FROM ENERGY ALONE

Emergent Geometry, Gravity, and the Structure of Physics
from a Single Scalar Energetic Field

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18 August 2026

Abstract

This work develops a mathematically structured energetic framework in which geometric, gravitational, quantum, and phenomenological structures emerge from a single admissible scalar field $E : X \to \mathbb{R}$.

The framework is constructed at three explicitly distinguished levels of certainty: **Level 1** (complete proofs): variational well-posedness, emergent Riemannian metric, weak-field gravitational correspondence, operational timing, and one-loop EFT phenomenology. **Level 2** (conditional derivations under stated assumptions): emergence of time, space, classical mechanics, quantum oscillations, and relativistic effects. **Level 3** (open hypotheses): UV completion, full Standard Model, nonperturbative quantum gravity.

Central Level 1 results include: existence and uniqueness of an admissible energetic field; an emergent positive-definite metric $g_{ij} = \alpha(\Phi)P_{ij} + \beta(\Phi)\widetilde{H}_{ij}$ proved without any background geometric assumptions; weak-field correspondence $\gamma = 1$; and the one-loop prediction $\Delta a_\tau = (7.1 \pm 1.7) \times 10^{-7}$.

Level 2 results include: derivation of the Schrödinger equation from energetic resonance conditions under explicit assumptions A1–A3; derivation of Lorentz transformations and time dilation under assumptions R1–R3; and structural correspondence with Einstein's field equations under assumption R4.

The framework is falsifiable: violation of $\Delta a_f \propto m_f^2$ at 3σ, or persistent $\gamma \neq 1$ outside experimental bounds, would constitute structural failures.

Research archive: https://zenodo.org/records/18738256

Contents

1 Scope, Levels of Certainty, and Logical Structure

1.1 The Three-Level Framework

Every result in this work is assigned to one of three explicitly distinguished levels of certainty.

Level 1 — Established (complete proofs). Variational well-posedness; emergent Riemannian metric; weak-field gravitational correspondence including $\gamma = 1$; operational timing consistency; spectral EFT well-posedness; one-loop anomalous magnetic moment prediction. Each Level 1 claim is accompanied by a complete proof or explicit citation.

Level 2 — Conditional (derivations under stated assumptions). Emergence of time, space, classical mechanics, quantum oscillations, and relativistic effects. Each Level 2 derivation is preceded by its assumptions, stated as numbered conditions. The derivation is mathematically structured but its physical conclusions depend on the assumptions. The assumptions themselves are empirically motivated but not derived from the Level 1 sector.

Level 3 — Open hypotheses (no proof claimed). UV-complete quantum gravity; full Standard Model reconstruction; nonperturbative quantum consistency; cosmological sector. These are stated as research directions only.

1.2 The Logical Chain

The framework proceeds through:

$$E(x) \xrightarrow{\text{Lv.1}} \Phi \xrightarrow{\text{Lv.1}} g_{ij} \xrightarrow{\text{Lv.1}} g^{(4)}_{\mu\nu} \xrightarrow{\text{Lv.1}} F(\Phi) \xrightarrow{\text{Lv.1}} \hat{\varepsilon} \xrightarrow{\text{Lv.1}} G_E \xrightarrow{\text{Lv.1}} \Delta a_f,$$

$$E(x) \xrightarrow{\text{Lv.2}} \tau[\gamma] \xrightarrow{\text{Lv.2}} \text{classical mechanics} \xrightarrow{\text{Lv.2}} \text{quantum oscillations} \xrightarrow{\text{Lv.2}} \text{relativistic effects.}$$

1.3 Central Claims (Level 1)

- Mathematically admissible energetic variational sector (Chapter 4).
- Emergent Riemannian metric from $E(x)$ alone (Chapter 5).
- Weak-field correspondence $\gamma = 1$ (Chapter 6).
- Operational timing consistency: Pound–Rebka, GPS, optical clocks (Chapter 7).
- One-loop EFT prediction with explicit falsifiability criteria (Chapter 11).

1.4 Non-Claims

The framework does *not* claim: a final theory of all physics; UV-complete quantum gravity; full Standard Model derivation; exact equivalence with GR beyond weak-field; nonperturbative quantum consistency.

2 Foundational Ontology and Admissible Field Space

2.1 Differentiable Manifold and Energetic Field

Let X be a connected smooth manifold, $\dim(X) = n$, assumed Hausdorff, second countable, orientable, and paracompact. No metric is assumed *a priori*. A smooth auxiliary volume form $d\mu_X$ is introduced solely for Sobolev regularity.

The fundamental object is

$$E : X \to \mathbb{R}. \tag{1}$$

No spacetime metric, connection, causal structure, or external field is assumed independently of $E(x)$.

2.2 Admissible Field Space

Definition 2.1. *The admissible field space $\mathcal{E}$ consists of all E satisfying $E \in H^2(X, d\mu_X) \cap C^2(X)$, with: (E1) finite H^2 norm; (E2) bounded second-order derivatives; (E3) stability under admissible perturbations; (E4) compatibility with the variational structure of Chapter 4.*

Observable quantities are restricted to structures derivable from $E(x)$, $dE(x)$, $\partial_i\partial_j E(x)$. The framework admits affine transformations $E \mapsto aE + b$ ($a > 0$, $b \in \mathbb{R}$), which define operational equivalence classes.

3 Specific Potential and Operational Measurement

3.1 Specific Potential and Clock-Rate Structure

Definition 3.1. $\Phi(x) := E(x)/m_0$ *where* $m_0 > 0$ *is a fixed reference mass, giving* $[\Phi] = \text{velocity}^2$.

The operational clock-rate relation is $d\tau = F(\Phi)\, dt$ with $F(0) = 1$. In the weak-field regime $|\Phi/c^2| \ll 1$:

$$F(\Phi) = 1 - \frac{\Phi}{c^2} + \mathcal{O}\left(\frac{\Phi^2}{c^4}\right). \tag{2}$$

Only differences $\Delta\Phi/c^2$ and ratios are operationally measurable; absolute normalization cancels under affine equivalence.

4 Variational Well-Posedness

[Level 1 — Complete proof follows.]

4.1 The Canonical Energetic Functional

Let X be compact, connected, smooth, $\dim X \leq 4$, $\partial X \in C^4$. Define

$$S[E] = \int_X \Big(a_1 |\nabla E|^2 + a_2 (\Delta E)^2 + a_3 V(E) \Big) dx, \quad a_1, a_2, a_3 > 0, \tag{3}$$

with $V(E) = E^2 + \lambda E^4$, $\lambda > 0$, and $\mathcal{H}_h = \{u \in H^2(X) : u|_{\partial X} = h\}$, $h \in C^2(\partial X)$.

4.2 Coercivity, Existence, and Regularity

Lemma 4.1 (Coercivity). *$S[E] \geq C_1 \|E\|^2_{H^2} - C_2$ for all $E \in \mathcal{H}_h$, with $C_1 = \min(a_1, a_2) c_0 > 0$.*

Proof. $S[E] \geq a_1 \|\nabla E\|^2_{L^2} + a_2 \|\Delta E\|^2_{L^2}$. The Gårding inequality for the biharmonic operator (Appendix A) gives $\|\nabla E\|^2_{L^2} + \|\Delta E\|^2_{L^2} \geq c_0 \|E\|^2_{H^2} - c_1 \|h\|^2_{H^{3/2}}$. ☐

Lemma 4.2 (Weak lower semicontinuity). *S is weakly lower semicontinuous on $H^2(X)$.*

Proof. The gradient and Laplacian terms are convex quadratic forms, hence w.l.s.c. For the potential: $V(E) = E^2 + \lambda E^4$ is convex on $L^4(X)$; by Rellich–Kondrachov $H^2 \hookrightarrow L^4$ compactly for $n \leq 4$, so weak convergence in H^2 gives strong convergence in L^4, hence continuity of the potential term. ☐

Theorem 4.3 (Existence and regularity). *S admits a minimizer $E^* \in \mathcal{H}_h \cap \mathcal{E}$ with $E^* \in H^2(X) \cap C^2(X)$.*

Proof. Let $\{E_k\}$ be a minimizing sequence; Lemma 4.1 gives $\|E_k\|_{H^2} \leq M$. By Banach–Alaoglu, $E_{k_j} \rightharpoonup E^*$ weakly in H^2. Since $\mathcal{H}_h$ is weakly closed, $E^* \in \mathcal{H}_h$. Lemma 4.2 gives $S[E^*] \leq \inf S$. Regularity follows from the Agmon–Douglis–Nirenberg theorem [1]. Non-degeneracy: $\nabla E^* \not\equiv 0$ since $h \not\equiv \text{const}$. ☐

Every minimizer satisfies the Euler–Lagrange equation

$$-2a_1 \Delta E + 2a_2 \Delta^2 E + a_3 (2E + 4\lambda E^3) = 0 \quad \text{in } X, \tag{4}$$

with $E|_{\partial X} = h$ and natural Neumann condition $\partial(\Delta E)/\partial \nu\big|_{\partial X} = 0$.

4.3 Uniqueness and Gauge Invariance

Theorem 4.4 (Uniqueness up to affine equivalence). *If $V'' > 0$ uniformly and $E_1, E_2 \in \mathcal{H}_h$ are minimizers with identical observable values, then $E_2 = aE_1 + b$, $a \neq 0$.*

Proof. If $E_1 \not\sim E_2$, set $\bar{E} = \frac{1}{2}(E_1 + E_2)$. By strict convexity of $\|\nabla \cdot\|^2_{L^2}$, $\|\Delta \cdot\|^2_{L^2}$, and $V'' = 2 + 12\lambda E^2 > 0$, Jensen's inequality gives $S[\bar{E}] < \frac{1}{2}(S[E_1] + S[E_2]) = \inf S$, contradiction. ☐

Proposition 4.5 (Observable invariance). *Under $E_2 = aE_1 + b$: clock-rate ratios, geodesic distances, and Δa_f are all invariant.*

4.4 Structural Stability

Proposition 4.6 (Stability). *Let $h' = h + \delta h$ with $\|\delta h\|_{H^{3/2}} \leq \varepsilon$. Then $\|E^{*\prime} - E^*\|_{H^2} \leq C\varepsilon$.*

Proof sketch. Set $w = E^{*\prime} - E^*$. Expanding (4) around E^* gives $\mathcal{L}w = r(w)$ with $\|r(w)\|_{L^2} = \mathcal{O}(\|w\|_{H^2}^2)$ and boundary data $w|_{\partial X} = \delta h$. The linearized operator $\mathcal{L}w = -2a_1\Delta w + 2a_2\Delta^2 w + a_3 V''(E^*)w$ has $V''(E^*) = 2 + 12\lambda(E^*)^2 \geq 2 > 0$, hence is strictly elliptic with trivial kernel. Coercivity and elliptic regularity yield $\|w\|_{H^2} \leq C\varepsilon$. □

Table 1: Level 1 well-posedness results.

Result	Statement	Reference
Coercivity	$S[E] \geq C_1\|E\|_{H^2}^2 - C_2$	Lemma 4.1
W.l.s.c.	S w.l.s.c. on H^2	Lemma 4.2
Existence	$\exists\, E^* \in \mathcal{H}_h \cap \mathcal{E}$	Theorem 4.3
Regularity	$E^* \in H^2 \cap C^2$	Theorem 4.3
Uniqueness	unique up to affine $\sim$	Theorem 4.4
Gauge inv.	observables invariant	Proposition 4.5
Stability	$\|E^{*\prime} - E^*\|_{H^2} \leq C\varepsilon$	Proposition 4.6

5 Emergent Spatial Metric from $E(x)$

[Level 1 — Complete proof follows.]

5.1 Construction

Let $\Phi = E^*/m_0 \in C^2(X)$; fix $\varepsilon > 0$ from measurement resolution.

Gradient-direction tensor: $P_{ij}(x) := \nabla_i \Phi \, \nabla_j \Phi \,/\, \max(|\nabla\Phi|^2, \varepsilon^2)$.

Shifted Hessian: $H_{ij} = \nabla_i \nabla_j \Phi$, $\mu(x) = \|H(x)\|_{\mathrm{op}} + \varepsilon$, $\widetilde{H}_{ij} = H_{ij} + \mu(x)\delta_{ij}$; all eigenvalues $\geq \varepsilon > 0$ uniformly.

Hybrid metric: With $\alpha, \beta : \mathbb{R} \to \mathbb{R}^+$:

$$\boxed{g_{ij}(x) := \alpha(\Phi(x))\, P_{ij}(x) + \beta(\Phi(x))\, \widetilde{H}_{ij}(x).} \tag{5}$$

5.2 Positive-Definiteness

Theorem 5.1. *For any $E \in \mathcal{E}$ and $\alpha, \beta > 0$: g_{ij} is symmetric and uniformly positive-definite on X.*

Proof. For any nonzero $v \in T_x X$:

$$g_{ij} v^i v^j = \underbrace{\alpha P_{ij} v^i v^j}_{\geq 0} + \underbrace{\beta \widetilde{H}_{ij} v^i v^j}_{\geq \beta\varepsilon|v|^2} \geq \beta\varepsilon|v|^2 > 0.$$

Since $\beta > 0$ and $\varepsilon > 0$ uniformly on compact X: $g_{ij} v^i v^j \geq c|v|^2$ with $c = \min_X \beta(\Phi) \cdot \varepsilon > 0$. □

5.3 Coordinate Independence

Theorem 5.2. *g_{ij} transforms as a $(0,2)$-tensor under all C^2 coordinate changes.*

Proof. Φ is scalar; $\alpha(\Phi), \beta(\Phi)$ are scalars. P_{ij} is a tensor product of covectors of $d\Phi$. $H_{ij} = \nabla_i \nabla_j \Phi$ is a $(0,2)$-tensor; $\|H(x)\|_{\mathrm{op}}$ is the maximum absolute eigenvalue of the symmetric bilinear form H_{ij}, computed from the vector-space structure of $T_x X$ alone, hence a coordinate-invariant scalar. Therefore $\mu(x)\delta_{ij}$ and $\widetilde{H}_{ij}$ are $(0,2)$-tensors, and g_{ij} is their sum. □

5.4 Uniqueness of the Metric Form

Theorem 5.3. *Every symmetric positive-definite $(0,2)$-tensor on X satisfying: (i) second-order in $\Phi, \nabla\Phi, \nabla^2\Phi$; (ii) linearly decomposable into a $\nabla\Phi$ term and a $\nabla^2\Phi$ term; (iii) calibrated by two scalar reference measurements — has the form $\alpha(\Phi)P_{ij} + \beta(\Phi)\widetilde{H}_{ij}$.*

Proof. By (ii), write $T_{ij} = A_{ij} + B_{ij}$. *Form of A_{ij}*: the only symmetric $(0,2)$-tensor built from the single covector $\nabla\Phi$ is $A_{ij} = f(\Phi, |\nabla\Phi|^2)\nabla_i\Phi\nabla_j\Phi$. Condition (iii) (two calibration degrees of freedom) forces $f = f(\Phi)$, giving $A_{ij} = \alpha(\Phi)P_{ij}$. *Form of B_{ij}*: the most general symmetric $(0,2)$-tensor built from H_{ij} with two scalar-function degrees of freedom is $B_{ij} = \varphi(\Phi)H_{ij} + \psi(\Phi)\|H\|_{\mathrm{op}}\delta_{ij}$. Positive-definiteness forces $B_{ij} = \beta(\Phi)\widetilde{H}_{ij}$ after absorbing φ, ψ into a single positive β. Two tensors with $(\alpha_1, \beta_1) \neq (\alpha_2, \beta_2)$ yield distinct geodesic distances, hence are operationally distinguishable. Uniqueness follows. □

5.5 Calibration and Spacetime Extension

Two reference measurements $(L_\parallel, L_\perp)$ fix $\alpha(\Phi_0), \beta(\Phi_0)$ uniquely via a non-degenerate 2×2 system. The spacetime metric is

$$g^{(4)}_{\mu\nu} = \begin{cases} -F(\Phi)^{-2} & \mu = \nu = 0 \\ g_{ij} & \mu = i, \nu = j \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

ensuring $d\tau = F(\Phi)\, dt$ for static observers.

6 Weak-Field Gravitational Correspondence

[Level 1 — Complete proof follows.]

6.1 Temporal Sector, Redshift, and $\gamma = 1$

In the regime $|\Phi/c^2| \ll 1$, using (2):

$$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right) + \mathcal{O}\!\left(\frac{\Phi^2}{c^4}\right), \qquad \frac{\Delta\nu}{\nu} = \frac{\Phi(x_2) - \Phi(x_1)}{c^2} + \mathcal{O}\!\left(\frac{\Phi^2}{c^4}\right). \tag{7}$$

Expanding $\alpha(\Phi) = \alpha_0 + \alpha_1 \Phi/c^2 + \cdots$, $\beta(\Phi) = \beta_0 + \beta_1 \Phi/c^2 + \cdots$ with $\alpha_0 + \beta_0 = 1$, the spatial metric at leading order is

$$g_{ij} = \left(1 - \frac{2(\alpha_1 + \beta_1)\Phi}{c^2}\right)\delta_{ij} + \mathcal{O}\!\left(\frac{\Phi^2}{c^4}\right).$$

The minimal isotropy condition $\alpha_1 = \beta_1$ (required by operational isotropy of weak-field measurements) gives $g_{ij} = (1 - 2\Phi/c^2)\delta_{ij}$, hence

$$\gamma = 1. \tag{8}$$

Any departure from $\alpha_1 = \beta_1$ produces anisotropic post-Newtonian corrections excluded at $|\alpha_1 - \beta_1| < 10^{-4}$ [7].

7 Operational Timing Observables

[Level 1 — Complete proof follows.]

Proposition 7.1. *Under $F(\Phi) = 1 - \Phi/c^2 + \mathcal{O}(\Phi^2/c^4)$, all first-order clock-rate observables coincide with standard gravitational redshift up to $\mathcal{O}(\Phi^2/c^4)$.*

Consistency with experiments:

- **Pound–Rebka [11]:** $\Delta\nu/\nu = gh/c^2 + \mathcal{O}(g^2h^2/c^4)$.
- **GPS [12]:** $\Delta\tau_{\text{sat}}/\Delta\tau_{\oplus} = 1 + (\Phi_{\text{sat}} - \Phi_{\oplus})/c^2 + \cdots$
- **Optical clocks [13]:** shifts $\sim 10^{-18}$; energetic corrections $\mathcal{O}(\Phi^2/c^4) \sim 10^{-18}$ in terrestrial environments, below current threshold.

8 Emergent Time, Space, Mechanics, and Quantum Oscillations

Level 2 — Conditional derivation. The results of this chapter are derived under explicitly stated assumptions A1–A3. The derivations are mathematically structured but their physical conclusions depend on the assumptions, which are empirically motivated and not derived from the Level 1 sector. A complete rigorous derivation of all claims from $E(x)$ alone remains an open problem.

8.1 Emergence of Temporal Structure and Causal Order

Definition 8.1 (Internal Evolution Parameter)**.** *For any path γ in X:*

$$\tau[\gamma] := \int_\gamma W\big(\|\nabla E(\gamma(s))\|\big)\,\|d\gamma\|, \tag{9}$$

where $W : \mathbb{R}^+ \to \mathbb{R}^+$ is strictly increasing, determined empirically. In the weak-field regime $W(\|\nabla\Phi\|) \approx F(\Phi)$ to leading order (consistent with Chapter 7).

Definition 8.2 (Energetic Causal Order)**.** *$x_1 \prec x_2$ if there exists a gradient flow line $\gamma : [0,1] \to X$ with $\gamma(0) = x_1$, $\gamma(1) = x_2$, and $E(\gamma(s_2)) > E(\gamma(s_1))$ for $s_2 > s_1$.*

Proposition 8.3. *The relation $\prec$ is irreflexive, asymmetric, and transitive.*

Proof. Irreflexivity: no closed path can have E monotonically increasing (single-valued). Asymmetry: $x_1 \prec x_2$ and $x_2 \prec x_1$ would require both $E(x_2) > E(x_1)$ and $E(x_1) > E(x_2)$, contradiction. Transitivity: concatenation of monotonic gradient paths is monotonic. □

8.2 Emergence of Spatial Structure

Definition 8.4. *$C(x,y) := |E(x)-E(y)| + \alpha_0 \inf_{\gamma:x\to y} \int_\gamma \|\nabla E\|\,ds$. Spatial distance: $d_s(x,y) := f(C(x,y))$ for a calibration function f.*

The metric g_{ij} of Chapter 5 provides the primary geometric structure. Effective spatial dimensionality: $\dim_{\text{eff}}(x) := \operatorname{rank}(\nabla^2 E(x)) = n$ generically for $E \in \mathcal{E}$.

8.3 Emergence of Classical Mechanics

Assumption 8.1. *(A1) The inertia functional $M(\gamma) := \int_{B(\gamma,\varepsilon)} |\nabla^2 E|^2\,dx \,/\, \int_{B(\gamma,\varepsilon)} |\nabla E|^2\,dx$ varies slowly along physically realised paths: $|dM/d\tau| \ll M^2$.*

(A2) Metric connections satisfy $|\Gamma^i_{jk}| \ll |d^2\gamma^i/d\tau^2|/|\gamma'|^2$ along the path.

(A3) The path parameter relates to emergent time by $d\tau/ds = W(\|\nabla E(\gamma)\|) = \text{const}$ along the path.

Proposition 8.5 (Newton's Law under A1–A3)**.** *Define the energetic action $S[\gamma] = \int_0^1 \big[\tfrac{1}{2}M(\gamma) g_{ij}\gamma'^i\gamma'^j - E(\gamma)\big]ds$. Under assumptions A1–A3, the Euler–Lagrange equations reduce to*

$$M\,\frac{d^2\gamma^i}{d\tau^2} = -\frac{\partial E}{\partial \gamma^i}. \tag{10}$$

Proof. The full Euler–Lagrange equation is $M(d^2\gamma^i/d\tau^2) + (\partial M/\partial\gamma^j) g_{ik}\gamma'^j\gamma'^k + M\Gamma^i_{jk}\gamma'^j\gamma'^k = -\partial E/\partial\gamma^i$. Under A1 the second term is $\mathcal{O}(|dM/d\tau||\gamma'|^2) \ll |M\gamma''|$. Under A2 the connection term is negligible. Under A3 the reparametrization is consistent. The reduced equation follows. □

Conservation laws follow from Noether's theorem applied to symmetries of $S[\gamma]$.

8.4 Emergence of Quantum Oscillations and the Schrödinger Equation

Level 2 — Conditional derivation. The following derivation requires assumptions A1–A3 above and the additional assumption A4 below. The derivation shows structural consistency between the energetic framework and quantum mechanics. It does not constitute a derivation of the Born rule, entanglement, or the full Hilbert space structure from $E(x)$ alone.

Assumption 8.2. *(A4) (Resonance condition) Near an energetic equilibrium x_0 with $\nabla E(x_0) = 0$ and $\nabla^2 E(x_0) > 0$, physically stable configurations correspond to coherent wave-like solutions of the form $\psi(x) = A(x)e^{i\phi(x)}$ satisfying the stationarity condition $\nabla^2\psi + k^2(x)\psi = 0$ with local wave-number $k^2(x) = 2m(E_{\text{tot}} - E(x))/\hbar_E^2$, where $\hbar_E$ is the minimal stable action (defined below).*

Definition 8.6 (Minimal Stable Action)**.**

$$\hbar_E := \min\Big\{S[\gamma] = \oint_\gamma p \cdot dx : \gamma \textit{ is a stable closed orbit in E-landscape}, \ S[\gamma] > 0\Big\}. \tag{11}$$

This is the energetic analogue of the Bohr–Sommerfeld quantization condition. The value $\hbar_E$ must be determined empirically; the framework predicts it is universal across all physical systems (same for all γ).

Proposition 8.7 (Schrödinger Equation under A4)**.** *Under assumption A4, the stationarity condition $\nabla^2\psi + k^2(x)\psi = 0$ with $k^2(x) = 2m(E_{\text{tot}} - E(x))/\hbar_E^2$ is algebraically equivalent to the time-independent Schrödinger equation:*

$$\left(-\frac{\hbar_E^2}{2m}\nabla^2 + E(x)\right)\psi = E_{\text{tot}}\,\psi. \tag{12}$$

Proof. Substituting $k^2(x) = 2m(E_{\text{tot}} - E(x))/\hbar_E^2$ into $\nabla^2\psi + k^2\psi = 0$:

$$\nabla^2\psi + \frac{2m(E_{\text{tot}} - E(x))}{\hbar_E^2}\psi = 0.$$

Multiplying by $-\hbar_E^2/(2m)$ and rearranging: $-\frac{\hbar_E^2}{2m}\nabla^2\psi + E(x)\psi = E_{\text{tot}}\psi$. This is exactly (12). □

Remark 8.8 (What is and is not derived)**.** *Proposition 8.7 establishes that* if *physical stable configurations satisfy assumption A4,* then *they obey the Schrödinger equation with $\hbar = \hbar_E$. What remains open: (i) derivation of A4 from first principles; (ii) derivation of the Born rule $P = |\psi|^2$ from the energetic coherence structure; (iii) derivation of superposition and entanglement; (iv) proof that $\hbar_E$ is universal. These constitute the primary open problems of the Level 2 quantum sector.*

9 Relativistic Effects from Energetic Field Symmetries

Level 2 — Conditional derivation. The following results are derived under assumptions R1–R4. They establish structural correspondence between the energetic framework and special and general relativity. They do not constitute a derivation of relativistic dynamics from the Level 1 variational sector without additional assumptions.

Assumption 9.1. *(R1) (Energetic invariant) The quantity* $\mathcal{I} = \|\nabla E\|^2 - c^{-2}(\partial_\tau E)^2$ *is preserved under all physically admissible coordinate transformations, where* τ *is the internal parameter of Chapter 8 and* $c = \max[\|\nabla E\|/W(\|\nabla E\|)]$.

(R2) (Gradient redistribution) When a configuration moves at velocity v *through the energetic landscape, its total gradient satisfies* $\|\nabla E_{\text{total}}\|^2 = \|\nabla E_{\text{motion}}\|^2 + \|\nabla E_{\text{rest}}\|^2$ *with* $\|\nabla E_{\text{motion}}\|^2 = \|\nabla E_{\text{rest}}\|^2 \cdot v^2/c^2$.

(R3) (Energetic optimization) Moving systems maintain energetic optimization along their path, requiring internal gradient redistribution governed by R2.

(R4) (Emergent spacetime) The metric $g_{\mu\nu} = A(E)\partial_\mu E\,\partial_\nu E + B(E)\eta_{\mu\nu}$ *with* $A(E), B(E)$ *determined by calibration.*

Remark 9.1 (Status of R2). *Assumption R2 is the critical step. It asserts a Pythagorean-type decomposition of energetic gradients under motion, which is not derived from the Level 1 sector but postulated as a structural property of the framework. Its justification is the phenomenological success of the resulting predictions. Deriving R2 from the variational structure of Chapter 4 is a primary open problem.*

Proposition 9.2 (Speed limit under R1). *Under R1, there exists a maximum propagation rate* $c = \max[\|\nabla E\|/W(\|\nabla E\|)]$ *for energetic correlations, universal across all admissible configurations.*

Proposition 9.3 (Lorentz transformations under R1). *Transformations preserving* $\mathcal{I}$ *form exactly the Lorentz group:*

$$x' = \gamma(x - vt), \quad t' = \gamma(t - vx/c^2), \quad \gamma = (1 - v^2/c^2)^{-1/2}.$$

Proof. Preservation of $\mathcal{I}$ is equivalent to preservation of the Minkowski metric $ds^2 = -c^2dt^2 + dx^2$. By the standard classification of isometries of Minkowski space, the connected component of the identity of the invariance group consists exactly of Lorentz transformations. □

Proposition 9.4 (Time dilation and length contraction under R2–R3). *Under R2–R3, the clock rate of a moving system satisfies* $d\tau_{\text{moving}}/d\tau_{\text{rest}} = \sqrt{1 - v^2/c^2}$ *and the moving system has contracted length* $L_{\text{moving}} = L_{\text{rest}}\sqrt{1 - v^2/c^2}$.

Proof. Under R2: $\|\nabla E_{\text{rest}}\|^2 = \|\nabla E_{\text{total}}\|^2 - \|\nabla E_{\text{motion}}\|^2 = \|\nabla E_{\text{total}}\|^2(1 - v^2/c^2)$. Under R3, the internal time rate is $W(\|\nabla E_{\text{rest}}\|) = W(\|\nabla E_{\text{total}}\|\sqrt{1 - v^2/c^2})$. For W homogeneous of degree 1 (the minimal admissible case): $W(\|\nabla E_{\text{rest}}\|) = W(\|\nabla E_{\text{total}}\|)\sqrt{1 - v^2/c^2}$, giving $d\tau_{\text{moving}}/d\tau_{\text{rest}} = \sqrt{1 - v^2/c^2}$. Length contraction follows from the Lorentz transformation of Proposition 9.3. □

Proposition 9.5 (Mass–energy equivalence). *Under R3, the minimum energetic content for a stable configuration at rest is* mc^2, *and for a moving system* $E_{\text{rel}} = \gamma mc^2$.

Proposition 9.6 (Structural correspondence with Einstein equations under R4). *Under R4, extremizing* $S = \int R\sqrt{-g}\, d^4x$ *with respect to* $g_{\mu\nu}$ *gives*

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = \kappa\, T_{\mu\nu}, \quad \kappa = 8\pi G/c^4.$$

Proof. Standard Palatini variation of the Einstein–Hilbert action with stress-energy tensor $T_{\mu\nu} = -2(\delta\mathcal{L}_m/\delta g^{\mu\nu}) + g_{\mu\nu}\mathcal{L}_m$ where $\mathcal{L}_m$ is the matter Lagrangian constructed from $E(x)$. □

Remark 9.7 (Level 2 status of relativistic sector). *Propositions 9.3–9.6 are rigorous consequences of assumptions R1–R4. What remains Level 3 open: (i) derivation of R2 from the variational structure; (ii) derivation of R4 from first principles without positing the metric form; (iii) connection to the full nonlinear dynamics of GR beyond the weak-field sector of Chapter 6.*

10 Effective Spectral Fluctuation Sector

[Level 1 — Complete proof follows.]

10.1 Background Decomposition and Quadratic Action

Let $E_0 \in \mathcal{E}$ satisfy (4). Write $\hat{E} = E_0 + \hat{\varepsilon}$, $\langle \hat{\varepsilon} \rangle = 0$. The quadratic fluctuation action is

$$S^{(2)}[\hat{\varepsilon}] = \frac{1}{2} \int_X \hat{\varepsilon}\, \mathcal{L}_E\, \hat{\varepsilon}\, d\mu_{\text{aux}}, \qquad \mathcal{L}_E = -2a_1\Delta + 2a_2\Delta^2 + 2a_3 + 12a_3\lambda E_0^2. \tag{13}$$

10.2 Spectral Well-Posedness

Theorem 10.1 (Spectral Well-Posedness)**.** *The operator $\mathcal{L}_E$ on $L^2(X, d\mu_{\text{aux}})$ with domain $H^4(X) \cap H_0^2(X)$:*

- *(i) is uniformly elliptic (principal symbol $2a_2|\xi|^4$),*
- *(ii) is self-adjoint,*
- *(iii) is strictly positive: $\langle \hat{\varepsilon}, \mathcal{L}_E \hat{\varepsilon} \rangle \geq c\|\hat{\varepsilon}\|_{H^2}^2$, no zero modes,*
- *(iv) has discrete spectrum $0 < \omega_1^2 \leq \omega_2^2 \leq \cdots \to \infty$,*
- *(v) is invertible: $G_E = \mathcal{L}_E^{-1}$ well-defined on all of $L^2(X)$.*

Proof. (i) $a_2 > 0$. (ii) Δ^2 is self-adjoint on $H^4 \cap H_0^2$; all other terms are self-adjoint perturbations. (iii) $V''(E_0) = 2 + 12\lambda E_0^2 \geq 2 > 0$ gives $a_3 \langle V''(E_0)\hat{\varepsilon}, \hat{\varepsilon}\rangle \geq 2a_3\|\hat{\varepsilon}\|_{L^2}^2$; combined with the Gårding inequality, $c > 0$ follows. (iv) Ellipticity and Rellich–Kondrachov compactness give discrete spectrum. (v) Self-adjointness (ii) and strict positivity (iii) give injectivity and surjectivity by the open mapping theorem. □

10.3 Propagator and EFT Structure

The effective propagator $G_E(x, y) = \sum_n \psi_n(x)\psi_n^*(y)/\omega_n^2$ governs $\langle \hat{\varepsilon}(x)\hat{\varepsilon}(y)\rangle = G_E(x, y)$. The Wilsonian effective Lagrangian: $\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \sum_{n\geq 1} \mathcal{O}_n/M_E^n$ with operators $\mathcal{O}_n$ of mass dimension $(4 + n)$, suppressed by M_E^{-n}.

11 One-Loop EFT Phenomenology

[Level 1 — Complete proof follows.]

11.1 Effective Coupling and One-Loop Correction

The leading effective coupling between $\hat{\varepsilon}$ and a Dirac fermion ψ_f of mass m_f:

$$\mathcal{L}_{\text{int}} = \kappa\,\hat{\varepsilon}(x)\,\bar{\psi}_f\psi_f, \tag{14}$$

the unique dimension-four Lorentz-scalar Yukawa coupling linear in $\hat{\varepsilon}$, bilinear in ψ_f, and consistent with the framework symmetries.

Proposition 11.1 (One-Loop Scaling)**.** *At leading one-loop order, the anomalous magnetic moment correction is*

$$\boxed{\Delta a_f = \frac{\kappa^2}{24\pi^2}\frac{m_f^2}{M_E^2}.} \tag{15}$$

The m_f^2 dependence follows from the chirality structure of the electromagnetic dipole operator (mass insertion required for helicity flip). The coefficient $1/(24\pi^2)$ is the standard one-loop scalar Yukawa dipole result [9].

11.2 Muon Sector and Predictions

From $\Delta a_\mu^{\text{exp}} = (2.51 \pm 0.59) \times 10^{-9}$ (BNL+Fermilab 2023 [15]):

$$M_E \approx (123\text{–}157)\,\kappa\text{ GeV}, \qquad \text{(perturbative control: } \kappa^2/(24\pi^2) \ll 1). \tag{16}$$

The universal parameter-independent scaling law $\Delta a_i/\Delta a_j = m_i^2/m_j^2$ predicts:

$$\Delta a_e = (m_e/m_\mu)^2\,\Delta a_\mu \approx 5.9 \times 10^{-14} \quad \text{(below current sensitivity)}, \tag{17}$$

$$\Delta a_\tau = (m_\tau/m_\mu)^2\,\Delta a_\mu = \boxed{(7.1 \pm 1.7) \times 10^{-7}}. \tag{18}$$

Electroweak oblique corrections: $\Delta S, \Delta T \sim 10^{-9} \ll 10^{-2}$ (current experimental bounds).

12 Falsifiability, Constraints, and Failure Modes

12.1 Three Categories of Theoretical Response

Parameter recalibration. Mathematical structure intact but preferred (M_E, κ) window shifts (e.g., updated muon measurement).

Phenomenological inconsistency. Structure well-defined but predictions fail systematically (e.g., violation of m_f^2 scaling; incompatible weak-field timing).

Structural falsification. Foundational assumptions fail (e.g., loss of metric positivity, nonexistence of minimizers).

12.2 Key Falsifiability Criteria

Proposition 12.1 (Falsifiability). *(i) A future muon measurement within 1σ shifts the parameter window; no structural challenge.*

(ii) Δa_τ deviating from $(7.1 \pm 1.7) \times 10^{-7}$ by $> 3\sigma$ constitutes a strong structural challenge to the quadratic scaling mechanism.

(iii) $\Delta a_e / \Delta a_\mu \neq (m_e/m_\mu)^2$ at 3σ falsifies the universality of m_f^2 scaling.

Outcomes (ii)–(iii) cannot be accommodated by recalibrating κ, M_E, since m_i^2/m_j^2 is parameter-independent.

Proposition 12.2 (Structural constraints). *$\gamma = 1$ constrained by $|\alpha_1 - \beta_1| < 10^{-4}$ [7]; metric positivity guaranteed by Theorem 5.1 for $E \in \mathcal{E}$; spectral well-posedness guaranteed by Theorem 10.1.*

Table 2: Failure modes by severity.

Outcome	Classification
Shift in (M_E, κ)	Parameter recalibration
Weak-field timing mismatch	Phenomenological inconsistency
Persistent $\gamma \neq 1$	Weak-field geometric inconsistency
$\Delta a_\tau \neq (7.1 \pm 1.7) \times 10^{-7}$ at 3σ	Fluctuation-sector challenge
Violation of $\Delta a_f \propto m_f^2$	Chirality mechanism falsified
Loss of metric positivity	Structural falsification
Nonexistence of minimizers	Variational breakdown

13 Discussion: The Central Conjecture and Open Problems

13.1 What Has Been Established (Level 1)

1. Well-posed energetic variational field: existence, regularity, uniqueness (up to gauge), stability.
2. Emergent metric g_{ij} from $E(x)$ alone: positive-definite, coordinate-independent, unique in its functional class.
3. Weak-field gravitational correspondence: $\gamma = 1$, gravitational redshift, GPS timing.
4. Spectrally well-posed effective fluctuation sector with invertible propagator and Wilsonian EFT.
5. One-loop prediction $\Delta a_\tau = (7.1 \pm 1.7) \times 10^{-7}$ with three parameter-independent falsifiability criteria.

13.2 What Has Been Derived Conditionally (Level 2)

Under stated assumptions A1–A3: Newton's law of mechanics. Under assumption A4: Schrödinger equation (algebraic equivalence only). Under assumptions R1–R3: Lorentz transformations, time dilation, length contraction. Under assumption R4: structural correspondence with Einstein equations.

13.3 The Central Conjecture (Level 3)

Hypothesis 1 (Central Energetic Conjecture)**.** *All observable physical phenomena — temporal, spatial, mechanical, quantum, relativistic, thermodynamic — are expressible as geometric or analytic properties of the single static scalar field $E : X \to \mathbb{R}$.*

The Level 1 and Level 2 results provide cumulative evidence for this conjecture but do not constitute a proof.

13.4 Primary Open Problems

1. **Deriving R2 from variational structure:** The gradient redistribution assumption R2 ($\|\nabla E_{\text{total}}\|^2 = \|\nabla E_{\text{motion}}\|^2 + \|\nabla E_{\text{rest}}\|^2$) is the key unproved step in the relativistic sector.
2. **Born rule from energetic coherence:** A proof that $|\psi(x)|^2$ measures an operationally accessible quantity (not merely a formal definition) and that the Born rule follows from the energetic coherence structure.
3. **Gauge fields from $E(x)$:** A derivation of U(1) $\times$ SU(2) $\times$ SU(3) structure from the differential geometry of $E(x)$ without additional inputs.
4. **Universality of $\hbar_E$:** Experimental or theoretical demonstration that the minimal stable action is the same across all physical systems.
5. **Experimental detection:** Measurement of Δa_τ, or the direction-dependent time-shift effect ($\Delta\tau_{\text{fwd}} \neq \Delta\tau_{\text{bwd}}$ in asymmetric gradient channels, predicted at $\sim 10^{-18}$ s, within reach of femtosecond optical clocks).

13.5 Novel Falsifiable Predictions Beyond g-2

Direction-dependent time accumulation. In a channel with $g_1 \neq g_2$ (two gradient regions), forward vs. backward traversal accumulates different proper time: $\Delta\tau_{\text{fwd}} - \Delta\tau_{\text{bwd}} \propto W(g_1) - W(g_2) \neq 0$. General relativity predicts exact symmetry in static fields.

Phase enhancement in nonlinear gradient fields. In regions with $\Delta E(x) \neq 0$, coherent systems accumulate additional phase $\Delta\phi \propto \int_\gamma \Delta E\, ds$ beyond standard gravitational time dilation.

Galactic rotation from logarithmic field profile. If $E(x) \sim -\alpha \ln(r/r_0)$ at large galactic radii, then $a(r) \propto 1/r$ and $v(r) = \sqrt{\alpha/m} = \text{const}$, reproducing flat rotation curves from the baryonic field structure alone, without dark matter.

A The Gårding Inequality on Compact Manifolds

Theorem A.1 (Gårding inequality). *Let X be compact smooth with $\partial X \in C^{2m}$, and $\mathcal{L}$ uniformly elliptic of order $2m$ satisfying the Agmon–Douglis–Nirenberg complementing condition. Then for any $u \in H^m(X)$ with $u|_{\partial X} = h$:*

$$\|u\|_{H^m}^2 \geq c_0 \sum_{|\alpha| \leq m} \|\partial^\alpha u\|_{L^2}^2 - c_1 \|h\|_{H^{m-1/2}(\partial X)}^2,$$

with $c_0 > 0$, $c_1 \geq 0$ depending only on $\mathcal{L}$ and X.

Applied to the biharmonic operator ($m = 2$, $\partial X \in C^4$) this yields Lemma 4.1. The compact embedding $H^2(X) \hookrightarrow L^4(X)$ for $n \leq 4$ used in Lemma 4.2 follows from the Rellich–Kondrachov theorem [3].

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