

Proof of Legendre's Conjecture

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Abstract

This paper attempts to prove Legendre's conjecture through mathematical operations involving limiting cases.

Keywords: Legendre; conjecture; proof

Introduction

Legendre's conjecture is a significant problem in the field of number theory. This paper attempts to prove the conjecture by employing mathematical derivations and operations applicable to specific cases.

1. Mathematical Derivation Process

The mathematician Legendre proposed that for any positive integer $n \geq 1$. There exists at least one prime number between n^2 and $(n+1)^2$.^{1,2,3} This paper assumes that an arbitrary positive integer $n = +\infty$. Therefore, we can derive the following mathematical formula:

$$\begin{aligned} n^2 &= +\infty^2 \\ (n+1)^2 &= (+\infty+1)^2 \end{aligned}$$

Based on the mathematical formula above and Legendre's conjecture, we can conclude that:

$$+\infty^2 - (+\infty+1)^2 = +\infty^2 - +\infty^2 = 0$$

Based on the mathematical derivation above, we can conclude that when any positive integer equals $+\infty$, There is no number between n^2 and $(n+1)^2$, and n^2 and $(n+1)^2$ are equal. Therefore, it is possible that there is no prime number between n^2

and $(n+1)^2$, and that Legendre's conjecture does not hold.

2. Conclusion

This paper demonstrates that Legendre's conjecture does not hold when the arbitrary positive integer in the conjecture is taken to be positive infinity.

References

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