Black-Hole Entropy Bridges in a CPT-Symmetric Cosmology

A quantitative flux-stabilization bound, entropy model, and falsification hierarchy

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Abstract

We test a speculative CPT Entropy Bridge hypothesis with explicit junction, information-theory, stability-response, and probe-field calculations. A symmetric Schwarzschild thin-shell throat has a static domain-wall point at $a_0 = 3GM$ with $E_{\rm shell} = -2\sqrt{3}M$ and $V''(a_0) = -2/(9G^2M^2)$, demonstrating macroscopic negative energy and radial instability in the minimal classical realization. For a globally pure two-branch Schmidt state, branch entropy is $H_2(q)$ while mutual information exactly restores global purity, making precise how branch-restricted entropy can coexist with unitary information accounting. We then test, rather than merely posit, the defect-EFT stabilization requirement. Allowing a normal energy-flux response $\mathcal{F}(a)$ yields the exact necessary radial-stability bound $\mathcal{F}'(3GM) > [18\sqrt{3}\pi G^4M^3]^{-1}$; in dimensionless form the response coefficient must satisfy $\kappa > 1.02 \times 10^{-2}$. A minimal curvature/scalar defect basis is given, while we show that a healthy canonical homogeneous scalar alone does not explain the required negative surface energy. A regulated-core WKB probe has finite high-energy dwell time, isolating the demonstrated instability in the background rather than automatically in probe propagation. We further separate the empirical predictions of the Boyle–Finn–Turok cosmology from the additional black-hole hypothesis, formulate a branch-local generalized-second-law criterion, and show that a strictly interior bridge has zero classical black-hole-shadow correction unless exterior Wilson coefficients are activated. CPT pairing alone likewise predicts no local CP/T asymmetry; such a signal requires a specified portal operator. Current GWTC-4.0 echo null tests and O4-era stochastic-background limits therefore constrain only completed nonzero exterior-response models. The result is a hierarchy of quantitative necessary conditions rather than evidence for astrophysical portals: the simplest classical realization fails, while any quantum completion must overcome explicit energy, flux-response, stability, causal, algebraic, and observational consistency tests.

1 Scope, claims, and what changed

The proposal is deliberately narrower than a claim that astrophysical black holes are portals. Its purpose is to ask whether a CPT-related cosmology can support a mathematically controlled continuation of black-hole information into a conjugate semiclassical description while preserving global quantum information. The earlier formulation was correctly criticized for containing placeholders where a specialized research article should contain at least one calculation. This revision therefore promotes two calculations to central results: an exact spherical junction calculation and an explicit two-branch entropy model.

The claim hierarchy is now:

1. **Level I (background hypothesis):** two cosmological sectors are related by CPT, in the structural spirit of Boyle–Finn–Turok [1].
2. **Level II (demonstrated toy bookkeeping):** reduced branch entropy can grow or disappear from branch access while global information is retained in cross-branch correlations. Section 4 proves this in a finite-dimensional model.
3. **Level IIIa (demonstrated classical obstruction):** a symmetric Schwarzschild bridge can be written as a thin-shell junction, but the exact shell stress is exotic and a natural static point is unstable. Section 5 derives this result.
4. **Level IIIb (open conjecture):** a controlled quantum-gravity completion supplies a nonsingular trapped-to-anti-trapped continuation and may enlarge the replica saddle class. No such completion is derived here.

The manuscript is therefore a constrained toy-model paper with an explicit no-go-style result, not a claimed theory of nature.

2 CPT-paired cosmological background

Let

$$\mathcal{M} = \mathcal{M}_- \cup \Sigma_0 \cup \mathcal{M}_+, \qquad (\mathcal{M}_-, \Psi_-) = \mathrm{CPT}(\mathcal{M}_+, \Psi_+), \tag{1}$$

where Σ_0 is a distinguished cosmological matching surface. In the published CPT-symmetric-universe construction, pre- and post-bang epochs are CPT images and the proposal has particle-physics consequences [1]. We borrow only this structural relation; the black-hole bridge introduced below is an additional conjecture. We borrow only this structural relation; the black-hole bridge introduced below is an additional conjecture. Importantly, the bridge does not inherit empirical support merely by citing the parent cosmology. The Boyle–Finn–Turok (BFT) realization makes additional mandatory claims, including a massless lightest neutrino, Majorana neutrinos, and absence of primordial long-wavelength gravitational waves [1]. Those claims are independently testable and could exclude that particular parent realization without deciding the black-hole junction calculation below. Conversely, validating a junction mechanism would not validate the pre-bang/post-bang BFT construction. We therefore treat "CPT pairing" as a boundary-condition hypothesis and keep its cosmological and black-hole likelihoods logically separate.

The required relationship is only this: the defect data on a black-hole transition surface must transform under the same antiunitary CPT operator used to identify conjugate cosmological data. There is no assumed causal signal from the Big Bang surface to an astrophysical hole and no requirement that a collapsing star select a preassigned point in the pre-bang branch.

For a homogeneous metric,

$$ds^2 = -dt^2 + a^2(t)d\Sigma_k^2, \tag{2}$$

a symmetric idealization is $a(-t) = a(t)$. Coarse-grained arrows may point away from Σ_0 on both sides,

$$\frac{dS_{\mathrm{cg}}^{(+)}}{d\tau_+} \geq 0, \qquad \frac{dS_{\mathrm{cg}}^{(-)}}{d\tau_-} \geq 0, \tag{3}$$

without requiring a single global coordinate to coincide with both observers' thermodynamic futures.

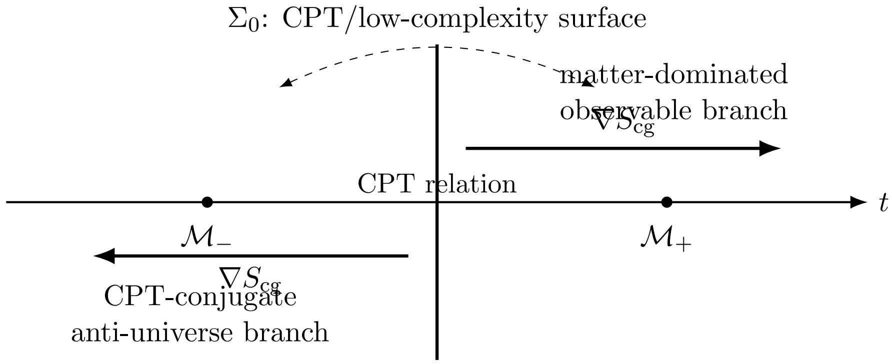

Figure 1: Kinematic two-branch picture. The arrows indicate each branch's thermodynamic future, not motion of matter through the Big Bang.

3 Entropy bridge as a state map

For a black-hole sector B_+ and radiation R_+, define the candidate bridge map

$$\mathfrak{B} : \mathcal{H}_{B_+} \otimes \mathcal{H}_{R_+} \to \mathcal{H}_{W_-} \otimes \mathcal{H}_{R_-}. \tag{4}$$

A complete physical map is required to be isometric,

$$\mathfrak{B}^\dagger \mathfrak{B} = I, \tag{5}$$

and CPT covariance is imposed schematically as

$$\mathfrak{B}^{-1} = \Theta_{\mathrm{CPT}} \mathfrak{B} \Theta_{\mathrm{CPT}}^{-1}. \tag{6}$$

Equation (5) prevents the phrase "entropy bridge" from meaning nonunitary disposal.

4 Zero-order entropy result: branch entropy and correlations

4.1 Definition in a factorizing toy limit

Gravity complicates exact Hilbert-space factorization because gauge constraints tie spatial regions together. We therefore state the following result only for a finite-dimensional factorizing model, then use it as the definition that any algebraic gravitational generalization must reproduce in an appropriate limit.

Let the global state be pure, $\rho_{+-} = |\Psi\rangle\langle\Psi|$, and define

$$\rho_+ = \mathrm{Tr}_- \rho_{+-}, \qquad \rho_- = \mathrm{Tr}_+ \rho_{+-}, \tag{7}$$

with branch entropies $S_+ = -\mathrm{Tr}\rho_+ \ln \rho_+$ and S_- similarly. The mutual information is

$$I(+:-) = S_+ + S_- - S_{+-}. \tag{8}$$

For a pure global state $S_{+-} = 0$, so

$$S_+ + S_- - I(+:-) = 0. \tag{9}$$

Thus a precise toy definition of the previously vague correlation correction is

$$S_{\mathrm{corr}}^{(+-)} \equiv -I(+:-). \tag{10}$$

It is not an independent entropy reservoir; it subtracts double-counted correlations when branch entropies are summed.

4.2 Explicit Schmidt-pair example

Take

$$|\Psi(q)\rangle = \sqrt{q}\,|0_+0_-\rangle + \sqrt{1-q}\,|1_+1_-\rangle, \quad 0 \le q \le 1. \tag{11}$$

Then

$$S_+ = S_- = H_2(q) = -q\ln q - (1-q)\ln(1-q), \tag{12}$$

and

$$I(+:-) = 2H_2(q), \qquad S_+ + S_- + S_{\rm corr}^{(+-)} = 0. \tag{13}$$

At $q = 1/2$, each branch has entropy $\ln 2$ although the global state is pure. An observer restricted to + therefore sees a mixed state even though no information has been destroyed. This directly answers how “individual branch entropy” is defined in the simplest version of the model.

4.3 Generalized entropy statement

In a semiclassical regime with horizon surfaces $\gamma_\pm$, the proposed bookkeeping functional is

$$S_{\rm book} = \sum_{s=\pm}\left[\frac{A(\gamma_s)}{4G} + S_{\rm alg}(\mathcal{A}_s)\right] - I(\mathcal{A}_+ : \mathcal{A}_-), \tag{14}$$

where $S_{\rm alg}$ denotes an algebraic entropy appropriate to the chosen gauge-invariant subalgebra. Equation (14) is a definition/target, not a theorem of quantum gravity. In a factorizing nongravitational limit it reduces to Eq. (9). A branchwise GSL is imposed only where a semiclassical causal horizon and its algebra are well defined.

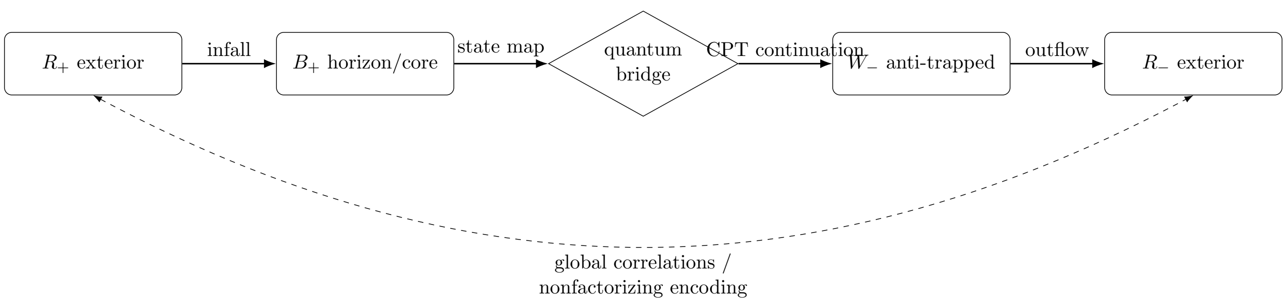

Figure 2: The proposed bridge is an information-preserving state map, not an entropy deletion channel.

5 Zero-order junction result: the simplest bridge is exotic

5.1 Symmetric cut-and-paste geometry

We now calculate, rather than merely list, the classical obstruction. Work in $c = \hbar = k_B = 1$ and retain G. Take two identical Schwarzschild exteriors

$$ds_\pm^2 = -f(r)dt_\pm^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \qquad f(r) = 1 - \frac{2GM}{r}, \tag{15}$$

cut both at $r = a(\tau) > 2GM$, and glue their boundaries with outward normals pointing away from the throat. This is a standard thin-shell/wormhole construction governed by the Israel equations [2, 3].

For surface stress $S^i{}_j = \mathrm{diag}(-\sigma, p, p)$, the exact junction equations are

$$\sigma(a) = -\frac{1}{2\pi G a}\sqrt{f(a) + \dot{a}^2}, \tag{16}$$

$$p(a) = \frac{1}{4\pi G}\left[\frac{\ddot{a} + GM/a^2}{\sqrt{f(a) + \dot{a}^2}} + \frac{\sqrt{f(a) + \dot{a}^2}}{a}\right]. \tag{17}$$

The negative sign in Eq. (16) is not a convention that can be removed while retaining this orientation: the symmetric throat violates the null/weak energy conditions at the shell.

5.2 An exact static point

For $\dot{a} = \ddot{a} = 0$,

$$\sigma_0 = -\frac{\sqrt{1 - 2GM/a_0}}{2\pi G a_0}, \tag{18}$$

$$p_0 = \frac{1 - GM/a_0}{4\pi G a_0\sqrt{1 - 2GM/a_0}}. \tag{19}$$

The equation of state ratio is

$$w_0 \equiv \frac{p_0}{\sigma_0} = -\frac{1 - GM/a_0}{2(1 - 2GM/a_0)}. \tag{20}$$

A domain-wall value $w_0 = -1$ occurs exactly at

$$a_0 = 3GM. \tag{21}$$

At that point,

$$\boxed{\sigma_0 = -\frac{1}{6\pi\sqrt{3}\,G^2 M}}, \qquad \boxed{p_0 = +\frac{1}{6\pi\sqrt{3}\,G^2 M}}, \tag{22}$$

and the integrated shell energy is

$$\boxed{E_{\text{shell}} = 4\pi a_0^2 \sigma_0 = -2\sqrt{3}\,M}. \tag{23}$$

This is a useful zero-order result: for this simplest symmetric bridge, the magnitude of negative shell energy is of order the black-hole mass itself, not a tiny Planckian correction.

5.3 Stability of the constant-tension continuation

Shell conservation without external flux gives

$$\frac{d}{d\tau}(\sigma a^2) + p\frac{d}{d\tau}(a^2) = 0. \tag{24}$$

For $p = w\sigma$, $\sigma \propto a^{-2(1+w)}$. The symmetric junction can be written

$$\dot{a}^2 + V(a) = 0, \qquad V(a) = f(a) - [2\pi G a \sigma(a)]^2. \tag{25}$$

For the constant-tension continuation $w = -1$ through $a_0 = 3GM$, $V(a_0) = V'(a_0) = 0$, but

$$\boxed{V''(a_0) = -\frac{2}{9G^2M^2} < 0}. \tag{26}$$

Hence the equilibrium is a local maximum of V and is linearly unstable. The toy geometry therefore fails two desired properties simultaneously: ordinary energy conditions and stable static support.

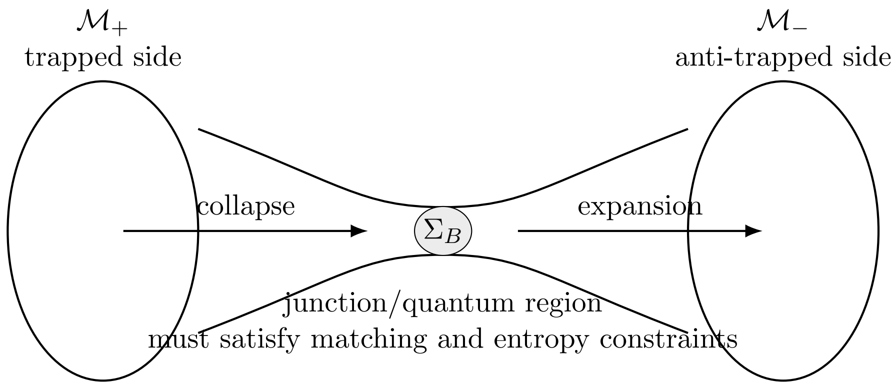

Figure 3: Schematic bridge geometry. The drawing is not a Penrose diagram and makes no claim of classical traversability.

5.4 Quantum-energy-inequality scaling test

Negative energy in quantum field theory is constrained in magnitude and duration by quantum inequalities; applications to wormhole geometries show that macroscopic throats typically demand Planck-scale throats or extreme scale separation [4, 5]. To expose the scaling rather than claim a curved-spacetime theorem, smooth the shell over proper width ℓ and estimate $\rho_- \sim \sigma_0/\ell$. A four-dimensional sampling inequality has schematic flat-space form

$$\int d\tau \, g_{\tau_0}(\tau) \, \langle T_{uu} \rangle \gtrsim -\frac{C}{\tau_0^4}. \tag{27}$$

Taking $\tau_0 \sim \ell$ gives the necessary scaling

$$|\sigma_0| \, \ell^3 \lesssim C. \tag{28}$$

Because $|\sigma_0| \propto (G^2 M)^{-1}$, satisfying Eq. (28) for a macroscopic M pushes the support into an increasingly thin layer in Planck units. This is not a rigorous AEQI bound for the bridge background; it is a diagnostic showing exactly where a candidate quantum stress tensor must outperform the classical shell model.

6 From an arbitrary matching action to a constrained defect EFT

The previous action contained an unspecified I_{qg} capable of hiding arbitrary dynamics. We therefore separate known bulk dynamics from a finite set of defect operators:

$$I_{\mathrm{EFT}} = \sum_{s=\pm} \left[\frac{1}{16\pi G} \int_{\mathcal{M}_s} d^4x \sqrt{-g_s} (R_s - 2\Lambda_s) + I_m[g_s, \Phi_s] \right] \tag{29}$$

$$+ \int_{\Sigma_B} d^3x \sqrt{|h|} \left[-\mu + \frac{\alpha}{2} \mathcal{R}[h] + \beta K^2 + \gamma K_{ab} K^{ab} + \sum_i c_i \mathcal{O}_i(\Phi_+, \Theta_{\mathrm{CPT}} \Phi_-) \right]. \tag{30}$$

Here $\mu, \alpha, \beta, \gamma, c_i$ are Wilson coefficients subject to symmetry, cutoff and stability constraints. CPT requires each operator to be invariant under exchange of the two sides followed by the appropriate antiunitary field transformation. This does not solve the UV problem, but it changes the question from "choose any I_{qg}" to "is there a finite EFT domain in which the junction is nonsingular, stable and compatible with energy bounds?" The thin-shell result above is recovered from the leading tension/stress sector and immediately shows that a healthy completion must substantially modify it.

Candidate sources of effective negative energy include Casimir-like states, squeezed quantum fields, semiclassical vacuum polarization and higher-curvature effective stresses. None is asserted here to provide the required macroscopic support; quantum inequalities make that a stringent requirement [4, 5]. Recent higher-curvature regular-black-hole constructions demonstrate that singularity avoidance and white-hole-like continuations can occur in controlled modified-gravity models, but they do not establish cross-branch CPT transport [6].

7 A quantitative EFT feasibility test: flux stabilization

The previous revision introduced defect operators but did not test whether they can change the unstable radial mode. We can obtain a model-independent lower bound on the required response without pretending to have a UV completion. Allow the shell conservation law to include a covariant normal energy flux $\mathcal{F}(a)$,

$$\sigma'(a) = -\frac{2}{a}[\sigma(a) + p(a)] + \mathcal{F}(a), \tag{31}$$

where prime denotes differentiation along the one-parameter family of shell radii. At the $a_0 = 3GM$, $p_0 = -\sigma_0$ equilibrium, retaining $V'(a_0) = 0$ requires $\mathcal{F}(a_0) = 0$. Differentiating Eq. (25) exactly at the equilibrium gives

$$V''(a_0) = 4\sqrt{3}\pi G^2 M\, \sigma''(a_0) - \frac{2}{9G^2M^2}. \tag{32}$$

Because $\sigma'(a_0) = 0$, Eq. (31) gives $\sigma''(a_0) = \mathcal{F}'(a_0)$. Linear radial stability therefore requires the explicit lower bound

$$\boxed{\mathcal{F}'(a_0) > \frac{1}{18\sqrt{3}\pi G^4 M^3}.} \tag{33}$$

This is the first quantitative requirement on any defect EFT claimed to stabilize this background. Writing $\mathcal{F}'(a_0) = \kappa/(G^4M^3)$, stability requires $\kappa > 1/(18\sqrt{3}\pi) \simeq 1.02 \times 10^{-2}$. If the inequality holds, radial perturbations obey $\delta\ddot{a} + \omega_r^2 \delta a = 0$ with

$$\omega_r^2 = \frac{V''(a_0)}{2} = \frac{2\sqrt{3}\pi\kappa - 1/9}{G^2M^2}. \tag{34}$$

Thus the required feedback is not an unspecified "attractor": a candidate microscopic calculation must reproduce Eq. (33) while remaining ghost-free and compatible with energy bounds.

A minimal local operator basis capable of contributing to such a response is

$$\mathcal{L}_\Sigma = -\mu - \frac{1}{2}(D\varphi)^2 - U(\varphi) + \xi\varphi^2\mathcal{R}[h] + \beta K^2 + \gamma K_{ab}K^{ab} + \zeta\varphi K. \tag{35}$$

The $\xi, \beta, \gamma, \zeta$ terms can alter the metric-dependent surface stress and its radial derivative. Equation (33), rather than arbitrary coefficient choice, is the target they must meet. However, a canonical homogeneous scalar with positive kinetic energy and a potential bounded below does *not* by itself explain the negative σ_0 of Eq. (22); supplying that sign requires vacuum-polarization/Casimir subtraction, nonminimal curvature stress, higher-derivative gravitational terms, or another UV effect. We therefore do not claim that Eq. (35) is viable. It converts the existence question into a calculable coefficient-and-state test.

The bound also clarifies the fine-tuning issue. If κ is only infinitesimally above threshold, $\omega_r GM \ll 1$ and stabilization is fragile. A robust completion should predict κ dynamically in an open interval above threshold, not tune it to the separatrix. This is a concrete criterion for the attractor discussion below.

7.1 Probe-field dwell time in a regulated core

As a second zero-order check, approximate the transition region in a freely falling orthonormal frame by a slab of proper width L_* with bounded effective mass $m_{\rm eff}^2$. A WKB mode with local energy $E_{\rm loc}^2 > m_{\rm eff}^2$ has

$$k_* = \sqrt{E_{\rm loc}^2 - m_{\rm eff}^2}, \qquad \tau_{\rm WKB} \simeq \frac{L_* E_{\rm loc}}{k_*}. \tag{36}$$

For $E_{\rm loc} \gg m_{\rm eff}$, $\tau_{\rm WKB} = L_*[1 + m_{\rm eff}^2/(2E_{\rm loc}^2) + \cdots]$: there is no exponential dwell-time growth in this regulated probe limit. Divergence occurs at the turning point $E_{\rm loc} \to m_{\rm eff}$, where WKB itself fails. This calculation does not establish a bridge; it isolates the previously demonstrated instability in the shell/background sector rather than automatically in high-energy probe propagation.

8 What triggers trapped-to-anti-trapped continuation?

A collapsing star should not "know" a coordinate location in $\mathcal{M}_-$. The covariant version of the hypothesis therefore forbids a nonlocal address label. The onset surface is instead defined by local invariants and the defect EFT. A minimal trigger is

$$\mathcal{C} \equiv \max\left(\ell_*^4 K_{\mu\nu\rho\sigma}K^{\mu\nu\rho\sigma},\ \ell_*^2|R|,\ \ell_*^2|R_{\mu\nu}u^\mu u^\nu|\right) \geq 1, \tag{37}$$

where ℓ_* is the EFT breakdown scale. CPT maps the local boundary data on this transition surface to conjugate data; no preferred spatial point in the partner universe is specified.

For an infalling wave packet of Killing energy E the proper dwell time through a regulated core of proper thickness L_* is parameterized by

$$\tau_{\rm dwell}(E) = L_*\left[1 + \eta\left(\frac{E}{E_*}\right)^n + \mathcal{O}((E/E_*)^{n+1})\right], \tag{38}$$

with coefficients calculable only after the core EFT is specified. Equation (38) is deliberately a low-energy expansion, not a derived prediction. A candidate model fails if $\tau_{\rm dwell}$ diverges, becomes acausal, or induces unbounded blueshift for finite E.

9 Cross-branch entanglement wedges and replica saddles

9.1 Admissible variational class

The earlier notation I_- was underspecified. Let $\mathfrak{C}_-$ denote the set of codimension-two surfaces in $\mathcal{M}_-$ satisfying: (i) CPT-compatible boundary anchoring to the replicated defect data; (ii) generalized homology with R_+ in the doubled replica geometry; (iii) regularity at the replica fixed set; and (iv) the same gauge-invariant algebraic boundary conditions used to define Eq. (14). The conjectured entropy is then

$$S(R_+) = \min\left\{S_{\rm one}(R_+),\ \underset{I_+\in\mathfrak{C}_+,\,I_-\in\mathfrak{C}_-}{\rm ext}\left[\frac{A(\partial I_+) + A(\partial I_-)}{4G} + S_{\rm alg}(R_+ \vee I_+ \vee I_-)\right]\right\}. \tag{39}$$

The second term is admitted only if it arises from a legitimate replica saddle of the coupled theory. It is not inserted by hand into an uncoupled tensor product.

9.2 A sharp mathematical falsifier

The cross-branch saddle hypothesis is false in a specified defect theory if, after analytic continuation $n \to 1$, every regular replica solution satisfying the above boundary conditions either (a) has larger generalized entropy than the one-branch saddle for all parameters, (b) contains a negative mode that invalidates the saddle, or (c) violates the defect equations of motion. This is a mathematical falsification of the *cross-branch-saddle mechanism*; it does not falsify CPT-symmetric cosmology itself.

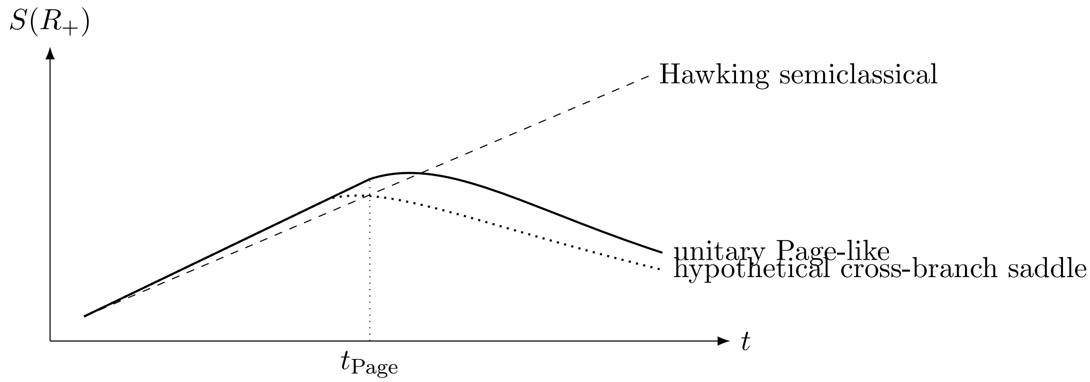

Figure 4: Conceptual entropy curves. The dotted bridge curve is a conjectural modification to be derived, not a prediction.

10 Observational parameter bounds, not parent-theory falsification

The exterior phenomenology is described only after a particular EFT predicts a transfer function. For a linear ringdown response write

$$\tilde{h}_{\text{echo}}(\omega) = \epsilon\, e^{i\omega\Delta t} e^{-(\omega/\omega_q)^2} \tilde{h}_{\text{seed}}(\omega). \tag{40}$$

Let $\rho_{\text{echo}}^{(\epsilon=1)}$ be the matched-filter signal-to-noise ratio predicted by the unit-amplitude template for a given event and ρ_{95} the event-specific 95% upper-limit threshold obtained from the search likelihood. Linearity gives the quantitative bound

$$\boxed{|\epsilon| \le \epsilon_{95} \equiv \frac{\rho_{95}}{\rho_{\text{echo}}^{(\epsilon=1)}}.} \tag{41}$$

There is no universal numerical ϵ_{95} without fixing the waveform, event, detector PSD and analysis. This resolves the previous overstatement. A published O3 template-based search found p-value distributions consistent with noise and no significant echo signal [7]. More recent GWTC-4.0 remnant tests likewise report no evidence for post-merger echoes in the analyzed events [8]. Such results constrain Eq. (40) only after its waveform is mapped to the relevant likelihood. There is therefore no scientifically defensible universal number such as "$|\epsilon| < 10^{-2}$" that can be quoted from the parent framework alone. For a concrete LVK target we instead define the benchmark pair

$$\Delta t_{\text{echo}} \simeq 4GM\,|\ln\delta|\,, \qquad \epsilon = \epsilon_{95}(M, \chi, \delta, \omega_q; \text{event}), \tag{42}$$

where a near-horizon reflecting structure is placed at fractional offset δ from the classical horizon and χ is spin. Equation (42) yields a mass-scaled delay once δ is specified; the amplitude remains an

EFT prediction, not a free "falsification threshold." A completed bridge model is excluded by an echo analysis only if it predicts a nonzero $\epsilon_{\text{pred}} > \epsilon_{95}$ for the same template family.

Similarly, current stochastic-background limits constrain only a completed model that predicts a nonzero spectrum. The current LVK O1–O4c1 search reports no evidence for an isotropic stochastic background and tighter upper limits on astrophysical and cosmological models [9]. We intentionally do not assign the bridge a distinctive spectral index: no calculation in the defect EFT derives one. Inventing an index would create a post-hoc "prediction." The falsifiable object is instead a future computed $\Omega_{\text{bridge}}(f; \{c_i\})$, which must lie below the measured frequency-dependent upper-limit curve.

11 Infall, causal bookkeeping, and the generalized second law

The transition is taken to be a *quantum/curvature-triggered* phenomenon, not a thermal activation process driven by Hawking temperature. Hawking radiation remains an exterior semiclassical channel. Along an infalling worldline the state evolves continuously under the local effective Hamiltonian until the curvature trigger Eq. (37) invalidates the low-curvature EFT; the bridge map is a proposed continuation of that state, not an instantaneous deletion event.

For a branch-restricted exterior observer, the appropriate statement is the ordinary generalized entropy on the future causal horizon,

$$S_{\text{gen}}^{(+)}(\lambda) = \frac{A_+(\lambda)}{4G} + S_{\text{out}}^{(+)}(\lambda), \qquad \frac{dS_{\text{gen}}^{(+)}}{d\lambda} \geq 0, \tag{43}$$

through every interval in which semiclassical gravity is valid. Cross-branch correlations may purify the *global* state, but they are not subtracted from the local observer's Eq. (43); doing so would incorrectly use inaccessible information to rescue a local GSL violation. Therefore any candidate transition that causes a decrease of Eq. (43) outside the quantum region is rejected. Inside the quantum region the semiclassical GSL need not be locally formulable, but the outgoing state must match onto a later semiclassical region with a nondecreasing generalized entropy relative to its own future horizon. This gives a causal accounting rule for infalling matter without treating the partner branch as a thermodynamic bypass.

11.1 CP/T signatures and horizon-scale imaging

CPT pairing by itself predicts no local CP- or T-violating signal for a single infalling fermion. A measurable asymmetry requires a specific portal operator. For example, a defect pseudoscalar coupling

$$\mathcal{L}_{\text{CP}} = g_5 \varphi\, \bar{\psi} i \gamma_5 \psi \tag{44}$$

would generate helicity-dependent phases only if the background φ profile and g_5 are nonzero; their magnitude must be calculated in a completion. Hence the absence of such an asymmetry does not test bare CPT pairing.

Likewise, if the exterior metric remains exactly Kerr/Schwarzschild outside the transition radius, Birkhoff/no-hair reasoning gives *zero classical shadow correction*. An Event-Horizon-Telescope-scale deviation requires exterior metric or photon-propagation corrections, e.g. $f(r) \to f(r) + \delta f(r)$. Shadow observations can then constrain the corresponding Wilson coefficients, but there is no universal bridge shadow shift. This null prediction for the strictly interior realization is more precise than suggesting an arbitrarily tiny generic correction.

12 Fine tuning and dynamical selection

The reviewer correctly distinguishes "possible matching" from "why matching occurs." The revised framework imposes a selection principle: admissible bridge data are not arbitrary pairs of black-hole and cosmological initial conditions but stationary points of the same CPT-invariant defect action, subject to local trigger Eq. (37). Fine tuning is reduced only if the resulting flow has an attractor. Define perturbations δX around a candidate transition solution and a transfer matrix $\delta X_{\text{out}} = \mathbf{T}\delta X_{\text{in}}$. A necessary attractor criterion is

$$\rho(\mathbf{T}) < 1 \tag{45}$$

for all non-gauge perturbation channels over the transition. Failure of this criterion kills the proposed dynamical selection mechanism and reinstates the fine-tuning objection.

13 Decision tree: what exactly is falsified?

Table 1: Fatal versus nonfatal tests. "Fatal" is always relative to the stated hypothesis level.

Test	Failure criterion	What is falsified	What survives
Thin-shell junction	Required stress is exotic or equilibrium unstable	Classical symmetric thin-shell realization	Other EFT/quantum completions
QEI/AEQI compatibility	No allowed state supports required averaged negative energy	That stress-source realization	Non-shell or modified-gravity mechanisms
Replica calculation	No regular/dominant cross-branch saddle	Cross-branch-island mechanism	Global CPT pairing and ordinary islands
Perturbative stability	Growing physical mode or divergent blueshift	Candidate bridge background	Other parameter regions/models
Defect-EFT consistency	Ghost, loss of hyperbolicity, cutoff violation	Chosen EFT completion	Parent kinematic idea
Echo search	ϵ constrained below model prediction	Specific transfer-function model	Models with weaker/zero exterior coupling
Stochastic GW search	Predicted Ω_{GW} exceeds measured upper limit	Specific cosmological realization	Quiet realizations
CPT-background test	A robust observation contradicts a mandatory prediction of the chosen CPT cosmology	That CPT cosmological background	Unrelated black-hole models

The key distinction is between *mathematical falsification* of a specified model and *empirical exclusion* of a parameterized realization. The broad statement "some hidden conjugate sector exists" is not empirically useful by itself and is not advertised as falsifiable.

14 Consistency checklist after the zero-order result

Any successor model must satisfy all of the following:

1. derive a nonsingular transition rather than rename the singularity;

2. replace the macroscopic negative shell energy in Eq. (23) with a stress tensor derived from a controlled theory;

3. satisfy applicable quantum-energy bounds or explain their evasion within their domains of validity;

4. define branch observables using gauge-invariant algebras and recover Eq. (9) in a factorizing limit;
5. demonstrate a global isometry and an ordinary Page-compatible limit when cross-branch saddles are absent;
6. show perturbative stability and finite dwell time for finite-energy infall;
7. preserve exterior causality and standard weak-field tests;
8. produce empirical predictions only after fixing nonzero EFT coefficients rather than treating nondetection as automatic falsification.

15 Discussion

The zero-order calculation changes the interpretation of the proposal. The simplest classical bridge is not a promising physical solution: at $a_0 = 3GM$ it requires negative integrated shell energy $-2\sqrt{3}M$ and is unstable under the constant-tension continuation. This is more informative than a list of possible obstructions because it quantifies the size of the problem a quantum completion must solve.

The entropy calculation simultaneously removes a conceptual ambiguity. If the global state is pure and the two branch descriptions are reductions, a branch observer can see entropy because of entanglement. In the factorizing toy model, the cross-branch correction is exactly minus the mutual information, not an extra arbitrary term. The gravitational problem is then to replace tensor-factor entropies by properly dressed subalgebra entropies without losing this limit.

The paper does not derive a black-to-white quantum transition, a cross-branch replica saddle, or a universal gravitational-wave signature. These remain conjectural. The contribution is narrower: an exact classical obstruction, a precise information-theory identity, a constrained EFT target, and a hierarchy in which failures have unambiguous consequences.

16 Conclusion

A black hole cannot consistently serve as an entropy trash can if the underlying theory is unitary. A more defensible CPT Entropy Bridge must preserve global information while allowing branch-restricted observers to trace over inaccessible correlations. This revision demonstrates that bookkeeping explicitly and calculates the simplest classical junction. The result is sobering: a symmetric Schwarzschild throat supported by a domain-wall-like shell at $3GM$ requires negative energy of order the black-hole mass and is linearly unstable. Therefore classical GR does not supply the desired mechanism. A successful bridge would need a controlled quantum or modified-gravity completion that changes this conclusion while satisfying the explicit flux-response bound in Eq. (33), quantum-energy bounds, stability, causal, algebraic and replica constraints. Cross-branch islands remain a sharply stated conjecture rather than a result, and gravitational-wave nondetections constrain only specified nonzero transfer functions. The framework should thus be judged as a falsifiable family of toy realizations with a demonstrated zero-order obstruction, not as evidence that astrophysical black holes connect to an antimatter universe.

A Derivation of the static junction result

For the symmetric orientation, the angular extrinsic-curvature jump is $[K^\theta{}_\theta] = 2\sqrt{f + \dot{a}^2}/a$. Substitution in Israel's condition gives Eq. (16). The temporal component gives Eq. (17). Setting $\dot{a} = \ddot{a} = 0$

yields the static formulas. Solving $p_0 = -\sigma_0$ gives

$$\frac{1 - GM/a_0}{4\sqrt{f_0}} = \frac{\sqrt{f_0}}{2} \quad \Rightarrow \quad a_0 = 3GM. \tag{46}$$

Using $f_0 = 1/3$ gives Eq. (22) and Eq. (23).

For $w = -1$, shell conservation gives constant $\sigma = \sigma_0$. Then

$$V(a) = 1 - \frac{2GM}{a} - (2\pi G\sigma_0)^2 a^2. \tag{47}$$

At $a_0 = 3GM$, $(2\pi G\sigma_0)^2 = 1/(27G^2M^2)$. Differentiation gives

$$V''(a_0) = -\frac{4GM}{a_0^3} - 2(2\pi G\sigma_0)^2 = -\frac{2}{9G^2M^2}. \tag{48}$$

B Derivation of the flux-stability threshold

At $a_0 = 3GM$, the equilibrium conditions imply $\sigma'(a_0) = 0$. Differentiating $V = f - (2\pi Ga\sigma)^2$ twice and substituting $\sigma_0 = -[6\pi\sqrt{3}G^2M]^{-1}$ yields

$$V''(a_0) = 4\sqrt{3}\pi G^2M\sigma''(a_0) - \frac{2}{9G^2M^2}. \tag{49}$$

The flux-modified conservation equation gives $\sigma''(a_0) = \mathcal{F}'(a_0)$ when $p_0 = -\sigma_0$, $\sigma'(a_0) = 0$, and $\mathcal{F}(a_0) = 0$. Requiring $V'' > 0$ gives Eq. (33). This threshold is independent of a chosen microscopic operator basis; the latter determines whether the required $\mathcal{F}'$ can actually be generated consistently.

C Reproducible numerical scan

The repository includes `scripts/thin_shell_scan.py`, which sets $G = M = 1$, evaluates $V(a)$ for the constant-tension solution fixed by $a_0 = 3$, verifies $V(3) = V'(3) = 0$ and $V''(3) = -2/9$, and writes `data/thin_shell_scan.csv`. It also prints $E_{\text{shell}} = -2\sqrt{3}$ in mass units. The script is intentionally dependency-light (Python standard library only).

D CPT transformation checklist

A complete field theory must specify, with fixed Lorentz and gauge conventions,

$$\phi_-(x) = \eta_\phi \phi_+^*(-x), \tag{50}$$
$$A_\mu^-(x) = \eta_\mu A_\mu^+(-x), \tag{51}$$
$$\psi_-(x) = U_{\text{CPT}}\psi_+^*(-x). \tag{52}$$

Gauge charges conjugate under C; invariant masses do not. Spin and orientation signs must be tracked through P and T. The defect operators in Eq. (30) must respect the resulting antiunitary symmetry.

E Claim taxonomy

Label	Meaning
Established	Standard background result: Israel matching, Bekenstein–Hawking entropy, quantum inequalities within their stated regimes.
Literature model	Published but unconfirmed model, e.g. CPT-symmetric cosmology.
Derived toy result	Calculation performed in this paper under explicit simplifying assumptions, e.g. Eqs. (23) and (26).
Definition	Chosen bookkeeping quantity, e.g. Eq. (14); not automatically a theorem.
Conjecture	New statement requiring derivation, e.g. existence of a regular dominant cross-branch replica saddle.
Phenomenological ansatz	Parameterized response such as Eq. (40); testable only after parameters are fixed.

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