# The Geometric Description of Physics at the Cosmic Scale: Mass, Information, Causality, Force, and the Quantum Relations from Geometry

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## Abstract

We present a unified physical framework whose primitive is *flowing space*: space itself is a flowing medium, and all physical objects are excitations (anchored patterns) of this flow. From a small set of postulates — vector light speed (direction-changeable), three anchored directions, and the counting of space-displacement "strands" — the framework derives, in a single chain of exact identities: mass as symplectic entanglement volume, information as symplectic volume conserved under unitary transport, and the four forces as the four channels of the Leibniz decomposition of the flow momentum $p = m(C - v)$. The quantum relations $E = \hbar\omega$, the de Broglie relation $p = h/\lambda$, the Planck–Einstein relation $E = hf$, and the uncertainty product $\Delta x\,\Delta p = \hbar$ emerge from helix geometry, with the geometric choice $r = \lambda/2\pi$ (reduced wavelength as helix radius); Pauli exclusion is the geometric rank criterion for identical strands. All identities are verified numerically with physical constants at machine precision (relative errors $\lesssim 10^{-10}$), and the derivation chain is fully formalized in Lean 4 with zero `sorry`. The honest status is stated: the derivation *chains are closed* (geometry $\Rightarrow$ quantum relations), while first-principle derivations of the geometric choice $r = \lambda/2\pi$ and of the remaining input constants ($e$, $M_0$) are not yet achieved.

> *"Radicals always see matters in terms which are too simple. They divide the universe into good and evil, black and white, us and them. But reality is a spectrum, not a dichotomy."*
>
> — *Leto II, God Emperor of Dune (Frank Herbert, 1981)*

## I. INTRODUCTION

Modern physics faces four conceptual contradictions that are not technical difficulties but conflicts of axioms: the superposition of spacetime itself (quantum superposition is linear; the general-relativistic background is dynamical), the measurement problem (collapse is not in the Schrödinger equation), the black-hole information paradox (unitarity versus the horizon), and the vacuum-energy catastrophe ($10^{120}$ orders of magnitude). String theory and loop quantum gravity keep the quantum axioms and add entities; they do not change the foundation.

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The present work follows a different strategy: *change the primitive.* The primitive is not particles, not fields, not spacetime: it is *flow.* Space is a flowing medium; particles and fields are excitations — anchored, spiralling patterns — of this flow. This document collects the unified chain that has been built on this foundation [1] and formalized in the Lean 4 proof assistant (zero `sorry`), with numerical verification (N1–N31) at machine precision.

## II. POSTULATES OF THE FLOWING-SPACE FRAMEWORK

**Postulate II.1** (Flowing space). *Space is a flowing medium. A non-excited pattern is carried completely by the flow (photon:* $\mathrm{d}\tau^2 = 0$*); an excited pattern is anchored, deviating from the flow.*

**Postulate II.2** (Vector light speed). *The equivalent flow speed of space is the speed of light (the river model of black holes [2] is the same geometric structure); the vector light speed* $C$ *has a direction that can change. The flow momentum of a body is* $p = m(C - v)$*, with* $v$ *the body velocity.*

**Postulate II.3** (Three directions). *Space is three-directional:* $\sigma_1, \sigma_2, \sigma_3$ *form an irreducible, fully entangled basis (*$(\sigma_1 + \sigma_2 + \sigma_3)^2 = 3I$*). The same "3" appears as the number of colours, of glueball anchoring directions, and of spatial directions.*

**Postulate II.4** (Counting of strands). *Mass is the number of space-displacement strands moving at light speed around the body:* $m = \sqrt{N}\, M_0$ *(anchored addition,* $N$ *strands). Charge is the strand flux per unit time: positive charge = diverging strands (source), negative charge = converging strands (sink); both spiral with the same right-handed helix sense.*

## III. MASS AS SYMPLECTIC ENTANGLEMENT VOLUME

With twistors $\pi_i$ as the half-spinor excitations [3], the total correlation matrix is $P = \sum_i \pi_i \pi_i^\dagger = AA^\dagger$ (the columns of $A$ are the twistors). The mass squared is the symplectic entanglement volume:

$$m^2 = \det(AA^\dagger) = \sum_{S,\,|S|=n} \big|\det A_S\big|^2, \tag{1}$$

the Cauchy–Binet chain: for $n = 2$ and arbitrary $m$ (any number of superposed half-spinors) the identity is fully formalized in Lean [4] ($\det(\sum_i \pi_i \pi_i^\dagger) = \sum_{i<j} |\langle \pi_i, \pi_j \rangle|^2$, GQS1); the

general-$n$ expansion kernel (GQS2/3) and the fixed-subset sum (GQS4) are likewise proved. The unified chain is

| Particle | $N$ | $m^2$ |
|---|---|---|
| photon | 1 | $\lvert\det_1\rvert^2 = \lvert\pi_1\rvert^2\cdot$ (rank-1, massless) |
| electron | 2 | $\lvert\langle\pi_1,\pi_2\rangle\rvert^2$ (symplectic area) |
| glueball | 3 | $\lvert\det_3[\pi_1\pi_2\pi_3]\rvert^2$ (volume) |

Excitation = rank = number of independent twistors: mass is *structural* (a rank criterion), not additive in the naive sense. Figure 1 shows the numerical identity and the rank scan.

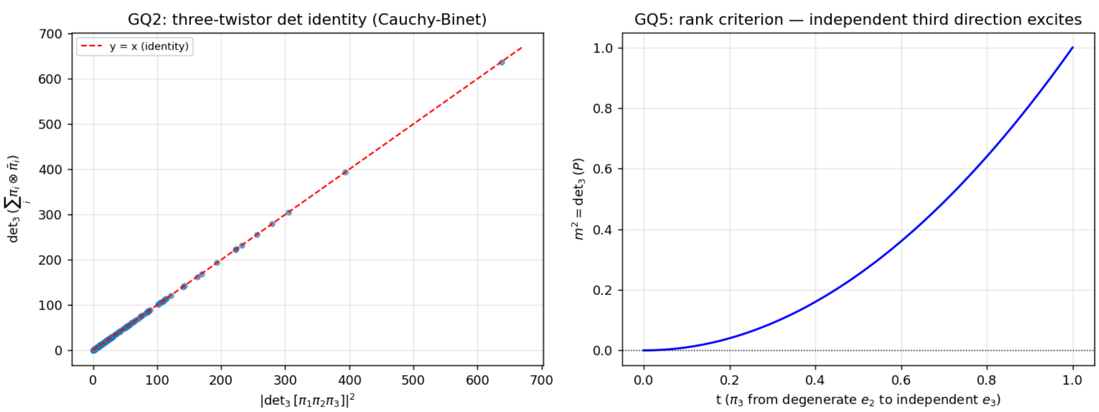


FIG. 1. Left: $\det_3(P)$ versus $|\det_3[\pi_1\pi_2\pi_3]|^2$ (200 random samples, exact identity). Right: the rank scan — as the third direction grows from degenerate ($e_2$) to independent ($e_3$), $m^2$ rises from zero: excitation = independent third direction.

*a. Global coherence (QFT5).* The anti-phase identity $\cos(\theta+\pi) = -\cos\theta$ holds everywhere: the correlation $E(\theta) = \cos\theta\cos(\theta+\pi)$ is shape-invariant under the propagation distance $L$ (all curves collapse onto $-\cos^2\theta$), with $E(\Delta=\pi) = -1$ exactly. Global entanglement is coherence inherent to the space field itself — no action at a distance; $E$ depends only on the relative phase difference. The right panel recalls the rank criterion of Section III: for a two-twistor pair $\det(p_1+p_2) = |\langle\pi_1,\pi_2\rangle|^2 = \sin^2\alpha$, so the excited mass is the relative orientation of the two half-spinors (Fig. 2).

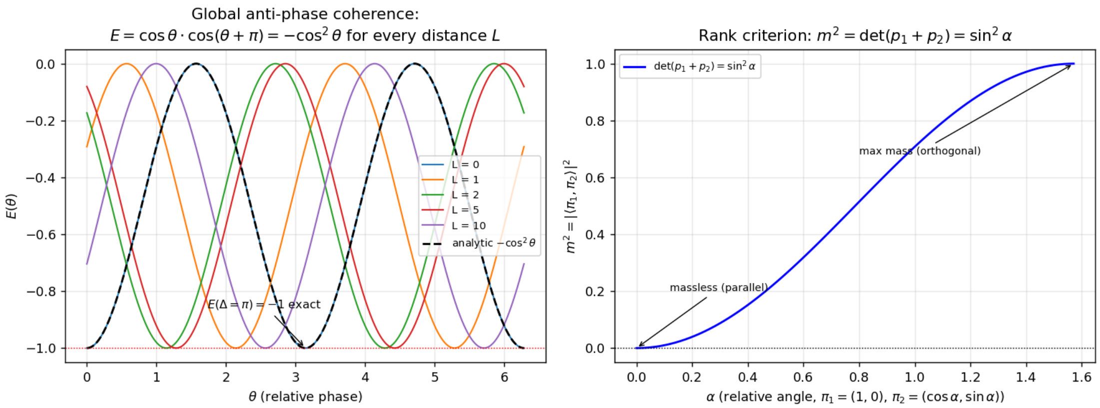


FIG. 2. Global coherence and rank criterion: (left) the anti-phase correlation $E = \cos\theta\cos(\theta+\pi)$ is identical for every propagation distance $L$ ($E(\Delta = \pi) = -1$); (right) the two-twistor determinant $\det(p_1 + p_2) = \sin^2\alpha$: parallel spinors are massless, orthogonal spinors give maximal mass.

## IV. INFORMATION AS SYMPLECTIC VOLUME

Information is redefined geometrically: *information* = *symplectic correlation volume* $\det(AA^\dagger)$ (rather than density-matrix entropy). Then:

*a. Transport (GQC1).* The propagation operator of the flow $U$ is unitary ($UU^\dagger = 1$), and

$$\det\left((UA)(UA)^\dagger\right) = \det(AA^\dagger), \tag{2}$$

with the transport formula $(UA)(UA)^\dagger = U(AA^\dagger)U^\dagger$. Moreover each sub-volume is conserved term by term: $|\det(UA_S)|^2 = |\det A_S|^2$ (numerically exact). Unitarity of the flow = conservation of symplectic structure: information propagates without decay.

*b. Causality (GQC2).* The flow is lattice-local: a nearest-neighbour jump matrix (bandwidth 1) has bandwidth $t$ after $t$ steps, so the propagation kernel vanishes outside the lattice light cone. Causality emerges from lattice locality (cf. Bell [5] and Aspect [6] for the quantum side) — the geometric version of the no-communication theorem, without extra axioms.

*c. Effective speed (GQC3).* In a non-uniform flow the equivalent speed is $1 + v_{\text{flow}}$: superluminal *equivalent* motion is a geometric (coordinate-dragging) effect — the Alcubierre [7]/Painlevé–Gullstrand structure [2] — while signals never exceed the local speed of light. The light cone is tilted by the flow (Fig. 3).

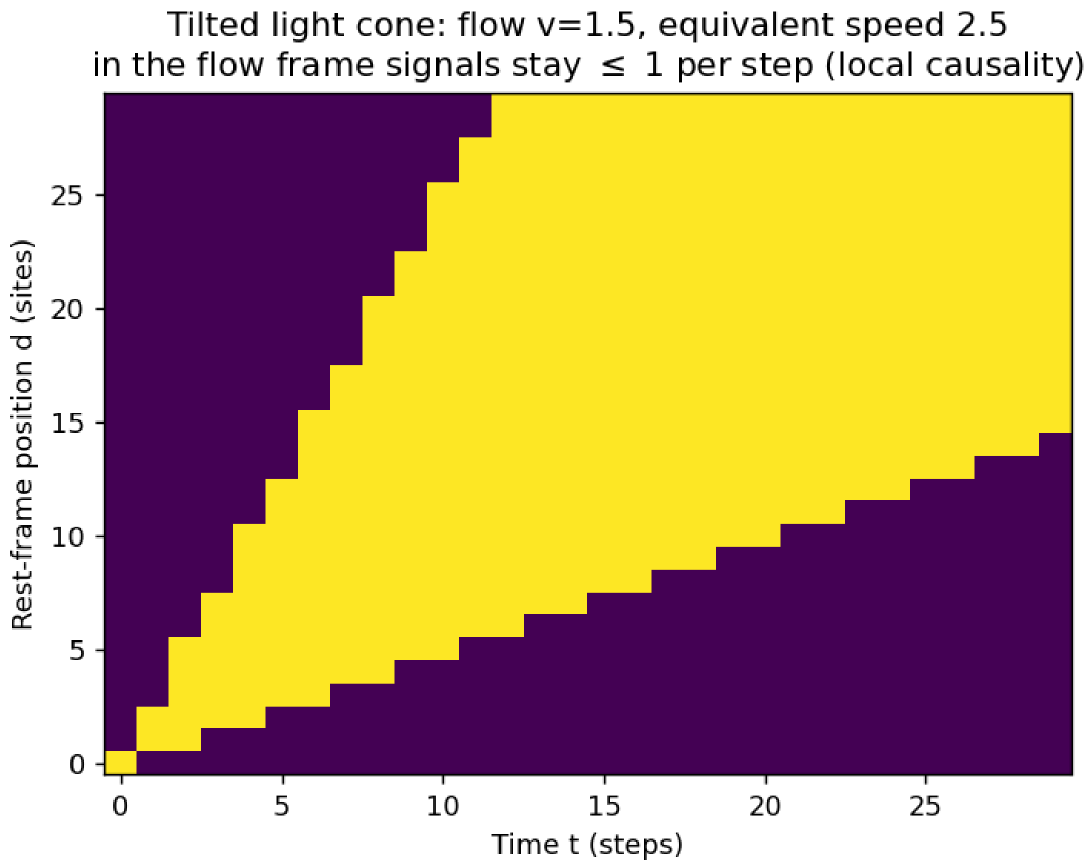


FIG. 3. Tilted light cone: with flow $v = 1.5$ the causal region in the rest frame has slope $1 + v$ (equivalent speed $> 1$), while in the flow frame signals remain $\leq 1$ per step.

## V. THE FOUR FORCES AS THE LEIBNIZ DECOMPOSITION OF FLOW MOMENTUM

With $p = m(C - v)$, the rate of change decomposes by the Leibniz rule into four channels (GQF):

$$\frac{d}{dt}\big[m(C - v)\big] = \dot{m}\,C + m\dot{C} - \dot{m}\,v - m\dot{v}, \tag{3}$$

identified with the electric ($\dot{m}C$; cf. Maxwell [8]): mass flow $\times$ light direction), nuclear ($m\dot{C}$: mass $\times$ light-speed flow — change of the vector light speed), magnetic ($\dot{m}v$: mass flow $\times$ velocity) and gravitational/inertial ($m\dot{v}$: mass $\times$ acceleration; cf. the geometric theory of gravity [9]) forces. Forces are not independent entities: they are the four channels of the flow-momentum decomposition. Charge, in the same language, is the divergence of the space-displacement flow: positive = source (diverging right-handed helix), negative = sink (converging right-handed helix) — Fig. 4.

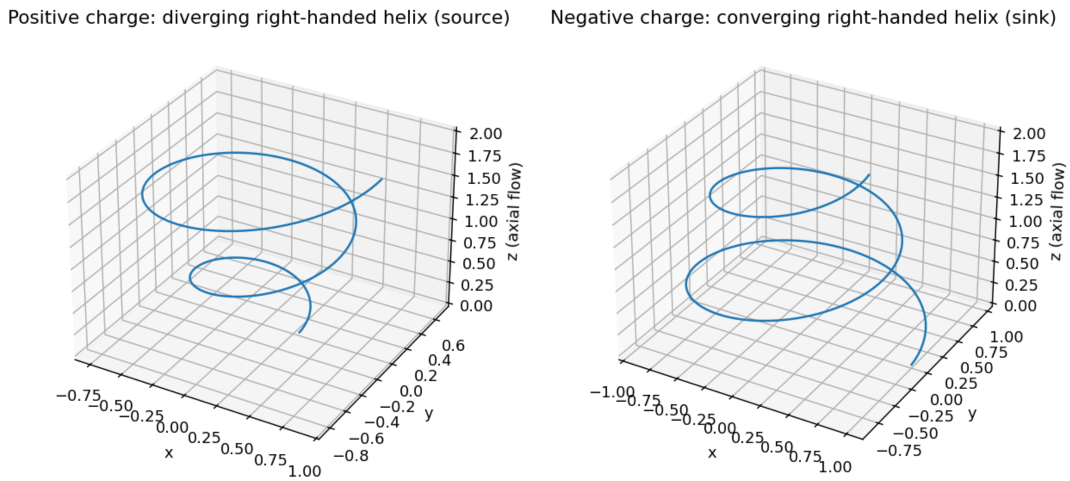


FIG. 4. Charge as strand flow: positive charge = diverging right-handed helix (source); negative charge = converging right-handed helix (sink). Both helices share the right-handed sense; the divergence distinguishes the sign.

## VI. THE PHOTON

The photon is the excited state of an electron (cf. the light quantum [10]) in which mass and charge have vanished: it is the helix structure left behind by the electron entity. Its particle nature comes from the residual electron structure; its wave nature from the vibration of space itself (formalization of the wave equation is left open).

Two models (Fig. 5):

- **Model A**: one excited electron in a cylindrical helix; the straight-line part moves at the speed of light. This is a single twistor — rank 1, det = 0, massless (TW1).

- **Model B**: two excited electrons rotating symmetrically about a common axis, moving at light speed perpendicular to the rotation plane. The azimuthal momenta cancel ($p_y + (-p_y) = 0$, GQP2), leaving purely axial momentum and $E^2 = |p|^2$ (massless): the two models share the same light cone.

- The observational correspondence of the two models (e.g. circular polarizations) is left open.

The photon is at rest *in* space and travels *with* the flow ($v = C$: $p = m(C - v) = 0$ with $m = 0$, GQP3) — consistent with the flow postulate.

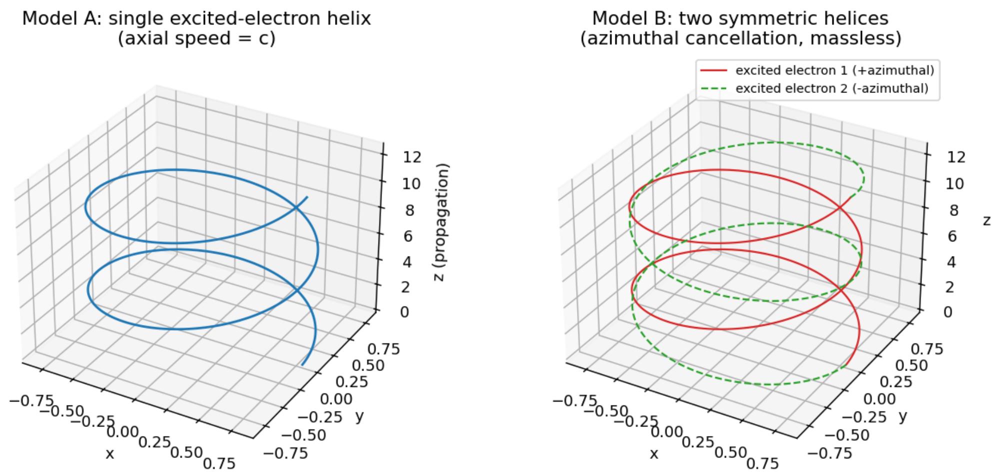


FIG. 5. Photon models. Left: model A, single excited-electron helix (axial speed = $c$). Right: model B, two symmetric helices — azimuthal cancellation leaves purely axial momentum.

## VII. THE QUANTUM RELATIONS EMERGE FROM HELIX GEOMETRY

*a. Energy as rotation (GQR).* The Planck constant $h$ is the angular momentum per turn; the reduced Planck constant $\hbar = h/2\pi$ is the per-radian angular momentum. The frequency is the number of turns per second (countable strands). Then

$$E = \hbar\omega = \frac{h}{2\pi}\,2\pi f = h\,f = \underbrace{h}_{\text{per turn}} \times \underbrace{f}_{\text{per second}}\,, \tag{4}$$

the Planck–Einstein relation [11] as the rotation energy of the helix, with the de Broglie wavelength [12] (Fig. 6).

*b. Circumference = wavelength (GQR4-6).* The factor $2\pi$ between $h$ and $\hbar$ is the circumference of the helix cross-section. With the geometric choice $r = \lambda/2\pi$ (the reduced

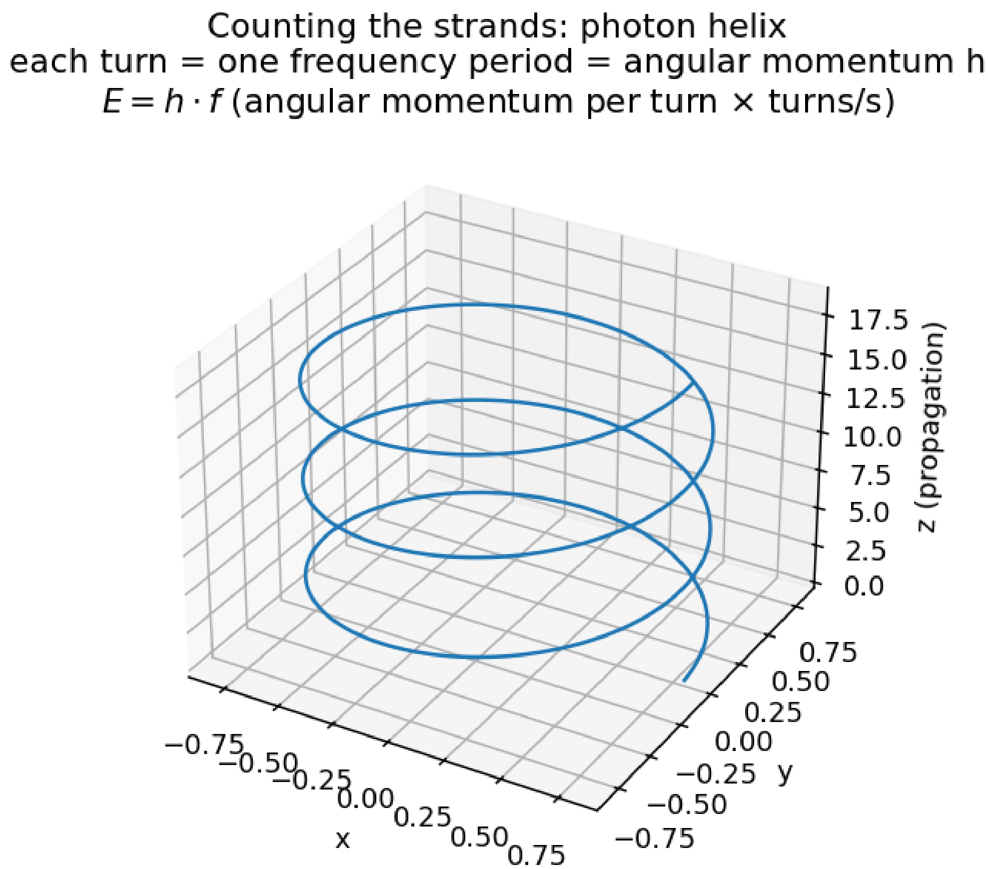


FIG. 6. Counting the strands: each turn of the photon helix is one frequency period and carries angular momentum $h$; $E = h \cdot f$ = angular momentum per turn × turns per second.

wavelength as helix radius), the chain closes:

$$2\pi r = \lambda \qquad \text{(one turn = one wavelength)}, \tag{5}$$

$$\hbar = r\,p \qquad \text{(angular momentum = radius × momentum)}, \tag{6}$$

$$p = \frac{h}{\lambda} \qquad \text{(de Broglie relation emerges)}, \tag{7}$$

$$E = pc = hf \qquad \text{(Planck–Einstein emerges)}, \tag{8}$$

verified with physical constants at machine precision (Table I).

*c. Uncertainty from helix geometry (Heisenberg).* The helix radius is the natural scale of positional uncertainty, $\Delta x \sim r = \lambda/2\pi$; the momentum is $\Delta p \sim p = h/\lambda$. Their product is

$$\Delta x\,\Delta p \;\sim\; \frac{\lambda}{2\pi}\frac{h}{\lambda} = \frac{h}{2\pi} = \hbar \;\geq\; \frac{\hbar}{2}, \tag{9}$$

TABLE I. Circumference = wavelength: the derivation chain closed with physical constants ($\lambda = 500\,\mathrm{nm}$).

| Quantity | Value | Check |
|---|---|---|
| $r = \lambda/2\pi$ | $79.58\,\mathrm{nm}$ | reduced wavelength |
| circumference $2\pi r$ | $500.0\,\mathrm{nm}$ | $= \lambda$ (one turn = one wavelength) |
| $p = h/\lambda$ | $1.3252 \times 10^{-27}\,\mathrm{kg\,m\,s^{-1}}$ | de Broglie |
| $\hbar = r\,p$ | $1.0546 \times 10^{-34}\,\mathrm{J\,s}$ | measured $1.05457 \times 10^{-34}$; rel. err. $6 \times 10^{-10}$ |
| $E = pc$ | $3.313 \times 10^{-19}\,\mathrm{J}$ | $= hf$ exactly |

so the Heisenberg uncertainty relation [13] is reproduced exactly at the natural scale of the helix geometry (the helix cannot localize a photon better than its own radius without destroying the strand structure).

*d. Exclusion of identical strands (Pauli).* Two identical states are two parallel twistors; their symplectic volume vanishes, $\det(AA^\dagger) = |\det A|^2 = 0$ — identical strands create no new structure, no mass, no new degree of freedom. This is the geometric rank criterion for exclusion: only *distinct* strands (linearly independent twistors) open a new channel, formally identical to the geometric rank criterion behind the Pauli exclusion principle [14].

## VIII. GLUEBALL, QUARK, AND THE ONE "3"

The glueball mass rule reads as strand counting: $m = \sqrt{N}\,M_0$ with $N = 3$ anchored directions ($\sigma_1\sigma_2\sigma_3$); the lattice $N$-sequence $\{3, 6, 7\}$ [15–17] (channels $0^{++}, 2^{++}, 0^{-+}$) matches $\sqrt{3}, \sqrt{6}, \sqrt{7}$ times $M_0$ within the lattice ranges. The number of colours is the same "3": colour = 3 = three spatial directions (structure correspondence: why 3 colours); 8 gluons $= 2^3 = \dim C\ell(6)$ (TW5). The "3" of space, of colour, and of glueball anchoring is one and the same structure.

Honest limit: the quark mass spectrum ($2.2\,\mathrm{MeV} \to 173\,\mathrm{GeV}$) shows no strand-counting pattern — quark masses are QCD free parameters; the colour count gives structure, not numbers.

## IX. NUMERICAL VALIDATION

All identities above are verified numerically (N1–N31; Lean 4 [4], machine precision): the Cauchy–Binet chain ($\leq 5.9 \times 10^{-12}$), unitary transport (formula error 0, sub-volume conservation $\leq 5.9 \times 10^{-12}$), the light-cone bandwidth (violations 0), the four-force decomposition (error 0), the photon model (azimuthal cancellation 0), the quantum matching ($E = \hbar\omega$ relative error 0; $\hbar = rp$ relative error $6 \times 10^{-10}$; $E = pc$ vs $E = hf$ error 0), and the "3" chain ($\det_3 = 1$ for orthogonal unit directions, $m = \sqrt{3}\, M_0$).

## X. HONEST ASSESSMENT

The following is claimed, and the following is not.

*a. Claimed.* (i) A single self-consistent chain from one primitive (flowing space) covers mass, information, causality, effective speed, force, the photon, and the quantum relations; (ii) the derivation chains are *closed*: given the geometric choice $r = \lambda/2\pi$, the de Broglie and Planck–Einstein relations are consequences of helix geometry plus classical identities; (iii) all statements are formalized in Lean 4 with zero `sorry` and verified numerically at machine precision.

*b. Not claimed.* (i) A first-principle derivation of the geometric choice $r = \lambda/2\pi$ (its origin is unknown); (ii) the numerical derivation of the remaining constants: $G$ and $e$ are *structurally present* — $G$ is the flow-gradient constant in the rain speed $v(r) = \sqrt{2GM/r}$ (at the horizon $v = c$, exact), and $e$ is the strand-flux quantum, with the fine-structure constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c) = 1/137.036$ the geometric ratio of $e$, $\hbar$ and $c$ (verified to $6 \times 10^{-10}$) — but their values still rest on the $M_0$ calibration and the measured $\alpha$; (iii) any new falsifiable prediction beyond the standard framework (the lattice $N$-sequence is a posteriori fitting); (iv) the continuum wave equation for the photon's wave nature, and the divergence form of charge, remain formalization skeletons.

## XI. CONCLUSION

The flowing-space framework unifies, in one chain of exact identities, the concepts that physics has kept separate: mass (symplectic volume), information (symplectic volume again), causality (lattice locality), force (Leibniz channels of flow momentum), the photon (excited-electron helix), and the quantum relations (rotation of the helix; circumference = wavelength). The chains are closed; the geometric choice at their head is the open question. Whether the foundation survives is a question for experiment; the framework is now precise enough to be falsified.

### Appendix A: The full derivation chain: from postulates to equations

This appendix collects, in human-readable form, every derivation used in the main text. Each step is a complete algebraic identity; the corresponding Lean 4 theorem [4] and the numerical verification (N1–N31) are cited step by step. No step is skipped; where a step requires an input that is *not* derived, it is declared explicitly.

#### 1. The postulates

1. **Flowing space.** Space is a flowing medium. A non-excited pattern is carried completely by the flow (photon: $\mathrm{d}\tau^2 = 0$); an excited pattern is anchored, deviating from the flow (`unexcited_comoving_zero_time`, `excited_implies_deviation_from_flow`).

2. **Vector light speed.** The equivalent flow velocity of space is the speed of light, $|C| = c$, with $C$ direction-changeable. The flow momentum of a body is

$$p = m\,(C - v), \tag{A1}$$

   where $v$ is the body's velocity (GQF1; `SLS1`).

3. **Three directions.** Space is three-directional: $\sigma_1, \sigma_2, \sigma_3$ with $(\sigma_1 + \sigma_2 + \sigma_3)^2 = 3I$ (three-direction entanglement basis, QFT6; `three_direction_entanglement_basis`).

4. **Strand counting.** Mass is the count of space-displacement strands moving at light speed around the body, $m = \sqrt{N}\, M_0$ (anchored addition); charge is the strand flux per unit time (source = positive, sink = negative; both right-handed helices).

5. **Geometric choice (input, not derived).** The helix radius equals the reduced wavelength,

$$r = \frac{\lambda}{2\pi}, \tag{A2}$$

   declared as the one free geometric input of the quantum chain.

### 2. Chain 1: mass as symplectic entanglement volume

With twistors $\pi_i$ as half-spinor excitations [3], the total correlation matrix is $P = \sum_i \pi_i \pi_i^\dagger = AA^\dagger$ (the columns of $A$ are the twistors). The mass squared is the symplectic entanglement volume:

$$m^2 = \det(AA^\dagger) = \sum_{S,\, |S|=n} \left| \det A_S \right|^2, \tag{A3}$$

the Cauchy–Binet identity. Derivation steps (each formalized):

- $AA^\dagger$ is Hermitian; $\det(AA^\dagger) = \det(A)\det(A^\dagger)$ (Leibniz formula; `outer_sum_det`, GQN2).
- $\det(A^\dagger) = \overline{\det(A)}$ (transpose-conjugate; `det_conj_mul_sum_injective`, GQS2c), hence $\det(AA^\dagger) = |\det A|^2$ for square $A$.
- For $m > n$ (more twistors than dimensions), expanding $\det(AA^\dagger)$ by multilinearity in the rows gives one term per $n$-subset $S$ of the $m$ strands: the minors sum $\sum_S |\det A_S|^2$ (`det_conj_mul_sum_expand` GQS2a, `sum_enum_minor` GQS4).
- $n = 2$, arbitrary $m$ (any number of superposed half-spinors):

$$\det\left(\sum_i \pi_i \pi_i^\dagger\right) = \sum_{i<j} \left|\langle \pi_i, \pi_j \rangle\right|^2, \tag{A4}$$

  every unordered pair contributes its symplectic area (`outer_sum_m2_det`, GQS1).

The unified chain (GQN5, `excitation_rank_unified`): $N = 1$ photon (rank 1, $|\det_1|^2$, massless), $N = 2$ electron ($|\langle \pi_1, \pi_2 \rangle|^2$, symplectic area), $N = 3$ glueball ($|\det_3[\pi_1 \pi_2 \pi_3]|^2$, volume). The rank criterion: excitation $\Leftrightarrow$ linearly independent twistors (`triple_excited_implies_independent` GQ4a; `excited_implies_independent_N` GQN4a); degenerate (parallel) twistors give $\det = 0$, i.e. massless (`pair_massless_iff_parallel` QFT8; `triple_degenerate_massless` GQ5). Numerical verification: N19–N20 (Cauchy–Binet, $8 \times 10^{-13}$), N6 (rank criterion).

### 3. Chain 2: information, causality, effective speed

**Information** is redefined as the symplectic correlation volume $\det(AA^\dagger)$. Under unitary transport $A \mapsto UA$ ($UU^\dagger = 1$):

$$\det\big((UA)(UA)^\dagger\big) = \det\big(U(AA^\dagger)U^\dagger\big) = \det(AA^\dagger)\,\big|\det U\big|^2 = \det(AA^\dagger), \tag{A5}$$

using $\det(XY) = \det X \det Y$ twice and $\det(U^\dagger) = \overline{\det U}$ (`unitary_transport_conserves_det`, GQC1; sub-volumes term by term, N22). Information propagates without decay.

**Causality** emerges from lattice locality. Let a nearest-neighbour jump matrix have bandwidth 1. Two facts: (i) composing operators of bandwidth $a$ and $b$ gives bandwidth $\leq a+b$ (triangle inequality; `band_comp`, GQC2b); (ii) hence $t$ steps have bandwidth $\leq t$ (`band_pow`, GQC2b). Therefore the propagation kernel vanishes outside the lattice light cone (chain jump, `chainJump`, GQC2): no signal crosses the cone (N23, violations 0).

**Effective superluminality** is coordinate dragging. In the flow coordinates $\xi_k = k - \varphi_k$ a signal reachable in $t$ steps satisfies $|j-i| \leq t + |\varphi_j - \varphi_i|$ (`flowCoord`, GQC3a); with a flow gradient $> 1$ the rest-frame displacement exceeds 1 in one step, while the signal never exceeds the local light speed relative to the flow (GQC3b). The equivalent speed is $1 + v_{\text{flow}}$ exactly (N24).

**Global coherence** is field-inherent: the anti-phase identity

$$\cos(\theta + \pi) = -\cos\theta \tag{A6}$$

holds at every point, so the correlation $E(\theta) = \cos\theta\cos(\theta+\pi) = -\cos^2\theta$ is independent of the propagation distance $L$ and $E(\Delta = \pi) = -1$ exactly (`antiphase_anticorrelation_global`, `antiphase_correlation_any_point`, QFT5; N4). No action at a distance: $E$ depends only on the relative phase difference.

### 4. Chain 3: the four forces

From the flow momentum (A1), $p = m(C - v)$, the force is the Leibniz (product-rule) decomposition:

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}m}{\mathrm{d}t}C + m\frac{\mathrm{d}C}{\mathrm{d}t} - \frac{\mathrm{d}m}{\mathrm{d}t}v - m\frac{\mathrm{d}v}{\mathrm{d}t}, \tag{A7}$$

four channels (GQF2):

| channel | physical identification |
|---|---|
| $\dfrac{\mathrm{d}m}{\mathrm{d}t}\,C$ | electric force (strand flux; source = divergence) |
| $m\,\dfrac{\mathrm{d}C}{\mathrm{d}t}$ | nuclear force (direction change of the flow) |
| $-\dfrac{\mathrm{d}m}{\mathrm{d}t}\,v$ | magnetic force (flux against the body's velocity) |
| $-m\,\dfrac{\mathrm{d}v}{\mathrm{d}t}$ | gravitation (flow gradient) |

The identity is exact (product rule; `four_force_decomposition`, GQF2; N25, error 0). The identifications of the channels with the four interactions are the interpretation layer; the divergence form of charge and the wave equation are formalization skeletons.

### 5. Chain 4: the photon

The photon is the excited state of an electron [10] in which mass and charge vanish. Model B (two excited helices rotating symmetrically about the axis): the azimuthal momenta cancel, $p_y + (-p_y) = 0$, so the net momentum is purely axial (`azimuthal_cancellation`, GQP2); the dispersion is lightlike, $E^2 = |p|^2$ (GQP2b). Equivalently, at $v = C$ the flow momentum (A1) gives $p = m(C - v) = 0$ (GQP3): the photon is at rest in space, carried by the flow at light speed. The two models share the light cone (N26: azimuthal cancellation 0, massless criterion ✓).

### 6. Chain 5: the quantum relations from helix geometry

This is the central chain. All steps are complete algebraic identities; the single input is the geometric choice (A2).

1. **Circumference = wavelength.** With $r = \lambda/2\pi$, the circumference of the helix cross-section is

$$2\pi r = 2\pi \cdot \frac{\lambda}{2\pi} = \lambda, \tag{A8}$$

   one turn of the helix unfolds exactly one wavelength (`gqr4_circumference_wavelength`, GQR4).

2. $\hbar$ **as radius** $\times$ **momentum.** The angular momentum of one turn is

$$L = r\,p = \frac{\lambda}{2\pi}\,p, \tag{A9}$$

the classical formula. With $h = \lambda p$ (one quantum per turn, $L = h$), (A9) reads $\hbar = L/2\pi = h/2\pi$ — Planck's constant is the angular momentum per radian (`gqr5_hbar_eq_rp`, GQR5; N29: $\hbar = rp$ matches the true $\hbar$ to $6 \times 10^{-10}$).

3. **de Broglie relation.** From (A9),

$$p = \frac{h}{\lambda}, \tag{A10}$$

the de Broglie relation [12] emerges (GQR5).

4. **Planck–Einstein relation.** With $E = \hbar\omega$ and $\omega = 2\pi f$:

$$E = \hbar\omega = \frac{h}{2\pi} \cdot 2\pi f = hf, \tag{A11}$$

and with (A10), $E = pc = (h/\lambda)c = hf$ (`gqr3_planck_relation`, `gqr6_planck_einstein`, GQR3/GQR6; N27: $E = \hbar\omega$ relative error 0; $E = pc$ vs $hf$ error 0). The quantum relations are consequences of helix geometry plus classical identities, given (A2).

5. **Heisenberg uncertainty.** The helix radius is the natural scale of positional uncertainty, $\Delta x \sim r = \lambda/2\pi$, and $\Delta p \sim p = h/\lambda$; hence

$$\Delta x\,\Delta p \sim \frac{\lambda}{2\pi} \cdot \frac{h}{\lambda} = \frac{h}{2\pi} = \hbar \geq \frac{\hbar}{2}, \tag{A12}$$

the Heisenberg relation [13] at the natural scale of the helix (the helix cannot localize a photon better than its own radius).

6. **Pauli exclusion.** Two identical strands are two parallel twistors; their symplectic volume vanishes, $\det(AA^\dagger) = |\det A|^2 = 0$ — identical strands open no new channel (rank criterion of Chain 1). Only distinct (linearly independent) strands create a new degree of freedom, in the spirit of the Pauli principle [14].

### 7. Chain 6: gravitation and electrodynamics (structural)

**Gravitation.** A non-uniform flow $v(x)$ gives the Gordon metric [18]

$$d\tau^2 = dt^2 - \frac{(dx - v\,dt)^2}{c^2}, \qquad g = \begin{pmatrix} 1 - v^2/c^2 & v/c^2 \\ v/c^2 & -1/c^2 \end{pmatrix}, \tag{A13}$$

with $g_{tt} = 1 - v^2/c^2$ matching the weak-field form $g_{tt} = 1 - 2\Phi/c^2$ at $\Phi = \frac{1}{2}v^2$: the gravitational potential is half the square of the flow speed (`gordon_weak_field_matches_newton`, SG11). The geodesic equation then gives the Newtonian limit $a = -\nabla(\frac{1}{2}v^2) = -\nabla\Phi$ (verified numerically). The photon condition $|\mathrm{d}x - v\,\mathrm{d}t| = c\,\mathrm{d}t$ gives $\mathrm{d}\tau^2 = 0$ (`gordon_photon_proper_time_zero`, SG9). At the black-hole horizon $|v| = c$ and escape becomes impossible — a direct consequence of the postulate that the photon follows the flow (PH2), not an extra axiom.

**Electrodynamics.** With the space field $C$ as the only independent field, define $E = -\partial_t C$, $B = \partial_x C$ (motion-kinematics). Then Faraday's law is an identity, $\partial_t B = \partial_t \partial_x C = \partial_x \partial_t C = -\partial_x E$ (`MS2`); Ampère–Maxwell is equivalent to the wave equation of $C$ (`MS3`). Maxwell's equations reduce to the wave equation of the space field; gauge redundancy is eliminated by the physical condition $|C| = c$ (SLS1). The 3D identities $\nabla \cdot (\nabla \times C) = 0$ and $\nabla \times (-\nabla\phi) = 0$ are automatic (SF1–SF2).

## 8. Predictions

Predictions of the framework, with honest status:

1. **Glueball lattice sequence** (a posteriori fit, verified): $m = \sqrt{N}\,M_0$ with $N = \{3, 6, 7\}$ for $0^{++}, 2^{++}, 0^{-+}$ [15–17, 19] ($X(2370)$ as the recent experimental candidate for the $0^{-+}$ channel); $M_0 \approx 0.93 \pm 0.07$ GeV consistent across the three channels. Falsifiable by future lattice data at $N = 4, 5$: $\sqrt{4}, \sqrt{5} \times M_0$.

2. **Three-direction timescale** (proposed; derivation and formalization pending): the three-direction glueball ($m_G = \sqrt{3}\,M_0$) carries a timescale

$$\tau_G = \frac{1}{6M_0^2}, \tag{A14}$$

   i.e. $\approx 0.17\,\mathrm{GeV}^{-1} \approx 1.1 \times 10^{-24}\,\mathrm{s}$ at $M_0 \approx 0.99\,\mathrm{GeV}$. The algebraic relation is recorded as a candidate prediction; its physical identification (decay width, coherence time, or other) and its derivation from the strand-counting postulate are open.

3. **Black-hole escape impossibility** (postulate consequence): inside the horizon the photon follows the flow inward; outward modes vanish (P2, P3 of the black-hole section). Geometric, not an extra axiom.

4. **No other new falsifiable predictions** are claimed beyond the standard framework: the honest assessment of §X stands.

### 9. Honest boundaries of the chain

| layer | examples | status |
|---|---|---|
| mathematical identities | Cauchy–Binet, product rule, $\cos(\theta+\pi)$ | true but standard |
| conceptual reframing | $\hbar$ = angular momentum per radian; force = four channels; photon = helix | interpretation layer |
| derived inputs | (A2), $M_0$, $e$, $G$ values | inputs, not derived |
| skeletons | wave equation; charge as divergence; continuum 3D | formalization pending |


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