 60 same-parity pairs, with least period $\text{lcm}(10, 12) = 60$.

Proof. Repetition requires a step difference divisible by both ten and twelve, with minimum positive value 60. The simultaneous congruences are solvable precisely when $g \equiv z \pmod{2}$, selecting 60 of 120 formal pairs. $\square$

This is kinematics of counters, not a real time unit, force, energy, or precise astronomical parameter relation. A bijection on nine Lo Shu positions cannot produce 60 physical states without extra variables. Adding an eight-position counter advancing one place each step gives joint period 120. Conversely, no homomorphism $C_{60} \rightarrow C_8$ sends a generator to a generator: its image order divides both 60 and eight and is at most four. The 120-period is a consequence of the chosen update, not a new Lo Shu prediction.

5.2 Adopted Five-Phase configurations (H) and different dictionaries

Number Wood, Fire, Earth, Metal, Water as zero to four in the generating order. Generation is $j \mapsto j + 1$ and overcoming is $j \mapsto j + 2$ modulo five; see *Wuxing dayi*, volume two [8]. These define directed cycles, not interaction strengths.

Assigning Jia/Yi to Wood, Bing/Ding to Fire, Wu/Ji to Earth, Geng/Xin to Metal, and Ren/Gui to Water, with stem yin–yang pairing, encodes ten stems as five classes times two classes. This set decomposition does not mean that stem-order updating increments both components at every step. Although $C_5 \times C_2 \cong C_{10}$ abstractly, a chosen isomorphism need not preserve the traditional order and phase dictionary.

The River-Chart dictionary in Hu Fangping [6] assigns 1/6 Water, 2/7 Fire, 3/8 Wood, 4/9 Metal, and 5/10 Earth. The common Later-Heaven nine-cell dictionary assigns 1 Kan/Water, 2 Kun/Earth, 3 Zhen/Wood, 4 Xun/Wood, 5 central Earth, 6 Qian/Metal, 7 Dui/Metal, 8 Gen/Earth, and 9 Li/Fire. The same numbers need not have the same attributes. The latter is an explicitly adopted common input whose every historical attribution has not been traced here; subsequent mathematics is conditional on that dictionary. The two dictionaries are not mixed.

5.3 Why dynamics is not uniquely determined

A generating/overcoming graph admits permutation updates, continuous Markov processes, oscillatory networks, or nonlinear reaction equations. For example, if P is a five-cycle permutation, the selected model

$$\dot{y} = \alpha(P - I)y + \beta(P^2 - I)y, \quad \alpha, \beta \geq 0 \quad (8)$$

preserves the total and nonnegative initial values. Fourier-mode eigenvalues are $\alpha(\zeta^k - 1) + \beta(\zeta^{2k} - 1)$, where ζ is a fifth root of unity. For $\alpha + \beta > 0$, every nonconstant mode has strictly negative real part, since neither root factor is one for $k = 1, \dots, 4$; the state mixes toward uniformity. If both rates vanish, every state stays fixed. Pure permutation updates are periodic without dissipation. Both live on the same symbolic graph yet have different dynamics.

Established interfaces are synchronization periods, trigram encodings and permutations, and phase-relation graphs. Lo Shu has not been shown to uniquely determine their physical dynamics. A physical theory must add state variables, time scales, strengths, driving, and observables.

6 Three-dimensional unit cells: distinguish three line conditions

6.1 Three-point lines and collinearity graphs

Definition 2. Let $V_d = \{-1, 0, 1\}^d$ for $d = 2, 3$. A three-point line is $p - v, p, p + v$, where coordinates of v lie in $\{-1, 0, 1\}$, $v \neq 0$, opposite directions are identified, and $p_j = 0$ whenever $v_j \neq 0$. Connect every pair on a line to obtain the simple collinearity graph G_d .

Theorem 6 (line and edge counts; T). There are $(5^d - 3^d)/2$ three-point lines: eight lines and 24 edges in two dimensions, and 49 lines and 147 edges in three.

Proof. For a direction with k nonzero coordinates, choose the coordinates in $\binom{d}{k}$ ways and signs in 2^{k-1} ways after identifying reversal. Each remaining fixed coordinate has three choices, giving $\binom{d}{k} 2^{k-1} 3^{d-k}$ lines. Summing gives the formula. Two points determine a line, so no unordered pair occurs on two different lines. Each contributes three edges. $\square$

The 49 lines comprise 27 axis-parallel, 18 face-diagonal-direction, and four body-diagonal lines. The identity $a + c = 2b$ first concerns position vectors, with endpoints a, c and midpoint b . It is not by itself conservation of energy or equal sums of numerical labels.

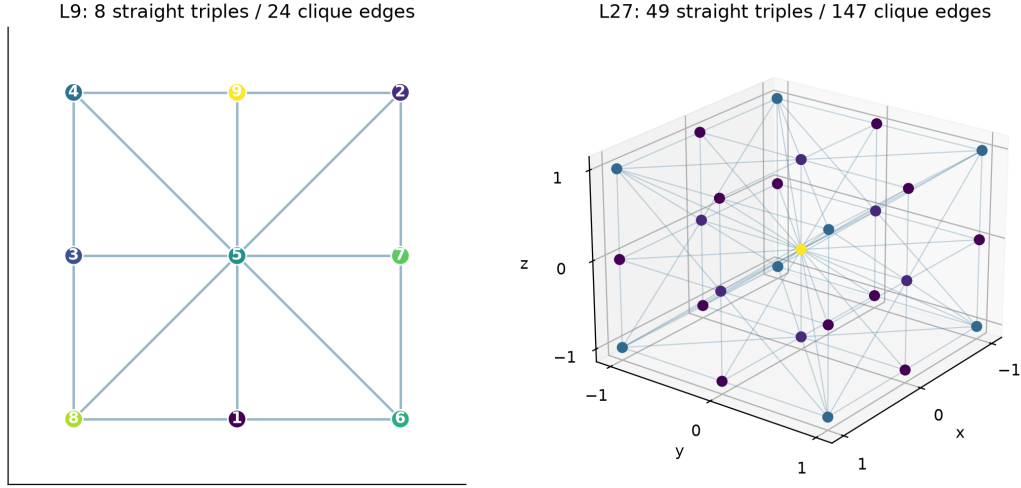


Figure 1: Collinearity graphs of the specified real-coordinate grids (T): eight lines in two dimensions and 49 in three; colors indicate vertex degrees. This is a Euclidean enumeration, not the set of all affine lines over $\mathbb{F}_3$, and specifies neither energy nor time evolution.

6.2 Different solution spaces for equal sums and midpoint constraints

Let S_3 be the 49×27 line-sum matrix, with three coefficients one in each row. Let D_3 be the midpoint-difference matrix, with coefficients $(1, -2, 1)$ in endpoint-midpoint-endpoint order. They cannot be interchanged over the reals.

Theorem 7 (all-line equal sums force constant labels; T/E). If all 49 lines have common sum S , every vertex label is $S/3$. Equivalently, $\ker S_3 = \{0\}$ and $\text{rank } S_3 = 27$ over the reals.

Proof. Subtract $S/3$ to make all sums zero. Each axis-parallel 3×3 section satisfies row, column, and both diagonal constraints and is therefore a zero-sum matrix of Theorem 2, with center zero. Edge midpoints in the three central coordinate planes are centers of other sections and thus vanish. If the middle row and column of a zero-sum 3×3 matrix vanish, its parameterization forces $a = b = 0$; hence the three central planes vanish. The middle rows and columns of the outer planes $z = -1, 1$ lie in those central planes, so the outer planes also vanish. $\square$

Only the 27 axis and 18 face-diagonal lines are needed; body-diagonal conditions are redundant. The result is stronger than nonexistence of a distinct $1, \dots, 27$ cube under this definition, but applies specifically to the stated all-section double-diagonal conditions, not every use of “perfect” in the literature.

Theorem 8 (midpoint constraints give affine fields; T/E). If every three-point line in V_d satisfies $f(p - v) - 2f(p) + f(p + v) = 0$, then f is affine. Solution dimensions are three and four for $d = 2, 3$, with ranks $\text{rank } D_2 = 6$ and $\text{rank } D_3 = 23$.

Proof. Axis second differences make f separately linear in each coordinate. In three dimensions it is a combination of $1, x, y, z, xy, xz, yz, xyz$. The two face-diagonal directions, with the third coordinate fixed, eliminate xy and its third-coordinate-dependent xyz term; the other faces eliminate xz, yz . Only $1, x, y, z$ remain, and affine fields satisfy every midpoint equation. The two-dimensional proof is analogous. $\square$

A centered two-dimensional Lo Shu field generally does not lie in this affine kernel. Its zero-sum kernel has dimension two, whereas the midpoint kernel has dimension three. Real midpoint and line-sum constraints are therefore different; flow conservation additionally requires edge variables. We register the operator domains below.

6.2a Unified classification of constraint spaces

Theorem 9 (constraint spaces on the specified grids; T/E). Fix Definition 2 and its eight or 49 Euclidean triples, with coefficient field $K = \mathbb{R}$ or $\mathbb{F}_3$. Let S_d, D_d denote line sums and midpoint differences. Choose edge orientations and let B_d be the vertex–edge incidence matrix and $G_d = B_d^\top$ the potential gradient. The homogeneous kernel dimensions are as follows. “Two/three dimensions” refers to grid dimension: scalar fields belong to K^9, K^{27} and edge flows to K^{24}, K^{147} . The finite-field columns retain the same Euclidean triples, not all finite-field affine lines. *Proof and computational certificates.* Theo-

Structure / equation	$d = 2, \mathbb{R}$	$d = 3, \mathbb{R}$	$d = 2, \mathbb{F}_3$	$d = 3, \mathbb{F}_3$
Position identity $a + c = 2b$	No constraint on labels specified			
Line sums $S_d f = 0$	2	0	3	4
Midpoint differences $D_d f = 0$	3	4	3	4
Potential gradient $B_d^\top f = 0$	1	1	1	1
Flow divergence $B_d j = 0$	16	121	16	121
Graph Laplacian $B_d B_d^\top f = 0$	1	1	4 (E)	8 (E)

rems 2, 7, and 8 give the real scalar kernels. Since $-2 = 1$ in $\mathbb{F}_3$, $S_d = D_d$ there and the affine-field proof gives dimension $d + 1$. Vanishing gradient equates endpoints along every edge; connectivity forces constants over either field. Every incidence column sums to zero, while a reduced spanning-tree incidence minor has determinant ± 1 . Thus B_d has rank $n - 1$ over either field, with flow-kernel dimension $m - n + 1 = 16, 121$. The real Laplacian energy identity gives only constants in its kernel. Exact modulo-three elimination gives ranks five and 19 in the last row, hence dimensions four and eight, recorded as E; integer matrices and code accompany the paper. $\square$

The position identity alone specifies no scalar-field equation and has no invented “geometric kernel.” Not all kernels are distinct, nor is there a universal orthogonality statement covering them. Over $\mathbb{F}_3$, S and D coincide. Over the reals in two dimensions, the zero-sum kernel is orthogonal to the affine midpoint kernel, because zero row and column sums give zero inner products with $1, x, y$; this specific result does not establish general orthogonality of geometry and numbers. Real positive-definiteness arguments do not transfer to $\mathbb{F}_3$, where connected graphs can have extra Laplacian zero modes. The classification prevents undeclared identification of different constraints, while allowing further energies, driving, or axioms in subsequent models.

6.3 An explicit normal magic cube under the 31-line definition

Requiring only 27 axis lines and four body diagonals permits a normal cube containing one to 27 exactly once. Let $\beta(t)$ be the balanced modulo-three representative $-1, 0, 1$ and set

$$C(x, y, z) = 14 + 9\beta(x + y + z) + 3\beta(x + y - z) + \beta(x - y + z). \quad (9)$$

Proposition 5 (feasibility; T/E). This C takes exactly the values one to 27, and every specified axis or body-diagonal sum is 42.

Proof. The coefficient determinant of the three forms is -4 , nonzero modulo three. Their balanced digits thus run through all 27 triples and the weighted combination through $-13, \dots, 13$. Along an axis each form has nonzero coefficient, so its balanced representatives sum to zero. Body diagonals pass through the origin; oddness of β gives complementary endpoints and center 14, hence sum 42. $\square$

This is an explicit instance under the 31-line definition, not a uniqueness claim, and its common vertex count does not identify its labeling with the L27 geometric graph.

6.4 Real translations versus finite-field states: two ternary arithmetics

Stacks of normal numerical Lo Shu layers and ternary coordinate cells are different constructions. The author's supplied same-title framework manuscript (public program record [13], with versions distinguished in the data statement) uses $\{0, 1, 2\}^3$ and the endpoint/midpoint condition. Section 7.2's numerical-average test does not refute that geometry. As real coordinates, $\{0, 1, 2\}$ and $\{-1, 0, 1\}$ differ by translation one; adopting modulo-three operations is a separate modeling step.

Proposition 5a (ternary geometric interface; T/E). For endpoints A, C and midpoint B of any three-point line, $A + C = 2B$. Translation preserves this relation. Every real affine readout satisfies $f(A) + f(C) = 2f(B)$, equivalently zero sum of the three deviations from the local midpoint value. All 49 lines satisfy it, including face and body diagonals.

The three original values sum to three times their local midpoint value, generally varying with the line. This geometric midpoint interface is not a common global sum of all raw labels, and is consistent with Theorem 7.

Proposition 5b (modulo three versus real arithmetic; T/E). Over $\mathbb{F}_3$, the midpoint difference and three-point zero sum are the same equation. The 49-line sum matrix has rank 23, kernel dimension four, and precisely the fields $f = c + \alpha x + \beta y + \gamma z \pmod{3}$, totaling $3^4 = 81$. The same real sum matrix has rank 27; its real constant-label conclusion must not be imposed on finite-field states.

Proof. Axis constraints give a multi-affine expansion. Face-diagonal midpoint conditions eliminate cross terms as in Theorem 8, since two is invertible in $\mathbb{F}_3$. The four remaining coefficients are arbitrary. $\square$

The real embedded grid has 49 three-point lines, whereas $\mathbb{F}_3^3$ has 117 affine lines including wraparound. "Ternary" alone does not select adjacency. The decomposition in Section 3.2 also gives an encoding interface: nine pairs (q, r) and an independent ternary s form 27 internal states, with readout $L = 5 + 3q + r$. The permutation into (q, r) does not preserve all original real collinear triples: only four of eight lines map to real collinear triples. Digital and Euclidean-geometry interfaces must therefore be distinguished.

7 Two-/three-dimensional projections and subsequent unit-cell interfaces

7.1 Slices, sums, and averages

For a three-dimensional scalar field define the middle slice $R_z f(x, y) = f(x, y, 0)$, sum $\Sigma_z f = \sum_z f$, and mean $P_z f = \frac{1}{3} \sum_z f$. They differ. The mean has rank nine and kernel dimension 18, so one two-dimensional image loses 18 degrees of freedom.

Theorem 10 (obstruction from equal-sum axis projection; T). If each vertical triple sums to S , then $\Sigma_z f = SJ$ and $P_z f = (S/3)J$, precluding a nonconstant Lo Shu image.

This follows directly from the column definition. The mean of the normal cube in Section 6.3 is therefore constant 14. A slice, nonuniform depth weights, or a different internal observable can preserve a pattern, but each adds a readout rule and is not ordinary sum projection.

If only $P_z f = L$ is imposed, every solution is $f = L + \eta$ with $\sum_z \eta = 0$. The 18-dimensional freedom prevents unique reconstruction. Even all three orthogonal averages have combined rank only 19 and common kernel dimension eight, the tensor product of the three two-dimensional zero-mean factors.

7.1a Finite-field quotient and matching integer readout for a given Lo Shu

Proposition 5c (quotient reproduction of a given Lo Shu; T/E, projection and readout are construction inputs). On $V = \mathbb{F}_3^3$ with representatives $\{0, 1, 2\}^3$, choose

$$\pi(x, y, z) = (i, j) = ((x - z + 1) \bmod 3, (y - z + 1) \bmod 3).$$

Its linear part has rank two and kernel generated by $(1, 1, 1)$. Each target has three preimages $(i - 1 + k, j - 1 + k, k)$ for $k = 0, 1, 2$, all modulo three, compressing 27 states into nine positions.

Choose digits compatible with known L :

$$u = (y - x + 1) \bmod 3, \quad v = (2(x + y + z) + 1) \bmod 3.$$

Take representatives zero, one, two and then use integer $N = 1 + 3u + v$. Modulo-three readout would erase $3u$ and could not yield nine values. The readout is constant on fibers and becomes $u = (j - i + 1) \bmod 3$, $v = 2(i + j) \bmod 3$ at the target, giving exactly standard L , with eight sums 15. Each target digit occurs once and has three three-dimensional preimages. The center $(1, 1, 1)$ maps to $(1, 1)$ with readout five; the middle layer $z = 1$ is standard L . Also $N(x, y, z) + N(2 - x, 2 - y, 2 - z) = 10$.

Proof. Adding (k, k, k) changes neither quotient nor digits modulo three. Those three values exhaust each fiber. The target-to-digit determinant is -4 , nonzero modulo three, so digit pairs differ. Substitution or the Q, R decomposition checks all eight sums. $\square$

Retaining $s = z$ gives $K = 1 + 9s + 3u + v$, taking one to 27 exactly once. Compression $N = 1 + ((K - 1) \bmod 9)$ explicitly discards one ternary component. This is complete digital encoding, not an all-49-line real equal-sum claim.

For any real field let C be the nine-fiber 0/1 membership matrix. In real matrix arithmetic,

$$P_\pi = C/3, \quad T_\pi = C^\top, \quad CC^\top = 3I_9, \quad P_\pi T_\pi = I_9.$$

Finite-field fiber labels do not make averaging division in that field. The projection has rank nine and kernel dimension 18 and returns Lo Shu on the constructed fiber-constant N . It does not require every original layer to be a normal square and does not belong to Section 7.2's layer-average assumptions.

This is a target-matching existence construction, not automatic selection. Only one of the nine fibers is a straight line in the finite real cube; the others wrap modulo three, so the quotient is geometrically distinct from Euclidean orthogonal projection. Known Lo Shu and its digit decomposition preceded the choice of π, N ; there was no blind prediction or prior registration. Publication of the rules enables recalculation, not retroactive independent discovery. Indeed every target array g satisfies $P_\pi T_\pi g = g$. Even center five and inversion sum ten allow the nonmagic row-major array $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Those conditions still do not select Lo Shu.

Set $U_\pi = C^\top / \sqrt{3}$. Its effective Laplacian is

$$U_\pi^\top L_3 U_\pi = CL_3 C^\top / 3.$$

For the original 49-line real graph this is a nine-node complete graph with weights one or $7/3$, not $(5/3)L_2$. The factor-five identity of Section 7.3 concerns coordinate replication instead. Both exact matrices are supplied. The quotient also fails dynamical closure: $P_\pi L_3 - (CL_3 C^\top / 3)P_\pi$ has maximum absolute entry $8/3$. A supplied hidden field satisfies $P_\pi \eta = 0$ but $P_\pi L_3 \eta \neq 0$.

7.1b Exact closure-defect spectra and a conditional error bound

Proposition 6 (quantitative defects of two compressions; T/E). All arithmetic and norms here are real, with unit vertex masses and standard Euclidean norms. For quotient membership C , set $P = C/3$,

$T = C^\top$, $L_{\text{eff}} = CL_3C^\top/3$, and $D = PL_3 - L_{\text{eff}}P$. Its rank is seven, and the nine eigenvalues of $DD^\top$ are

$$0 \ (2), \quad \frac{2}{3}, \quad \frac{74}{27} \ (2), \quad \frac{14}{3} \ (2), \\ \frac{175 - \sqrt{29977}}{27}, \quad \frac{175 + \sqrt{29977}}{27},$$

where parentheses denote multiplicities. Thus

$$\|D\|_2 = \sqrt{\frac{175 + \sqrt{29977}}{27}} \approx 3.590825, \quad \|D\|_F^2 = \frac{256}{9}.$$

Since D is 9×27 , this is its squared singular-value spectrum, not its eigenvalue spectrum.

Exact verification. The integer matrix $E = 3CL_3 - (CL_3C^\top)C$ satisfies $D = E/9$ and $EC^\top = 0$. Integer computation gives

$$\chi_{EE^\top}(\lambda) = \lambda^2(\lambda - 54)(\lambda - 222)^2(\lambda - 378)^2(\lambda^2 - 1050\lambda + 5832).$$

Divide the roots by 81 to obtain the stated spectrum. Exact elimination verifies rank seven; the code checks expanded characteristic polynomials and matrix identities. Floating-point values only approximate analytic roots. This is a fixed finite-matrix certificate, not experimental uncertainty.

For the coordinate-average defect $D_z = P_zL_3 - (5/3)L_2P_z$, the analogous computation gives rank eight and squared singular values

$$0, \quad \frac{2}{3}, \quad \frac{98}{27}, \quad \frac{20 \pm 6\sqrt{11}}{9} \text{ (each twice),} \\ \frac{334 \pm 4\sqrt{4306}}{27} \text{ (each once).}$$

Consequently $\|D_z\|_2 = \sqrt{(334 + 4\sqrt{4306})/27} \approx 4.700199$ and $\|D_z\|_F^2 = 1024/27$. Its integer certificate is also supplied. Norms are comparable only with the same Euclidean input/output normalization; reading out with $U^\top$ instead of P changes them by $\sqrt{3}$.

Model-conditional corollary (M/T). For selected diffusion $\dot{f} = -\kappa L_3 f$, $\kappa > 0$, let $g = Pf$, $\eta = (I - TP)f$, and $\hat{g} = -\kappa L_{\text{eff}}\hat{g}$, with $\hat{g}(0) = g(0)$. From $DT = 0$,

$$\dot{g} = -\kappa L_{\text{eff}}g - \kappa D\eta.$$

The effective semigroup is contractive, yielding

$$\|g(t) - \hat{g}(t)\|_2 \leq \kappa \|D\|_2 \int_0^t \|\eta(s)\|_2 ds.$$

This quantifies the interface between hidden amplitude and projection error but is not a small-error guarantee without control of η . Even $\eta(0) = 0$ generally fails to imply $\eta(t) = 0$ unless the replication subspace is invariant under L_3 or additional constraints are imposed. The physical elimination question of Section 12.2 remains open.

7.2 Equal-weight averages of three normal numerical layers

The earlier internal L27 stack (see data statement) uses L , its quarter-turn, and its half-turn. Normal individual layers do not ensure equal three-dimensional axis sums. The average is $(10J + \text{rot}_{90^\circ} L)/3$, an affine rescaling of a magic square, not a normal square containing one to nine.

This section classifies only normal one-to-nine layers whose equal-weight average is also normal. It does not classify all ternary cells, conservation laws, or projections.

Proposition 7 (identical layers are necessary and sufficient for a normal average; T/E). Three layers, each one of the eight normal squares, have a normal equal-weight average if and only if all three are identical.

Proof. Each squared Frobenius norm is 285, as is a normal average. Strict convexity gives

$$\begin{aligned} & \frac{\|X_1\|_F^2 + \|X_2\|_F^2 + \|X_3\|_F^2}{3} - \left\| \frac{X_1 + X_2 + X_3}{3} \right\|_F^2 \\ &= \frac{1}{9} \sum_{i < j} \|X_i - X_j\|_F^2. \end{aligned} \quad (10)$$

The left side is zero, so the layers coincide. Sufficiency is immediate. Of all $8^3 = 512$ stacks, exactly eight meet the condition. $\square$

Proposition 8 (line-condition counts for 512 normal-layer stacks; E). Of the 512 ordered stacks, zero have all 27 axis sums 15, eight have all 18 face-diagonal sums 15, 64 have all four body-diagonal sums 15, and zero have all 49 equal sums. These separately counted properties need not be disjoint. Each layer independently ranges over eight choices; the program tests the 27/18/4 line groups and normality of the average. It covers all 512 stacks and accepts eight normal averages, recording line classes and counts.

Axis failure also follows by parity: every normal layer has even corners, so a vertical corner column sums three evens and cannot equal 15. Conversely, three identical layers make every body diagonal follow a planar principal diagonal, summing to 15; vertical sums are $3L_{ij}$ and vary with position. Thus it is false that one-to-nine layers necessarily violate body diagonals; the correct obstruction concerns simultaneous full three-dimensional equality. Ternary coordinate midpoint conditions provide a different interface in Section 6.4.

2D Lo Shu	Rotated-layer mean	31-line normal cube: z-mean																											
<table> <tr><td>4</td><td>9</td><td>2</td></tr> <tr><td>3</td><td>5</td><td>7</td></tr> <tr><td>8</td><td>1</td><td>6</td></tr> </table>	4	9	2	3	5	7	8	1	6	<table> <tr><td>4</td><td>5.67</td><td>5.33</td></tr> <tr><td>6.33</td><td>5</td><td>3.67</td></tr> <tr><td>4.67</td><td>4.33</td><td>6</td></tr> </table>	4	5.67	5.33	6.33	5	3.67	4.67	4.33	6	<table> <tr><td>14</td><td>14</td><td>14</td></tr> <tr><td>14</td><td>14</td><td>14</td></tr> <tr><td>14</td><td>14</td><td>14</td></tr> </table>	14	14	14	14	14	14	14	14	14
4	9	2																											
3	5	7																											
8	1	6																											
4	5.67	5.33																											
6.33	5	3.67																											
4.67	4.33	6																											
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14	14	14																											
14	14	14																											

Figure 2: Ordinary coordinate means of specified three-dimensional label models (M/T). Rotated normal layers yield an affinely rescaled magic square; the 31-line normal cube yields constant 14. The finite-field quotient of Section 7.1a is not shown, nor is natural selection of Lo Shu asserted.

7.3 Exact compression of the graph Laplacian

Let L_2, L_3 be the unweighted combinatorial Laplacians of Section 6.1. Replicate a planar field by $(Tg)(x, y, z) = g(x, y)$ and set $P = T^\top/3$.

Theorem 11 (Galerkin interface; T/E).

$$T^\top L_3 T = 5L_2, \quad U^\top L_3 U = \frac{5}{3}L_2, \quad U = T/\sqrt{3}. \quad (11)$$

Proof. Vertical edges have zero differences on replicated fields. Each planar three-point line lifts to three constant-height lines and two tilted-height lines, so every planar edge has exactly five three-dimensional

edges projecting onto it. Thus $(Tg)^\top L_3(Tg) = 5g^\top L_2g$ for every g , proving the identity. Normalization gives the second equation. $\square$

This is a usable operator interface, not autonomous dynamical reduction: generally $PL_3 \neq (5/3)L_2P$. The defect has maximum entry magnitude $32/9$; Proposition 6 supplies the full squared singular-value spectrum and norm. Layer locking, a closure assumption, or a restricted variational model is needed to adopt the effective planar evolution.

7.4 Interface protocol for graphs and internal algebra

The three-dimensional degrees are: one body center of degree 26, eight corners of degree 14, six face centers of degree ten, and twelve edge centers of degree eight. Some earlier reports had the correct degree multiset but misassigned coordinate categories; coordinates are re-enumerated here.

The node space $\mathbb{R}^{27}$ and the 27-dimensional Albert algebra merely admit a chosen linear bijection by dimension. Neither Albert multiplication nor automorphism or gauge groups follows from the number 27. The exceptional-algebra literature [9] concerns a Lie-algebra “magic square,” distinct from numerical Lo Shu. Attaching a 27-dimensional internal algebra to every node gives $\mathbb{R}^{27} \otimes J$ of dimension 729, not 27.

Comparing “the same operator” across studies requires vertex order and matrix permutation. Equality of kernels requires the same coefficient field and variable space, row-space comparison, or an explicit isomorphism. A shared gap, rank 23, or dimension four is not sufficient by itself.

Subsequent calculations must specify vertices and order; edges/hyperedges and boundaries; weights and units; internal space and multiplication/operators; and projection with closure error. The package provides the geometric and projection matrices, without inheriting unverified charge, mass, vacuum-selection, or exceptional-group-emergence conclusions. Earlier internal negative records concerning adjacency and Jordan multiplication are addressed in the data statement. Multiplication must be separately defined and checked, not inferred from node count or adjacency.

8 Dynamics: what static Lo Shu does and does not imply

8.1 Static equal sums do not determine time evolution

Proposition 9 (nonunique dynamics; T). In a real state space, requiring $A_2x(t) = 0$ for all times still permits infinitely many evolutions.

Proof. By Theorem 2, $x(t) = M(a(t), b(t))$ satisfies the constraint for arbitrary differentiable a, b . Constant parameters, exponential decay, and periodic rotation give stationary, dissipative, and periodic motions. The static equation selects none of them. $\square$

Equal sums alone therefore do not make Lo Shu a unique attractor. Restriction to nine distinct digits leaves eight permutations, but identity updates, cyclic D_4 rotation, and stochastic transitions with different rates all preserve the finite magic set. Allowed transitions, rates, and energies remain model inputs.

8.2 Constraint relaxation and local transport

A selected relaxation model is $E(x) = \|A_2x\|^2/2$ and $\dot{x} = -A_2^\top A_2x$, with $\dot{E} = -\|A_2^\top A_2x\|^2 \leq 0$. Its equilibria comprise the whole two-dimensional zero-sum kernel, not one Lo Shu. This is hyperedge-constraint dynamics with fixed lines; it generally does not conserve the sum of vertex values and cannot be called source-free local graph transport.

For graph transport, let B be oriented vertex–edge incidence and j edge flow. The continuity equation is $\dot{\rho} = -Bj$. Choosing $j = \kappa B^\top \rho$ gives $\dot{\rho} = -\kappa L\rho$. Opposite endpoint signs conserve total density. For $\kappa > 0$ on a finite connected graph, source-free steady states are constant, so a nonconstant Lo Shu

is generally not a steady state of pure diffusion. Sources, potentials, nonlinear interactions, or other conserved variables are needed to sustain a pattern.

The rate κ sets a time scale not determined by the magic sum 15. The L27 Laplacian has gap $7 - \sqrt{6} \approx 4.550510$ of multiplicity two and 13 distinct eigenvalues. Its exact characteristic polynomial is

$$\begin{aligned}\chi_{L_3}(\lambda) &= \lambda(\lambda - 9)^3(\lambda - 15)(\lambda - 27)(\lambda^2 - 14\lambda + 43)^2 \\ &\quad \times (\lambda^2 - 20\lambda + 97)(\lambda^2 - 24\lambda + 133)^3 \\ &\quad \times (\lambda^3 - 35\lambda^2 + 387\lambda - 1329)^3.\end{aligned}$$

Integer expansion agrees with direct matrix computation. This spectrum is an input for specified vibration or diffusion models, not a match to undefined particle frequencies.

A zero mode is not automatically an attractor. Undamped $\ddot{q} = -c^2 L_3 q$ for $c > 0$ conserves $(\|\dot{q}\|^2 + c^2 q^T L_3 q)/2$. Nonzero modes oscillate and the constant zero mode can move at constant velocity. Adding $\gamma \dot{q}$, $\gamma > 0$, damps nonzero modes but leaves a line of constant equilibria. Unique convergence to zero also requires zero-mean position and velocity constraints or pinning. Attractor properties must be proved for the actual equations, constraints, and initial data.

9 Fourier readout under a specified scattering-weight model (M)

9.1 Model and analytic intensities

A structure factor is a weighted phase sum; its interpretation and units depend on the scattering channel [10]. We choose ideal positions $(i/3, j/3)$ and weights $X_{ij} = c + M_{ij}(a, b)$, without identifying integers with measured atomic scattering factors.

Set $\omega = \exp(-2\pi i/3)$ and $F(h, k) = \sum_{i,j=0}^2 X_{ij} \omega^{hi+kj}$. Equal row/column sums extinguish nonzero axis frequencies. Substitution gives the basic modulo-three intensity array

$$I(h, k) = |F(h, k)|^2 = \begin{pmatrix} 81c^2 & 0 & 0 \\ 0 & 27a^2 & 27(a+b)^2 \\ 0 & 27(a+b)^2 & 27a^2 \end{pmatrix}. \quad (12)$$

For standard $a = -1, b = 4, c = 5$, nonzero intensities are 2025, 27, and 243 with multiplicities one, two, two. Parseval gives total intensity $2565 = 9 \times 285$. The 27 and 243 follow from the parameterization, with no extra quantum interpretation of 27, one, or nine.

Peak counts depend on the display window. For integer $h, k = -8, \dots, 8$, there are $12^2 = 144$ off-axis-frequency weak peaks with neither coordinate divisible by three, merely modulo-three repetitions. The equally sized window $0, \dots, 16$ instead gives $11^2 = 121$, not 144 intrinsic cell states. Changed weights may activate extinctions of a uniform reference, without establishing new particles or universal quantization.

9.2 Which signals are specific to Lo Shu?

Proposition 10 (scope of transpose-intensity symmetry; T). Any real 3×3 matrix with a common sum for every row and column satisfies $I(h, k) = I(k, h)$ at integer modulo-three reciprocal points. Diagonal equal sums are unnecessary.

Proof. Axis amplitudes vanish except at the origin, which is fixed by transpose. Remaining frequencies are $(1, 1), (1, 2), (2, 1), (2, 2)$. Transpose fixes the two diagonal points and exchanges $(1, 2)$ with $(2, 1)$, opposite frequencies. Real-weight Fourier conjugacy makes their intensities equal. $\square$

Generic random weights are consequently insufficient controls. Include semimagic nonmagic arrays with matching marginal sums, such as $\begin{pmatrix} 3 & 5 & 7 \\ 4 & 9 & 2 \\ 8 & 1 & 6 \end{pmatrix}$, whose row/column sums are 15 and diagonals 18 and

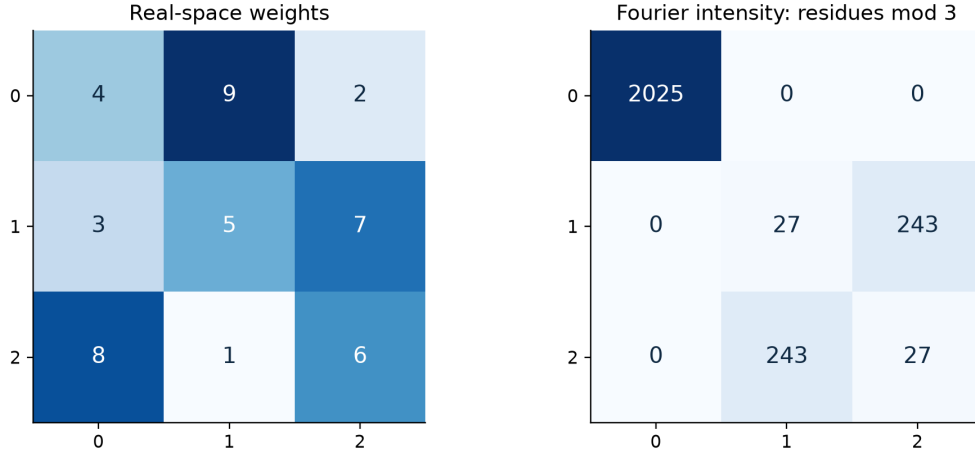


Figure 3: Discrete Fourier intensities for given standard Lo Shu and a specified linear scattering-weight model (M; T/E). Frequencies repeat modulo three and amplitudes are complex. Transpose symmetry is also present in a nonmagic semimagic control. No experimental scattering data are used.

24, yet which has the same intensity symmetry. Noninteger frequencies do not obey the modulo-three simplification. Between discrete points, asymmetry is a continuous extension of a finite motif's form factor; actual diffuse scattering requires disorder, thermal motion, finite-size effects, or other physical sources.

9.3 Three-dimensional projection and Fourier sections

For three-dimensional weights define $F_3(h, k, l) = \sum f_{ijk} \exp[-2\pi i(hi + kj + lk)/3]$. Then

$$F_3(h, k, 0) = F_2 \left[\sum_z f \right] (h, k). \quad (13)$$

This exact interface follows by exchanging finite sums. For slices $z_0 = 0, 1, 2$,

$$F_2[f(\cdot, \cdot, z_0)](h, k) = \frac{1}{3} \sum_{l=0}^2 \exp(2\pi i l z_0 / 3) F_3(h, k, l).$$

Centered indices require the corresponding phase shift. A real-space slice is not the $l = 0$ Fourier section. Transpose-symmetric layer intensities also do not force every three-dimensional frequency section to be symmetric, because cross-layer phases depend on amplitudes. Three-dimensional readout claims must use the full complex amplitude.

10 Vibration interfaces under specified spring models (M)

This section retains models directly connecting an operator with mechanical energy. Tight-binding spectral pairing is a general consequence of inversion-odd potentials and bipartite hopping and is placed in Appendix A as a check of assumptions and nonspecificity.

10.1 Acoustic duplication and elastic stability

Both models use the four nearest-neighbor bonds of an infinite periodic square grid, each undirected bond once, microscopic spacing one, $\kappa > 0$, and masses $m_i > 0$. Neither uses the 24-edge collinearity graph or the quotient's weighted complete graph.

For two-component displacement u_i , choose vector-difference energy $\kappa \sum_{\langle i,j \rangle} \|u_i - u_j\|^2/2$. Each bond uses the same scalar operator for both polarizations, making the dynamical matrix the scalar mass-weighted Laplacian tensored with I_2 . Each scalar mode is copied twice, with possible extra accidental degeneracy; no hidden Lo Shu point group is needed. There are two translational zero modes at Γ . This coupling penalizes rigid rotation: a small angle θ on a unit x -bond gives difference θe_y and energy $\kappa \theta^2/2$, whereas an unstressed central-force spring has zero quadratic extension energy. The vector-difference model is therefore not directly free, unstressed central-force elasticity.

For genuine nearest-neighbor central-force energy $\kappa \sum_{\langle i,j \rangle} [(u_i - u_j) \cdot e_{ij}]^2/2$, horizontal and vertical bonds constrain their own components. A 3×3 supercell has six zero modes at Γ , including flexible slips. These two models cannot share an ambiguous “scalar spring” label. Stable shear needs diagonal bonds, angular forces, or other explicit elastic terms.

In the first model, long-wave effective mass is the cell’s average mass. If $m_i = w_i m_0$ and stiffness is uniform, then $v^2 = \kappa/(\bar{w} m_0)$ at unit microscopic spacing; $\bar{w} = 5$ gives $v^2 = \kappa/(5m_0)$. Any other arrangement with the same mean has the same leading value. This is a homogenization interface, not a new fundamental constant inferred from the center five.

11 Revision of the earlier argument and audit of existing interfaces

The earlier work [12] retains the general solution, minimum integer order under specified normalization, and eight normal squares. If “inevitability” means that conditional classification, Theorems 1–4 prove it here. If it means a derivation from arbitrary local flow conservation or five single states, the earlier chain is unproved and single-state sufficiency has a counterexample. The revisions are:

1. Signed generators with $e_- = -e_+$ and $v_- = -v_+$ give a quotient of $\mathbb{Z}^4$ isomorphic to $\mathbb{Z}^2$. The finite alphabet $\{0, \pm e, \pm v\}$ is not an additively closed group.
2. The claim that five single states suffice for nine distinct fixed readouts is withdrawn. A fixed single-valued map plus common translation gives at most five values, and only three at the earlier $w = 1$. Allowing combinations changes the single-occupancy assumption and needs a new model.
3. Selective weighted projection does not preserve zero sums. The earlier W changes only the entries $b, -b$, giving first/last row sums $\pm b(w - 1)$. Filtering only range and distinctness accepts four nonmagic arrays at $w = 4$. Repair must change the rule, not merely relabel the output.
4. Center five and sum 15 follow from Theorem 1 and total sum. Translations to $[0, 8]$ and $[2, 10]$ have sums 12 and 18.
5. “Local conservation” becomes specified line equal sums. Graph transport, midpoint differences, and Laplacians are separately defined; a row in ordinary eight-neighbor adjacency is not called a three-clique.
6. A D_4 orbit is distinguished from one labeling’s point group; corner, face-center, and edge-center degrees are assigned from coordinates.
7. Integer diffraction symmetry is a property of real order-three semimagic arrays, not uniquely Lo Shu. Intensities 27 and 243 are derived analytically.
8. Gap statements distinguish direct/indirect gaps, sampling domain, filling, and model inputs. Acoustic pairing is tensor duplication in the selected polarization model.

The author’s supplied same-title unit-cell manuscript (public program record [13]) also discusses Albert algebra, gauge groups, flavor fits, vacuum actions, and multicell evolution. We supply independently

checkable geometric/operator interfaces without treating those other conclusions as proved here. Existing two-parameter flavor fitting is excluded from this no-observational-fit evidence chain.

“No fitting” means not inferring parameters from observations. It does not mean no assumptions, choices, dimensional parameters, or target-based constructions. Section 7.1a explicitly designs readout for known Lo Shu and is not independent physical prediction. Dictionaries, adjacency, projection, and intensity ratios must remain visible.

12 Conclusions and testable research exits

Connections to traditional symbolism and modern discrete models yield specified interfaces and specified obstructions. Under integer, zero-sum, distinctness, and minimization assumptions, complete parameterization, minimum ternary prototypes, and normal classification hold. Given standard Lo Shu and a matching readout, a 27-to-nine finite-field quotient reproduces it without selecting it. Given traditional dictionaries, encodings, and synchronous updates, trigram pairings and stem–branch periods are computable. Full three-dimensional equal-sum labeling, ordinary axis projection, and direct projection of eight cube vertices face the proved obstructions. Compression identities, finite Fourier projection, and specified vibration models provide reproducible objects for later unit-cell research.

Theorem 9 registers constraint spaces; Propositions 2 and 8 classify the 432 affine readouts and 512 normal-layer stacks; Proposition 6 upgrades nonclosure from entrywise counterexamples to exact defect spectra and a conditional error bound. Subsequent work can choose operator models and controls without asserting unique generation of numerical Lo Shu from general flow conservation.

Static equal sums have not uniquely determined physical stem–branch dynamics; planar Lo Shu has not uniquely reconstructed a three-dimensional cell; dimension 27 has not generated exceptional multiplication; and numerical readout has not established fundamental physics. Dictionaries, adjacency, readout, controls, and falsification criteria should be fixed before future calculations.

12.1 O1: a selection principle without supplying target Lo Shu

Publish the candidate projection/readout family, objective, symmetry equivalences, and no-/multiple-solution criteria in advance, without minimizing distance to known L . Section 2.5 selects the normal orbit conditional on two minimum ternary components with eight zero sums, integer distinctness, and minimum range. It does not derive those premises from independent physics or choose a unique π, N . Only an independently sourced rule with proved optima and robustness can support prediction beyond reproduction.

12.2 O2: when hidden channels can be controllably neglected

An 18-dimensional projection kernel means invisibility, not high energy, fast decay, or decoupling. Specify complete dynamics, initial data, visible–hidden coupling, a gap or time-scale separation, and an error bound on a stated interval. Layer-locking energy, dissipation, or scale separation needs physical origins and parameters. A Galerkin identity alone is not error control of this kind.

12.3 O3: connecting traditional topology with a physical action

Fix the phase/stem–branch configuration, then define states, units, time, interactions, driving or dissipation, and verify conservation, stability, and observable consequences. Directed relations are not automatically equilibrium potentials; irreversible models need an energy budget. Section 5.3 is an abstract stable cyclic mixing example, not a unique physical interpretation. No action meeting all these conditions and independently derived from Lo Shu has been established here.

All three retain status O. Future calculations should include non-Lo-Shu controls with matching marginal sums, inversion oddness, or mean mass to identify added magic-square constraints. Model-conditional stability or symmetry does not itself establish natural causation for traditional correspondences.

Data and materials availability

Reproducible programs, JSON, and NPZ provide the finite geometry (eight lines/24 edges and 49 lines/147 edges), sum and midpoint matrices, incidence and Laplacian matrices, two projections, and closure defects. Enumeration domains, vertex orders, and coefficient fields are explicit. No observational data or regression fitting is used. Integer enumeration and exact elimination are finite computational evidence; floating spectra and momentum grids do not replace continuous-domain proofs.

Earlier readout, graph-audit, and negative-result records are author-supplied internal archives, not external validation references. Relevant definitions and values are independently recalculated here. Public [13] is AiXiv v1.0, whereas the supplied manuscript used to check geometry is labeled v1.5; identity of versions is not assumed. Internal originals are not distributed, so conclusions must be supported by the definitions and evidence given here.

Programs cover $9!$ digit permutations, 8^3 ordered normal stacks, 432 affine bijections, ranks over the rationals and modulo three, compression identities, and exact characteristic polynomials. Manuscript checking and finite verification are not absolute correctness guarantees or external peer review.

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A Spectral pairing and gap limits in a specified tight-binding model

A.1 Inversion-odd potential and spectral pairing

Choose real nearest-neighbor hopping on an infinite square lattice, $H = -tA + \Delta \text{diag}(v)$ with real t, Δ . Put the cell center at the origin: $r = (i-1, j-1)$, $i, j = 0, 1, 2$, and periodically extend $v(r) = L_{ij} - 5$ with period three. Complementarity gives $v(-r) = -v(r)$; with uncentered indices inversion instead means $(i, j) \mapsto (2-i, 2-j)$. Inversion P reverses potential and preserves hopping. Sublattice sign $S\psi(r) = (-1)^{r_x+r_y}\psi(r)$ reverses hopping and preserves potential. Thus SP anticommutes with H and connects different Bloch momenta.

At unit microscopic spacing the reciprocal period is $2\pi/3$, with momentum shift $Q = (\pi/3, \pi/3)$:

$$E_n(k) = -E_{10-n}(Q - k). \quad (14)$$

The spectra at Γ and the supercell corner Q are opposite in reverse order. For $t \neq 0$, the identity holds for any Δ/t ; at $t = 0$ pairing holds directly. It needs bipartite nearest-neighbor hopping and an inversion-odd potential, and is not unique to Lo Shu. Same-sublattice hopping or broken inversion oddness may destroy it.

An independently built 9×9 Bloch matrix has zero-potential folding error about 2.9×10^{-15} and pairing error 5.3×10^{-15} : implementation checks, not experimental error bars. A second Bloch gauge

with momentum phase on every bond verifies pairing and uses a nonmagic inversion-odd potential as control.

Corollary 1 (restriction on a zero-containing gap; T/E). At $2k = Q$ modulo a reciprocal vector, the nine-band spectrum pairs with itself, so $E_5(k) = -E_5(k) = 0$. In the fundamental zone there are four such points, $k = (\pm\pi/6, \pm\pi/6)$, for all real t, Δ . Hence the model has no complete spectral gap containing zero. This does not exclude gaps at other integer fillings or require each zero to be a band crossing; it limits insulating interpretations near zero. Twenty momentum/parameter checks agree.

A.2 Gaps, filling, and the no-fit discipline

Distinguish the minimum direct separation $\min_k (E_{r+1}(k) - E_r(k))$ from the indirect gap $\min_k E_{r+1}(k) - \max_k E_r(k)$. Nondegeneracy at each fixed momentum is insufficient for an insulator; filling, positive global indirect gap, and the relevant approximation assumptions are also needed.

At selected $\Delta/t = 1$, a 61×61 grid gives about 0.477627 direct separation and 0.369343 indirect gap between bands four and five, in units of t . These are grid evidence, not certified continuous-zone extrema or material claims. The dimensionless ratio is a model input even when set to one, not a freedom eliminated by units.

A purely analytic existence result follows from $\|H_0\| \leq 4|t|$ and onsite spacing $|\Delta|$. Weyl bounds put the nine ordered bands in disjoint intervals of radius $4|t|$ around onsite values when $|\Delta| > 8|t|$, with global neighboring gaps at least $|\Delta| - 8|t|$. This proves high-contrast model gaps, not parameter-free selection of a natural Lo Shu amplitude.

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