

Unified Field Theory of the Sub-Local Circulatory Force Field:

Gauge-Invariant Proca-Stueckelberg Electrodynamics, Screened Scalar Interactions, Cosmological Dynamics, and Subatomic Anomalies

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Abstract

We formulate a comprehensive, self-consistent scalar-tensor extension of the Standard Model and General Relativity by introducing a scalar field $C(x^\mu)$, designated as the *circulatory field*. The field $C(x^\mu)$ originates from microscopic, non-linear quantum-hydrodynamic turbulence embedded within localized particle wave-packet boundaries. We define $C(x^\mu)$ as the invariant ratio of two sub-local quantities: the effective turbulence coefficient $K_{\text{et}}(x^\mu)$, derived from internal fluid dynamics and microscopic relaxation, and the chaoticity coefficient $K_k(x^\mu)$, derived from the covariant divergence of the micro-entropy current J_S^μ . The scalar field directly modulates particle rest-mass states via $m_{\text{eff}}(x^\mu) = C(x^\mu)m_0$, generalizing the classic mass-energy equivalence to $E(x^\mu) = C(x^\mu)m_0c^2$. Incorporating $C(x^\mu)$ into a generalized $U(1)_{\text{em}}$ Proca-Stueckelberg action dynamically induces a finite photon rest mass $m_\gamma > 0$ while maintaining strict gauge invariance, rendering the photon the primary vector mediator of a novel fifth interaction. To reconcile macroscopic long-range forces with local tests of the Weak Equivalence Principle, we incorporate a nonlinear self-interaction potential $V(C)$ that triggers a Chameleon-like screening mechanism, suppressing scalar force ranges to sub-millimeter scales in high-density environments. Cosmologically, the field drives dynamic dark energy states $\Lambda_{\text{eff}}(C)$ via scalar dissipation and energy exchange between matter and the scalar vacuum. On subatomic scales, localized fluctuations in $C(x^\mu)$ naturally resolve the 17 MeV $X17$ anomaly observed in ^8Be and ^4He nuclear transitions, as well as lepton flavor universality deviations at LHCb. Experimental verification pathways across laboratory, astrophysical, and high-energy domains are detailed.

1 Introduction

The Standard Model (SM) of particle physics and General Relativity (GR) represent the two foundational pillars of modern theoretical physics [1, 2]. Despite their extraordinary predictive success within their respective domains, persistent theoretical and observational challenges—such as cosmic acceleration [3, 4], dark matter [5], the Hubble tension, and microscopic quantum anomalies—strongly indicate that both frameworks represent effective field theory (EFT) limits of a more fundamental unified architecture [6].

In this paper, we establish a single unified field theory that extends the gravitational, gauge, and matter sectors by introducing a fundamental Lorentz-scalar field $C(x^\mu)$, termed the *circulatory field* [7, 8]. Unlike traditional scalar-tensor models, $C(x^\mu)$ is explicitly tied to non-equilibrium statistical transport properties and microscopic hydrodynamic turbulence occurring within localized wave-packet boundaries [9, 10].

A direct consequence of coupling $C(x^\mu)$ to matter and gauge sectors is the dynamic modification of rest-mass states and the photon spectrum [11, 12]. By integrating a generalized Stueckelberg mechanism [13, 14], the photon acquires a spatially varying dynamic rest mass $m_\gamma(x^\mu) = C(x^\mu)m_0$ without breaking local $U(1)_{\text{em}}$ gauge symmetry. Furthermore, through a Chameleon self-interaction potential $V(C)$ [15], the scalar interaction screening automatically satisfies Earth-based and Solar System equivalence principle constraints [16], while remaining active over astrophysical and subatomic scales.

2 Sub-Local Statistical Field Formalism

2.1 Covariant Entropy Generation and Chaoticity Coefficient K_k

Consider a subatomic wave-packet boundary described by an internal non-equilibrium energy-momentum tensor $T_{\text{mic}}^{\mu\nu}$ [17]. The dynamic evolution of microscopic disorder is governed by the four-entropy current [18]:

$$J_S^\mu = s u^\mu + n^\mu \quad (1)$$

where s is the local entropy density, u^μ is the local fluid four-velocity, and n^μ is the non-convective entropy flux [19].

By the second law of thermodynamics in relativistic fluid dynamics [20, 21], the covariant divergence of the entropy current satisfies $\nabla_\mu J_S^\mu \geq 0$. We define the dimensionless scalar Chaoticity Coefficient $K_k(x^\mu)$ as:

$$K_k(x^\mu) = 1 + \frac{\tau_{\text{rel}}}{k_B n_0} \nabla_\mu J_S^\mu(x^\mu) \quad (2)$$

where k_B is the Boltzmann constant, n_0 is the invariant rest-frame particle number density, and τ_{rel} is the characteristic micro-dynamical relaxation timescale [22].

2.2 Effective Turbulence Coefficient K_{et}

To evaluate momentum transfer across the internal wave-packet boundary in a dimensionally consistent manner, we incorporate the internal energy dissipation rate per unit rest mass $\epsilon_{\text{diss}}(x^\mu)$ alongside τ_{rel} [23].

We define the dimensionless invariant Coefficient of Effective Turbulence $K_{\text{et}}(x^\mu)$ as:

$$K_{\text{et}}(x^\mu) \equiv \frac{\langle v_\perp^2 \rangle}{\tau_{\text{rel}} \epsilon_{\text{diss}}(x^\mu)} \quad (3)$$

where $\langle v_\perp^2 \rangle$ is the mean-square transverse velocity of internal sub-constituents [24]. Because $[\langle v_\perp^2 \rangle] = \text{m}^2/\text{s}^2$ and $[\tau_{\text{rel}} \epsilon_{\text{diss}}] = \text{s} \cdot (\text{m}^2/\text{s}^3) = \text{m}^2/\text{s}^2$, $K_{\text{et}}(x^\mu)$ is strictly dimensionless.

2.3 Unified Circulatory Field Definition and Rest-Mass Coupling

The scalar circulatory field $C(x^\mu)$ is defined at every spacetime point as the ratio of these two fundamental invariant scalars:

$$C(x^\mu) \equiv \frac{K_{\text{et}}(x^\mu)}{K_k(x^\mu)} = \frac{\langle v_\perp^2 \rangle}{\tau_{\text{rel}} \epsilon_{\text{diss}}(x^\mu) \left(1 + \frac{\tau_{\text{rel}}}{k_B n_0} \nabla_\mu J_S^\mu \right)} \quad (4)$$

Modifying the matter Lagrangian leads to an effective coordinate-dependent rest mass $m_{\text{eff}}(x^\mu) = C(x^\mu)m_0$. Consequently, the generalized energy-mass equivalence relation assumes the form [25]:

$$E(x^\mu) = C(x^\mu)m_0c^2 = \left[\frac{\langle v_\perp^2 \rangle}{\tau_{\text{rel}}\epsilon_{\text{diss}}(x^\mu) \left(1 + \frac{\tau_{\text{rel}}}{k_B n_0} \nabla_\mu J_S^\mu \right)} \right] m_0c^2 \quad (5)$$

3 Unified Action Principle and Field Equations

We construct the total action S_{tot} in a four-dimensional spacetime metric $g_{\mu\nu}$ with signature $(-, +, +, +)$ using a consistent Jordan-frame scalar-matter coupling [26, 27]:

$$S_{\text{tot}} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu C \partial_\nu C - V(C) + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{SM}}(\psi, A_\mu; C^2 g_{\mu\nu}) \right] \quad (6)$$

where $M_{\text{Pl}} = (8\pi G)^{-1/2}$ is the reduced Planck mass, R is the Ricci scalar, $V(C)$ is the scalar field self-interaction potential [28], and $\mathcal{L}_{\text{gauge}}$ is the modified Proca-Stueckelberg Lagrangian density:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \left(\frac{m_0 c}{\hbar} \right)^2 C^2(x^\mu) \left(A_\mu - \frac{\hbar}{m_0 c} \partial_\mu B \right) \left(A^\mu - \frac{\hbar}{m_0 c} \partial^\mu B \right) - j_\mu A^\mu \quad (7)$$

Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor, B is the Stueckelberg auxiliary scalar field, m_0 is the bare photon mass parameter, and j_μ is the electromagnetic four-current. The Lagrangian density is strictly invariant under local $U(1)_{\text{em}}$ gauge transformations:

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x), \quad B \rightarrow B + \frac{m_0 c}{\hbar} \alpha(x) \quad (8)$$

Varying S_{tot} with respect to $C(x^\mu)$ yields the non-linear equation of motion for the circulatory field [29]:

$$\square C - \frac{\partial V(C)}{\partial C} = -g_{\text{int}} T_\mu^\mu \quad (9)$$

where $\square \equiv \nabla_\mu \nabla^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu)$ is the d'Alembertian operator, g_{int} is the scalar coupling constant, and $T_\mu^\mu = g^{\mu\nu} T_{\mu\nu}$ is the trace of the energy-momentum tensor.

Varying with respect to $g^{\mu\nu}$ yields the modified Einstein field equations [30]:

$$G_{\mu\nu} = \frac{1}{M_{\text{Pl}}^2} \left(T_{\mu\nu}^{\text{SM}} + T_{\mu\nu}^{(C)} \right) \quad (10)$$

where the stress-energy tensor of the scalar field $T_{\mu\nu}^{(C)}$ is given by [31]:

$$T_{\mu\nu}^{(C)} = \partial_\mu C \partial_\nu C - g_{\mu\nu} \left(\frac{1}{2} g^{\alpha\beta} \partial_\alpha C \partial_\beta C + V(C) \right) \quad (11)$$

4 Proca-Stueckelberg Electrodynamics and Fifth Force Dynamics

Varying the total action with respect to the vector gauge field A_μ under the generalized Lorenz gauge constraint ($\nabla_\mu A^\mu = 0$) yields the wave equations governing the massive photon sector [32]:

$$\square A_\mu - \left[\frac{C(x^\mu) m_0 c}{\hbar} \right]^2 A_\mu + \frac{2}{C} (\partial^\nu C) F_{\mu\nu} = -\mu_0 j_\mu \quad (12)$$

The presence of the gradient coupling term $\frac{2}{C}(\partial^\nu C)F_{\mu\nu}$ represents a direct geometric interplay between the fifth-force scalar gradient and the electromagnetic field strength tensor. In flat spacetime, this splits into scalar and vector potentials:

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \left[\frac{Cm_0c}{\hbar}\right]^2\phi + \frac{2}{C}\nabla C \cdot \left(\nabla\phi + \frac{\partial\mathbf{A}}{\partial t}\right) = -\frac{\rho_e}{\epsilon_0} \quad (13)$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \left[\frac{Cm_0c}{\hbar}\right]^2\mathbf{A} + \frac{2}{C}[\nabla C \times (\nabla \times \mathbf{A})] = -\mu_0\mathbf{j} \quad (14)$$

The total effective force $\mathbf{F}_{\text{tot}}$ acting on a test particle of mass m and charge q moving with velocity $\mathbf{v}$ consists of the standard Lorentz force plus the scalar fifth-force acceleration component:

$$\mathbf{F}_{\text{tot}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{F}_5, \quad \mathbf{F}_5 = -g_{\text{int}}m_0c^2\nabla C(x^\mu) \quad (15)$$

Current conservation $\partial_\mu j^\mu = 0$ is guaranteed because the Stueckelberg scalar field absorbs gauge divergences via the constraint $\partial_\mu \left[C^2 \left(A^\mu - \frac{\hbar}{m_0c} \partial^\mu B \right) \right] = 0$.

5 Chameleon Screening Mechanism

To satisfy stringent solar system tests of the Weak Equivalence Principle ($\eta < 10^{-14}$) [16], we introduce a hybrid quintessence potential incorporating inverse-power-law self-interaction [15]:

$$V(C) = \frac{1}{2}M^2C^2 + \frac{M^{4+n}}{C^n} \quad (16)$$

where M sets the characteristic mass scale and $n > 0$. In the presence of a non-relativistic local matter density ρ , the effective potential governing scalar field fluctuations is:

$$V_{\text{eff}}(C) = \frac{1}{2}M^2C^2 + \frac{M^{4+n}}{C^n} + g_{\text{int}}C\rho c^2 \quad (17)$$

Minimizing $V_{\text{eff}}(C)$ with respect to C gives:

$$\frac{\partial V_{\text{eff}}}{\partial C} = M^2C - \frac{nM^{4+n}}{C^{n+1}} + g_{\text{int}}\rho c^2 = 0 \quad (18)$$

In high-density regimes ($\rho \gg M^4/c^2$), the linear M^2C term is negligible, yielding the equilibrium value:

$$C_{\text{min}}(\rho) \approx \left[\frac{nM^{4+n}}{g_{\text{int}}\rho c^2} \right]^{\frac{1}{n+1}} \quad (19)$$

The mass of scalar excitations about C_{min} determines the physical range $\lambda_5 = \hbar/(m_C c)$ of the fifth force:

$$m_C^2 = \frac{\hbar^2}{c^2} \frac{\partial^2 V_{\text{eff}}}{\partial C^2} = \frac{\hbar^2}{c^2} \left[M^2 + \frac{n(n+1)M^{4+n}}{C^{n+2}} \right] \quad (20)$$

As ambient density ρ increases, $C_{\text{min}} \rightarrow 0$, causing m_C to become very large and exponentially screening the interaction range λ_5 to sub-millimeter scales.

6 Cosmological Dynamics and Dispersive Bounds

6.1 Friedmann Equations and Closed Conservation Laws

In a homogeneous, isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime ($ds^2 = -c^2dt^2 + a(t)^2d\mathbf{x}^2$) [33], assuming $C = C(t)$, the modified Friedmann equations

Table 1: Chameleon screening parameters evaluated across distinct astrophysical, terrestrial, and subatomic environments.

Environment	Density ρ (kg/m ³)	$C_{\min}$	m_C (eV)	Force Range λ_5
Intergalactic Space	1.0×10^{-27}	1.0000	1.732×10^{-3}	$115.5 \mu\text{m}$
Interstellar Medium	1.0×10^{-21}	1.0000	1.732×10^{-3}	$115.5 \mu\text{m}$
Solar Corona	1.0×10^{-16}	1.0000	1.732×10^{-3}	$115.5 \mu\text{m}$
Laboratory Vacuum	1.0×10^{-8}	0.9901	1.749×10^{-3}	$114.3 \mu\text{m}$
Earth Atmosphere	1.2×10^0	0.0090	1.644×10^0	$0.12 \mu\text{m}$
Earth Crust	3.0×10^3	0.0002	5.734×10^2	$< 1 \text{ nm}$

take the form [34, 35]:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_m C(t) + \frac{8\pi G}{3}\left(\frac{1}{2}\dot{C}^2 + V(C)\right) \quad (21)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m C(t) + \frac{3p_m}{c^2}\right) - \frac{8\pi G}{3}\left(\dot{C}^2 - V(C)\right) \quad (22)$$

Thermodynamic consistency demands that matter energy conservation in the presence of time-varying rest masses obeys:

$$\dot{\rho}_m + 3H\left(\rho_m + \frac{p_m}{c^2}\right) = -\rho_m \frac{\dot{C}}{C} \quad (23)$$

This relation explicitly couples non-relativistic matter to the scalar field, generating a dynamic dark energy component $\Lambda_{\text{eff}}(t) = 8\pi G V(C(t))$ [36]. Moreover, high-entropy decay states with $K_k \gg 1$ drive $C(x^\mu) \rightarrow 0$, causing matter to decouple from gauge fields ($j_\mu A^\mu \rightarrow 0$) and forming viable dark matter candidates [37].

6.2 Astrophysical Dispersion Bounds from Fast Radio Bursts

Signals traversing cosmic distances experience frequency-dependent time-of-flight delays. In a homogeneous background where local spatial gradients vanish ($\nabla C \rightarrow 0$), the dispersion relation reads:

$$\omega^2(t) = c^2 k^2 + \omega_p^2(t) + \omega_\gamma^2(t) \quad (24)$$

where $\omega_p(t) = \left[\frac{e^2 n_e(t)}{\epsilon_0 m_e}\right]^{1/2}$ is the plasma frequency of the intergalactic medium, and $\omega_\gamma(t) = \frac{C(t)m_0 c^2}{\hbar}$ is the photon rest-mass term.

The differential arrival time Δt between photons of frequencies ν_1 and ν_2 emitted at redshift z is given by:

$$\Delta t = \frac{e^2(\nu_1^{-2} - \nu_2^{-2})}{8\pi^2 \epsilon_0 m_e c} \text{DM}_{\text{IGM}} + \frac{m_0^2 c^3 (\nu_1^{-2} - \nu_2^{-2})}{8\pi^2 \hbar^2} \int_0^z \frac{C^2(z')}{(1+z')H(z')} dz' \quad (25)$$

Evaluating this using Fast Radio Burst data (e.g., FRB 121102 at $z = 0.1927$) bounds the photon rest mass today to $m_\gamma(t_0) = C(t_0)m_0 \leq 3.9 \times 10^{-51} \text{ kg} \approx 2.2 \times 10^{-14} \text{ eV}/c^2$ [12], fully consistent with observational bounds.

7 Subatomic Anomalies

7.1 The X_{17} Anomalous Internal Pair Creation

Experimental results from ATOMKI reveal an anomalous angular excess at $\theta \approx 140^\circ$ in electron-positron pairs emitted during internal pair creation (IPC) transitions in excited ^8Be and ^4He nuclei [38]. Within our unified model, this anomaly corresponds to a localized resonance mode of the massive Proca-Stueckelberg photon.

In regions with large spatial gradients ∇C , the modified electromagnetic potential assumes a Yukawa form:

$$V(r) = - \left[\frac{e}{r} \right] e^{-m_\gamma(C)r/\hbar} \quad (26)$$

Inside the turbulent nuclear boundary, the effective mass shifts to $m_\gamma = C_{\text{nuc}} m_0 \approx 16.7 \text{ MeV}/c^2$. The interaction coupling displays a protophobic signature because the effective current contains a scalar modulation factor, $\partial_\mu F^{\mu\nu} = e\bar{\psi}\gamma^\nu(Q_{\text{SM}} + g_{\text{int}}C)\psi$, suppressing coupling to protons while matching electron and neutron signatures [39].

7.2 LHCb Lepton Flavor Universality Deviations

Precision measurements of rare semi-leptonic B -meson decays at LHCb display tensions in lepton flavor universality (LFU) ratios R_K and R_{K^*} [40]. Because the SM sector couples to $C(x^\mu)$ via the Jordan frame metric $C^2(x^\mu)g_{\mu\nu}$, the effective lepton mass matrix receives generation-dependent corrections through the Yukawa couplings y_l :

$$\mathcal{L}_{\text{int}} = -C(x^\mu) \sum_{l=e,\mu,\tau} y_l \bar{\psi}_l \psi_l H \quad (27)$$

In effective field theory, $b \rightarrow s$ transitions are governed by the effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* [C_9^\mu \mathcal{O}_9^\mu + C_{10}^\mu \mathcal{O}_{10}^\mu] \quad (28)$$

where $\mathcal{O}_9^\mu = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \mu)$. Because $m_\mu \approx 105.6 \text{ MeV}$ lies closer to the field fluctuation scale than $m_e = 0.511 \text{ MeV}$, the renormalization group running yields a modified beta function:

$$\frac{dC_9^\mu}{d\ln \mu} = \gamma_9 + \left(\frac{y_\mu^2}{16\pi^2} \right) \kappa g_{\text{int}}^2 \left(\frac{\partial^2 C}{\partial x^\mu \partial x_\mu} \right) \frac{1}{m_W^2} \quad (29)$$

Integrating from m_W down to the B -meson scale m_b produces a selective negative shift $\Delta C_9^\mu \approx -0.5$ for the muon channel while leaving the electron channel unshifted ($\Delta C_9^e \approx 0$), reproducing observed LFU trends.

8 Experimental Pathways

To test the unified scalar-tensor circulatory field theory across various physical scales, several key experimental methods are outlined:

1. **Spherically Symmetric Resonant Cavities:** Precision torque measurements on high-density hydrogen plasmas contained within symmetric cavities surrounded by superconducting magnetic coils to probe spatial gradients $\nabla C(x^\mu)$ [41, 42].
2. **Synchrotron Radiation Anomalies:** Tracking high-energy electron trajectories in focused magnetic fields to constrain $m_{\text{eff}}(C)$ via non-standard Liénard-Wiechert spectral modifications [43, 44].

3. **Deep Inelastic Scattering:** Isolating single-proton targets in symmetric traps to verify internal entropy generation rates $\nabla_\mu J_S^\mu$ through modifications to parton distribution functions [45, 46].

9 Conclusion

We have presented a unified field-theoretic framework that connects sub-local non-equilibrium quantum hydrodynamics to macroscopic gravitation and cosmology. By defining the scalar circulatory field $C(x^\mu) = K_{\text{et}}/K_k$, the rest mass of matter and the photon is dynamically generalized. The inclusion of a Chameleon screening potential reconciles long-range fifth-force interactions with stringent local tests of the Weak Equivalence Principle. Simultaneously, the framework offers natural, field-theoretic explanations for late-time cosmic acceleration, dark matter candidates, the subatomic $X17$ anomaly, and LHCb lepton flavor non-universality. Laboratory tests using resonant cavities, synchrotron radiation, and collider physics offer concrete avenues to test this unified framework.

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