Quantum Measurement? ... MEH

Non-Markovian Information Accumulation, Residual Extremalization, and Born-Weight Conservation

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Abstract

We formulate a residual-and-information framework for continuous quantum measurement in which non-Markovian record history is represented by an exact anchored accumulation and conditional measurement ambiguity is represented by a residual whose zero set is the extremal record manifold. The construction is deliberately layered on established quantum instruments, nondemolition filtering, and quantum-trajectory theory rather than presented as a replacement for them. The central objects are the branch log-likelihood accumulators and the normalized posterior weights. Under an objective record filtration, the weights are bounded martingales. For a QND measurement, the conditional pointer variance has a universal negative predictable drift; its martingale coefficient depends on the spectrum and is written in full generality. Divergent accumulated record information then forces the posterior to an extremal simplex vertex. Martingale conservation yields the terminal Born probabilities, while the projective case gives the Lüders state. Counting and diffusive records, finite-memory Gaussian environments, and completely positive detector dilations are treated in a common framework. We also isolate what is and is not structurally new: the paper proves uniqueness only inside explicitly defined AAC/RCC admissible classes, and it separates that internal uniqueness from any historical priority claim. Finally, a no-free-actualization theorem shows that exact history re-expression, conditional filtering, and residual coordinates cannot by themselves turn deterministic closed-system unitary dynamics into an objectively unique outcome. The remaining ontological input is therefore made explicit rather than hidden.

Keywords: quantum measurement; quantum instruments; quantum trajectories; non-Markovian dynamics; quantum filtering; martingales; Lyapunov residuals; information accumulation; AAC; RCC; Born rule; Lüders rule.

1 Introduction

Quantum measurement combines several logically distinct questions: suppression of inter-sector coherence, identification of a preferred record algebra, conditional state purification, terminal outcome statistics, macroscopic record stability, and the ontological status of a single realized outcome. Quantum filtering and instrument theory already provide rigorous mathematical descriptions of conditional states for continuous observations; quantum trajectory theory supplies stochastic conditioned evolutions; and open-system theory provides a large body of Markovian and non-Markovian models. [2, 3, 4, 5, 6, 7]

The purpose of the present work is narrower and more structural. We organize the measurement dynamics around two objects. The first is an exact, causal, history-preserving representation of accumulated branch likelihood, denoted Anchored Accumulation Calculus (AAC). The second is a residual closure construction, denoted Residual Closure Calculus (RCC), whose primary scalar is the conditional uncertainty of the measured pointer observable. The intent is not to claim that conditional expectations, innovations, likelihood ratios, or stochastic Lyapunov functions are new. The intended contribution is a precise decomposition of the measurement calculation into an information clock, a residual, and a terminal-zero-set theorem.

The main chain is

$$\begin{aligned} &\text{microscopic channel} \longrightarrow \text{history likelihood} \xrightarrow{\text{AAC}} \text{posterior} \\ &\xrightarrow{\text{RCC}} \text{extremal record} \longrightarrow \text{Born weights}. \end{aligned} \tag{1.1}$$

A central result is that, for an ideal QND measurement, the conditional branch weights are bounded martingales while the pointer variance has a strictly negative predictable drift away from the distinguishable record sectors. The combination yields a clean proof of asymptotic conditional extremalization. The Born law then appears as the preserved barycentric coordinate of the terminal random vertex.

The paper also makes a hard foundational distinction. Conditional reduction relative to a physical record is a theorem of the corresponding instrument and filtering structure. The existence of one objectively realized record trajectory is not derived from a closed deterministic unitary model by changing notation. This boundary is proved explicitly.

2 Mathematical setting

2.1 Pointer decomposition

Let

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_E \tag{2.1}$$

and let the measured pointer observable be

$$S = \sum_{\alpha=1}^{r} s_\alpha P_\alpha, \qquad P_\alpha P_\beta = \delta_{\alpha\beta} P_\alpha, \qquad \sum_\alpha P_\alpha = I. \tag{2.2}$$

The projectors may have arbitrary rank. Let ρ_0 be any normalized density operator and define

$$p_\alpha = \mathrm{Tr}(P_\alpha \rho_0), \qquad p_\alpha \geq 0, \qquad \sum_\alpha p_\alpha = 1. \tag{2.3}$$

For a rank-one projector and a pure state, $p_\alpha = |c_\alpha|^2$. No purity assumption is needed in the principal theorems.

2.2 Record algebra and filtration

Let $\mathcal{A}_Y(t)$ be the commutative algebra generated by the experimentally accessible output record through time t. For a continuous record Y,

$$\mathcal{F}_t = \sigma\{Y_s : 0 \leq s \leq t\}, \tag{2.4}$$

while for a counting record N,

$$\mathcal{F}_t = \sigma\{N_s : 0 \leq s \leq t\}. \tag{2.5}$$

The use of a commutative output algebra is the standard mathematical route by which the observed output admits a classical probability representation. [1, 2, 3]

2.3 Branch laws and posterior weights

Let P^t_α denote the branch-conditioned record law on $\mathcal{F}_t$, and let P^t_* be a common dominating reference measure. Define

$$L_\alpha(t) = \frac{d\mathrm{P}^t_\alpha}{d\mathrm{P}^t_*}(Y_{0:t}), \qquad \Lambda_\alpha(t) = \log L_\alpha(t). \tag{2.6}$$

The physical mixture measure is

$$\mathrm{P}^t = \sum_\alpha p_\alpha \mathrm{P}^t_\alpha. \tag{2.7}$$

The normalized posterior sector weight is

$$W_\alpha(t) = \frac{p_\alpha L_\alpha(t)}{\sum_\beta p_\beta L_\beta(t)} = p_\alpha \frac{d\mathrm{P}^t_\alpha}{d\mathrm{P}^t}. \tag{2.8}$$

3 Anchored Accumulation Calculus (AAC)

3.1 Definition

AAC is an exact, causal, anchored representation of a history-dependent accumulation operator. In discrete time, its normal form is

$$\Lambda_{\alpha,n} = \Lambda_{\alpha,0} + \sum_{k=0}^{n-1} \Delta\Lambda_{\alpha,k}, \qquad \Delta\Lambda_{\alpha,k} = \mathrm{AACStep}_\alpha(Y_{0:k}; K_\alpha). \tag{3.1}$$

The defining requirement is exactness on the chosen model class: if the physical branch likelihood is representable by a causal history kernel, then the AAC state reproduces the same log-likelihood without changing the record measure.

For a linear causal history operator with kernel a_k,

$$A_n[Y] = \sum_{k=0}^{n-1} a_{n-1-k} Y_k, \tag{3.2}$$

and the exact anchored increment is

$$A_{n+1} - A_n = a_0 Y_n + \sum_{k=0}^{n-1} (a_{n-k} - a_{n-1-k}) Y_k. \tag{3.3}$$

The residual is the exact discrepancy between the full next-step history functional and the update represented by the retained memory state.

3.2 Structural uniqueness inside the AAC class

We now state precisely what is meant by uniqueness. It is an internal mathematical statement, not a historical priority claim.

Definition 3.1 (AAC-admissible normal form)**.** Fix a causal kernel $a = \{a_k\}_{k\geq 0}$. An AAC-admissible realization is causal, linear in the retained history, time-translation covariant, anchored at the initial datum, exact on finitely supported histories, and equipped with an explicit residual for any truncation or approximation of the retained history state.

Theorem 3.2 (Coefficient uniqueness)**.** *Within the AAC-admissible normal-form class, the physical lag coefficients are uniquely fixed by the input-output kernel. Any two minimal finite-*

dimensional exact state realizations of the same kernel are equivalent by an invertible similarity transformation.

Proof. Apply the accumulation operator to a unit impulse at the origin. Causality and translation covariance identify the response at each lag with the corresponding coefficient a_k. Exactness therefore fixes the entire coefficient sequence. For minimal finite-dimensional realizations, standard realization theory gives uniqueness up to an invertible change of state coordinates. The residual transforms covariantly with the chosen state representation. □

The theorem does not claim that convolution, state-space realizations, or memory kernels were invented here. It states only that, once the AAC admissible normal form is fixed, there is no additional coefficient freedom.

Proposition 3.3 (Genuine-memory obstruction)**.** *Suppose two histories can have the same instantaneous posterior W_t and current local observation but different memory states ξ_t, and suppose the future likelihood increment depends on ξ_t. Then no exact deterministic posterior-only update $W_{t+dt} = G(W_t, Y_{t+dt})$ exists.*

Proof. The two histories have identical arguments for G but different future likelihood increments, so exactness would require two different outputs for the same input. Hence a posterior-only closure is not single-valued. An augmented state is necessary. □

This proposition is important for the non-Markovian claim: AAC is not merely a renamed instantaneous filter equation. Its defining state carries history information that the instantaneous posterior need not contain.

4 Posterior martingale and barycentric invariance

Theorem 4.1 (Posterior martingale)**.** *For every pointer sector α, $(W_\alpha(t), \mathcal{F}_t)$ is a bounded martingale under the physical mixture measure* P*.*

Proof. From Eq. (2.8), $W_\alpha(t)$ is the conditional density of $p_\alpha \mathrm{P}_\alpha$ relative to P. Therefore the conditional Radon–Nikodym identity gives, for $s \le t$,

$$\mathrm{E}[W_\alpha(t) \mid \mathcal{F}_s] = W_\alpha(s). \tag{4.1}$$

Also $0 \le W_\alpha(t) \le 1$, so the martingale is uniformly integrable. Doob's convergence theorem yields an almost-sure and L^1 limit $W_\alpha(\infty)$. Hence

$$\mathrm{E}[W_\alpha(\infty)] = W_\alpha(0) = p_\alpha = \mathrm{Tr}(P_\alpha \rho_0). \tag{4.2}$$

□

Equation (4.2) is the barycentric invariant. It is the central probabilistic backbone of the theory.

5 Residual Closure Calculus (RCC)

5.1 Pointer variance as the primary residual

Define

$$V_S(t) = \mathrm{Tr}(\rho_t^c S^2) - \left[\mathrm{Tr}(\rho_t^c S)\right]^2. \tag{5.1}$$

After the AAC coherence residual has vanished, the conditional state is block diagonal in the pointer algebra and

$$V_S(t) = \sum_\alpha W_\alpha s_\alpha^2 - \left(\sum_\alpha W_\alpha s_\alpha\right)^2 = \frac{1}{2}\sum_{\alpha,\beta} W_\alpha W_\beta (s_\alpha - s_\beta)^2. \tag{5.2}$$

Thus $V_S \geq 0$. For a nondegenerate spectrum,

$$V_S = 0 \quad \Longleftrightarrow \quad W \in \{\mathbf{e}_1, \dots, \mathbf{e}_r\}. \tag{5.3}$$

For degenerate spectra, zero variance identifies one eigenspace rather than a unique vector inside that eigenspace.

5.2 Structural uniqueness of the quadratic residual

Definition 5.1 (RCC pairwise quadratic class)**.** Consider residuals of the form

$$R(W; s) = \sum_{\alpha<\beta} W_\alpha W_\beta q(s_\alpha - s_\beta), \tag{5.4}$$

where q is the same even, nonnegative function for every pair and is homogeneous of degree two.

Theorem 5.2 (Quadratic RCC uniqueness)**.** *Within the RCC pairwise quadratic class,*

$$q(\delta) = C\delta^2, \qquad C \geq 0, \tag{5.5}$$

and hence

$$R(W; s) = 2C\, V_S. \tag{5.6}$$

Proof. Evenness gives $q(-\delta) = q(\delta)$. Homogeneity of degree two gives $q(a\delta) = a^2 q(\delta)$. Taking $a = \delta$ relative to unit scale yields $q(\delta) = q(1)\delta^2$. Set $C = q(1) \geq 0$. Substitution into the pairwise form and Eq. (5.2) gives the result. □

The uniqueness is intentionally scoped. Outside the stated class there are other legitimate Lyapunov or residual functionals.

5.3 Diffusive measurement equation

After whitening a scalar QND output, write

$$dY_t = 2\sqrt{\kappa_{\mathrm{AAC}}(t)}\, \bar{s}_t\, dt + dB_t, \qquad \bar{s}_t = \sum_\alpha W_\alpha(t) s_\alpha, \tag{5.7}$$

with innovation

$$dI_t = dY_t - 2\sqrt{\kappa_{\mathrm{AAC}}(t)}\, \bar{s}_t dt, \qquad dI_t^2 = dt. \tag{5.8}$$

The posterior weights satisfy

$$dW_\alpha = 2\sqrt{\kappa_{\mathrm{AAC}}(t)}\, W_\alpha (s_\alpha - \bar{s}_t) dI_t. \tag{5.9}$$

The coefficient $\kappa_{\mathrm{AAC}}(t)$ is the information-production rate induced by the physical detector after the AAC reduction; it is not introduced as a free collapse constant.

5.4 General finite-dimensional variance identity

Let

$$m_k(t) = \sum_\alpha W_\alpha(t) s_\alpha^k. \tag{5.10}$$

Then Eq. (5.9) gives

$$d\bar{s}_t = 2\sqrt{\kappa_{\mathrm{AAC}}(t)}\, V_S(t)\, dI_t. \tag{5.11}$$

Applying Itô's rule to Eq. (5.1) yields the general finite-dimensional identity

$$dV_S = -4\kappa_{\mathrm{AAC}}(t) V_S^2\, dt + 2\sqrt{\kappa_{\mathrm{AAC}}(t)}\, [m_3 - \bar{s}\, m_2 - 2\bar{s} V_S]\, dI_t. \tag{5.12}$$

The first term is universal predictable RCC dissipation. The second is a spectrum-dependent local martingale. Simplified noise formulas valid for a binary observable should not be promoted to the general finite-dimensional case.

5.5 Almost-sure extremalization

Theorem 5.3 (RCC extremalization)**.** *Assume*

$$\kappa_{\mathrm{AAC}}(t) \geq 0, \qquad \int_0^\infty \kappa_{\mathrm{AAC}}(t)\, dt = \infty. \tag{5.13}$$

For a nondegenerate pointer observable, the posterior weight vector satisfies

$$W(t) \longrightarrow \mathbf{e}_K \qquad \textit{almost surely} \tag{5.14}$$

for a random $K \in \{1, \ldots, r\}$.

Proof. By Eq. (5.12), V_S is bounded, nonnegative, and has nonpositive predictable drift. Hence it is a bounded supermartingale and converges almost surely and in L^1 to V_∞. Taking expectations gives

$$\frac{d}{dt}\mathrm{E}[V_S(t)] = -4\kappa_{\mathrm{AAC}}(t)\mathrm{E}[V_S(t)^2]. \tag{5.15}$$

Therefore

$$4\int_0^\infty \kappa_{\mathrm{AAC}}(t)\mathrm{E}[V_S(t)^2] dt \leq V_S(0) < \infty. \tag{5.16}$$

Because $V_S(t)^2$ has an almost-sure limit and is bounded, its expectation converges to $\mathrm{E}[V_\infty^2]$. Condition (5.13) then forces $\mathrm{E}[V_\infty^2] = 0$, hence $V_\infty = 0$ almost surely. Equation (5.3) gives $W(\infty) = \mathbf{e}_K$. □

6 Born statistics and Lüders closure

Theorem 6.1 (Born-weight conservation)**.** *Under the hypotheses of the posterior martingale and RCC extremalization theorems,*

$$\boxed{\mathrm{P}(K = \alpha) = \mathrm{Tr}(P_\alpha \rho_0).} \tag{6.1}$$

Proof. RCC extremalization gives $W_\alpha(\infty) = \mathbf{1}_{\{K=\alpha\}}$. Equation (4.2) and L^1 convergence then give

$$\mathrm{P}(K = \alpha) = \mathrm{E}[\mathbf{1}_{\{K=\alpha\}}] = \mathrm{E}[W_\alpha(\infty)] = W_\alpha(0) = \mathrm{Tr}(P_\alpha \rho_0). \tag{6.2}$$

□

The logically precise statement is that the Born rule is the terminal barycentric invariant of the conditional branch process. The Hilbert-space probability assignment $\mathrm{Tr}(P_\alpha \rho_0)$ remains part of the standard quantum state/instrument structure rather than being derived from no quantum probabilistic premise whatsoever.

Theorem 6.2 (Conditional Lüders state)**.** *If the measurement is projective and QND on S, and sector K is realized, then*

$$\boxed{\rho_K = \frac{P_K \rho_0 P_K}{\mathrm{Tr}(P_K \rho_0)}.} \tag{6.3}$$

Proof. The asymptotic conditional weight concentrates on K. Projective conditioning removes inter-sector blocks while retaining the normalized state within the selected sector, yielding Eq. (6.3). □

For rank-one P_K, Eq. (6.3) reduces to $\rho_K = P_K$. For a degenerate sector, the theory does not manufacture distinctions absent from the measured record.

The unconditional state is

$$\mathrm{E}[\rho_c(\infty)] = \sum_\alpha P_\alpha \rho_0 P_\alpha, \tag{6.4}$$

which reconciles the conditional definite record with the decohered ensemble state.

7 Microscopic non-Markovian measurement channel

7.1 Quantum bath kernels

Consider

$$H_{\mathrm{int}} = gS \otimes B \tag{7.1}$$

with a stationary Gaussian environment and zero mean $\langle B(t) \rangle = 0$. Define

$$\nu(\tau) = \frac{1}{2}\langle \{B(\tau), B(0)\} \rangle, \qquad \chi(\tau) = \frac{i}{\hbar}\theta(\tau)\langle [B(\tau), B(0)] \rangle. \tag{7.2}$$

The symmetrized kernel controls the fluctuation/noise sector; the causal response kernel controls response, phase, and back-action. In a thermal stationary state these are related by the fluctuation–dissipation relation. [6, 7, 8]

For pointer sectors α, β, the real coherence-suppression exponent has the generic form

$$\Gamma_{\alpha\beta}(t) = \frac{g^2}{\hbar}(s_\alpha - s_\beta)^2 \int_0^t ds \int_0^s du\, \nu(s-u), \tag{7.3}$$

with convention-dependent prefactors. AAC retains this finite-history object as an explicit accumulation rather than replacing it immediately by a delta kernel.

7.2 Exponential-memory benchmark

For

$$\nu(\tau) = \nu_0 e^{-|\tau|/\tau_c}, \tag{7.4}$$

one obtains

$$\Phi(t) = \nu_0 \left[\tau_c t - \tau_c^2 \left(1 - e^{-t/\tau_c}\right)\right]. \tag{7.5}$$

Hence

$$\Phi(t) \sim \frac{\nu_0}{2} t^2, \quad t \ll \tau_c, \qquad \Phi(t) \sim \nu_0 \tau_c t, \quad t \gg \tau_c. \tag{7.6}$$

The same kernel therefore produces a memory-dominated early regime and a linear asymptotic information clock.

7.3 Exact completely positive memory dilation

A Lorentzian bath admits an exact pseudomode-type representation in an enlarged system. A representative dilation is

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_S + H_{\mathrm{pm}}, \rho] + \lambda \mathcal{D}[a]\rho, \tag{7.7}$$

with

$$H_{\mathrm{pm}} = \hbar\omega_0 a^\dagger a + gS(a + a^\dagger). \tag{7.8}$$

The enlarged dynamics are CPTP. Monitoring the auxiliary output therefore produces a legitimate instrument. This establishes physical admissibility at the dilation level instead of guessing a non-Markovian reduced equation. Related exact and trajectory-based constructions are standard in open-system theory. [6, 9, 7]

7.4 AAC information clock

For a Gaussian record with branch means μ_α and covariance operator N_t, the branch distinguishability is

$$D_{\alpha\beta}(t) = \frac{1}{2}\langle \mu_\alpha - \mu_\beta, N_t^{-1}(\mu_\alpha - \mu_\beta)\rangle. \tag{7.9}$$

If $\mu_\alpha - \mu_\beta = (s_\alpha - s_\beta)h$, define

$$D_{\alpha\beta}(t) = (s_\alpha - s_\beta)^2 \mathcal{I}_{\mathrm{AAC}}(t), \qquad \mathcal{I}_{\mathrm{AAC}}(t) = \frac{1}{2}\langle h, N_t^{-1}h\rangle. \tag{7.10}$$

The associated information clock is

$$\tau_{\mathrm{AAC}}(t) = \int_0^t \kappa_{\mathrm{AAC}}(u)\, du, \tag{7.11}$$

with κ_{AAC} determined by the physical detector response and noise after the AAC history reduction.

8 Counting records and record-class universality

8.1 Counting posterior

For a counting record with branch intensity $\lambda_\alpha(t)$, define

$$\bar{\lambda}_t = \sum_\alpha W_\alpha(t)\lambda_\alpha(t), \qquad \mathrm{E}[dN_t \mid \mathcal{F}_t] = \bar{\lambda}_t dt. \tag{8.1}$$

The compensated innovation is

$$dI_t = dN_t - \bar{\lambda}_t dt. \tag{8.2}$$

The exact posterior update is

$$dW_\alpha = W_\alpha \left(\frac{\lambda_\alpha}{\bar{\lambda}_t} - 1\right)(dN_t - \bar{\lambda}_t dt). \tag{8.3}$$

Its conditional drift is zero, so the bounded martingale theorem applies exactly as in the diffusive case. Counting and diffusive records are standard components of quantum trajectory theory. [4, 5, 3]

8.2 Point-process information

For sectors α, β,

$$\Lambda_{\alpha\beta}(t) = \int_0^t \log \frac{\lambda_\alpha(s)}{\lambda_\beta(s)}\, dN_s - \int_0^t (\lambda_\alpha(s) - \lambda_\beta(s))ds. \tag{8.4}$$

Under branch α, the predictable relative-entropy density is

$$\mathcal{J}_{\alpha\beta}(t) = \lambda_\alpha \log \frac{\lambda_\alpha}{\lambda_\beta} - \lambda_\alpha + \lambda_\beta \geq 0, \tag{8.5}$$

with equality iff $\lambda_\alpha = \lambda_\beta$. Thus asymptotic record identification is equivalent to divergent accumulated point-process information between distinguishable sectors.

Theorem 8.1 (AAC–RCC record universality)**.** *Let a physical record admit an exact branch log-likelihood representation* $\Lambda_\alpha(t) = \mathrm{AACAcc}_\alpha[Y_{0:t}]$. *Suppose the corresponding instrument is valid, the record filtration is objective, inter-sector coherence is asymptotically extinguished, and every physically distinguishable pair accumulates infinite relative information. Then the posterior weights converge almost surely to a single distinguishable record sector, and the terminal probabilities are* $\mathrm{Tr}(P_\alpha \rho_0)$.

Proof. The posterior is a bounded martingale. Infinite pairwise information forces posterior odds between distinct observable sectors to separate. The limiting posterior is therefore an extremal point of the distinguishable-sector simplex. The martingale identity fixes the probability of that terminal sector. □

9 Why AAC and AAC–RCC are structurally distinct

9.1 Three different originality questions

Three claims must be separated.

1. Historical priority: whether the underlying mathematical ideas appeared earlier is a literature question.

2. Internal mathematical uniqueness: whether the proposed normal form is fixed inside its stated admissible class is an internal theorem.

3. Non-derivation: whether the framework contains information that cannot be recovered from a restricted instantaneous description is a structural statement, proved here by the genuine-memory obstruction.

Accordingly, this paper makes no claim that martingales, conditional expectations, quantum filters, quantum instruments, innovations, or stochastic Lyapunov functions originated here. These are established components of the subject. [2, 3, 4, 5]

9.2 AAC versus standard filtering

A standard quantum filter starts from an observation model and produces the conditional state or observable. [2, 3] AAC instead starts from the accumulated history operator that generates the branch likelihood and demands an exact anchored representation of that history state. Symbolically,

$$\text{filter:} \qquad \text{observation law} \to \text{conditional state}, \tag{9.1}$$

$$\text{AAC}: \qquad \text{history kernel} \to \text{exact accumulated state} + \text{residual}. \tag{9.2}$$

In the Markov limit, the AAC memory state can reduce to the local innovation representation. Away from that limit, the defining AAC object retains explicit memory information. Proposition 3.2 shows why this can be necessary: a posterior-only description need not be exact when two histories share the same posterior but possess different future likelihood increments.

9.3 RCC versus generic Lyapunov analysis

Generic stochastic stability theory allows many Lyapunov functions. RCC chooses its principal residual from the physical measurement geometry: the residual must vanish on precisely the indistinguishable pointer-sector manifold. Theorem 5.2 proves uniqueness, up to an overall scale, within the explicitly stated pairwise-even-degree-two class. The actual measurement filter then supplies the negative predictable drift in Eq. (5.12).

9.4 The composite normal form

The proposed composite is

$$\begin{aligned}&\text{microscopic detector} \to \text{history kernel} \to \text{AAC likelihood state}\\ &\to \text{record posterior} \to \text{RCC residual} \to \text{extremal record.}\end{aligned}$$

The novelty claim is therefore about the decomposition and its exact residual/clock bookkeeping, not about replacing the standard theory of conditional quantum dynamics.

9.5 What would count as derivative

The framework would be derivative in the strong sense if the AAC memory state were shown to be no more than a cosmetic reparameterization with no retained history variable, or if the RCC residual were merely a renamed standard Lyapunov function with no physical zero-set selection. The genuine-memory obstruction and the scoped uniqueness theorem are included precisely to make those possibilities testable.

9.6 What is actually proved

The provable structural statements are:

1. the AAC coefficient sequence is uniquely fixed by its causal kernel within the admissible exact normal form;
2. the quadratic RCC residual is unique up to scale inside its explicitly stated pairwise class;
3. exact non-Markovian posterior closure may require a memory state that is not reconstructible from the instantaneous posterior;
4. the same physical information clock controls finite-time residual reduction while the terminal barycentric law remains invariant;
5. conditional extremalization and Born weights follow jointly from the posterior martingale and the residual-zero endpoint.

None of these propositions is a claim of historical priority. They are internal mathematical statements about the proposed formulation.

10 Quantum instruments, complete positivity, and no-signalling

The correct positivity statement belongs at the instrument level. Let a record-conditioned instrument be

$$\mathcal{I}_y(\rho) = M_y \rho M_y^\dagger, \qquad \int M_y^\dagger M_y \, d\mu(y) = I. \tag{10.1}$$

Then the unconditional channel

$$\mathcal{E}(\rho) = \int \mathcal{I}_y(\rho) d\mu(y) \tag{10.2}$$

is completely positive and trace preserving. The normalized conditional state is

$$\rho_y = \frac{M_y \rho M_y^\dagger}{\mathrm{Tr}(M_y^\dagger M_y \rho)}. \tag{10.3}$$

The conditional state is nonlinear because of normalization; the underlying instrument is linear.

For a local instrument on subsystem A,

$$\mathcal{E}_{AB}(\rho_{AB}) = (\mathcal{E}_A \otimes I_B)(\rho_{AB}). \tag{10.4}$$

Tracing over A after averaging over the local record leaves the unconditional reduced state of B unchanged. Hence the AAC–RCC formulation must be implemented through a legitimate instrument or an equivalent dilation; an arbitrary nonlinear reduced equation is not sufficient for physical consistency.

10.1 General POVMs

For a POVM $\{E_y\}$,

$$E_y \geq 0, \qquad \int E_y \, dy = I, \qquad E_y = M_y^\dagger M_y, \tag{10.5}$$

and

$$p(y) = \mathrm{Tr}(E_y \rho_0), \qquad \rho_y = \frac{M_y \rho_0 M_y^\dagger}{\mathrm{Tr}(E_y \rho_0)}. \tag{10.6}$$

The projective/Lüders case is recovered for sharp effects $E_i = P_i$.

11 Macroscopic amplification and record objectivity

Let $\mathcal{B}_\alpha^{(N)}$ be the macroscopic basin associated with sector α for an apparatus with N amplifying degrees of freedom. A sufficient stability condition is a large-deviation estimate

$$\mathrm{P}_\alpha(\mathcal{B}_\beta^{(N)}) \lesssim \exp(-N\Delta_{\alpha\beta}), \qquad \Delta_{\alpha\beta} > 0, \qquad \alpha \neq \beta. \tag{11.1}$$

This implies exponentially small misidentification and switching probabilities. It provides a quantitative route to record robustness, while not claiming that a globally unitary state is literally reduced to one branch.

If the environment is partitioned into fragments E_r carrying redundant information about the same terminal sector K, then sufficiently informative observers have

$$W_\alpha^{(r)}(t) \longrightarrow \mathbf{1}_{\{K=\alpha\}}. \tag{11.2}$$

Agreement across many fragments is an operational criterion for record objectivity. AAC can represent the history-dependent buildup of this redundancy, while RCC controls the unresolved branch residual.

12 Falsifiable dynamical content

The central separation is

$$\text{terminal statistics} \quad \neq \quad \text{finite-time dynamics.} \tag{12.1}$$

The terminal ideal law is

$$\mathrm{P}(K = \alpha) = \mathrm{Tr}(P_\alpha \rho_0), \tag{12.2}$$

whereas the finite-time residual depends on coupling strength, detector bandwidth, efficiency, memory kernel, and record class. For the exponential kernel in Eq. (7.5), the early-time information accumulation is quadratic in time and the long-time clock is linear.

Thus the experimentally meaningful quantity is not a new terminal probability law but a memory-dependent measurement transient, for example the time-to-threshold

$$\tau_\varepsilon = \inf\{t : V_S(t) \leq \varepsilon\}. \tag{12.3}$$

Different detector spectra can give different $V_S(t)$ and information clocks while preserving the same terminal ideal Born probabilities.

12.1 Finite-information regime

If

$$\int_0^\infty \kappa_{\mathrm{AAC}}(t)dt < \infty, \tag{12.4}$$

then the sufficient condition for complete extremalization fails. The theory therefore allows imperfect measurement with a nonzero residual rather than forcing an artificial collapse endpoint.

12.2 Quantum detector limits

A microscopic detector has imprecision, back-action, cross-correlation, and response functions subject to quantum constraints. In the frequency domain these impose a positive-semidefinite noise matrix together with commutator/response constraints. AAC reorganizes the temporal accumulation of information but does not remove these physical limits. A quantum-limited detector corresponds to saturation of the relevant detector inequality; an inefficient detector has a smaller observed information rate.

13 Discrete event actualization and detector memory

A counting detector with branch-dependent intensity $\lambda_\alpha(t)$ admits an especially transparent microscopic interpretation. If the detector output arises from monitoring a physical field channel with jump operator L, then

$$\lambda_t = \mathrm{Tr}(L^\dagger L \rho_t^c). \tag{13.1}$$

For $L = \sum_\alpha \ell_\alpha P_\alpha$,

$$\lambda_\alpha = |\ell_\alpha|^2. \tag{13.2}$$

The event record is then a physical counting process rather than an abstract random symbol. History dependence enters through the detector memory state; AAC compresses that history into a sufficient state when an exact finite realization exists.

For a Lorentzian bath, one may use the pseudomode amplitude $\alpha_\alpha(t)$ obeying

$$\dot{\alpha}_\alpha = -(\lambda + i\omega_0)\alpha_\alpha - igs_\alpha. \tag{13.3}$$

The output intensity is a function of α_α. In a phase-sensitive or locally displaced detector, the intensity can be made linearly sensitive to s_α, whereas an undisplaced intensity detector may respond only to s_α^2. This distinction is physical and prevents AAC–RCC from claiming more branch information than the detector actually carries.

14 Foundational boundary: no-free-actualization

Conditional purification and objective actualization are different statements. The former is a theorem about a record-conditioned instrument. The latter asserts that one particular record trajectory is physically realized.

Theorem 14.1 (No-free-actualization)**.** *No deterministic affine map compatible with closed linear unitary quantum dynamics can map every nontrivial mixture $\sum_\alpha p_\alpha P_\alpha$ to one of its extreme components while leaving every P_α fixed.*

Proof. If F is affine and $F(P_\alpha) = P_\alpha$, then

$$F\left(\sum_\alpha p_\alpha P_\alpha\right) = \sum_\alpha p_\alpha F(P_\alpha) = \sum_\alpha p_\alpha P_\alpha. \tag{14.1}$$

For a nontrivial mixture this is not an extreme point. Therefore deterministic affine unitary-compatible evolution cannot supply objective single-outcome selection under these conditions. □

AAC does not evade this obstruction by re-expressing history, and RCC does not evade it by changing residual coordinates. Hence one must specify either an objective record-realization principle, genuinely stochastic/nonlinear dynamics, or additional ontology if objective single-outcome actualization is required.

14.1 Precise closure statement

The mathematically closed implication of the present paper is

$$\begin{aligned}&\text{objective physical record} + \text{ valid instrument}\\&+\text{AAC history closure} + \text{RCC information dissipation}\\&\Longrightarrow \text{definite conditional sector} + \text{Born/Lüders statistics.}\end{aligned}$$

The paper does not claim

$$\text{closed deterministic unitary dynamics alone} \implies \text{one objectively realized outcome.} \tag{14.2}$$

15 Discussion

The framework yields a clean separation of roles. AAC retains and organizes the causal memory of the physical record. The record likelihood becomes a finite or augmented state when the chosen kernel admits such a realization. RCC identifies the physical residual left unresolved by that record and proves, for the QND class, that the pointer variance has a negative predictable drift. The terminal state is determined by the zero set of the residual. The posterior martingale then preserves the initial barycentric weights, which become the probabilities of the terminal extremal records.

Three points are especially important. First, the framework is compatible with standard filtering and instrument theory; it is not a proposal to replace them. Second, its originality claims are deliberately internal: uniqueness is proved only within explicit admissible classes.

Third, the ontological boundary is not hidden. Objective actualization is isolated as the only ingredient that cannot be manufactured from a deterministic affine reparameterization of closed unitary dynamics.

A useful experimental programme is therefore to compare detectors with identical pointer projectors but different memory spectra. The predicted terminal statistics remain unchanged, while the finite-time information clock, residual decay, and time-to-resolution can differ. This separates the universal statistical endpoint from memory-sensitive dynamical signatures.

16 Conclusions

The AAC–RCC construction provides a compact structural closure for non-Markovian continuous measurement. The branch likelihoods form a bounded posterior martingale. The conditional pointer variance is a nonnegative RCC residual whose predictable drift is strictly negative away from the distinguishable-sector manifold. Divergent accumulated information forces the residual to zero and the posterior to an extremal sector. Martingale invariance then yields

$$\mathrm{P}(K = \alpha) = \mathrm{Tr}(P_\alpha \rho_0), \tag{16.1}$$

while projective conditioning yields the corresponding Lüders state.

The principal claim is therefore not that the elementary mathematics of filtering is new. The claim is that an exact anchored memory state and a residual-zero closure can be used as a unified normal form for non-Markovian measurement dynamics, with explicit structural uniqueness statements and a sharply isolated foundational boundary. The remaining ontological question is whether the realized record trajectory is fundamental or derivable from a deeper theory. That question is left explicit rather than hidden inside the notation.

A Binary observable as a special case

For a two-level observable with

$$s_1 = +s, \qquad s_2 = -s, \tag{A.1}$$

write $W_1 = w$, $W_2 = 1 - w$. Then

$$V_S = 4s^2 w(1 - w). \tag{A.2}$$

The general variance identity collapses to the familiar binary form, and the residual vanishes only at $w = 0$ or $w = 1$. This is a special simplification, not the general finite-dimensional law.

B Mixed and degenerate measurements

For an arbitrary mixed state ρ_0 and a degenerate spectral sector P_α, the terminal conditional state is

$$\rho_\alpha = \frac{P_\alpha \rho_0 P_\alpha}{\mathrm{Tr}(P_\alpha \rho_0)}, \qquad \mathrm{P}(K = \alpha) = \mathrm{Tr}(P_\alpha \rho_0). \tag{B.1}$$

The measured record cannot distinguish states inside the same observational equivalence class. Thus the residual-zero manifold is the quotient by physically accessible record information.

C Assumption audit

Ingredient	Mathematical status	Physical status
AAC history representation	Exact construction within chosen model class	No independent probability postulate
Pointer coherence closure	Theorem under kernel/information conditions	Depends on physical coupling
Posterior martingale	Theorem from conditional Radon–Nikodym structure	No
RCC variance dissipation	Theorem for the stated measurement filter	No
Born terminal weights	Theorem from martingale + extremalization	Standard initial quantum weighting remains in the state
CPTP instrument	Required physical admissibility condition	Yes, as detector model structure
Objective record realization	Not derivable from deterministic affine unitary re-expression	Irreducible foundational input in the present framework

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