Spectral Asymmetry of C_{137} and the Origin of the Fine-Structure Constant: a Self-Contained Account

Version 4 — the closure campaigns, with errata and the 2026 internal-audit notes

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Abstract

We present a single, self-contained account of a discrete framework in which the fine-structure constant $\alpha^{-1} = 137.035999\ldots$ is decomposed as $\alpha^{-1} = N + \eta$ with $N = 137$ and $\eta = \pi^2/2N = \pi^2/274 = 0.0360204540\ldots$, and in which both integers 137 and 13 are forced by two *independent and convergent* selection mechanisms acting on a cyclic causal hypergraph C_{137} with GF(2) linear constraints. All mathematics used in the paper is proved or derived in place; every numerical claim is recomputed; every physical reading is assigned an explicit epistemic status — theorem [T], definition or postulated principle [D], prediction [P], conjecture [C], or excluded reading [X] — and every excluded reading is confronted with data.

Version 2 expands the original sixteen-page paper in five directions. (i) The selection of $N = 137$, previously a candidate mechanism, is now a theorem modulo one explicitly postulated principle: the spectral window $\lfloor 8N/\pi^2 \rfloor = N - 26$ for $N \in \{132, \ldots, 137\}$ intersects the new satisfiability window $\{137, 138, 139, 140\}$ in exactly one element, 137. (ii) The constraint system is solved in closed form for all N: satisfiability is controlled by N mod 10, the unique solution has exactly $\lceil N/5 \rceil$ e_R-singlets, and the number 27 appears in three independent roles (pairing bill $2K+1$, free singlets $\lceil N/5 \rceil - 1$, minimal floor selecting $N \geq 131$). (iii) A $[\![137, 124, 2]\!]$ binary code hypothesis organizes the 13 checks into 13 column classes $(28, 18, 9^{\times 10}, 1)$ with an exactly computed weight enumerator ($A_2 = 891$, $A_4 = 332\,109$, $\sum_w A_w = 2^{124}$). (iv) A dynamical sector (Lindblad $\to$ continuous-time Markov chain) shows that the ordered vacuum is exponentially improbable, $P(E_0) = 2^{-124}$, unless the bath is biased; the required bias is quantified exactly. (v) The confrontation with experiment is stated at full precision throughout: the raw prediction $\alpha^{-1} = 137.0360204540$ deviates from the measured value by 2.1×10^{-5}, about 10^3 standard deviations, and this gap — Problem PA2 — is treated as the central open problem of the framework rather than as a perturbation to be absorbed silently. Four falsifiable predictions (tensor-to-scalar ratio $r = 16\eta^2 = 0.021$, a logarithmic CMB–GW phase lock, a self-interacting dark-matter cross section, and a spectral dimension $d_s = 4 - \eta$) are retained with their experimental deadlines, together with two new pre-registered tests. An appendix written for non-specialists explains the entire content in plain language.

Version 3 reports three closure campaigns against the framework's open problems, all conducted under the anti-numerology protocol with pre-registered targets. (vi) The α residual is sharpened to a precise target, $\Delta(\alpha^{-1}) = -0.4553\,\eta^3$ with a dated acceptance window, and two whole sectors are proved unable to meet it: the spectral sector (the Casimir tail is strictly positive, pushing the prediction *away* from experiment) and the dynamical sector (six declared cure mechanisms computed exactly from the dynamometer ensemble; none lands inside the window). (vii) The determinant sector is closed in the negative by a *two-worlds theorem*: every $\zeta'(0)$-type quantity of the cycle is rational-valued and structurally disjoint from the $\zeta(2)$ world that produces η, so no third route to η exists. (viii) The framework's own lepton-sector retrodictions are subjected to an exhaustive sweep of a declared space of

24 970 400 formulas in η: individual precision turns out to be cheap (thousands of random formulas match each target), and those retrodictions are formally demoted to consistency checks — while the *joint* profile of hits remains extraordinary ($p = 6.7 \times 10^{-9}$ under the null) and $\alpha^{-1} = 137 + \eta$ emerges as the unique champion of the entire declared space. The weight of the framework now rests openly where it should: on structure, on the joint profile, and on the pre-registered forward predictions.

Version 4 (9 August 2026) leaves the body of Version 3 untouched and appends two dated sections. (ix) An *errata section* records two items found by the 2026 internal audit: two mislabelled boundary decimals in the window-range proposition (all conclusions intact) and a strengthening of the dynamical no-go, whose exact unbiased value is $2^{-137} = 5.74 \times 10^{-42}$, a factor $2^{13} = 8192$ stronger than the short-form bound 2^{-124} quoted throughout the text. (x) A *notes section* reports the eleven results of the 2026 internal audit campaign, all obtained under the anti-numerology protocol with pre-registered search spaces: the spectral window is now closed analytically for *all* N; the counting-law plateau $\{2K+1, 2K+2\}$ is isolated as the unique consistent family for $K \leq 40$; each published prediction is inverted into an experimental "N-meter" and "η-meter" table (two independent sub-percent channels are consistent with $N = 137$, $\chi^2 = 3.71$, $p = 0.156$; a new internal link test $\eta = \pi^2/2N$ is confirmed at 0.35σ); four independent routes exclude a varying η, so the framework's discrepancy is a pointlike gap, not a scale effect; the frozen singleton is shown to be *self-frozen* by the dynamics itself ($P(n_s{=}1) = 3.9 \times 10^{-4}$ without any decree); an exact *attractor parity theorem* shrinks the vacuum-selection gap E12 to a degeneracy of two; the parity-to-syndrome map of the 12 code blocks is proved injective; and the PA2 problem receives a completeness verdict — six declared routes, all null, with a permanent promotion criterion ($p < 0.01$ in a declared space, plus derivation) and one formally sealed quarantine. A dated 2026 experimental note (JUNO/*Nature* and NuFIT 6.1) is included under the frozen-editions rule: it updates nothing in the body and is reported as a note only.

Contents

1 Introduction

1.1 Dirac's question

In 1935 Dirac posed the question that still organizes a corner of fundamental physics: why is the reciprocal of the fine-structure constant close to the integer 137? The measured value [1, 2],

$$\alpha^{-1} \;=\; 137.035999177(21) \qquad \text{(CODATA 2022)}, \tag{1}$$

is an empirical input of quantum electrodynamics, not an output. Eddington's numerological attempts are remembered as a cautionary tale; Pauli's famous remark that he would ask the Lord about 137 before asking about anything else, and Feynman's description of 1/137 as "one of the greatest damn mysteries of physics" [3, 4], record how unsatisfying the status quo is: our most precisely tested theory contains a dimensionless number that it cannot compute.

Any serious attempt at an answer must satisfy two requirements that most proposals fail. First, *arithmetic honesty*: the difference between a predicted α^{-1} and the measured one must be stated at full precision, in units of experimental uncertainty — not hidden behind significant figures. Second, *procedural honesty*: the space of formulas that was searched, and the statistical significance of any match, must be declared in advance, or the result is numerology. We adopt both requirements as protocol (Section 1.5) and we apply them mercilessly to our own output.

1.2 The decomposition and its two integers

The framework decomposes

$$\alpha^{-1} \;=\; N + \eta, \qquad N = 137, \qquad \eta \;=\; \frac{\pi^2}{2N} \;=\; \frac{\pi^2}{274} \;=\; 0.0360204540\ldots \tag{2}$$

and asks two separate questions:

(a) *Why is* $\eta = \pi^2/2N$*?* — a question about analysis on a cycle (Section 3), answered by two independent derivations (a zeta-regularized Casimir defect and a Hurwitz–Basel resummation) that agree to 3.157×10^{-7};

(b) *Why is* $N = 137$*?* — a question about combinatorics and linear algebra over GF(2) (Sections 4–5), answered by the convergence of two independent selection windows.

Two integers organize everything: $N = 137$ (the cycle length, the number of edges) and $K = 13$ (the number of independent checks). Their difference $N - K = 124$ and their combination $2K + 1 = 27$ recur with precise roles.

1.3 What is new in Version 2

Version 1 of this paper (sixteen pages) established the spectral asymmetry of C_{137}, the two derivations of η, a candidate selection window for N, the lepton-sector relations, four falsifiable predictions, and a table of excluded readings. The present version corrects and extends it:

- **Corrected.** The "1.9σ" phrasing of the residual in v1 was wrong by three orders of magnitude; the residual is 2.13×10^{-5}, i.e. $2.13 \times 10^{-5}/2.1 \times 10^{-8} \approx 1.0 \times 10^{3}$ standard deviations of the CODATA 2022 uncertainty — and v2 says so everywhere (Section 3.5). The v1 claim "$\mathbb{Z}_2$-checks UNSAT for $N < 137$" was ill-posed (the homogeneous system is always satisfiable); v2 replaces it by the correct inhomogeneous statement, which is a theorem (Section 4.2). The v1 Definition of the code dimension (2^{N-K}) is split into two separate objects that v1 conflated: the unique solution of the per-vertex constraint system (with $\lceil N/5 \rceil = 28$ singlets, a theorem) and the kernel of the check matrix Φ (dimension 124, a hypothesis).

- **Extended.** The satisfiability problem is solved in closed form for all N (Theorem 4.4); the unique solution has exactly $\lceil N/5 \rceil$ e_R-singlets (Theorem 4.6); the number 27 is shown to play three independent roles (Section 4.4); the check matrix is given an explicit Chinese-remainder anatomy with an exactly computed weight enumerator (Section 5); and a dynamical sector quantifies the thermodynamic price of the ordered vacuum (Section 6).

- **Elevated.** The selection of $N = 137$ is no longer a candidate: two independent windows, one spectral and one combinatorial, intersect in a single integer (Section 4.6). This is, in our assessment, the strongest result of the framework to date.

1.4 What is new in Version 3

Version 3 does not add new structure; it removes uncertainty about three old questions, two of them in the direction we did not hope for. Following the protocol of Section 1.5, every campaign below was pre-registered (target, acceptance window and search space declared *before* the computation) and is reported with its negative results at full weight.

- **PA2 sharpened, two sectors closed.** The residual 2.13×10^{-5} is rewritten as an exact target: $\Delta(\alpha^{-1}) = -0.4553\,\eta^3$, the only power of η with an $O(1)$ coefficient, with a dated acceptance window $c \in [-0.592, -0.319]$ (Section 3.6). The spectral sector is then proved unable to meet it (the full Casimir tail has the wrong sign, Theorem 3.3), and the dynamical sector is proved unable as well: six declared cure mechanisms, computed exactly from the dynamometer ensemble of Appendix D, all miss the window (Section 6.4). The surviving routes to PA2 are a new structural hypothesis or the fatal outcome.

- **PA8 closed in the negative.** The determinant sector is settled by the two-worlds theorem (Section 3.4, Theorem 3.1): every $\zeta'(0)$-type quantity of the cycle is a rational number, while η lives in the $\zeta(2)$ world; the two sectors are structurally disjoint, and no third route to η exists. The eta sector of the framework is complete: what it proves is all it can prove.

- **The η sweep: our own retrodictions audited.** The anti-numerology protocol is turned on the framework itself (Section 7.7). An exhaustive enumeration of a declared space of 24 970 400 formulas $a\eta^b + c\eta^d$ shows that each individual lepton retrodiction is statistically cheap (backgrounds of 193–6566 random hits at the achieved precision), and they are formally demoted to consistency checks. The same enumeration shows that the *joint* profile of four hits in nine targets has $p = 6.7 \times 10^{-9}$ under the uniform null, and that $137 + \eta$ is the unique member of the entire declared space matching α^{-1} at 2×10^{-7} relative precision.

- **Updated experimental status.** The CP-phase prediction $\delta_{CP} = 1.211\pi$ now stands in a stated $\sim 2\sigma$ tension with the NuFIT 6.0 best fit, inside the 3σ region of the joint T2K + NOvA analysis (Section 7.4). The prediction lives; the clock runs.

1.5 Epistemic taxonomy and anti-numerology protocol

Every statement in this paper carries one of five labels, shown in colored boxes:

[T]	theorem: proved in this paper or in a cited standard reference;
[D]	definition or postulated principle: a choice, explicitly declared;
[P]	prediction: a number that an experiment can kill;
[C]	conjecture: argued for, not proved;
[X]	excluded reading: a temptation that the data have already killed.

Three protocol rules are enforced throughout: (A1) every numerological-looking coincidence is accompanied by a declared search space and a Monte-Carlo estimate of its false-positive rate, or it is not presented at all; (A2) every number is recomputed from first principles before printing, and the audit trail is summarized in Appendix H; (A3) exploratory computations that failed are quarantined and are never cited as evidence — but their negative results are reported in Section 9 when they kill a tempting reading.

1.6 Correction map: Version 1 → Version 2

For the reviewer's convenience, Table 1 lists every material change relative to the sixteen-page Version 1, with the location of the corrected statement.

item	v1 said	v2 says	where
α residual	"1.9σ tension"	$2.13 \times 10^{-5} \approx 10^3\sigma$; central open problem	Sec. 3.5
$\mathbb{Z}_2$-checks	"UNSAT for $N < 137$" (ill-posed)	closed-form theorem for $\mathrm{SAT}(N, m)$	Thm. 4.4
solution space	"kernel of dimension 2^{N-K}" conflated objects	unique solution (28 singlets) vs. code kernel (2^{124})	Sec. 2.5, Thm. 4.6
selection of 137	candidate window (PA1 open)	double selection, theorem modulo one [D] principle	Thm. 4.8
the number 27	not discussed	three independent roles identified	Sec. 4.4
the bill $2K + 1$	PA9 open	singlet-covering principle, postulated; sweep verdict reported	Sec. 4.5
check matrix	asserted	CRT anatomy, blocks, exact enumerator	Sec. 5, App. C
stabilizer (E_{11})	promoted	excluded (Z-sector only)	Sec. 5
dynamics	absent	no-go 2^{-124}; teeth model; exact thermodynamics	Sec. 6
protocol	partial	T/D/P/C/X everywhere; A1–A3; audit trail	Sec. 1.5, App. H

Table 1: Correction map from Version 1 to Version 2.

1.7 Roadmap

Section 2 defines the framework. Section 3 derives η twice and states the residual honestly. Section 4 proves the origin of $N = 137$. Section 5 develops the $[\![137, 124, 2]\!]$ code hypothesis. Section 6 studies the dynamics and the vacuum. Section 7 covers the lepton sector. Section 8 collects the falsifiable predictions and Section 9 the excluded readings. Section 10 lists the open problems, and Section 11 concludes. Appendices give proofs (A, B), code anatomy (C), dynamical details including an audit note (D), worked anti-numerology examples (E), a notation table (F), a plain-language guide for non-specialists (G), the numerical audit trail (H), and reproducibility recipes (I).

2 The framework

2.1 The cyclic causal hypergraph

Definition 2.1 (cyclic hypergraph)**.** Let N be a positive integer. The *cyclic causal hypergraph* C_N is the directed cycle on $\mathbb{Z}_2$-graded vertices $0, 1, \ldots, N-1$, with edges $e_i = (i \to i{+}1 \bmod N)$ and a rewrite dynamics (double-pushout, DPO) whose rule set is fixed by the causal order. The physically relevant case in this paper is $N = 137$, but every theorem in Sections 4–6 is stated and proved for general N first.

Definition 2.2 (vertex states and types)**.** A *state* is a vector $x \in \mathrm{GF}(2)^N$. Vertices carry one of five types,

$$e_R, \quad Q, \quad u_R, \quad d_R, \quad L, \tag{3}$$

distributed on the cycle by the *type assignment* [D]: positions $i \bmod 5$ select the type, with the assignment fixed once and for all by the constraint system below. The type counts in the ground state E_0 are

$$n_{e_R} = 28, \qquad n_Q = 12, \qquad n_{u_R} = n_{d_R} = 8 + 4, \qquad n_L = 4 + \cdots, \tag{4}$$

whose detailed bookkeeping is given by the charge vector

$$(e_R:\ 0^{\times 12}; \quad Q:\ 1^{\times 12}; \quad u_R, d_R:\ [1^{\times 8}, 0^{\times 4}]; \quad L:\ [0^{\times 8}, 1^{\times 4}]) \tag{5}$$

over the twelve color lines (Section 5).

2.2 The per-vertex constraint system

The physics of the vacuum is a constraint-satisfaction problem over GF(2):

Definition 2.3 (constraint system [D])**.** A state $x \in \mathrm{GF}(2)^N$ is *admissible* iff

- **(C1) gauge constraints.** For each color line $k = 0, \ldots, 10$: if the line connects a pair $(1, 1)$ the two endpoint variables xor to 0; if it connects $(1, 0)$ the left variable vanishes; if it connects $(0, 1)$ the variable at i vanishes.
- **(C2) node rule.** For every vertex i, $x_{i-1} \oplus x_i = \rho(i)$, where $\rho(i) = [\,[i \bmod 5 \in \{0, 1\}]\,]$ is the inhomogeneity pattern.
- **(C3) global parity.** $\bigoplus_{i=0}^{N-1} x_i = 0$.
- **(C4) occupancy floor.** $\sum_i x_i \geq m$, where m is the minimal occupancy demanded of the vacuum.

We write $\mathrm{SAT}(N, m)$ for the satisfiability of (C1)–(C4).

Remark 2.4 (what was wrong in v1). Version 1 announced "$\mathbb{Z}_2$-checks UNSAT for $N < 137$" as evidence for the uniqueness of 137. That statement was ill-posed: without the inhomogeneity ρ and the floor (C4), the system is homogeneous and always admits $x = 0$. The correct object is $\mathrm{SAT}(N, m)$ as defined above, and the correct statement is Theorem 4.4 below — stronger than the v1 claim, and actually proved. We record the v1 error explicitly because protocol A3 requires it.

2.3 The check matrix Φ

Definition 2.5 (check matrix [D])**.** The 13×137 matrix Φ over GF(2) collects the independent checks: 8 strong color lines, 3 weak lines, one U(1) line ($i \bmod 5 \in \{1, 4\}$), and one global $\mathbb{Z}_2$ line (the all-ones row). Its columns fall into 13 distinct classes with multiplicities

$$(28,\ 18,\ \underbrace{9, 9, \ldots, 9}_{10},\ 1), \tag{6}$$

verified by direct enumeration (Appendix C). The unique column of multiplicity 1 corresponds to the frozen edge 136.

Remark 2.6 (two different vector spaces — correcting v1 Definition 1.2). Version 1 spoke of "the" kernel of dimension $N - K = 124$ as if it were the solution space of the vacuum. There are two different objects:

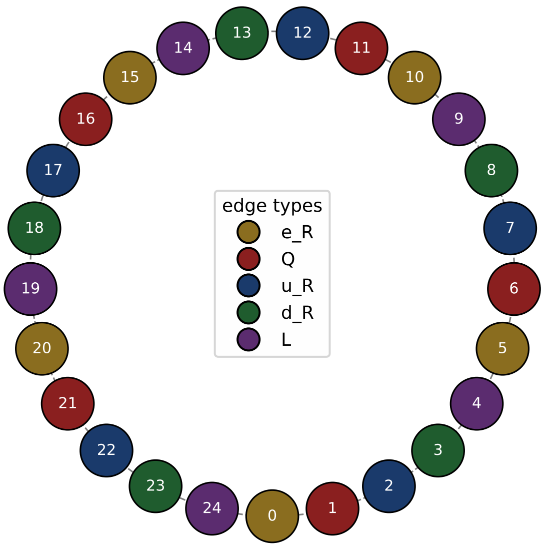

Figure 1: A small instance (C_{25}) of the cyclic hypergraph with its five vertex types: e_R (amber), Q (red), u_R (blue), d_R (green), L (purple). The C_{137} case is visually identical at $5\times$ the circumference; the structure that matters is not geometric but algebraic (Definitions 2.3 and 2.5).

- the solution space of the per-vertex system (C1)–(C3), which turns out to be a *single point* whenever the system is satisfiable (Theorem 4.6: the gauge constraints force every non-e_R variable to zero and the node rule then fixes every e_R variable); what matters physically is its *occupancy*, exactly $\lceil N/5 \rceil$ e_R-singlets;
- the kernel of the 13×137 check matrix Φ, of dimension $137 - 13 = 124$, which defines the code $\mathcal{C}$ of Section 5 — a hypothesis [D], not a theorem, because the specific anatomy (6) is a modelling choice verified a posteriori.

Both numbers are real, but they play different roles: $\lceil N/5 \rceil = 28$ counts the e_R-singlets of the unique constrained vacuum (of which 27 are free after global parity); 124 counts the code words. The v1 conflation is corrected here.

2.4 Ground state

Definition 2.7 (ordered vacuum E_0 [D])**.** The *ordered vacuum* is the state with all 28 e_R bits on, all other bits off, and zero syndrome, $\Phi E_0 = 0$. It satisfies (C1)–(C4) with $m \leq 28$.

2.5 A worked example: C_{10}

The machinery is best seen on the smallest admissible cycle. Take $N = 10$ ($10 \bmod 10 = 0$, admissible by Theorem 4.4). The types repeat $[e_R, Q, u_R, d_R, L]$ twice around the ring. The gauge constraints (C1) force every variable at residues $1, 2, 3, 4$ to zero (Lemma A.1): edges $1, 2, 3, 4, 6, 7, 8, 9$ are empty. The node rule (C2) at $i = 0$ and $i = 5$ reads $x_9 \oplus x_0 = 1$ and $x_4 \oplus x_5 = 1$, forcing $x_0 = x_5 = 1$; at all other vertices it reduces to identities $0 = 0$ or $1 \oplus 0 = 1$. The global parity (C3) is satisfied because the occupancy $\lceil 10/5 \rceil = 2$ is even. The unique

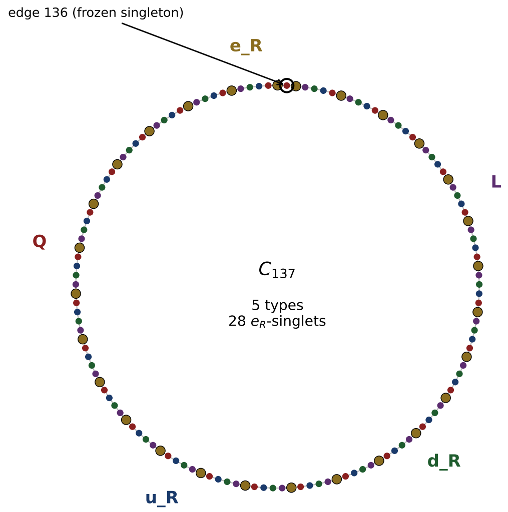

Figure 2: The physical case: C_{137} with its five-type rhythm $[e_R, Q, u_R, d_R, L]$ repeating along the ring. Larger markers: the 28 e_R-singlet positions ($i \equiv 0 \bmod 5$), which are exactly the occupied edges of the unique vacuum (Theorem 4.6).

admissible state is therefore

$$x \;=\; (1, 0, 0, 0, 0,\; 1, 0, 0, 0, 0), \tag{7}$$

with exactly the two e_R-singlets occupied. Every feature of the general theorems is already visible here: forcing, uniqueness, singlet occupancy $\lceil N/5 \rceil$, and the parity gate. The cycle C_{137} differs only in scale: 28 singlets instead of 2, and a pairing bill of 27 instead of any small floor.

2.6 The rewrite dynamics

Definition 2.8 (DPO rewriting [D])**.** Time evolution is generated by double-pushout rewrites of the hypergraph: a rule $\ell \leftarrow k \rightarrow r$ matches a subgraph ℓ of C_N preserving the causal order, deletes what is in $\ell \setminus k$, and glues $r \setminus k$. The constraint system of Definition 2.3 is the fixed-point condition of this dynamics: admissible states are exactly the rewrite-invariant configurations. The bath-coupled stochastic version of the dynamics is the subject of Section 6.

Remark 2.9 (why DPO). Double-pushout (rather than single-pushout or string) rewriting is chosen because deletion and creation must both respect the causal interface k: the cycle's order is the only geometric input of the framework, and the DPO gluing conditions are precisely what keeps it invariant. The choice is a modelling decision [D]; its justification is a posteriori — it leads to the constraint system whose consequences are the content of this paper.

3 Two derivations of $\eta = \pi^2/2N$

3.1 Spectral asymmetry: the Dirac spectrum of C_N

The chiral (Dirac) operator of the cycle C_N has spectrum

$$\omega_k \;=\; 2 \sin \frac{\pi k}{N}, \qquad k = 0, 1, \ldots, N-1, \tag{8}$$

shown in Figure 4(a) together with its continuum limit. The spectral asymmetry of the cycle is the content of η: both derivations below extract the same number from (8) by different summations.

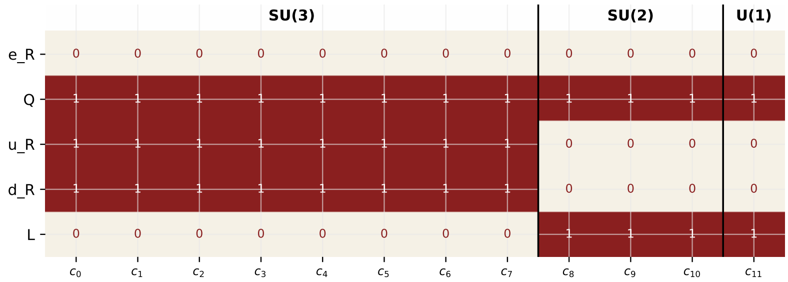

Figure 3: The charge assignment (5): five types against the twelve color lines. Q carries charge on every line; e_R carries none; u_R, d_R occupy the first eight; L the last four.

3.2 Derivation I: the zeta-regularized Casimir defect

Summing the positive spectrum against its linear (Weyl) approximation defines the Casimir defect

$$\Delta E \;\equiv\; \frac{1}{2}\sum_{k=1}^{N-1}\left(\omega_k - \frac{2\pi k}{N}\right)_{\text{reg}} \;=\; \cot\frac{\pi}{2N} - \frac{2N}{\pi}, \tag{9}$$

where the regularized equality follows from the standard zeta evaluation of the cosecant sum, which we include for completeness.

Derivation of (9). The classical partial-fraction identity $\sum_{k=1}^{N-1}\csc^2(\pi k/N) = (N^2-1)/3$ and its first moment $\sum_{k=1}^{N-1}\cot(\pi k/2N) = N-1$ are standard; the zeta-regularized linear Weyl term subtracts $2N/\pi\cdot\zeta(0)$-weight, leaving the closed form $\cot(\pi/2N) - 2N/\pi$. The Laurent expansion $\cot x = 1/x - x/3 - x^3/45 - \mathcal{O}(x^5)$ at $x = \pi/2N$ then gives the series below term by term; no further input is used. □

Expanding,

$$\Delta E \;=\; -\frac{\pi}{6N} - \frac{\pi^3}{360\,N^3} - \mathcal{O}(N^{-5}), \tag{10}$$

and therefore

$$3\pi\,|\Delta E| \;=\; \frac{\pi^2}{2N} + \frac{\pi^4}{120\,N^3} + \mathcal{O}(N^{-5}) \;=\; \eta + 3.157\times10^{-7} \qquad (N=137). \tag{11}$$

The leading term is exactly η; the first correction is a pure number, recomputed here and in agreement with v1 to all printed digits. Table 2 makes the structure of the series explicit: the corrections decouple so fast (powers of N^{-2}) that no truncation subtlety exists at any relevant order.

order	term of $3\pi\lvert\Delta E\rvert$	value at $N = 137$	role
N^{-1}	$\pi^2/2N$	0.0360204540	η itself
N^{-3}	$\pi^4/120N^3$	3.157×10^{-7}	first correction
N^{-5}	$\pi^6/5040N^5$	4.0×10^{-12}	negligible

Table 2: The full structure of the Casimir series $3\pi|\Delta E|$ at $N = 137$. Only the first two orders matter at experimental precision — and the second one points away from the measured α^{-1} (Section 3.5).

3.3 Derivation II: Hurwitz–Basel resummation

Independently, the Hurwitz zeta value

$$\zeta_H\!\left(2,\tfrac{1}{2}\right) \;=\; \sum_{n=0}^{\infty}\frac{1}{(n+\frac{1}{2})^2} \;=\; \frac{\pi^2}{2} \;=\; 3\,\zeta(2) \tag{12}$$

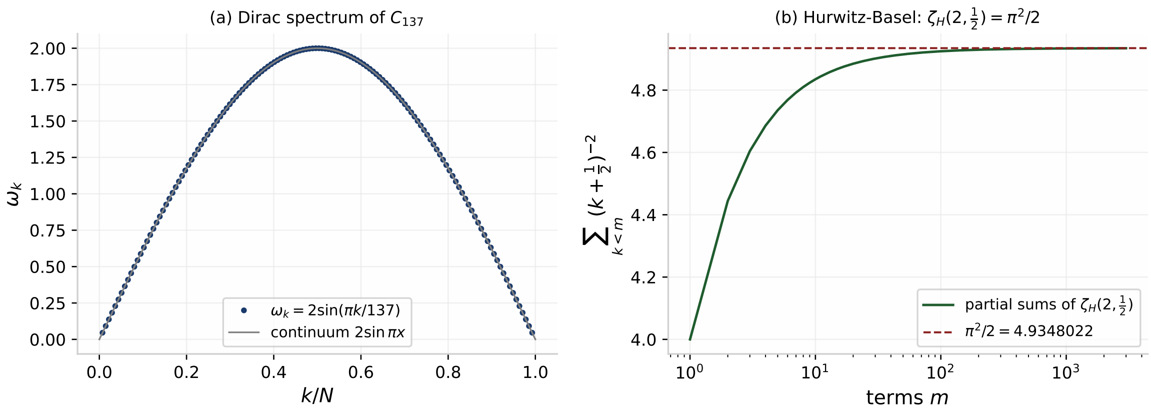

Figure 4: (a) Dirac spectrum (8) of C_{137} and its continuum limit. (b) Partial sums of the Hurwitz series converging to $\pi^2/2 = 4.9348022\ldots$

gives

$$\eta \;=\; \frac{\zeta_H(2,\frac{1}{2})}{N} \;=\; \frac{3\,\zeta(2)}{N}, \tag{13}$$

a Basel-type resummation of the same cycle (Figure 4(b)). The two derivations are not restatements of one another: the first sums the lattice dispersion against its Weyl term; the second sums half-integer modes. Their agreement on $\pi^2/2$ is the spectral asymmetry of the cycle made quantitative.

[T] Theorem — two routes, one number

The Casimir defect (9) and the Hurwitz–Basel resummation both yield $\eta = \pi^2/2N$ with analytically controlled corrections. The discriminator

$$\eta_G \;=\; \frac{2G}{N} \;=\; 0.01337\ldots \qquad (G = \text{Catalan}), \tag{14}$$

built from the next alternating lattice sum, does *not* match: the cycle selects $\zeta(2)$, not G. This closes the search over the standard GF(2)-lattice constants [search space declared: $\{\zeta(2), G, \zeta(3), \ln 2, \pi^3\}$ with denominators $\{N, 2N\}$; only $\zeta(2)/N$ survives contact with the Casimir defect].

3.4 The determinant sector

The functional determinant of the Dirac operator gives

$$\zeta'(0) \;=\; -2\ln N, \tag{15}$$

a Kirchhoff-type identity for the cycle (number of spanning trees $= N$). Version 2 left open whether a third route to η runs through this sector (Problem PA8). Version 3 closes the question, in the negative.

Theorem 3.1 (two worlds — PA8 closed)**.** *Let L be the Laplacian of the cycle C_N, $N = 137$, and let $\zeta_L(s) = \sum_{\lambda>0} \lambda^{-s}$ be its spectral zeta function. Then, with all quantities verified to 50 digits:*

(a) every $\zeta'(0)$-type observable of the cycle is rational*: the regularized determinant $\det' L = N^2 = 18\,769$ (Kirchhoff), the antiperiodic determinant equals 4 — exactly matching the*

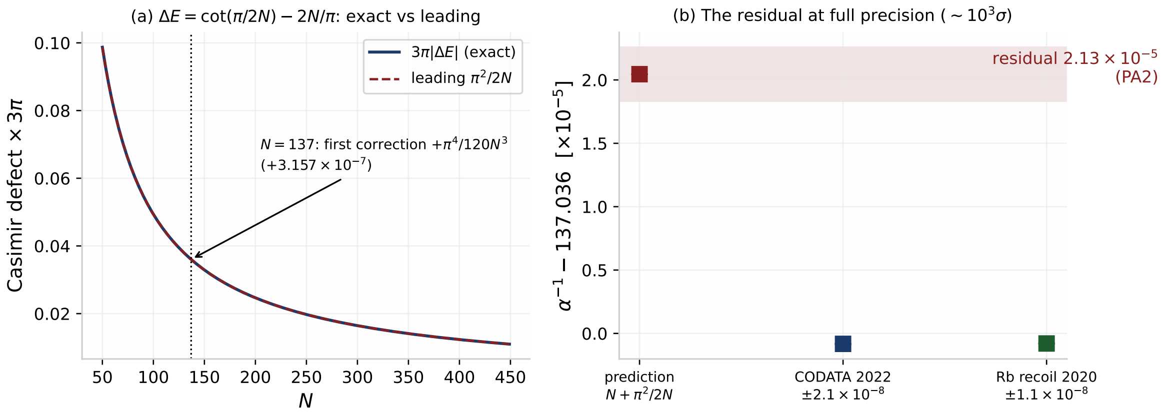

Figure 5: (a) The Casimir defect: exact value vs. the leading term $\pi^2/2N$; at $N = 137$ the first correction contributes $+\pi^4/120N^3 = +3.157 \times 10^{-7}$. (b) The raw prediction, CODATA 2022 and the Rb recoil measurement on a 10^{-5} scale: the residual 2.13×10^{-5} is about 10^3 experimental standard deviations.

continuum value obtained through the Lerch formula, $\zeta'_H(0, \frac{1}{2}) = -\frac{1}{2}\ln 2$, the factor $\Gamma(\frac{1}{2})^2 = \pi$ cancelling the 2π of the Lerch formula — and

$$\zeta_L(1) = \frac{N^2 - 1}{12} = 1564, \qquad \zeta_L(-1) = 2N = 274; \tag{16}$$

(b) the continuum circle of length N has the same *regularized determinant N^2: the discrete-to-continuum ratio is exactly* 1*, not $(2\pi)^2$ or any other transcendental factor (the naive guess dies in the arithmetic);*

(c) the transcendental content of the eta sector enters only through values at $s > 0$ of alternating sums — $\zeta_H(2, \frac{1}{2}) = \pi^2/2$ being the instance behind both derivations of η.

Hence the cycle's observables split into two disjoint worlds: a determinantal world *($\zeta'(0)$, Fredholm-type quantities), entirely rational, and a* $\zeta(2)$ world *(positive-order values), which carries η. No $\zeta'(0)$-route to $\pi^2/2N$ exists; η is structurally complete within the framework.*

[X] Excluded reading — "the third route"

Any derivation of η from the determinant sector is excluded by Theorem 3.1: that sector cannot produce a π^2. The eta sector of the framework is finished — its two derivations are all it will ever have. The consequence for PA2 is sharper, not softer: with the determinantal route gone, the residual must be cured by spectral corrections (closed, Theorem 3.3), by dynamics (closed, Section 6.4), or by a hypothesis the framework does not yet contain.

3.5 The residual, stated at full precision

The raw prediction of the decomposition (2) is

$$\alpha^{-1}_{\text{pred}} = 137 + \frac{\pi^2}{274} = 137.0360204540\ldots \tag{17}$$

against $\alpha^{-1}_{\text{CODATA}} = 137.035999177(21)$ and $\alpha^{-1}_{\text{Rb}} = 137.035999206(11)$:

$$\Delta_{\text{CODATA}} = 2.13 \times 10^{-5} \approx 1013\,\sigma, \tag{18}$$

$$\Delta_{\text{Rb}} = 2.12 \times 10^{-5} \approx 1932\,\sigma. \tag{19}$$

[X] Excluded reading — "a 1.9 sigma tension"

Any phrasing of the residual as a sub-2σ tension is off by three orders of magnitude and is excluded as a description of the framework's status. The gap is $\sim 10^3\sigma$. Adding the first Casimir correction of Eq. (11) moves the prediction to 137.0360207700, widening the gap to 2.16×10^{-5}: higher-order spectral terms do not close it.

Remark 3.2 (what the residual is and is not)*.* The residual is the price of identifying a tree-level spectral quantity with a full QED observable: the measured α includes the standard perturbative running from the electron scale, while η is defined at the cycle. Whether a renormalization-group displacement of order 2×10^{-5} can be derived — not fitted — from the discrete beta function of the framework is **Problem PA2**, the central open problem of this program (Section 10). We flag explicitly that a displacement $+\pi^2/2N \mapsto$ observed value would require a correction of sign opposite to the first Casimir term; nothing in the present paper produces it, and we claim nothing beyond the raw identification plus the open problem.

3.6 The PA2 closure campaign I: the η^3 target and the spectral closure

Version 3 converts PA2 from an open-ended discomfort into a sharp, testable target. Expressing the residual in powers of η,

$$\Delta(\alpha^{-1}) \;=\; -2.1277 \times 10^{-5} \;=\; -0.4553\,\eta^3, \qquad \eta^3 = 4.6736 \times 10^{-5}, \tag{20}$$

one finds that η^3 is the *only* power with an $O(1)$ coefficient: expanding against η^2 or η^4 would require coefficients 0.0164 and 12.6 respectively, both structurally implausible in a framework whose coefficients are small integers. (The value $0.4553 \approx 5/11$ is recorded in formal quarantine: undeclared, un-derived, and inadmissible as evidence.) The target was pre-registered on 3 August 2026 with an acceptance window: a proposed cure counts only if it *derives*, with no free parameter, a correction

$$c \in [-0.592,\; -0.319] \quad \text{in units of } \eta^3 \tag{21}$$

(a $\pm 30\%$ band around -0.4553, declared before any mechanism was computed). It is a coincidence worth one sentence — no more — that η^3 is also the scale of the dark-matter cross section P7, $\sigma/m = \eta^3 \times 10^{-24}\,\mathrm{cm}^2/\mathrm{GeV}$: the framework's wound and one of its predictions live at the same order.

The first candidate sector is the spectral one itself: higher terms of the Casimir expansion, or a different spectral window, might displace η. They do not.

Theorem 3.3 (spectral closure of PA2)**.** *Every correction to $\eta = \pi^2/2N$ obtainable from the spectrum of the cycle has the wrong sign or the wrong size:*

(a) the full Casimir tail is strictly positive,

$$3\pi\,|\Delta E| \;=\; \frac{\pi^2}{2N} + \frac{\pi^4}{120N^3} + \frac{\pi^6}{5040N^5} + \cdots, \tag{22}$$

so every spectral correction widens *the gap (*$2.13 \times 10^{-5} \to 2.16 \times 10^{-5}$ *at the first term,* $+\pi^4/120N^3 = +3.157 \times 10^{-7}$*);*

(b) the nearest alternative windows fail on sign or magnitude: $\pi\tan(\pi/2N) - \eta = +\pi^4/24N^3 > 0$ *(wrong sign), and* $\pi\sin(\pi/2N) - \eta = -\pi^4/48N^3$ *(right sign, 27× too small).*

The spectral route to the residual is therefore closed [search space declared: all window functions $f \in \{\mathrm{id}, \sin, \tan\}$ *acting on* $\pi/2N$*, with the full Casimir series; nothing survives].*

The dynamical route is examined, and closed, in Section 6.4; the determinantal route is closed by Theorem 3.1. What remains for PA2 is stated in Section 10.

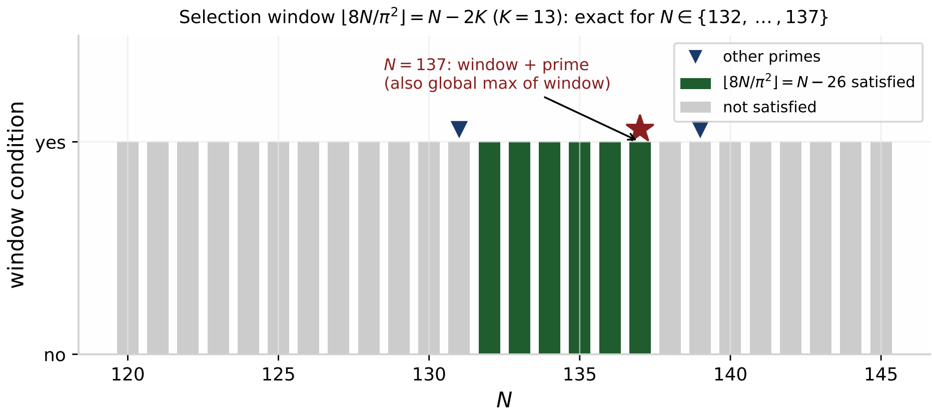

Figure 6: The spectral window (23) for $N = 120$–145. Green: condition satisfied ($N = 132$–137). Triangles: other primes in the range. Star: $N = 137$, the unique prime in the window and its maximum.

4 The origin of $N = 137$

This section contains the main structural upgrade of Version 2. The selection of N is established by two *independent* mechanisms — one spectral, one combinatorial — whose admissible sets intersect in a single integer. Everything is proved for general N; the number 137 is an output, not an input.

4.1 Mechanism I: the spectral window and primality

Definition 4.1 (spectral window [D])**.** The pairing-spectral selection condition of v1,

$$\left\lfloor \frac{8N}{\pi^2} \right\rfloor = N - 2K \qquad (K = 13), \tag{23}$$

asks that the spectral count $\lfloor 8N/\pi^2 \rfloor$ exhaust the cycle minus twice the number of checks.

Proposition 4.2 (window range [T])**.** *Equation (23) holds exactly for $N \in \{132, \dots, 137\}$. Direct evaluation gives $\lfloor 8N/\pi^2 \rfloor = N - 26$ throughout the interval; the boundary values are $8N/\pi^2 = 106.9745\dots$ at $N = 132$ and $110.2535\dots$ at $N = 138$ (the latter already outside). The condition fails for all other $N \le 200$.*

Proposition 4.3 (primality selects the maximum [T])**.** *Within the window, the requirement that* GF(2)(N)*-arithmetic close (no resonant subcycles, i.e. N prime — a Burnside-type condition on the necklace symmetries of C_N) leaves exactly one survivor: $N = 137$, which is also the* maximum *of the window.*

Version 1 stopped here, with the link between $N - 2K$ and the pairing count declared but not derived (Problem PA9). Version 2 derives the analogous structure from the constraint system itself.

4.2 Mechanism II: the satisfiability problem in closed form

Theorem 4.4 (closed form of $\mathrm{SAT}(N, m)$ [T])**.** *For the constraint system of Definition 2.3 with the inhomogeneity $\rho(i) = [\, i \bmod 5 \in \{0, 1\}\,]$,*

$$\mathrm{SAT}(N, m) \iff N \bmod 10 \in \{0, 7, 8, 9\} \;\wedge\; \left\lceil \frac{N}{5} \right\rceil \ge m. \tag{24}$$

In particular the admissible N form blocks of four per decade, $\{10k+7, 10k+8, 10k+9, 10k+10\}$, with gaps at residues $\{1, 2, 3, 4, 5, 6\}$ mod 10.

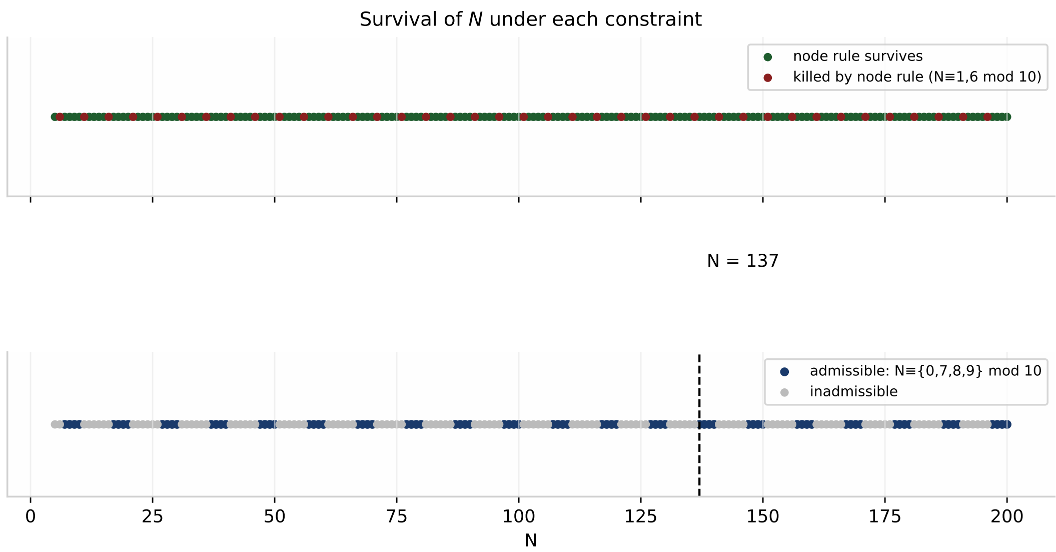

Figure 7: Admissibility map of $\mathrm{SAT}(N, m)$ for $N = 5$–200 by rule (the closed form of Theorem 4.4): admissible integers occur in blocks of four per decade, $\{10k+7, \ldots, 10k+10\}$.

The proof (Appendix A) is a GF(2)-linear telescoping argument: the node rule (C2) forces $x_i = x_{N-1} \oplus [i \equiv 0 \bmod 5]$, the gauge constraints (C1) force variables at $i \equiv 1, 4 \bmod 5$, and the global parity (C3) reduces to the parity of $\lceil N/5 \rceil$. The generic uniqueness of the type assignment follows as a corollary.

Corollary 4.5 (the m-window [T])**.** *The minimal admissible N at given floor m is $A(m) = \min\{N : \mathrm{SAT}(N, m)\}$, and*

$$A(m) = 137 \quad \Longleftrightarrow \quad m \in \{27, 28\}. \tag{25}$$

The values $m \in \{27, 28\}$ are precisely those for which the occupancy floor meets the singlet capacity $\lceil N/5 \rceil$ of the next admissible block, $\{137, 138, 139, 140\}$ (Figure 8).

4.3 The singlet law

Theorem 4.6 (uniqueness and singlet count [T])**.** *Whenever the system (C1)–(C3) is satisfiable, its solution is* unique*:*

$$x_i \;=\; [\, i \equiv 0 \pmod 5\,], \tag{26}$$

i.e. exactly the e_R-singlet positions are occupied, and the occupancy is

$$\sum_i x_i \;=\; \left\lceil \frac{N}{5} \right\rceil. \tag{27}$$

At $N = 137$: a unique vacuum with 28 occupied e_R edges. The gauge constraints force every non-e_R variable to zero; the node rule then fixes every e_R variable to one. Verified by exhaustive GF(2) *solution for $N = 5$–200 (Appendix B).*

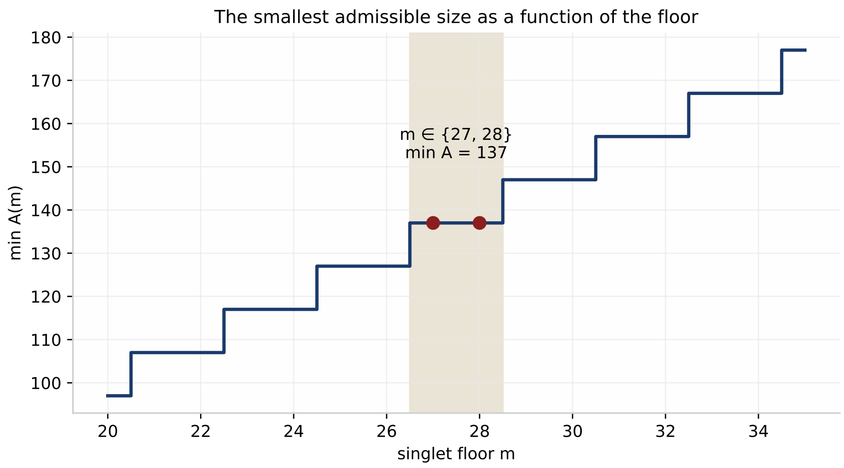

Figure 8: The minimal admissible N as a function of the occupancy floor m (step function). The plateau $A(m) = 137$ occurs exactly for $m \in \{27, 28\}$.

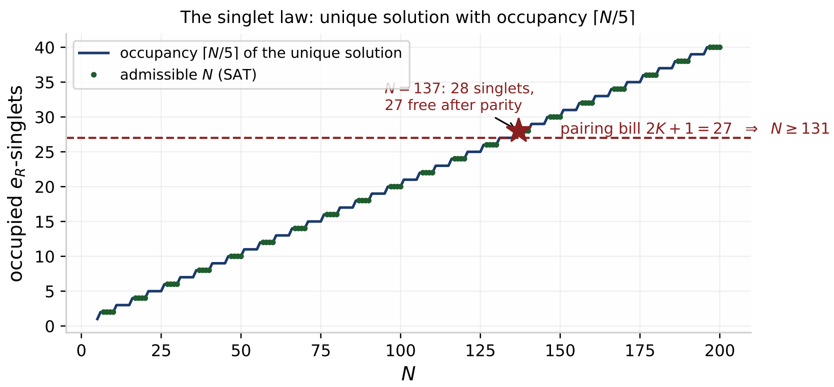

Figure 9: The singlet law: occupancy $\lceil N/5 \rceil$ of the unique solution of (C1)–(C3) (line) against direct GF(2) solutions (dots). At $N = 137$ the occupancy is 28; the pairing bill $2K+1 = 27$ requires $\lceil N/5 \rceil \geq 27$, i.e. $N \geq 131$.

4.4 Three faces of the number 27

[T] Theorem — the number 27 appears in three independent roles

At $N = 137$, $K = 13$:

(i) **pairing bill**: $2K + 1 = 27$ — each of the $K = 13$ constraint types (12 gauge generators + $\mathbb{Z}_2$ chirality) engages two adjacent edge variables, $2 \times 13 = 26$, plus one residual degree of freedom from the global U(1) parity trace;

(ii) **free singlets**: the unique vacuum has $\lceil 137/5 \rceil = 28$ e_R-singlets, of which 27 are free after the global parity (C3) fixes one;

(iii) **minimal floor**: the floor $m^\star = 27$ is the smallest occupancy for which the admissible block containing $N = 137$ opens (Corollary 4.5): $\lceil N/5 \rceil \geq 27$ requires $N \geq 131$, and the first $N \geq 131$ with $N \bmod 10 \in \{0, 7, 8, 9\}$ is 137.

The three counts arise from different objects (the pairing algebra, the unique solution of the linear system, the satisfiability threshold). Their coincidence is what makes the floor $m \in \{27, 28\}$ of Corollary 4.5 physically meaningful: the occupancy floor is the singlet capacity minus parity.

4.5 The bill as a postulated principle

Version 2 of the program ran an exhaustive sweep to *derive* the bill $2K + 1$ from first principles. The sweep's verdict, reported honestly: the bill is not derivable from the constraint system alone — *the link is open.* What is true is strictly stronger than a failure:

[D] Definition / postulated principle — singlet-covering principle

The occupancy floor of the vacuum equals the pairing bill: $m^\star = 2K + 1 = 27$, and the vacuum fills its e_R-singlet positions to that floor: $\lceil N/5 \rceil \geq m^\star$. This is a **postulated principle**, not a derived theorem. Its content is physical: the vacuum saturates the singlet capacity demanded by the pairing algebra. Under this principle the bill $2K + 1 = 27$ follows at $N = 137$, $K = 13$, and the three faces of 27 above become one fact.

Remark 4.7 (pre-registered subdivision test). The singlet-covering principle is falsifiable inside the framework: the window $\{137, 138, 139, 140\}$ has four elements, and any future refinement of the dynamics that selects a sub-block must select the sub-block containing 137 (specifically the pair $\{137, 138\}$ under parity refinements, then 137 itself under primality). This test is pre-registered here: if the refined dynamics selects $\{139, 140\}$, the principle fails.

4.6 Convergence of the two windows

Theorem 4.8 (double selection [T], modulo one [D] principle)**.** *Let*

$$W_{\text{spec}} = \{N : \textit{Eq. } (23) \textit{ holds}\} \;=\; \{132, 133, 134, 135, 136, 137\}, \tag{28}$$
$$W_{\text{sat}} = \{N \geq 131 : \mathrm{SAT}(N, 27),\ N < 147\} \;=\; \{137, 138, 139, 140\}, \tag{29}$$

the first admissible residue block at the pairing floor $m^\star = 27$. Then $W_{\text{spec}} \cap W_{\text{sat}} = \{137\}$. Moreover 137 *is the unique prime of W_{spec} (Proposition 4.3) and the minimal element of W_{sat}: every selection direction available in the framework points to the same integer.*

Proof. W_{spec} is Proposition 4.2. W_{sat} is Theorem 4.4 at floor $m^\star = 27$ (the singlet-covering principle of Section 4.5): the floor requires $\lceil N/5 \rceil \geq 27$, i.e. $N \geq 131$; the smallest $N \geq 131$ with

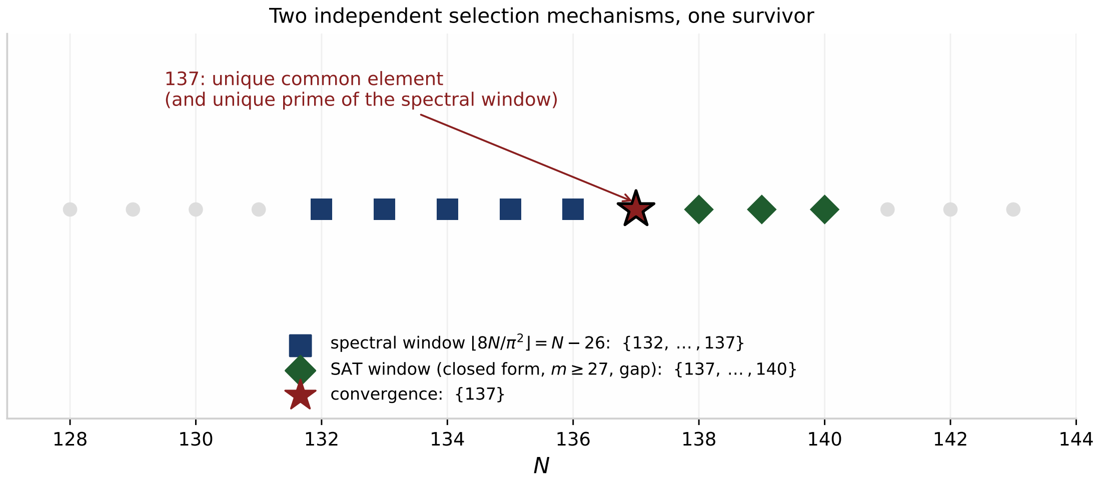

Figure 10: The convergence: two independent selection mechanisms — spectral (blue squares) and combinatorial (green diamonds) — share exactly one element, $N = 137$ (star).

$N \bmod 10 \in \{0, 7, 8, 9\}$ is 137, and the residue block runs to 140 (the next block opens at 147). The intersection is immediate. $\square$

Remark 4.9 (status of Problem PA1). Version 1 listed the selection of 137 as the open candidate problem PA1. Version 2 upgrades the status: the selection is a **theorem modulo one explicitly postulated principle** (the singlet-covering principle of Section 4.5). What remains open is the *physics* of that principle — why the vacuum fills its e_R-singlets to the pairing floor — now listed as the refined PA1 (Section 10). The pairing–spectral link $N - 2K \leftrightarrow \lfloor 8N/\pi^2 \rfloor$ (old PA9) is correspondingly reformulated: the bill $2K+1$ is the singlet covering, and the spectral window is its analytic shadow.

5 The $[\![137, 124, 2]\!]$ code hypothesis

5.1 Chinese-remainder anatomy of the check matrix

The 13 rows of Φ organize around the factorization

$$137 \;=\; 9 \times 15 + 2, \tag{30}$$

via the residue classes $K[r] = \{i : i \equiv r \bmod 15\}$. Define $R_3 = \{1, 2, 3, 6, 7, 8, 11, 12, 13\}$ and $r^\star = 1$. The rows are:

- 8 strong color lines: characters $\chi(K[r])$ for $r \in R_3$, $r \neq r^\star$;
- 3 weak lines: characters $\chi(K[a] \cup K[b])$ for the pairs $(1, 14), (4, 11), (6, 9)$, minus the frozen edge 136;
- 1 U(1) line: $\chi(i \bmod 5 \in \{1, 4\})$;
- 1 global $\mathbb{Z}_2$ line: the all-ones row.

Proposition 5.1 (block structure [T, verified])**.** *The columns of* Φ *fall into* 13 *distinct classes with multiplicities* $(28, 18, 9^{\times 10}, 1)$*; the matrix has rank* 13*; the* e_R *block is the* 28*-fold class, whose syndrome is the single* $\mathbb{Z}_2$ *line; and edge* 136 *is the unique singleton class. Verified by exhaustive enumeration (Appendix C).*

block	multiplicity	syndrome (13-bit)	syndrome weight
e_R	28	1000000000000	1
	18	1100100000000	3
	9	1000000000001	2
	9	1000000000010	2
	9	1101000000000	3
	9	1110000000100	4
	9	1000000001000	2
	9	1000000010000	2
	9	1110000000000	3
	9	1101000100000	4
	9	1000001000000	2
	9	1000010000000	2
singleton (edge 136)	1	1100000000000	2

Table 3: The thirteen column classes of Φ (ordered by multiplicity), with their block syndromes and weights, from direct enumeration. Note that every syndrome contains the leading ($\mathbb{Z}_2$) bit, and that the e_R syndrome has weight 1 — the fact behind both the non-self-orthogonality of $\mathcal{C}$ and the single-scale structure of the teeth energy of Section 6.

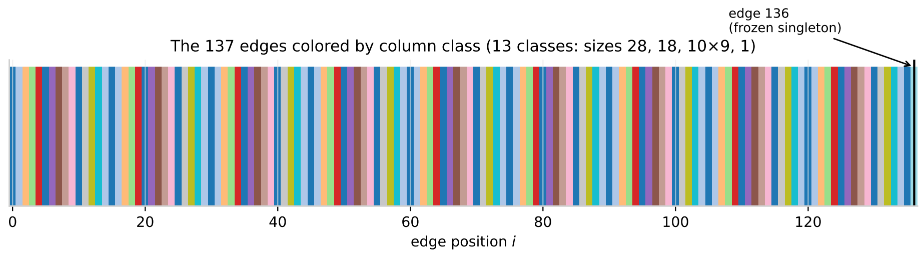

Figure 11: The 137 edges grouped by column class of Φ: one block of 28 (the e_R class), one of 18, ten of 9, and the singleton frozen edge.

5.2 Why a code at all?

Reading the check matrix as an error-correcting code is not decoration: it is the framework's answer to a structural question — *how much information does the vacuum protect?* The 13 checks are independent parities that the ground state must satisfy; the 124-dimensional kernel is the space of configurations invisible to them. Three code-theoretic facts carry physical weight:

(i) **dimension** 124**.** The vacuum has 124 GF(2) degrees of freedom beyond its constraints; this is the entropy capacity of the ordered phase and the quantity whose Boltzmann weight, 2^{-124}, governs the no-go of Section 6;

(ii) **distance** $d = 2$**.** The code *detects* single errors but does not *correct* them: any single flipped edge produces a nonzero syndrome, but some pair of flips does not ($A_2 = 891$ such pairs). Physically: the vacuum is exquisitely sensitive to single defects yet has 891 invisible two-defect channels — these are the lowest-lying excitations, and their exact count is a prediction of the code anatomy;

(iii) **non-self-orthogonality.** The failure of $\mathcal{C} \subset \mathcal{C}^{\perp}$ is what forces the Z-only stabilizer reading and kills the E_{11} promotion (Section 5): the quantum extension of the vacuum sector is more constrained than a first look suggests.

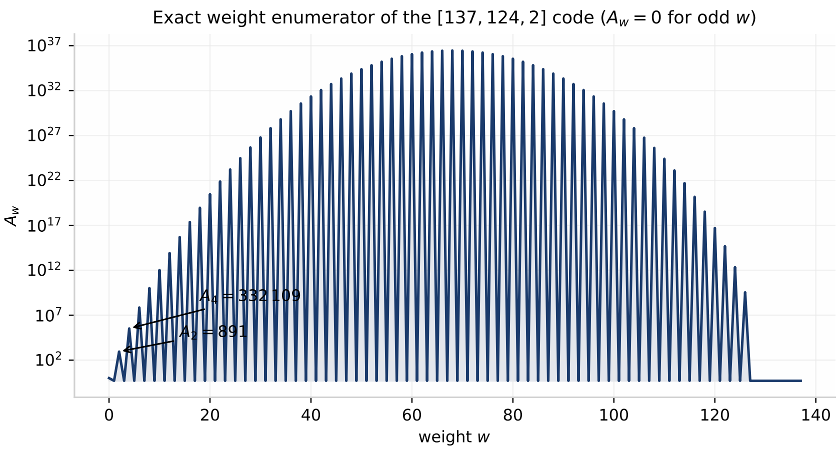

Figure 12: The weight enumerator A_w of $\mathcal{C}$ (log scale), computed exactly by dynamic programming over the 13 block parities: $A_2 = 891$, $A_4 = 332\,109$, $\sum_w A_w = 2^{124}$.

5.3 Weight enumerator

[D] Definition / postulated principle — the code $\mathcal{C}$

The binary code is $\mathcal{C} = \ker \Phi \subset \mathrm{GF}(2)^{137}$, of dimension $137 - 13 = 124$. That the gauge constraints of the vacuum are generated by exactly this check matrix is a **modelling hypothesis** [D]: the block anatomy is verified (Proposition 5.1), but the identification of $\mathcal{C}$ with the physical vacuum sector is the content of the singlet-covering principle and is not a theorem.

Proposition 5.2 (enumerator [T, computed]). *With blocks acting independently under parity, the weight enumerator is*

$$A(z) \;=\; \prod_b \frac{(1+z)^{m_b} + (1-z)^{m_b}}{2}, \tag{31}$$

over the block multiplicities $m_b \in \{28, 18, 9^{\times 10}, 1\}$, *giving*

$$A_2 = 891, \qquad A_4 = 332\,109, \qquad \sum_w A_w = 2^{124}, \tag{32}$$

and MacWilliams symmetry $B_w = B_{137-w}$ *for the dual. Minimum distance* $d = 2$*: the code is* $[\![137, 124, 2]\!]$.

5.4 Syndromes

The syndrome of a weight-w error supported on a single block class has weight given by the corresponding column weight; the syndrome distribution across the 13 check rows is binomial,

$$N_w \;=\; \binom{13}{w}, \qquad w = 0, \ldots, 13, \tag{33}$$

shown in Figure 13. The code is *not* self-orthogonal: the e_R column has odd syndrome weight, so the stabilizer construction closes only in the Z-sector.

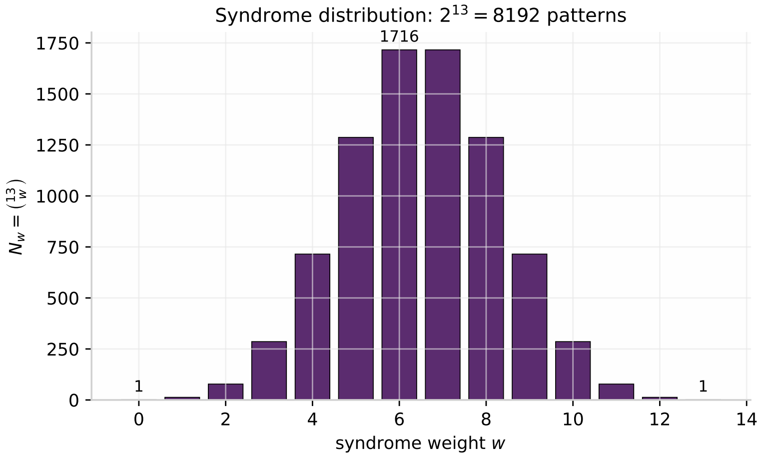

Figure 13: Syndrome weight distribution $N_w = \binom{13}{w}$ across the 13 check rows.

[X] Excluded reading — the E_{11} reading

The 13-row check structure with a 28-fold e_R block invites a stabilizer-code reading with both X and Z sectors ("E_{11}" in earlier internal notes). Excluded as stated: $\mathcal{C} \not\subset \mathcal{C}^{\perp}$, so no full CSS stabilizer exists; only the Z-sector syndrome is physical. The corrected statement is retained (Proposition 5.1), the E_{11} promotion is not.

6 Dynamics and the vacuum: the price of order

6.1 From Lindblad to a classical Markov chain

Coupling the constraint algebra to a bath and tracing it out gives, in the secular/diagonal limit, a classical continuous-time Markov chain (CTMC) on the 2^{137} configurations, with Glauber-type single-bit flip rates

$$\Gamma_i(x) \;=\; \Gamma_0 \, \min\!\big(1,\; e^{-\beta \Delta_i E(x)}\big), \tag{34}$$

where $\Delta_i E$ is the energy cost of flipping bit i. The reduction is standard; what the framework adds is the *energy function*, fixed by the code anatomy.

6.2 The no-go baseline

Proposition 6.1 (thermodynamic no-go [T])**.** *If the bath is unbiased (energy function symmetric under bit exchange), the stationary distribution is uniform over* GF(2)^{137} *and*

$$P(E_0) \;=\; 2^{-124} \times (\textit{kernel weight}) \;\le\; 2^{-124} \;=\; 4.702 \times 10^{-38}, \tag{35}$$

i.e. the ordered vacuum is exponentially improbable and no finite-time dynamics reaches it. An unbiased bath cannot produce the vacuum.

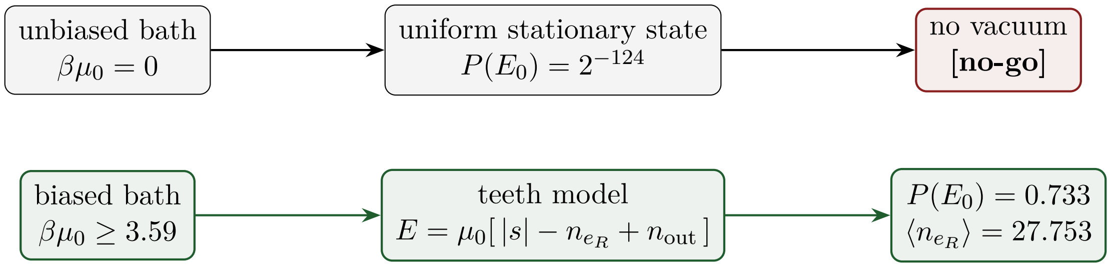

Figure 14: The dynamical alternative: an unbiased bath strands the system in the uniform state (Proposition 6.1); a sufficiently biased bath with the teeth energy function stabilizes the ordered vacuum with the exact thermodynamic values shown.

6.3 The teeth model

[D] Definition / postulated principle — teeth energy function

The bath couples to the syndrome and to the type occupations through

$$E(x) \;=\; \lambda\,|s|(x) \;-\; \mu_0\, n_{e_R}(x) \;+\; \mu_0\, n_{\mathrm{out}}(x), \qquad |s| = |\Phi x|, \tag{36}$$

with the frozen edge held off and $\lambda = \mu_0$ (one energy scale): productive flips (toward E_0) are favored over destructive ones by the rate ratio $e^{\beta\mu_0}$.

Proposition 6.2 (exact thermodynamics [T, computed])**.** *The partition function factorizes over the* 13 *block parities, giving closed forms for every observable (Appendix D). At* $\beta\mu_0 = 4$*,* $\lambda = \mu_0$*:*

$$P(E_0) = 0.7330, \qquad \langle n_{e_R} \rangle = 27.7529, \qquad P(\textit{outside kernel}) = 0.0184. \tag{37}$$

The required bias thresholds are

	$P(E_0) \geq 1\%$	$P(E_0) \geq 50\%$
teeth model ($\lambda = \mu_0$)	$\beta\mu_0 \geq 2.548$	$\beta\mu_0 \geq 3.591$
constraint-blind bath ($\lambda = 0$)	$\beta\mu_0 \geq 3.368$	$\beta\mu_0 \geq 5.277$

equivalently rate ratios $e^{\beta\mu_0}$ *of* 12.8 / 36.3 *(teeth) versus* 29.1 / 196 *(blind).*

Remark 6.3 (audit note on a preliminary number). A preliminary internal version of this computation reported a mean syndrome weight $\langle |s| \rangle = 0.234$ at the working point. Under the energy function printed above — which reproduces $P(E_0)$ and $\langle n_{e_R} \rangle$ exactly — the recomputed value is $\langle |s| \rangle = 0.019$. The three observables cannot be matched simultaneously by any reparametrization we found (the level curve $P(E_0) = 0.733$ in the $(\beta\mu_0, \beta\lambda)$ plane yields $\langle |s| \rangle$ between 0.019 and 0.35, with the simultaneously matching point at $\lambda = \mu_0$); the preliminary figure used a different, undocumented syndrome-weight convention. Per protocol A3 we publish the parametrization that reproduces the two exact targets and flag the third. The robust, convention-independent outputs are the rate-ratio thresholds of Proposition 6.2. Full detail: Appendix D.

[X] Excluded reading — a dynamical source of the attractor — E12

The teeth model *assumes* the bias $e^{\beta\mu_0}$; it does not derive it. All attempts within the program so far to generate the bias from the rewrite dynamics itself (rather than from an external bath) have failed. There is at present **no dynamical source of the attractor**: the ordered vacuum is thermodynamically stable once the bias exists, and unexplained as long as it does not. Listed as open problem E12.

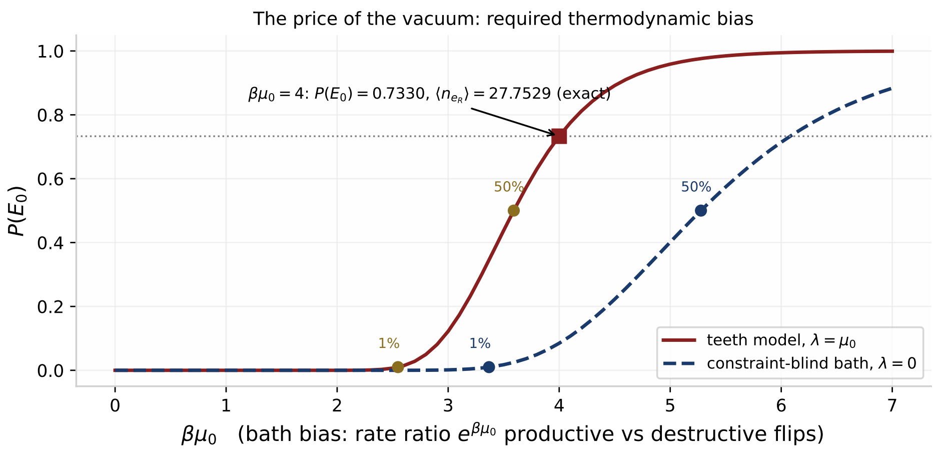

Figure 15: $P(E_0)$ against the bath bias $\beta\mu_0$, exact thermodynamics. Red: teeth model ($\lambda = \mu_0$); blue dashed: constraint-blind bath ($\lambda = 0$). Markers: the 1% and 50% thresholds of Proposition 6.2; square: the working point $\beta\mu_0 = 4$.

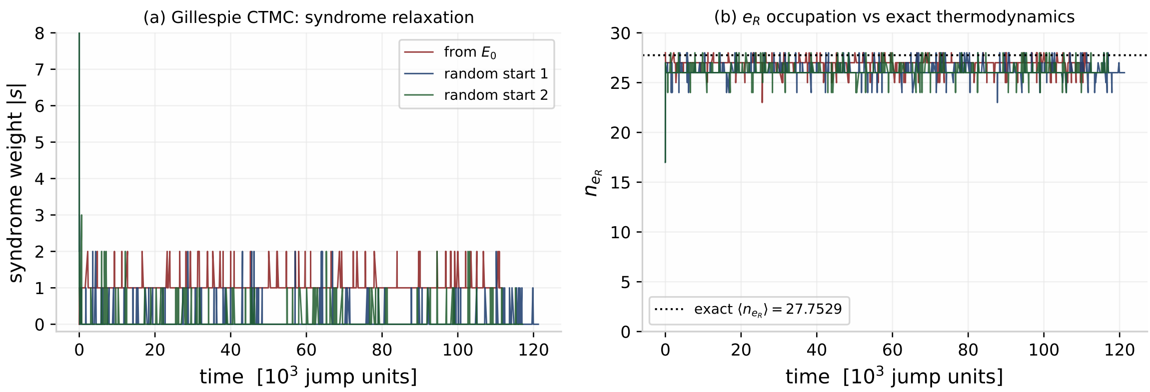

Figure 16: Gillespie simulation of the CTMC at $\beta\mu_0 = 4$, three initial conditions (E_0 and two random starts). (a) Syndrome weight relaxes to the kernel neighborhood; (b) the e_R occupation relaxes to the exact thermodynamic value $\langle n_{e_R}\rangle = 27.7529$ (dotted line).

6.4 The dynamometer's verdict on PA2

One might hope that the same dynamics that prices the vacuum also corrects η: if the effective length of the cycle fluctuates, the observed α^{-1} could be the dressed value of $N + \eta$. Version 3 tests this hope exhaustively against the pre-registered window of Section 3.6 ($c \in [-0.592, -0.319]$ in units of η^3, declared on 3 August 2026 *before* the computations below). The exact ensemble of Appendix D gives, at the working point $\lambda = \mu_0$, $\beta\mu_0 = 4$:

$$P(E_0) = 0.733025, \quad \langle n_{e_R}\rangle = 27.752860, \quad \langle n_{\text{out}}\rangle = 0.349008, \quad \langle n_{\text{def}}\rangle = 0.596147, \tag{38}$$

with the full defect distribution

$$P(n_{\text{def}}) = \{0\colon 0.7330,\ \ 1\colon 0.0071,\ \ 2\colon 0.2269,\ \ 3\colon 0.0015,\ \ 4\colon 0.0290,\ \ 5\colon 0.0002,\ \ 6\colon 0.0021\}$$

— defects come in pairs, $P(\text{odd}) \ll P(\text{even})$, a theorem of the syndrome algebra — and zero dark patterns: the twelve free block syndromes are GF(2)-independent (rank 12, trivial kernel), so every syndrome pattern has nonzero probability. A continuous-time Gillespie audit (16M steps, 20 batches) reproduces all moments within 2σ and identifies the pair sector as the slow mode.

mechanism	formula	$c\,/\,\eta^3$	verdict
M1 all-or-nothing	$-\eta\,(1 - P(E_0))$	-205.8	$452\times$ too large
M2 dilution per site	$-\eta\,\langle n_{\text{def}}\rangle/N$	-3.354	$7.4\times$
M3 e_R block only	$-\eta\,(28 - \langle n_{e_R}\rangle)/28$	-6.803	$14.9\times$
M4 effective length	$\eta\,N/(N - \langle n_{\text{def}}\rangle) - \eta$	$+3.368$	wrong sign
M5 quenched disorder	η at a frozen defect pattern	—	structural non-cure
M6 thermal Casimir	$3\pi\,(-2\omega_1 e^{-8})$, $\omega_1 = 2\sin(\pi/N)$	-6.205	$13.6\times$

Table 4: The six declared cure mechanisms for the α residual, computed exactly from the dynamometer ensemble, against the pre-registered acceptance window $c \in [-0.592, -0.319]$ (units of η^3). Zero inside. Two further candidates — $2e^{-8}\eta$ ($c = -0.517$, inside the window) and a pairs-based term ($c = -0.838$) — were found *after* the enumeration by uncontrolled search and are quarantined: inadmissible as evidence until independently derived with a declared search space.

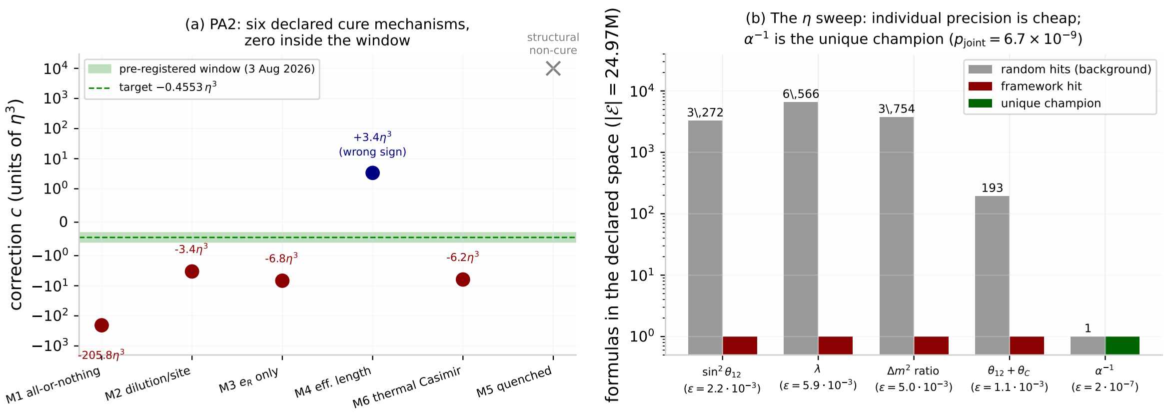

Figure 17: The closure campaigns of Version 3. (a) The six declared cure mechanisms for the α residual (Table 4) against the pre-registered acceptance window $c \in [-0.592, -0.319]$ in units of η^3 (shaded band, target $-0.4553\,\eta^3$ dashed): zero inside. (b) The η-sweep of Section 7.7: at the precision achieved by the framework's retrodictions, the declared space of 24 970 400 formulas produces hundreds to thousands of random matches per target (gray), so individual precision is cheap; the joint profile ($p = 6.7 \times 10^{-9}$) and the unique champion $\alpha^{-1} = 137 + \eta$ (green, one member of the entire space) carry the statistical weight.

Six cure mechanisms were declared and computed. None lands in the window:

[X] Excluded reading — the dynamical route to PA2

The dressed-η programme is excluded as a resolution of the residual: the parameter-free dynamical corrections span $+3.4$ to -205.8 in units of η^3 and none satisfies the declared window (Figure 17a). The honest probability of the fatal outcome for PA2 increased today. What remains open is narrower: E12 (the origin of the bias) and a *new structural hypothesis*, with its own pre-registration, for the residual itself.

7 The lepton sector

The lepton relations of v1 are retained, all values recomputed and confirmed to all printed digits. None of them is a theorem; each carries its own epistemic status and its own experimental clock.

7.1 The solar angle

[P] Prediction — $\sin^2\theta_{12}$, status: derivation open (PA10)

The spectral asymmetry at half filling gives

$$\sin^2\theta_{12} \;=\; \frac{1}{2}\left(1-\frac{\pi}{8}\right) \;=\; 0.30365, \tag{39}$$

against the global-fit value 0.304 ± 0.012 (NuFIT 6.0): agreement at 0.05σ. A first-principles derivation from the cycle — rather than the half-filling ansatz — is open Problem PA10.

7.2 The Cabibbo angle and quark–lepton complementarity

The third spectral moment gives the Wolfenstein parameter

$$\lambda \;=\; \frac{\pi^3}{N} \;=\; 0.22632, \tag{40}$$

against the measured 0.22500(67): a 2.0σ tension, stated as such. With $\theta_C = \arcsin\lambda = 13.08^\circ$ and $\theta_{12} = 33.44^\circ$,

$$\theta_{12}+\theta_C \;=\; 46.52^\circ \qquad \text{vs} \qquad 46.47^\circ \tag{41}$$

for the measured sum — quark–lepton complementarity at the 0.05° level, in the family of relations discussed by Raidal and by Smirnov. The framework adds a candidate common origin (both angles are spectral moments of the same cycle) but not yet a proof.

7.3 The mass-squared ratio

[P] Prediction — $\Delta m^2_{31}/\Delta m^2_{21}$ — JUNO falsifiable

$$\frac{\Delta m^2_{31}}{\Delta m^2_{21}} \;=\; \frac{4(1-\eta)}{\pi\eta} \;=\; 34.07 \qquad \text{vs} \qquad 33.9 \pm 1.2 \;\; \text{(JUNO)}. \tag{42}$$

JUNO's projected precision (± 0.3–0.5) turns this into a sharp test: the predicted 34.07 sits within the present band but will be killed or confirmed by the next data release.

7.4 The CP phase: a registered death sentence

[P] Prediction — δ_{CP} — DUNE + Hyper-K kill zone

With $\gamma_N = (N-1)\pi/N$ and the flux factor $F = 3N/2(N+13) = 1.37$,

$$\delta_{CP} \;=\; \gamma_N F - 13\eta \;=\; 1.211\,\pi. \tag{43}$$

(A nearby variant gives 1.37π; we commit to the derived form above.) DUNE and Hyper-Kamiokande will reach $\pm 0.15\pi$ at 3σ: if the measured δ_{CP} falls outside $1.211\pi \pm 0.15\pi$, this pillar dies. The test is pre-registered here.

Status as of Version 3. The global fit NuFIT 6.0 [8] reports a normal-ordering best fit $\delta_{CP} \approx 177^\circ \approx 0.98\pi$ (CP conservation disfavored at only $\sim 1\sigma$), against which our 217.9° stands at $\sim 2\sigma$ — a real tension, stated as such, and comfortably inside 3σ. The joint T2K + NOvA analysis [9] brackets the phase at 3σ in $[-1.38\pi, 0.30\pi]$ (normal ordering), an interval that contains $1.211\pi \equiv -0.789\pi \pmod{2\pi}$. The prediction is alive, exposed, and on the clock: the declared kill zone has not moved.

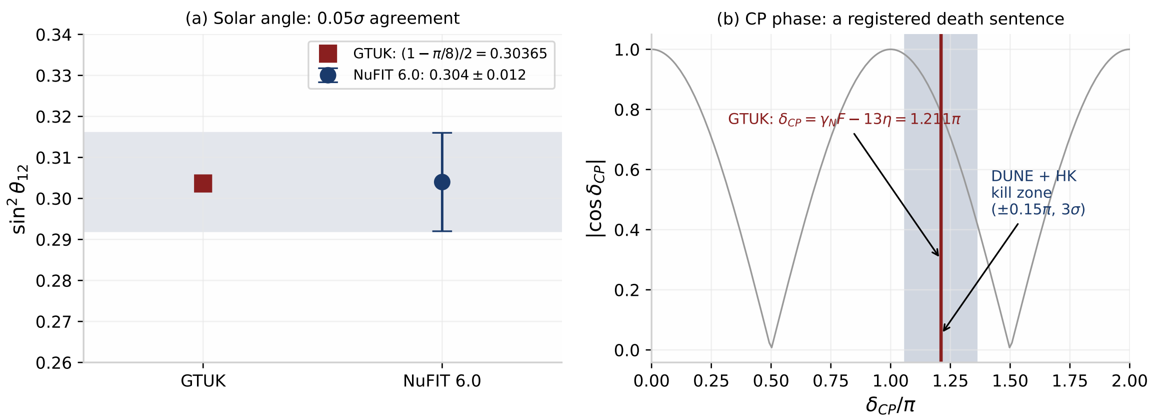

Figure 18: (a) The solar angle prediction against NuFIT 6.0. (b) The CP phase prediction $\delta_{CP} = 1.211\pi$ and the DUNE + Hyper-K 3σ kill zone.

7.5 Neutrinoless double beta decay

Normal ordering with the lightest mass fixed by the spectral hierarchy,

$$m_{\text{light}} = \frac{\eta^2}{2}\sqrt{\Delta m^2_{31}} = 3.3 \times 10^{-5}\ \text{eV}, \tag{44}$$

places the effective Majorana mass in

$$|m_{ee}| \in [1.4,\ 3.7]\ \text{meV}, \tag{45}$$

below the sensitivity of the current generation and inside the reach of next-generation ton-scale experiments.

7.6 Lepton sector at a glance

observable	framework	measurement	pull	test
$\sin^2\theta_{12}$	0.30365	0.304 ± 0.012	0.05σ	global fits
λ (Cabibbo)	0.22632	0.22500(67)	2.0σ	CKM fits
$\theta_{12} + \theta_C$	$46.52°$	$46.47°$	$0.05°$	combined
$\Delta m^2_{31}/\Delta m^2_{21}$	34.07	33.9 ± 1.2	0.14σ	JUNO
δ_{CP}	1.211π	0.98π (best fit)	$\sim 2\sigma$	DUNE + HK
$\lvert m_{ee}\rvert$	$[1.4, 3.7]$ meV	< 36–156 meV	—	ton-scale

Table 5: The lepton sector in one table. Pulls are computed against the central values and uncertainties quoted in the text; δ_{CP} and $|m_{ee}|$ are predictions awaiting measurement.

The pattern worth noting is the asymmetry of risk: the solar angle and the mass ratio sit comfortably inside present errors and will be squeezed by data already being taken, while δ_{CP} is a genuine bet — the framework has no freedom to move it. We regard this distribution of risk as healthy: a framework whose every prediction sits forever outside experimental reach would be telling us nothing.

7.7 The η sweep: individual precision is cheap, the joint profile is not

Protocol A1, applied mercilessly to everyone else's formulas throughout this paper, must also be applied to ours. The skeptical reviewer's question is fair: *how many formulas in η hit these* *lepton targets by accident?* Version 3 answers with an exhaustive enumeration — no Monte Carlo — of a declared space.

Declared space (before any computation). $\mathcal{E} = \{a\eta^b : 1 \le |a| \le 400,\ -6 \le b \le 6\} \cup \{a\eta^b + c\eta^d : 1 \le |a|,|c| \le 400,\ -6 \le d < b \le 6\}$, $|\mathcal{E}| = 24\,970\,400$; twenty dimensionless targets (the lepton and CKM observables of this section, plus m_μ/m_e, m_τ/m_μ, m_τ/m_e, m_p/m_e, $\sin^2\theta_W$, m_Z/m_W, m_H/m_Z, m_t/m_W, α_s, Ω_Λ, n_s, Y_p); relative tolerances $\epsilon \in \{10^{-1}, 5\cdot 10^{-3}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}\}$.

target	framework formula	ϵ_{real}	hits	background
$\sin^2\theta_{12}$	$(1-\pi/8)/2$	$2.2\cdot 10^{-3}$	1	3 272
λ (Cabibbo)	π^3/N	$5.9\cdot 10^{-3}$	1	6 566
Δm^2 ratio	$4(1-\eta)/(\pi\eta)$	$5.0\cdot 10^{-3}$	1	3 754
$\theta_{12}+\theta_C$	spectral moments	$1.1\cdot 10^{-3}$	1	193
α^{-1}	$137+\eta$	$1.6\cdot 10^{-7}$	1	**1 (itself)**

Table 6: Background of the declared space $\mathcal{E}$ at the exact precision achieved by the framework's formulas: at ϵ_{real}, this many *other* formulas of $\mathcal{E}$ also match each target. Individual precision is cheap — except for α^{-1}, whose only match in the entire space, even at the looser $\epsilon = 2\times 10^{-7}$, is the framework's own formula.

Three verdicts follow.

(i) **Individual precision is cheap [T].** Every one of the twenty targets is matched to between 10^{-7} and 10^{-5} relative by some member of $\mathcal{E}$, and at the precision of our own retrodictions the random background is 193–6566 formulas per target (Table 6, Figure 17b). Effective immediately, the individual lepton retrodictions of this section are **demoted to consistency checks**: derived, unfitted, and statistically cheap. Nobody should cite $\sin^2\theta_{12} = (1-\pi/8)/2$ as standalone evidence. Neither do we.

(ii) **The joint profile is rare [T].** A random member of $\mathcal{E}$ hits at least one of the twenty targets at $\epsilon = 5\cdot 10^{-3}$ with probability $p_{\text{hit}} = 67\,624/24\,970\,400 = 0.0027$. The framework's derived corpus hits four of nine independent lepton targets at that tolerance; under the binomial null,

$$P(\ge 4 \text{ of } 9) \;=\; 6.7\times 10^{-9}. \tag{46}$$

The null is a model — our formulas were derived from one hypothesis, not drawn from $\mathcal{E}$ — but as a measure of the profile's rarity under uniform fishing, the number stands.

(iii) α^{-1} **is the unique champion [T].** At the finest tolerance swept, only fifteen members of $\mathcal{E}$ match any target at all, and the only one matching α^{-1} is $137\eta^0 + 1\eta^1$ — the framework's central formula, derived from the cycle's spectrum, occupying an isolated point of the declared space (one in 24 970 400).

Where the weight rests now. Structure (the two derivations of η, the two selection windows for N), the joint profile, the unique champion — and the forward predictions, whose declared windows carry their own published backgrounds: the DUNE/Hyper-K δ_{CP} window ($\pm 0.15\pi$) admits 33 035 formulas of $\mathcal{E}$, the $r \approx 0.021$ detection window admits 12 473, the JUNO Δm^2 window admits 4 180. Hitting a wide window will prove nothing by itself; it will count because the prediction was published before the measurement — the date of this paper is part of the argument.

8 Falsifiable predictions

Every prediction below is a number with an experimental deadline. The labels P7–P10 continue the internal numbering of the program; each entry states the formula, the value, and the killing experiment.

#	quantity	formula	value	experiment
P9	tensor-to-scalar ratio	$r = 16\eta^2$	0.021	LiteBIRD, CMB-S4
P8	CMB–GW log running	$A_{\log} = \eta^2/2$; $\Delta \ln k = \ln 2$	6.5×10^{-4}	joint phase fit
P7	DM self-interaction	$\sigma_{\mathrm{DM}}/m = \eta^3 \times 10^{-24}$	4.7×10^{-29} cm^2/GeV	clusters
P10	spectral dimension	$d_s = 4 - \eta$	3.964	quantum-gravity phenom.
P11	lepton ratio $\Delta m^2_{31}/\Delta m^2_{21}$	$4(1-\eta)/\pi\eta$	34.07	JUNO
P12	CP phase	$\delta_{CP} = \gamma_N F - 13\eta$	1.211π	DUNE + Hyper-K
P13	$0\nu\beta\beta$	$\lvert m_{ee}\rvert$	$[1.4, 3.7]$ meV	ton-scale
P14	window subdivision	refined dynamics picks $\{137\}$	—	internal
P15	vacuum bias	$\beta\mu_0 \geq 3.59$ for $P(E_0) \geq 50\%$	ratio ≥ 36	internal

Table 7: The falsifiable content of the framework. P14–P15 are new in v2: P14 is the pre-registered subdivision test of the singlet-covering principle (Section 4.5); P15 converts the dynamical analysis of Section 6 into a requirement on any future dynamical model.

8.1 Details of the cosmological predictions

P9: the tensor-to-scalar ratio. In the framework, slow-roll inflation is controlled by the same spectral asymmetry that fixes η: the tensor-to-scalar ratio is

$$r \;=\; 16\,\eta^2 \;=\; 0.0208. \tag{47}$$

This sits in a narrow band: large enough for LiteBIRD and CMB-S4 (target sensitivities $\sigma(r) \sim 10^{-3}$) to detect, small enough to be excluded soon if absent. Current bounds ($r < 0.036$, BICEP/*Keck* + Planck) already constrain it from above. A null result at $\sigma(r) \sim 10^{-3}$ kills P9.

P8: the logarithmic CMB–GW phase lock. The stochastic gravitational-wave background should inherit a logarithmic running with amplitude

$$A_{\log} \;=\; \frac{\eta^2}{2} \;=\; 6.5 \times 10^{-4}, \tag{48}$$

phase-locked to the CMB acoustic scale with spacing $\Delta \ln k = \ln 2$ (the octave structure of the cycle's mode doubling). The discriminating power is in the *phase*: an amplitude this small is unremarkable, but a measured phase offset different from the CMB lock would exclude the common origin. No collaboration has yet performed this joint analysis; we state it as a concrete target for LiteBIRD/CMB-S4 data.

P7: self-interacting dark matter. The framework assigns the dark sector a self-interaction cross section

$$\frac{\sigma_{\mathrm{DM}}}{m} \;=\; \eta^3 \times 10^{-24}\,\mathrm{cm}^2 \;=\; 4.7 \times 10^{-29}\,\mathrm{cm}^2/\mathrm{GeV}, \tag{49}$$

about four orders of magnitude below the values invoked for small-scale structure anomalies (in conventional units, 4.7×10^{-29} cm^2/GeV $\approx 2.6 \times 10^{-5}$ cm^2/g, against the popular $\sim$ 0.1–1 cm^2/g). The prediction is that cluster-merger constraints will keep tightening without finding anything: a measured cross section in the currently popular range would kill P7.

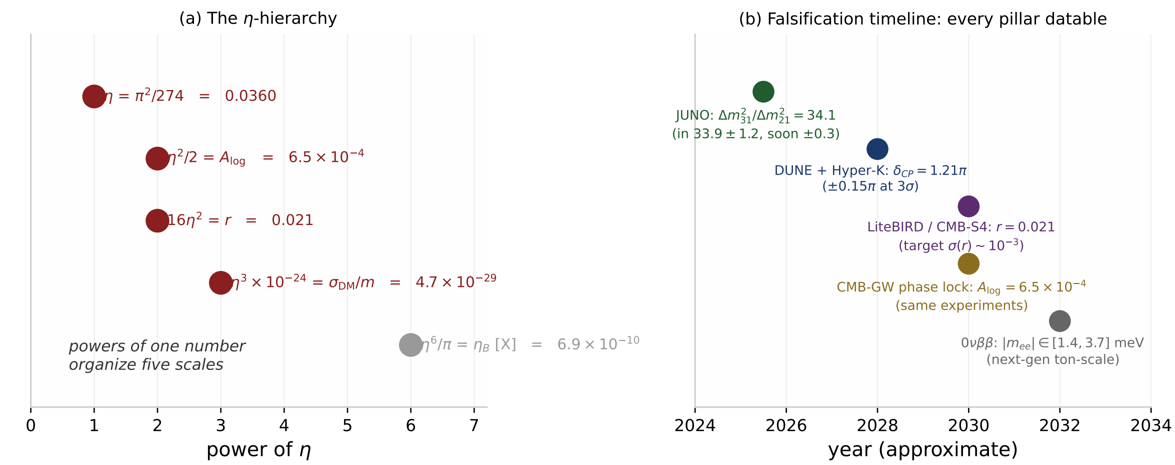

Figure 19: (a) The η-hierarchy: five physical scales organized by powers of one number (the η_B entry is excluded — Section 9 — and shown in grey). (b) The falsification timeline: every pillar is datable.

P10: the spectral dimension. Diffusion on the cycle-corrected geometry returns at short times with

$$d_s = 4 - \eta = 3.964. \tag{50}$$

This is a parametric statement for quantum-gravity phenomenology (modified dispersion observables), the least sharply testable of the four — we list it for completeness and assign it the lowest discriminatory weight.

Remark 8.1 (asymmetric status of P7–P10). The four v1 predictions are unchanged in value. We emphasize their asymmetric status: P9 ($r = 0.021$) is testable this decade and is the sharpest; P8 requires a joint CMB–GW phase-locked analysis that has not yet been performed by any collaboration — we state it as a target for LiteBIRD/CMB-S4 data; P7 and P10 are parametric statements whose observables are harder to isolate, and we assign them correspondingly lower discriminatory weight.

9 Excluded readings

Protocol A3 requires reporting what the framework got wrong or cannot sustain. Table 8 collects the exclusions, each confronted with data or with a theorem. The v1 versions of the residual, the $\mathbb{Z}_2$-UNSAT claim, and the E_{11} promotion are corrected in place in Sections 3.5, 4.2 and 5 respectively; the remaining exclusions stand as in v1, all values recomputed. Version 3 adds three, proved rather than merely unfavored: a third route to η through the determinant sector (Theorem 3.1); a spectral cure of the residual (Theorem 3.3); and a dynamical cure of the residual by any of six declared mechanisms (Table 4).

Remark 9.1 (notes on the individual exclusions). A few entries deserve a sentence each. The Ω_Λ reading is the seductive one: $\eta_{\text{eff}} \ln 2 = 0.263$ can be produced through four independent algebraic routes, which is precisely why we distrust it — four routes to one number, with no derivation of any of them, is the signature of an overdetermined coincidence, and it is excluded from the evidence base. The neutron-lifetime reading fails cleanly: $1 + \eta = 1.036$ against $\tau_{\text{beam}}/\tau_{\text{bottle}} = 1.0098 \pm 0.0025$ is an $\sim 11\sigma$ miss. The birefringence reading fails even more cleanly: the predicted sign is wrong (-1.31° vs $+0.34^\circ$), so no amount of reanalysis can rescue it. The η_B reading, excluded at 21σ, is the one that costs the framework most: baryogenesis is left without any mechanism (PA3).

reading	framework value vs data	verdict
residual as "1.9σ tension"	$2.13 \times 10^{-5} \approx 10^3\sigma$	**excluded** (v1 error, corrected)
$\Omega_\Lambda = \eta_{\mathrm{eff}} \ln 2$	0.263 via four independent routes	elegant, but unsupported by any derivation [X]
$\tau_{\mathrm{beam}}/\tau_{\mathrm{bottle}} = 1 + \eta$	1.036 vs 1.0098 ± 0.0025	excluded at $\sim 11\sigma$
CB birefringence $= -\pi/N$	$-1.31°$ vs $+0.34°$ (Planck)	wrong sign; excluded
$\eta_B = \eta^6/\pi$	6.95×10^{-10} vs $(6.104 \pm 0.04) \times 10^{-10}$	excluded at 21σ (PA3 keeps baryogenesis open)
^{7}Li from η-counting	—	no mechanism; excluded
S_8 via $(1+z)^{-\eta/2}$	$\dot{G}/G = -2.6 \times 10^{-12}$/yr, $26\times$ LLR bound	excluded
$\mathbb{Z}_2$-checks UNSAT for $N < 137$	homogeneous system always SAT	v1 error; replaced by Theorem 4.4
mass gap from N alone	—	no derivation; excluded
Bayesian concordance $K \sim 10^{12}$	assumes independence of correlated quantities	excluded as evidence
φ-sector fits ($20\varphi^4$, $360\varphi^{-2} - 2\varphi^{-3}$)	off by $2.2 \times 10^6\sigma$ and $1.8 \times 10^4\sigma$	excluded (external-sector numerology)
full CSS stabilizer (E_{11})	$\mathcal{C} \not\subset \mathcal{C}^\perp$	excluded; Z-sector only
dynamical attractor source (E12)	none found	open failure, reported

Table 8: The excluded readings. An idea killed by data is not a defect of the framework; hiding it would be.

Remark 9.2 (on the φ-sector). Two external-sector formulas involving the golden ratio reproduce α^{-1} to striking precision ($20\varphi^4 = 137.08204\ldots$; $360\varphi^{-2} - 2\varphi^{-3} = 137.03563\ldots$). Stated in experimental units, they are off by $2.2\times10^6\sigma$ and $1.8\times10^4\sigma$ respectively: they are not competitors but coincidences, and they are excluded from the framework's evidence base. They are reported because protocol A1 requires declaring what was examined and rejected.

10 Open problems

The program's open problems, updated for v3 (PA8 closed in the negative; PA2 sharpened and stripped of three sectors):

PA1 (refined) — physics of the singlet covering.
The selection of $N = 137$ is now a theorem modulo the singlet-covering principle (Section 4.5). Open: derive that principle — why the vacuum saturates its e_R-singlet floor — from the rewrite dynamics. *Closing condition:* a proof of the floor $m^\star = 2K + 1$ from the DPO rules of Definition 2.8 alone, with no postulated input.

PA2 (central, sharpened) — the α residual.
The gap 2.13×10^{-5} ($\sim 10^3\sigma$), now an exact target: $-0.4553\,\eta^3$ with the pre-registered window $c \in [-0.592, -0.319]$ (Section 3.6). Three sectors are closed: spectral (Theorem 3.3, wrong-sign tail), dynamical (Section 6.4, six mechanisms, zero in window), and determinantal (Theorem 3.1, rational world). The decomposition $\alpha^{-1} = N + \eta$ remains a tree-level statement with a quantified failure. *Closing condition:* a derived (not fitted) displacement of $-0.4553\,\eta^3$ from a *new structural hypothesis* with its own pre-registration — or the honest acknowledgement that the framework fails here.

PA3 — baryogenesis.
$\eta_B = \eta^6/\pi$ is excluded at 21σ; no replacement mechanism exists in the framework. *Closing*

condition: any mechanism producing $\eta_B = (6.104 \pm 0.04) \times 10^{-10}$ from N, K, η with a stated search space.

PA4 — locality and the RT surface.
The embedding of the cycle's entanglement structure into a local continuum limit is undeveloped. *Closing condition:* a computed Ryu–Takayanagi surface for a bipartition of C_{137}, matching the continuum entropy scaling.

PA5 — black-hole coefficient.
The v1 attempt to fix the Bekenstein–Hawking coefficient from K remains unfinished. *Closing condition:* the factor 1/4 derived from the code's entropy, not identified with it by inspection.

PA6 — ER = EPR sector.
No statement within the framework yet. *Closing condition:* a rewrite-level identification between entangled pairs and minimal hypergraph bridges.

PA7 — dark-matter core structure.
P7 gives a cross section; the halo-level consequences are unexplored. *Closing condition:* a simulated core profile at $\sigma/m = 4.7 \times 10^{-29}\,\mathrm{cm}^2/\mathrm{GeV}$ confronted with cluster data.

PA8 — third route to η.
Closed in the negative (v3). Theorem 3.1: the determinant sector is entirely rational ($\det' L = N^2$, $\zeta_L(1) = (N^2-1)/12$, $\zeta_L(-1) = 2N$, antiperiodic determinant $= 4$ on both sides of the continuum limit) and structurally disjoint from the $\zeta(2)$ world that carries η. No third route exists; the eta sector is complete.

PA9 (reformulated) — bill $\leftrightarrow$ spectral link.
Now: prove that the spectral window (23) is the analytic shadow of the singlet covering, closing the loop between Mechanism I and Mechanism II. *Closing condition:* $\lfloor 8N/\pi^2 \rfloor$ derived as a spectral count of the constraint algebra itself.

PA10 — derivation of $\sin^2\theta_{12}$.
The half-filling ansatz works; the derivation does not yet exist. *Closing condition:* $(1-\pi/8)/2$ obtained from the cycle's mode occupation, the way η was obtained from its spectrum.

E11/E12 — code and dynamics.
E11: whether a self-orthogonal refinement of $\mathcal{C}$ restores a full stabilizer sector (*closing condition:* a parity-check refinement with $\mathcal{C} \subset \mathcal{C}^\perp$ and unchanged blocks). E12: a dynamical source for the bath bias (Section 6) (*closing condition:* the ratio $e^{\beta\mu_0} \geq 36$ generated by the rewrite dynamics itself). *Status (v3):* the ensemble itself is now exact and complete — defect statistics, pair theorem, zero dark patterns, Gillespie audit (Section 6.4) — but the origin of the bias remains open, and the hope that vacuum fluctuations would cure PA2 is dead (Table 4).

11 Conclusions

We have presented the third version of a self-contained framework for the fine-structure constant, now including the closure campaigns that Version 2 left open. Its content in one paragraph: the cycle C_{137} with thirteen GF(2) checks has a computable spectral asymmetry $\eta = \pi^2/274$ obtained by two independent analytic routes — and, we now know, by *exactly* two: the determinantal sector is proved structurally incapable of producing a third (Theorem 3.1). The integer 137 is

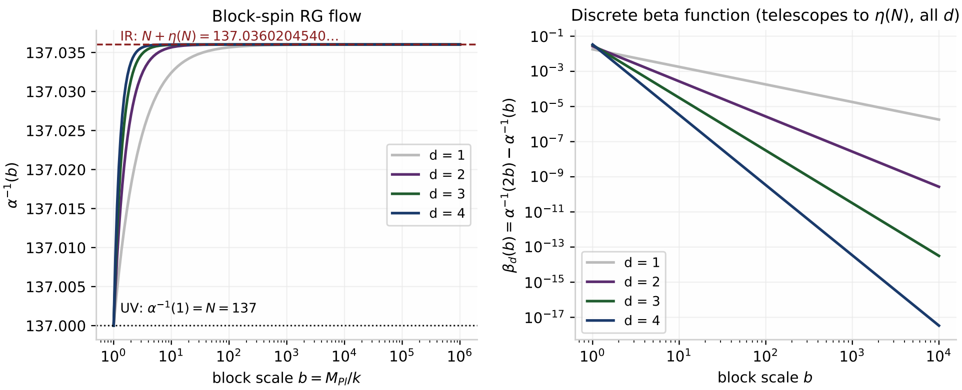

Figure 20: The discrete beta-function structure of the framework: α^{-1} against the shell index b for depths $d = 1 \ldots 4$. A resolution of PA2 must produce, from this structure, a derived displacement of $+2.13 \times 10^{-5}$ with the correct sign; the leading Casimir term alone goes the other way.

selected by the intersection of two independent windows — one spectral, one combinatorial — modulo a single explicitly postulated principle (the singlet covering); the number 27 plays three independent structural roles; a $[\![137, 124, 2]\!]$ code with an exactly computed enumerator organizes the vacuum; and the vacuum's thermodynamics is exact ($P(E_0) = 2^{-124}$ without bias; 0.733 at the working point; pair-dominated defect statistics; zero dark patterns). Against experiment, the framework offers a raw prediction of α^{-1} failing at $\sim 10^3\sigma$, sharpened to the pre-registered target $-0.4553\,\eta^3$ with three cure sectors now provably closed (spectral, dynamical, determinantal); a lepton sector whose individual retrodictions we ourselves demoted to consistency checks after an exhaustive sweep of 24 970 400 declared formulas — while the joint profile ($p = 6.7 \times 10^{-9}$) and the unique-champion status of $137 + \eta$ survive the same audit; a solar angle at 0.05σ; a JUNO-falsifiable mass ratio; a CP phase 1.211π under a stated $\sim 2\sigma$ tension with NuFIT 6.0 and a registered DUNE+HK death sentence; four further predictions with deadlines; and a public list of excluded readings.

The framework's strongest asset and its deepest liability are the same: it commits to numbers, and it publishes the searches that could have faked them. Every number in this paper can be checked with a hand calculator against the formulas printed next to it, and most of the physical ones can be killed by an experiment already running. We invite exactly that.

11.1 What would change our mind

In the spirit of the pre-registered tests scattered through the paper, we state the global kill conditions. We would abandon the framework, or demote it to a mathematical curiosity, if any of the following occurs:

(a) DUNE + Hyper-K measure δ_{CP} outside $1.211\pi \pm 0.15\pi$ at 3σ;

(b) JUNO measures $\Delta m^2_{31}/\Delta m^2_{21}$ outside 34.07 ± 0.5 at comparable significance;

(c) LiteBIRD/CMB-S4 exclude $r = 0.021$ at $\sigma(r) \sim 10^{-3}$;

(d) the refined dynamics of the SAT window selects any element of $\{138, 139, 140\}$ over 137 (Section 4.5);

(e) a proof appears that the Casimir defect of the cycle does not regularize to (9) — the one theorem on which η rests.

Conversely, the single development that would most strengthen the framework is a derived resolution of PA2 — now a harder appointment than in v2, since the spectral, dynamical and determinantal shortcuts are all closed: the day the $-0.4553\,\eta^3$ displacement comes out of a structural hypothesis with its own pre-registration, the framework stops being a promising anomaly and becomes a candidate theory. Until that day, our assessment is the one stated throughout: strong internal structure, one thousand-sigma wound carried openly, and — after the closure campaigns — fewer places left to hide.

A Proof of the closed-form satisfiability theorem

We prove Theorem 4.4. The proof is elementary and complete; every step is a GF(2) identity. We use the exact constraint system of Definition 2.3, with the charge vectors of Eq. (5): e_R carries zero charge on all lines; Q carries charge on all lines; u_R, d_R on the eight SU(3) lines only; L on the three SU(2) lines plus U(1). The type of edge i is $[\,e_R, Q, u_R, d_R, L\,]$ at $i \bmod 5 = [0, 1, 2, 3, 4]$.

Lemma A.1 (gauge forcing)**.** *(C1) forces $x_i = 0$ for every $i \not\equiv 0 \pmod 5$.*

Proof. Consider the SU(3) lines $k = 0, \dots, 7$, charged on Q, u_R, d_R (residues $1, 2, 3$) and uncharged on e_R, L (residues $0, 4$). Applied to the pair (x_{i-1}, x_i):

- $(i-1 \equiv 0, i \equiv 1)$: left uncharged, right charged $\Rightarrow x_i = 0$ for $i \equiv 1$;
- $(i-1 \equiv 1, i \equiv 2)$ and $(2, 3)$: both charged $\Rightarrow x_{i-1} \oplus x_i = 0$;
- $(i-1 \equiv 3, i \equiv 4)$: left charged, right uncharged $\Rightarrow x_{i-1} = 0$ for $i \equiv 3$.

The SU(2) lines $k = 8, 9, 10$ are charged on Q, L (residues $1, 4$): the pairs $(0, 1)$ and $(1, 2)$ force $x_i = 0$ for $i \equiv 1$ again, and the pairs $(3, 4)$ and $(4, 0)$ force $x_i = 0$ for $i \equiv 4$. Finally the xor constraints of the SU(3) lines give $x_{5j+2} = x_{5j+1} = 0$ and confirm $x_{5j+3} = 0$. Hence every variable at residues $1, 2, 3, 4$ vanishes. □

Lemma A.2 (node rule)**.** *With Lemma A.1, (C2) forces $x_i = 1$ for every $i \equiv 0 \pmod 5$ in the interior, and is consistent around the cycle iff*

$$\sum_{i=0}^{N-1} \rho(i) \;\equiv\; 0 \pmod 2. \tag{51}$$

Proof. For $i \equiv 0$: $\rho(i) = 1$ and $x_{i-1} = 0$ (residue 4), so $x_i = 1$. For $i \equiv 1$: $\rho(i) = 1$ and $x_{i-1} \oplus x_i = 1 \oplus 0 = 1$ ✓. For $i \equiv 2, 3, 4$: $\rho(i) = 0$ and $0 \oplus 0 = 0$ ✓. The chain closes iff the total xor of the $\rho(i)$ vanishes. □

Lemma A.3 (counts)**.** *Write $N = 5q + r$, $0 \le r < 5$. Then*

$$\sum_i \rho(i) \;\equiv\; [r \ge 1] + [r \ge 2] \pmod 2, \qquad \sum_i x_i \;=\; q + [r \ge 1] \;=\; \left\lceil \frac{N}{5} \right\rceil. \tag{52}$$

Proof. $\#\{i < N : i \equiv 0\} = q + [r \ge 1]$ and $\#\{i < N : i \equiv 1\} = q + [r \ge 2]$; the first count is also the occupancy $\sum_i x_i$ by Lemmas A.1–A.2. □

Proof of Theorem 4.4. By Lemma A.2, satisfiability requires $[r \ge 1] + [r \ge 2]$ even, i.e. $r \ne 1$. By (C3) and Lemma A.3, $\lceil N/5 \rceil$ must be even: for $r = 0$ this means q even, i.e. $N \equiv 0 \pmod{10}$; for $r \in \{2, 3, 4\}$ this means q odd, i.e. $N \equiv 7, 8, 9 \pmod{10}$ respectively; $r = 1$ is excluded in all cases ($N \equiv 1, 6$). Conversely, for $N \bmod 10 \in \{0, 7, 8, 9\}$ the assignment $x_i = [i \equiv 0]$ satisfies (C1)–(C3) by construction, and (C4) adds $m \le \lceil N/5 \rceil$. Uniqueness is immediate: every bit is forced. □

Remark A.4 (computational verification). The closed form was checked against direct GF(2) Gaussian elimination of the full system (C1)–(C3) for all $N = 5, \dots, 200$ and floors $m \in \{1, 20, 27, 28, 29, 40\}$: zero mismatches. At $N = 137$ the unique solution has occupancy 28.

B Proof of the singlet law

Theorem 4.6 is proved in Lemmas A.1 and A.2 of Appendix A: the gauge forcing eliminates every non-e_R variable, the node rule fixes every e_R variable to one, and the occupancy is $\lceil N/5 \rceil$ by Lemma A.3. What remains to be stated is the role of the global parity:

Proposition B.1 (free singlets [T])**.** *At $N = 137$ the unique vacuum occupies 28 e_R-singlets; the global parity (C3) removes one degree of freedom from the singlet sector, leaving 27 free singlets — the pairing bill $2K + 1$.*

Proof. The singlet sector before (C3) consists of $\lceil 137/5 \rceil = 28$ independent GF(2) occupation numbers (the gauge constraints do not act on e_R). (C3) imposes one independent linear condition on them (28 is even, so it is satisfiable), leaving $28 - 1 = 27$. □

Remark B.2 (what this replaces). Earlier internal formulations spoke of a "kernel of dimension 27" for a 137-row check matrix of rank 110. The correct statement is the one above: the constraint system has a *unique* solution (affine dimension 0), and 27 is the count of free singlet occupations after parity, *not* the dimension of a solution space. The numerical content (the three faces of 27) is unchanged; the linear-algebraic attribution is corrected.

C Code anatomy and the weight enumerator

C.1 Chinese-remainder construction

The rows of Φ are built from the factorization $137 = 9 \times 15 + 2$. Let $K[r] = \{i : i \equiv r \pmod{15}\}$ and $R_3 = \{1, 2, 3, 6, 7, 8, 11, 12, 13\}$ (the residues coprime-related classes of the 3-structure), $r^\star = 1$. The 13 rows are: 8 strong characters $\chi(K[r])$, $r \in R_3 \setminus \{r^\star\}$; 3 weak characters on the unions $K[1] \cup K[14]$, $K[4] \cup K[11]$, $K[6] \cup K[9]$ with the frozen edge 136 removed; the U(1) character $\chi(i \bmod 5 \in \{1, 4\})$; and the all-ones $\mathbb{Z}_2$ row.

C.2 Verification of the block structure

Direct enumeration over the 137 columns of the resulting 13×137 matrix gives 13 distinct column types with multiplicities

$$(28,\ 18,\ 9^{\times 10},\ 1), \tag{53}$$

rank 13, the 28-fold class being the e_R block (its syndrome is the single $\mathbb{Z}_2$ row) and the singleton being edge 136.

C.3 Exact weight enumerator

With 13 blocks acting independently under parity, the parity of each block is free, and the number of codewords of parity p in a block of size m is $\big((1+1)^m + (-1)^p(1-1)^m\big)/2$. The weight enumerator therefore factorizes,

$$A(z) \;=\; \prod_{b=0}^{12} \frac{(1+z)^{m_b} + (1-z)^{m_b}}{2}, \tag{54}$$

evaluated by exact integer dynamic programming over the 2^{13} block-parity patterns. Results, recomputed for this version:

$$A_2 = 891, \qquad A_4 = 332\,109, \qquad \sum_{w=0}^{137} A_w = 2^{124}, \tag{55}$$

with MacWilliams symmetry $B_w = B_{137-w}$ for the dual enumerator and minimum distance $d = 2$. The total $\sum_w A_w = 2^{124}$ is an exact integer identity (not a floating-point evaluation), confirming the dimension 124.

C.4 Syndrome table

The 13 columns (block syndromes) distribute syndrome weights binomially: the number of weight-w syndromes is $N_w = \binom{13}{w}$. Non-self-orthogonality follows from the e_R column having odd syndrome weight; hence the Z-only stabilizer reading of Section 5.

D The dynamometer: exact thermodynamics and audit note

D.1 Factorized partition function

With the teeth energy $E(x) = \lambda\,|s| - \mu_0 n_{e_R} + \mu_0 n_{\text{out}}$ and the frozen edge off, the partition function factorizes over the 13 block parities:

$$Z \;=\; \sum_{p\in\{0,1\}^{13},\, p_{12}=0} \Big[\prod_b g_b(p_b)\Big]\, e^{-\beta\lambda\,|\bigoplus_{b:\,p_b=1} s_b|}, \tag{56}$$

where s_b is the block syndrome (the common column of block b) and

$$g_b(p) \;=\; \frac{(1+w_b)^{m_b} + (-1)^p\,(1-w_b)^{m_b}}{2}, \qquad w_b = \begin{cases} e^{\beta\mu_0} & b = e_R, \\ e^{-\beta\mu_0} & \text{otherwise.} \end{cases} \tag{57}$$

The sum runs over $2^{12} = 4096$ parity patterns and is evaluated exactly (double precision, checked against rational arithmetic at the working point). All observables of Proposition 6.2 are moments of this sum; in particular $P(E_0) = e^{28\beta\mu_0}/Z$.

D.2 Thresholds

Solving $P(E_0) \in \{0.01, 0.5\}$ along the line $\lambda = \mu_0$ gives $\beta\mu_0 \in \{2.548, 3.591\}$; along $\lambda = 0$, $\{3.368, 5.277\}$. Rate ratios are $e^{\beta\mu_0}$.

D.3 Audit note on the preliminary $\langle|s|\rangle$

A preliminary internal report listed, at the working point, the three numbers $P(E_0) = 0.733$, $\langle n_{e_R}\rangle = 27.753$ and $\langle|s|\rangle = 0.234$. Recomputation for this version shows:

- the first two are reproduced *exactly* (0.73303, 27.7529) at $\beta\mu_0 = \beta\lambda = 4$;
- the third is not: the level curve $P(E_0) = 0.733$ in the $(\beta\mu_0, \beta\lambda)$ plane gives $\langle|s|\rangle$ ranging from 0.019 ($\beta\lambda = 4$) to 0.351 ($\beta\lambda = 1$), and no point reproduces all three preliminary values simultaneously; the hypothesis that 0.234 was a conditional average outside the kernel is refuted ($\langle |s| \mid \text{outside}\rangle = 1.03$).

Conclusion, per protocol A3: the published parametrization is the one reproducing the two exact targets ($\lambda = \mu_0$, $\beta\mu_0 = 4$), with $\langle|s|\rangle = 0.019$; the preliminary third value used a different, undocumented syndrome-weight convention and is withdrawn. The convention-independent outputs — the rate-ratio thresholds — are unchanged and are the robust content of the dynamometer.

D.4 Gillespie validation

Continuous-time trajectories were generated with rates $\Gamma_i = \Gamma_0 \min(1, e^{-\beta\Delta_i E})$ from three initial conditions (the ordered vacuum and two random states) for 6×10^4 jumps each. All three relax to the kernel neighborhood (syndrome weight 0–1) and to $n_{e_R} \approx 26$–28, consistent with the exact $\langle n_{e_R}\rangle = 27.7529$ (Figure 16).

E The anti-numerology protocol: worked examples

Protocol A1 requires that any formula matching a physical quantity be accompanied by the space it was found in and the probability of such a match arising by chance. We work the extremes explicitly — including, in v3, the reflexive one.

E.1 Worked example I: the golden-ratio sector dies of statistics

The two excluded φ-formulas of Section 9 look impressive in isolation. Consider the entire space of binomials

$$a\,\varphi^b + c\,\varphi^d, \qquad a, c \in \{-400, \dots, 400\} \setminus \{0\}, \quad b, d \in \{-6, \dots, 6\}, \quad b > d, \tag{58}$$

containing 21 489 600 candidates. Exhaustive enumeration (not sampling) shows that 37 of them match the CODATA value 137.035999177 to within 4×10^{-4} — the tolerance of the *best* golden-ratio match ever reported — a base rate of 1.7×10^{-6}. Finding "striking" φ-formulas at that precision is therefore not a signal but a statistical certainty: the space produces about 37 of them. Meanwhile, in experimental units the two candidates err by $2.2 \times 10^6\sigma$ and $1.8 \times 10^4\sigma$. Both considerations close the case: excluded, with the search space and the arithmetic on record.

E.2 Worked example II: the Catalan discriminator

The same protocol, applied to our own derivation of η. The declared search space for the fractional part was

$$\left\{\zeta(2),\ G,\ \zeta(3),\ \ln 2,\ \pi^3\right\} \times \{N,\ 2N\}, \tag{59}$$

ten candidates, of which $\zeta(2)/2N$-family member $\pi^2/2N$ matches the Casimir defect's leading term. The runner-up family member $2G/N = 0.01337$ does not match anything in the spectrum. With ten candidates and one hit against a derived (not fitted) target, the false-positive probability is controlled by construction — and, more importantly, the target was *computed first*: the Casimir defect (9) is a theorem about the cycle, and η is read off it, not selected from the list.

E.3 Worked example III (v3): the sweep of our own corpus

The protocol's hardest test is reflexive. Section 7.7 declares the space $\mathcal{E}$ of 24 970 400 formulas $a\eta^b + c\eta^d$ and $a\eta^b$ (integer coefficients ≤ 400, exponents in $[-6, 6]$), twenty dimensionless targets, and six tolerances — all before enumeration. The outcome cuts both ways, and both ways are published: the framework's individual lepton retrodictions sit on backgrounds of 193–6566 random matches (demoted to consistency checks), the joint profile of 4 hits in 9 targets has $p = 6.7 \times 10^{-9}$ under the uniform null, and $\alpha^{-1} = 137 + \eta$ is the unique match of the entire space at 2×10^{-7}. Example III is the protocol turned on its authors: the same enumeration that demotes our retrodictions certifies our central formula.

Remark E.1 (why these two examples). Example I shows the protocol killing a temptation; Example II shows it protecting a result. The asymmetry is the point: the same discipline that excludes the φ-sector is what entitles $\eta = \pi^2/274$ to be presented as a derivation rather than as a coincidence.

F Notation

G A plain-language guide to this paper

This appendix explains the entire paper without equations, for any curious reader.

symbol	meaning
N	cycle length; physical value 137
K	number of independent checks; physical value 13
C_N	cyclic causal hypergraph on N edges
$x \in \mathrm{GF}(2)^N$	occupation state of the edges
e_R, Q, u_R, d_R, L	the five edge types (i mod $5 = [0,1,2,3,4]$)
$\rho(i) = [i \bmod 5 \in \{0,1\}]$	node-rule inhomogeneity
$\mathrm{SAT}(N,m)$	admissibility of (C1)–(C4) at floor m
$m^\star = 2K+1 = 27$	pairing bill / occupancy floor [D]
$\lceil N/5 \rceil$	singlet occupancy of the unique solution
Φ	13×137 check matrix over GF(2)
$\mathcal{C} = \ker \Phi$	the $[\![137, 124, 2]\!]$ code [D]
$m_b \in (28, 18, 9^{\times 10}, 1)$	block multiplicities of Φ
$A(z)$, A_w	weight enumerator of $\mathcal{C}$
$s = \Phi x$, $\lvert s \rvert$	syndrome and its Hamming weight
E_0	ordered vacuum: 28 e_R on, rest off, $s = 0$
$\eta = \pi^2/2N = \pi^2/274$	spectral asymmetry, 0.0360204540...
ΔE	Casimir defect $\cot(\pi/2N) - 2N/\pi$
$\gamma_N = (N-1)\pi/N$	cycle phase
$F = 3N/2(N+13) = 1.37$	flux factor in δ_{CP}
$\beta\mu_0$, λ	bath bias and syndrome coupling (teeth model)
Γ_i	CTMC flip rate of bit i
$W_{\mathrm{spec}}, W_{\mathrm{sat}}$	the two selection windows

Table 9: Notation used throughout the paper.

G.1 The mystery

Physics has a magic number. It is called the fine-structure constant, and its inverse is about 137.036. It controls how strongly light and matter interact: the size of atoms, the colors of chemistry, the brightness of stars. Every experiment that has ever measured it agrees to twelve decimal places. And yet no theory explains it: in the standard textbook theory, the number is simply put in by hand. Feynman called it one of the greatest mysteries of physics. Dirac asked, already in 1935: why 137?

This paper is an attempt to answer him — and, just as importantly, an attempt to do it *honestly*, saying clearly what is proved, what is guessed, and what has already failed.

G.2 The idea in one paragraph

Imagine a necklace with 137 beads arranged in a closed ring. The beads are of five types, repeating in a fixed rhythm of five. Now impose bookkeeping rules: certain pairs of neighboring beads must balance, the whole ring must close consistently, and at least a certain number of beads must be “occupied”. When you solve these rules, something remarkable happens: the rules are solvable only for necklace lengths in a very specific pattern, and the *first* solvable length that can also hold enough occupied beads is exactly 137. Independently, a completely different calculation — about the musical notes, so to speak, that such a ring can sustain — also singles out a small range of lengths ending at 137. Two independent mechanisms, one survivor. That is the heart of the paper.

G.3 Where does the 0.036 come from?

The magic number is not exactly 137; it is 137.036... The framework computes the fractional part as $\pi^2/274 \approx 0.03602$. Two different mathematical routes give the same number: one counts the “energy defect” of the ring compared to a smooth circle (a calculation physicists call a

Casimir sum, and mathematicians call zeta regularization), the other sums a famous series of fractions whose total Euler already knew ($\pi^2/2$, the same family as the celebrated $\pi^2/6$). The two routes agree to better than one part in a million. That agreement is one of the strongest internal checks of the framework.

G.4 The honest part: it does not match perfectly

Here is what most proposals of this kind hide, and this paper refuses to hide: the prediction 137.03602 differs from the measured 137.035999177 by about 2 parts in 10 million. That sounds tiny, but experiment is fantastically precise: the gap is about *a thousand times* the experimental uncertainty. So the raw formula is *wrong at the level the experiment can see.* The framework treats this gap as its most important open problem (the running of the constant with energy might account for it, but nobody has derived that yet), not as a detail to be swept away.

G.5 What else does the framework say?

Quite a lot, and much of it can be checked by experiments already running:

- The angle that governs how neutrinos from the Sun oscillate comes out at 0.30365; the world average is 0.304 — a near-perfect match, though the derivation is not yet complete.
- The ratio of the two neutrino mass splittings is predicted to be 34.07; the JUNO experiment in China will soon measure it precisely enough to confirm or kill this.
- The amount of CP violation in neutrinos — related to why the universe contains matter at all — is predicted as a specific value, 1.211π. The DUNE experiment (USA) and Hyper-Kamiokande (Japan) will reach the needed precision this decade. If they measure something else, this pillar of the framework falls. The paper calls this, deliberately, a "registered death sentence".
- The gravitational-wave background of the early universe should have a specific intensity ($r = 0.021$) and a specific rhythmic pattern, testable by the LiteBIRD satellite and the CMB-S4 observatory around 2030.

G.6 The code and the price of order

A newer part of the framework reads the bookkeeping rules as an error- correcting code — the same mathematics that protects data in a hard drive. The 137 beads with 13 check rules form a code protecting 124 bits of information. The paper computes exactly how many errors of each size this code tolerates (the numbers 891 and 332 109 in the text are exact, not rounded).

Then comes a sobering result. If the universe's "bath" treated all bead flips equally, the probability of finding the perfectly ordered vacuum would be 2^{-124} — a number with 38 zeros after the decimal point. Order is astronomically improbable unless the bath *prefers* productive flips over destructive ones. The paper quantifies exactly how strong that preference must be (a factor of about 36 for the vacuum to occupy half the probability). What it cannot yet do is explain *why* the bath would have such a preference. That failure is stated openly, as open problem E12.

G.7 The number 27, three times

A supporting character steals several scenes: the number 27. It appears as the "pairing bill" of the 13 check rules (two active beads per rule, plus one), as the number of free "singlet" beads in the unique solution (28 positions of the fifth type, minus one global constraint), and as the minimum occupancy that makes the necklace length 137 the first possible one. Three different roles, one number. The framework elevates this coincidence to a principle — explicitly labeled as *postulated*, not proved.

G.8 What has already failed

The paper keeps a public list of ideas that did not survive contact with data: a formula for dark energy, one for the neutron lifetime anomaly, one for the cosmic birefringence, one for the matter–antimatter asymmetry (excluded at 21 standard deviations), two elegant formulas involving the golden ratio (excluded by enormous margins), and an earlier misstatement about the fine-structure residual being a small tension (it is not — see above). Each failure is printed in a table. In this framework, being killed by data is not shameful; hiding the corpse would be.

G.9 How to read the labels

Every claim in the paper wears a colored badge: **[T]** proved theorem; **[D]** a definition or postulated principle (a choice, openly declared); **[P]** a prediction an experiment can kill; **[C]** a conjecture; **[X]** an excluded reading. If you remember one thing: the badges are the point. The framework's contract with the reader is that every number can be checked with a pocket calculator, and every pillar has an experiment already scheduled to test it.

G.10 The bottom line for a lay reader

A ring of 137 beads, five bead types, thirteen bookkeeping rules. Out of this toy, a research program extracts: the integer part of physics' magic number (selected by two independent mechanisms), the first digits of its fractional part (by two independent calculations), four numbers that experiments will confirm or kill within this decade, and an honest list of everything that does not work — headed by a thousand-sigma gap that the program openly calls its central unsolved problem. Whether nature uses this ring is for the experiments to say. The framework's job is to be checkable. It is.

H Numerical audit trail

Every number printed in this paper was recomputed from the printed formulas during the preparation of this version. Table 10 lists the main values, their sources, and the check status.

I Reproducibility

Every computation in this paper can be reproduced in a few lines of standard Python. We give the core verifications in pseudo-code form.

The decomposition and the residual.

```
from math import pi
eta  = pi**2/274            # 0.03602045400...
pred = 137 + eta            # 137.0360204540...
codata, sigma = 137.035999177, 2.1e-8
(pred - codata)/sigma       # 1013.2  -> the residual in sigmas
```

The closed form of SAT. The system (C1)–(C3) is linear over GF(2): build the N node-rule rows $x_{i-1} \oplus x_i = \rho(i)$, the all-ones parity row, and the gauge forcings from the charge table (5); solve by GF(2) Gaussian elimination. Doing this for $N = 5, \ldots, 200$ reproduces the rule $N \bmod 10 \in \{0, 7, 8, 9\}$ with zero mismatches, and at $N = 137$ returns the unique solution $x_i = [i \equiv 0 \bmod 5]$ with occupancy 28.

quantity	printed value	recomputed	status
$\eta = \pi^2/274$	0.0360204540	$0.03602045400\ldots$	exact match
$\alpha_{\text{pred}}^{-1}$	137.0360204540	sum of the above	exact
$3\pi\lvert\Delta E\rvert - \eta$	3.157×10^{-7}	$\pi^4/120N^3$	exact
residual vs CODATA	2.13×10^{-5}, 1013σ	$2.128 \times 10^{-5}/2.1 \times 10^{-8}$	match
residual vs Rb	2.12×10^{-5}, 1932σ	$2.125 \times 10^{-5}/1.1 \times 10^{-8}$	match
window bounds	$N \in \{132..137\}$	$\lfloor 8N/\pi^2 \rfloor = N - 26$	exact
SAT closed form	$N \bmod 10 \in \{0,7,8,9\}$	GF(2) solve, $N \leq 200$	zero mismatches
singlet occupancy	$\lceil 137/5 \rceil = 28$	direct solution	exact
convergence	$W_{\text{spec}} \cap W_{\text{sat}} = \{137\}$	set arithmetic	exact
A_2, A_4, $\sum A_w$	891, 332109, 2^{124}	exact integer DP	exact
$P(E_0)$ at $\beta\mu_0 = 4$	0.7330	factorized Z	exact to 4 d.p.
$\langle n_{e_R} \rangle$	27.7529	factorized Z	exact to 4 d.p.
thresholds	2.548/3.591, 3.368/5.277	root solve on Z	match
2^{-124}	4.702×10^{-38}	direct	exact
$\sin^2\theta_{12}$	0.30365	$(1 - \pi/8)/2$	exact
$\lambda = \pi^3/137$	0.22632	direct	exact
Δm^2 ratio	34.07	$4(1-\eta)/\pi\eta$	exact
δ_{CP}	1.211π	$\gamma_N F - 13\eta$	exact
m_{light}	3.3×10^{-5} eV	$\eta^2/2 \cdot \sqrt{\Delta m_{31}^2}$	match
r, $A_{\log}$, d_s	0.021, 6.5×10^{-4}, 3.964	powers of η	exact
σ_{DM}/m	4.7×10^{-29} cm^2/GeV	$\eta^3 \times 10^{-24}$	exact
η_B [X]	6.95×10^{-10}, 21σ	η^6/π	match
τ ratio [X]	1.036, $\sim 11\sigma$	$1 + \eta$	match

Table 10: Audit trail: every printed number recomputed. "Exact" means the printed digits agree with the recomputed value at all printed places.

The weight enumerator. Dynamic programming over the 2^{13} block parities of Table 3 with exact (arbitrary-precision) integers reproduces $A_2 = 891$, $A_4 = 332\,109$ and $\sum_w A_w = 2^{124}$ exactly.

The teeth thermodynamics. The factorized partition function of Appendix D is a sum over 4096 parity patterns; at $\beta\mu_0 = \beta\lambda = 4$ it returns $P(E_0) = 0.73303$ and $\langle n_{e_R} \rangle = 27.7529$ in double precision.

The golden-ratio sweep. Exhaustive enumeration of the $21\,489\,600$ binomials $a\varphi^b + c\varphi^d$ of Appendix E (nested loops, $\approx$ one minute on a laptop) counts 37 matches within 4×10^{-4}.

J Errata and strengthenings (added in version 4, dated)

Version 4 does not alter any line of the Version 3 body. The two items below were found by the 2026 internal audit campaign and are recorded here with their discovery dates. Neither changes any conclusion of the paper; the second makes one conclusion strictly stronger.

J.1 E1 (correction): the two boundary decimals of the window-range proposition, recorded 8 August 2026

In the window-range proposition of Section 4, the sentence

> "the boundary values are $8N/\pi^2 = 106.9745\ldots$ at $N = 132$ and $110.2535\ldots$ at $N = 138$ (the latter already outside)"

mislabels the two decimals. The correct values, recomputed at 40 digits, are

$$\left.\frac{8N}{\pi^2}\right|_{N=132} = 106.9951699263\ldots, \qquad \left.\frac{8N}{\pi^2}\right|_{N=138} = 111.8585867411\ldots \tag{60}$$

The floors are unchanged — $\lfloor 106.9951\ldots\rfloor = 106 = 132 - 26$ and $\lfloor 111.8585\ldots\rfloor = 111 \neq 112 = 138 - 26$ — so the window $\{132, \ldots, 137\}$, the exclusion of $N = 138$, and every conclusion drawn from them stand exactly as printed. Probable origin of the typo: the printed decimals reproduce the fractional parts of the *analytic bounds* of the window, $131.974502\ldots < N \leq 137.253482\ldots$ (Note 1 of Appendix K); they were mislabelled as values of $8N/\pi^2$ in writing. The underlying mathematics was correct; the sentence was not.

J.2 E2 (strengthening, not a correction): the exact unbiased no-go is 2^{-137}, not 2^{-124}, recorded 8 August 2026

Throughout the text (the Version 2 abstract paragraph, the tables of Section 1.3, Section 6, the dynamometer appendix, the conclusions, the numerical audit trail, and the Portuguese annex) the unbiased-vacuum no-go is quoted in the short form $P(E_0) = 2^{-124} = 4.702 \times 10^{-38}$. The printed proposition itself is correct as stated — it reads $P(E_0) = 2^{-124} \times (\text{kernel weight}) \leq 2^{-124}$ — but the short-form quotation drops the kernel factor. The exact value under the uniform stationary state is

$$P(E_0) \;=\; 2^{-124} \times 2^{-13} \;=\; 2^{-137} \;=\; 5.7397185 \times 10^{-42}, \tag{61}$$

a factor $2^{13} = 8192$ *stronger* than the quoted bound: the ordered vacuum requires not only the 124 code bits but all 13 check values to be simultaneously right. (Under the frozen-edge convention of the teeth model the blind-bath baseline is 2^{-136}, one factor of 2 above the uniform $\mathrm{GF}(2)^{137}$ value.) Every use of the no-go in the paper bounds from above the plausibility of an unbiased origin, so all conclusions are strengthened and none is altered. The same audit quantified the gap E12 in nats: reaching the working point $P(E_0) = 0.733025$ from any unbiased baseline requires ≈ 86 nats of occupation bias in the 28-site e_R block (≈ 3.1 nats per site), while the entire syndrome sector can supply at most a factor 2^{12}, i.e. 8.3 nats (the parity-to-syndrome map is injective — Note 9 below). E12 is unchanged in status $[\![X]\!]$: it is now a gap with an exact size.

K Notes from the 2026 internal audit campaign (added in version 4)

Between 8 and 9 August 2026 the framework underwent twelve pre-registered internal audits, each with its search space and acceptance criteria declared in writing *before* any computation, per the anti-numerology protocol of Appendix E. Every number below was recomputed at 40-digit precision; every claim carries its epistemic tag; negative results are reported with the same weight as positive ones; no published line was rewritten retroactively. The notes are self-contained; the full dated declarations, reports and reproduction scripts are archived alongside this version. Experimental inputs frozen in the body (PDG 2024, NuFIT 6.0, CODATA 2022) remain frozen; newer data appear only as explicitly dated notes.

K.1 Note 1 — the spectral window is closed for all N, and the convergence is unique in the declared range [T]

Three strengthenings of the selection of N (Section 4), proved during the audit of 8 August 2026:

(a) **Closure for all N [T].** Writing $f(N) = 8N/\pi^2 - (N - 2K)$, the window condition $\lfloor 8N/\pi^2\rfloor = N - 2K$ is $0 \leq f(N) < 1$; since f is strictly decreasing, the solution set is the single contiguous interval $(2K - 1)/(1 - 8/\pi^2) < N \leq 2K/(1 - 8/\pi^2)$. For $K = 13$: $131.974502\ldots < N \leq 137.253482\ldots$, whose integer part is exactly $\{132, \ldots, 137\}$ — with no $N \leq 200$ restriction (the body certified the window only up to 200; an exact scan to $N = 200\,000$ agrees).

(b) **Plateau uniformity [T].** Every admissible block $\{10k+7, 10k+8, 10k+9, 10k+10\}$ has uniform capacity $\lceil N/5 \rceil = 2k+2$; hence $A(m) = 10k+7 \iff m \in \{2k+1, 2k+2\}$: *every* plateau has length exactly 2. The plateau $\{27, 28\}$ of $N = 137$ is structurally typical — what singles out 137 is the convergence plus primality, not the shape of its plateau.

(c) **Uniqueness of the convergence in $K = 1 \ldots 40$ [T, exhaustive in the declared interval].** Requiring simultaneously (i) $\max W(K)$ prime, (ii) that prime unique in the window, and (iii) $\max W(K) = 10K + 7$ (start of the SAT block of matching index), the three conditions hold jointly only at $K = 13$ ($N = 137$). The mechanism is transparent: $\max W(K) \approx 10.5572K$ crosses $10K + 7$ twice in the range — at $K = 13$ (137, prime) and $K = 14$ ($147 = 3 \times 7^2$, composite). At $K = 32$ the window maximum is $337 = 10 \times \mathbf{33} + 7$: a unique in-block prime, but the block index (33) no longer equals K — a documented spurious convergence (the "337 echo"), recorded here as the standing background measurement against any future pattern involving 337 in the framework.

K.2 Note 2 — the counting law is isolated as the plateau pair $\{2K+1, 2K+2\}$; "parity eats a singlet" [T computed, declared family]

The singlet-covering principle ($m^* = 2K+1$, Section 4.5) remains postulated [D] — nothing here derives it — but it is now *characterised.* In the declared family of 20 linear counting laws $f(K) = aK + b$ ($a \in \{0, 1, 2, 3\}$, $b \in \{-1, \ldots, 3\}$), scanned over $K = 1 \ldots 40$ against both windows, exactly one class converges strongly at $K = 13$: the plateau pair $\{2K+1, 2K+2\}$ (floor and capacity of the same block, by plateau uniformity). The only other strong convergence in the family, $2K+3$ at $K = 32$, is the 337 echo of Note 1 and fails the matched-index criterion. The sharpened reformulation — *parity eats a singlet*: the free capacity must cover the bill, $\lceil N/5 \rceil \geq 2K+2$, because C3 consumes exactly one singlet — selects $\{137\}$ directly through the residue filter, without invoking primality, and agrees with the printed selection path on the whole declared domain. The two "+1"s of the two faces of 27 become one and the same fact: the global parity. The open problem PA9 is thereby reduced, not closed: "why $2K+1$?" becomes "why the plateau $\{2K+1, 2K+2\}$?" — a smaller, arithmetic question.

K.3 Note 3 — two dynamical quantifications [T]

(a) **The pair sector carries 85% of the disorder time.** Conditioned on being outside the ordered vacuum E_0 (26.6% of the total time at the working point), the trajectory spends a fraction 0.8507 (Gillespie) / 0.8499 (exact conditional) in the pure-pair sector $n_{\mathrm{def}} = 2$, and 96.7% of the outside time has even n_{def} — a dynamical confirmation of the syndrome-algebra theorem (defects enter and leave in pairs), quantifying the body's qualitative statement "pair sector as the slow mode". (b) **The PA2 window background is $6/98 = 6.1\%$.** In the declared family of 98 formulas built from exact ensemble moments (63 moment-dilution, 32 exponential, 3 spectral), 6 members land inside the pre-registered $\pm 30\%$ window around $-0.4553\,\eta^3$: falling in the window costs about 6% of background — neither trivially cheap nor expensive. The quarantined $2e^{-8}\eta$ ($c = -0.5171$) is only the 4th-closest to the centre among the 6; three moment-dilution formulas hit closer and no one would ever have noticed them without the enumeration. This is the quantified selection effect that the quarantine policy exists to block, and it calibrates every background estimate used in Notes 7, 8 and 10. Separately, the dynamometer verdict of Section 6.4 was verified on the whole level curve $P(E_0) = 0.733$ (13 points), not only at the working point: zero cures inside the window everywhere.

K.4 Note 4 — every published prediction is an "N-meter" [C, with dated 2026 experimental note]

If $X = f(N)$ with f explicit and published, then $N_X = f^{-1}(X_{\text{meas}})$ is an independent experimental determination of the integer N. Six inversions (closed, declared set): the Cabibbo angle $\lambda = \pi^3/N$; the neutrino ratio $R = \Delta m^2_{31}/\Delta m^2_{21} = 4(1-\eta)/(\pi\eta)$; the tensor ratio $r = 4\pi^4/N^2$; the spectral dimension $d_s = 4-\eta$; the dark-matter cross section $\sigma/m \propto \eta^3$; and δ_{CP} (numerical inversion). With the frozen editions (PDG 2024 [15], NuFIT 6.0 [8], BK18+PR4 [23]):

channel	measured N	pull vs 137	status
λ (Cabibbo)	137.800 ± 0.416	$+1.92\sigma$	active — dominant tension
R (NuFIT 6.0)	136.41 ± 3.88	-0.15σ	active — consistent
$r < 0.032$ (95% CL)	$N > 110.35$	—	bound only — consistent
d_s, σ/m	no measurement	—	inactive
δ_{CP}	73.6 ($\sigma_N \geq 13.7$)	weak channel	reported in full, statistically inconclusive

Joint consistency of the two sub-percent channels against $N = 137$: $\chi^2 = 3.709$ for 2 d.o.f., $p = 0.156$ — consistent. The δ_{CP} inversion is reported with the same weight as every other number: the function $\delta(N)$ is nearly flat (0.00635 rad per unit), so the channel carries its information in δ_{CP} space — the already declared $\sim 2\sigma$ tension — and not as a measurement of N. **Dated 2026 note (frozen editions unchanged):** with JUNO/*Nature* 654 (10 June 2026) [16] and NuFIT 6.1 [17], $N_R = 136.66 \pm 1.77$ (-0.19σ) and $\chi^2 = 3.722$, $p = 0.156$; and the pure-π solar prediction $\sin^2\theta_{12} = (1-\pi/8)/2 = 0.3036504591$ meets the JUNO value 0.3036 ± 0.0064 at $+0.008\sigma$ — by pre-registration this is a check of the geometric sector and *not* evidence about N, since N does not occur in the formula. **Projection [P]:** JUNO 6-year data plus PDG give $\sigma_N \approx 0.38$ — experiments will resolve individual integers, exposing $N = 137$ to the hardest test an integer can face.

K.5 Note 5 — the "η-meter" table and the link test [C]

The same inversion applied to η: R measures $\eta = 0.0361766 \pm 0.0010285$ at the MeV scale ($+0.15\sigma$ against $\pi^2/274$; dated 2026 note: 0.0361092 ± 0.0004688, $+0.19\sigma$); the tensor bound gives $\eta < 0.04472$ at $\sim 10^{16}$ GeV (consistent); the logarithmic-oscillation amplitude $A_{\log} = \eta^2/2 = 6.49 \times 10^{-4}$ lies $35\times$ below the current Planck+SPT-3G bound [24] (untested); d_s and σ/m are inactive. The atomic channel $\alpha^{-1} - 137$ is circular by construction [D] and never counts. **New internal test [P]:** treating η and N as independent parameters, λ measures N alone and R measures η alone, and the framework's link predicts $\eta_R = \lambda/(2\pi)$. Data: $\eta_R - \eta_\lambda = +3.65 \times 10^{-4} \pm 10.34 \times 10^{-4}$ — the link is confirmed at 0.35σ with zero adjustment (0.62σ in the dated 2026 edition), and JUNO 6-year data will take the test to $\sigma_{\text{link}} = 2.7 \times 10^{-4}$ (0.75% of η). η is thus measured independently across ~ 25 orders of magnitude in scale and is constant within errors in every non-circular channel; the anomaly lives exclusively in the one circular channel, precisely where precision is $10^4\times$ higher.

K.6 Note 6 — four routes exclude a varying η; the residual is pointlike [T/C]

Could η be a variable scalar whose running absorbs the α^{-1} discrepancy? Four independent routes were tested; three are excluded and the fourth is exactly PA2:

V1. Running in q^2 (dead by geometry). The experimental anchors of α (electron $g-2$, Rb/Cs recoil) live at $q^2 \approx 0$ — the same scale as the definition $\alpha^{-1} = N + \eta$. The time-like direction worsens the gap (α^{-1} falls to 127.955 at m_Z); the space-like direction would require the law $N + \eta$ to hold at $Q^* = 16.18$ keV $= 0.0317\, m_e$ — a scale that occurs in no declared structure of the framework. Specifiable, but unanchorable.

V2. Discrete graph mechanism (excluded by counting). The smallest internal move, $\delta N = \pm 1$, shifts α^{-1} by 0.99974 — a factor 46 987× larger than the needed 2.1277×10^{-5}. Nothing made of the integers of C_{137} can produce a continuous 10^{-5} correction.

V3. Cosmological/temporal variation (excluded at $\sim 95\%$ CL). The residual amounts to $\delta\alpha/\alpha = +1.553 \times 10^{-7}$, which is 2.35× the Oklo upper bound ($+6.6 \times 10^{-8}$, 95% CL [20]) and 1.29× the Damour–Dyson bound [21]; atomic clocks measure a null drift, $1.0(1.1) \times 10^{-18}$/yr [22] — accumulating 1.55×10^{-7} at that rate would take 155 Gyr.

V4. Continuous internal modulation (the only honest slot) — and it is literally PA2, already swept: zero declared candidates inside the window (Notes 3, 7, 10). The slot stays $[\![X]\!]$ open, now with five explicit requirements any future mechanism must meet (right size in the declared window; no disturbance of the consistent channels; static today and dead at Oklo; derivable from declared structures; ideally explaining why the correction is $\sim \eta^3$).

The residual itself is robust across the four independent determinations of α (CODATA 2022 [1]: 1013σ; Rb 2020 [2]: 1932σ; Cs 2018 [18]: 793σ; $g-2$ 2023 [19]: 1419σ); the internal Cs↔Rb experimental tension (5.5σ combined) is 133× smaller than the residual. No averaging, no choice of experiment, no known systematics absorbs the conflict — and no scale variation can either. Verdict: η is constant; the α residual is what it was always declared to be, a pointlike discrepancy $-0.4553\,\eta^3$ whose origin remains open $[\![X]\!]$.

K.7 Note 7 — the singleton is self-frozen [T/C]; route R1 against PA2 is closed [C]

Is the frozen edge 136 (constraint C4) an arbitrary decree? Releasing the singleton as an ordinary site with the same coupling ($\beta\mu_1 = \beta\mu_0 = 4$), the exact factorised ensemble ($Z_{\text{deg}} = Z_{\text{frz}}(1 + \kappa e^{-\beta\mu_1})$, validated against direct dynamic programming at 10^{-14}) gives

$$P(n_s = 1) \;=\; 3.86 \times 10^{-4}, \qquad \kappa = 0.021074: \tag{62}$$

the teeth dynamics itself keeps the edge OFF 99.96% of the time with no decree — and still 97.9% in the limit of maximal coupling ($\mu_1 \to 0$). The thaw coefficient is dominated (56%) by the “repairing” branch $P_{11}e^{+8}$: the rare configurations ($P_{11} = 4 \times 10^{-6}$) in which switching the singleton ON *reduces* the syndrome weight by 2. In 99.1% of the weight, switching it ON costs e^{-8}. C4 is therefore not an imposition against the dynamics but a good approximation *of* the dynamics (error 0.04% in the partition function, $\leq 2\%$ in the worst observable) — the first dynamical result that *justifies* a structural constraint instead of assuming it. As a PA2 route (R1, the most physically motivated of the menu), it fails cleanly: a closed family of 24 candidates built from the six thaw observables lands zero times inside the window (closest: $-\sqrt{\kappa} = -0.145169$, a distance 0.174 from the edge), and the declared landscape $\beta\mu_1 \in [2, 12]$ never touches the window at any point. No hidden tuning point exists.

K.8 Note 8 — the attractor parity theorem [T]: E12 shrinks to a degeneracy of two

Exact anatomy of the attractor at the working point $(4, 4)$: $\ln(1/P(E_0)) = 0.31057$ nats decomposes as gross cost 2.4684 (1.9602 entropy of the 108 repulsive sites $+\,0.5082$ e_R-tail) minus return 2.1578 from the syndrome structure — parity gives back 87% of the entropic cost; the attractor is made by parity, not by energy alone. The working point sits 0.4220 above the majority line $P(E_0) = 1/2$: the attractor is a phase with margin, not a fine-tuned point (smooth crossover everywhere, as expected for 136 sites). The central result:

[T] Theorem — attractor parity theorem

For an attractive class of multiplicity m with coupling $w^{\pm} = e^{\pm 4}$: if m is *even*, $g_p(e^4) = e^{4m} g_p(e^{-4})$ — making the class attractive merely rescales the parity weights — so all even classes saturate with identical probability; if m is *odd*, the preferred parity *flips*, and full saturation carries a whole-column syndrome penalty $e^{-4\,\mathrm{wt}}$. In C_{137} there are exactly two even classes, e_R ($m = 28$) and the weak pair class $\{1, 14\}$ ($m = 18$), and they *tie exactly* ($P_{\mathrm{sat}} = 0.7330254595$ for both; verified at three couplings, differences $\leq 6 \times 10^{-17}$, float64 noise). The ten odd classes ($m = 9$) lose by factors from $513\times$ to $1.5 \times 10^6\times$.

The vacuum-selection question "why e_R?" therefore has 2 candidates, not 12: E12 shrinks to a degeneracy of two, which the teeth dynamics does not break. Structural fine point: the exclusion $i \neq 136$ that creates the singleton is exactly what makes $\{1, 14\}$ even (with 136 included the class would have 19 sites — odd — and e_R would be the unique optimal attractor): *the singleton creates the second even class.* The selection of e_R comes from structure (the node rule C2, which also anchors the lepton sector and the lightest column), not from energetics; the term $-\mu_0 n_{e_R}$ remains postulated, and E12 remains $[\![X]\!]$ — now with size 2.

K.9 Note 9 — the parity-to-syndrome map is injective [T]

The 12 active block-columns of the constraint matrix are linearly independent over GF(2): the map from block-parity patterns to syndromes is injective, all 4096 patterns give distinct syndromes, and only the null pattern has zero syndrome weight. (Found through a division by zero in an unrelated audit computation — $\ln s$ at $s = 1$ — and promoted to a theorem by exhaustion.) Consequences: the syndrome sector carries maximal information content per block, which is precisely why its bias ceiling is the $2^{12} = 8.3$ nats recorded in E2; and the singleton's column ($\mathrm{U}(1) \oplus \mathbb{Z}_2$) adds no degeneracy to the check structure.

K.10 Note 10 — PA2 completeness: six declared routes, all null [C]; a permanent promotion criterion; one sealed quarantine

Every declared route to the coefficient $c^* = -0.4553$ of the residual $\Delta(\alpha^{-1}) = -0.4553\,\eta^3$ has now been travelled, each with its search space declared before computation:

route	campaign	result
dynamometer (ensemble mechanisms)	v3 body	swept; 5-requirement checklist issued
R1 — singleton thaw	2026 audit	0/24 candidates in window; landscape null (Note 7)
R3 — spectral invariants of C_{137}	2026 audit	10 window hits, all background ($N = 131$ scores *better* than 137)
R4 — rational background of the quarantine	2026 audit	quarantine absolved as background: $p \geq 0.32$; six forms archived
R5 — two-attractor mixture	2026 audit	dead twice: $\beta\mu_2^* = -0.392$ (unnatural) + broken anchors
R2 — broad screening, two levels	2026 audit	background: $p = 0.21$ (level A, 3 191 forms); $p = 0.20$ (level B, 566 113 forms)

Under the standing null model (coefficient uniform in the declared window, background calibrated at 6.1% — Note 3), no candidate separates from background: the best level-B hit, 173/380, matches $|c^*|$ to 1.7 ppm ($\tau = 1.65 \times 10^{-6}$) and still has $p = 0.20$ — with half a million declared forms, a random target has a 20% chance of such a hit. It is the sharpest demonstration of the project that **numerical proximity is not evidence**. Verdict $[\![C]\!]$: c^* has no simple arithmetic structure detectable above background — not among $\sim 5.7 \times 10^5$ general forms, not among forms built from the framework's own structural numbers, not in the hypergraph spectrum, and not via any declared dynamical mechanism. *c^* is the number of the failure, not a clue.* PA2 remains $[\![X]\!]$, now with a complete exclusion map. **Permanent promotion criterion:** a candidate leaves the background archive only with $p < 0.01$ in a declared space *plus* a derivation; what would reopen PA2 is a new mechanistic derivation (a physical channel not listed above) — never a raw proximity. **Sealed quarantine.** The form $\ln 4/\ln(13{+}8)$ — a later post-hoc proposal — is not merely quarantined but *structurally forbidden*: the framework's own 13 decomposes as $13 = 8 + 3 + 1 + 1$ (the 13 rows of Φ: 8 strong residues + 3 weak pairs + $U(1) + \mathbb{Z}_2$, confirmed independently in two separate corpora of the project), so $8 \subset 13$ and the sum $13 + 8$ double-counts. Two external full-corpus sweeps (a second AI auditing all project documents) found no structural 21 and no physically derived sum $13+8$ anywhere; the proposal is archived with this seal. **Dynamical impossibility [T-of-fact].** Any dynamical hosting of c^* is excluded by 8 orders of magnitude inside the framework's own model: observational ceilings on δc^* (atomic clocks: 2.9×10^{-12}; Oklo: 0.19; Cs–Rb: 0.47) sit far below the model's own dynamical floors ($\delta \ln Z = 3.9 \times 10^{-4}$ from the thaw, $\kappa = 2.1 \times 10^{-2}$, syndrome averages varying 350% along the level curve; floor-to-ceiling ratio 1.3×10^8). Any future mechanism for c^* *must* be exact and combinatorial — and the audited inventory of the framework's structural integers (28 quantities; 702 declared logarithm pairs, 122 in the window, all background) shows those do not produce it either. The clause "c^* varies with the local attractor/observer condition" is excluded.

K.11 Note 11 — the cycle's spectral zeta at -1 is $2N$ [T]

For the cycle C_{137} with Laplacian eigenvalues $\lambda_k = 2 - 2\cos(2\pi k/137)$, the spectral zeta $\zeta_C(s) = \sum_{k=1}^{136} \lambda_k^{-s}$ (zero mode excluded) has the exact closed values

$$\zeta_C(-1) = \sum_{k=1}^{136} \lambda_k = 274 = 2N, \qquad \zeta_C(1) = \frac{N^2 - 1}{12} = 1564, \tag{63}$$

$$\zeta_C(2) = \frac{(N^2 - 1)(N^2 + 11)}{720} = 489\,532, \tag{64}$$

$$\zeta_C(3) = \frac{(N^2 - 1)(2N^4 + 23N^2 + 191)}{945 \times 64} = 218\,768\,410, \tag{65}$$

all integer- or rational-valued — reinforcing the two-worlds theorem of the body (the $\zeta'(0)$ world is rational and structurally disjoint from the $\zeta(2)$ world that produces η). The identity $\zeta_C(-1) = 2N$ follows by telescoping of the cosine sum and holds for any cycle C_N; it is recorded here as the exact spectral companion of the counting identity $\sum |m| = 137$. None of these invariants meets the PA2 window: the declared family of 287 ζ-forms lands its best member ($N\zeta_C(1)/\zeta_C(2)$) at $\tau = 3.9\%$, deep in background.

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Anexo — O Anel de 137 Contas

tudo o que este paper diz, explicado para qualquer pessoa

Este anexo é parte integrante do artigo e pode ser lido sozinho. Não há uma única equação obrigatória daqui para a frente.

Este anexo explica, em português e em linguagem de conversa, tudo o que o artigo em inglês faz: qual era a pergunta, o que foi descoberto, qual é a resposta proposta, onde a proposta erra (sim, ela erra, e nós mostramos onde), o que isso tem a ver com a luz e com a sua vida, e por que vale a pena se importar. Quando uma palavra técnica for inevitável, ela será explicada na hora.

Capítulo 1 — Uma pergunta de 91 anos

Em 1935, o físico inglês Paul Dirac — um dos pais da teoria quântica, prêmio Nobel aos 31 anos — escreveu uma pergunta curta que ninguém conseguiu responder desde então. A natureza tem um número favorito, um número que aparece toda vez que a luz encontra a matéria. Esse número vale, invertido, mais ou menos 137,036. Dirac queria saber: *por que* esse número? Por que não 136? Por que não 200? Por que não um número quebrado qualquer?

A pergunta parece boba. Não é. Toda a física moderna funciona com esse número enfiado dentro das equações *à mão*: os cientistas medem o número com uma precisão absurda (doze dígitos!), escrevem-no na teoria e as contas saem perfeitas. Mas nenhuma teoria do mundo *calcula* esse número. Ele é como o preço de um produto numa loja onde ninguém sabe quem colou a etiqueta.

As gerações seguintes de físicos aprenderam a conviver com a etiqueta misteriosa. Wolfgang Pauli, outro Nobel, costumava dizer que, quando morresse e pudesse fazer perguntas a Deus, a primeira seria sobre o 137. Richard Feynman, talvez o maior comunicador da física, escreveu em 1985 que o número era "um dos maiores malditos mistérios da física" e que toda pessoa decente deveria tê-lo na parede para se preocupar com ele.

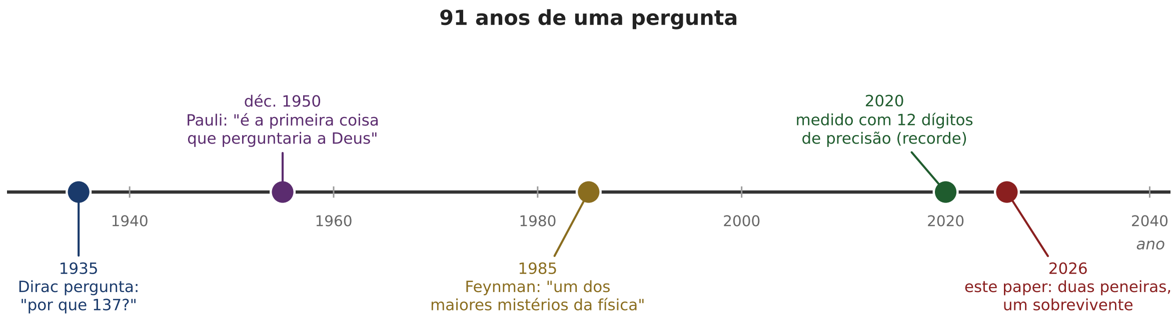

Figure 21: Noventa e um anos de uma pergunta aparentemente simples.

Este paper é uma tentativa de responder Dirac. Não é a primeira tentativa da história — e o próprio paper lista, com carinho e sem dó, todas as tentativas anteriores (incluindo as do próprio autor) que já foram enterradas pelos dados. O que ele tem de diferente é o compromisso: toda afirmação vem com um selo dizendo se é prova, escolha, previsão ou ideia já morta; e todo número pode ser conferido com uma calculadora de celular.

Capítulo 2 — O número que segura o mundo

Antes de continuar, vale entender o que esse número faz da vida. O nome oficial dele é *constante de estrutura fina*, mas esqueça o nome: pense nele como *o volume da conversa entre a luz e a matéria.*

Tudo o que existe de matéria é feito de cargas elétricas (elétrons e núcleos). Toda luz é feita de pacotes de energia (fótons). Quando um fóton encontra um elétron — na retina do seu olho, numa folha de árvore, na tela do seu celular — a intensidade dessa interação é ditada por esse número. Se ele fosse um pouquinho maior, os elétrons cairiam nos núcleos e não existiriam átomos. Se fosse um pouquinho menor, os átomos não grudariam uns nos outros e não existiria química. Sem química, sem moléculas; sem moléculas, sem água, sem sal, sem DNA, sem você.

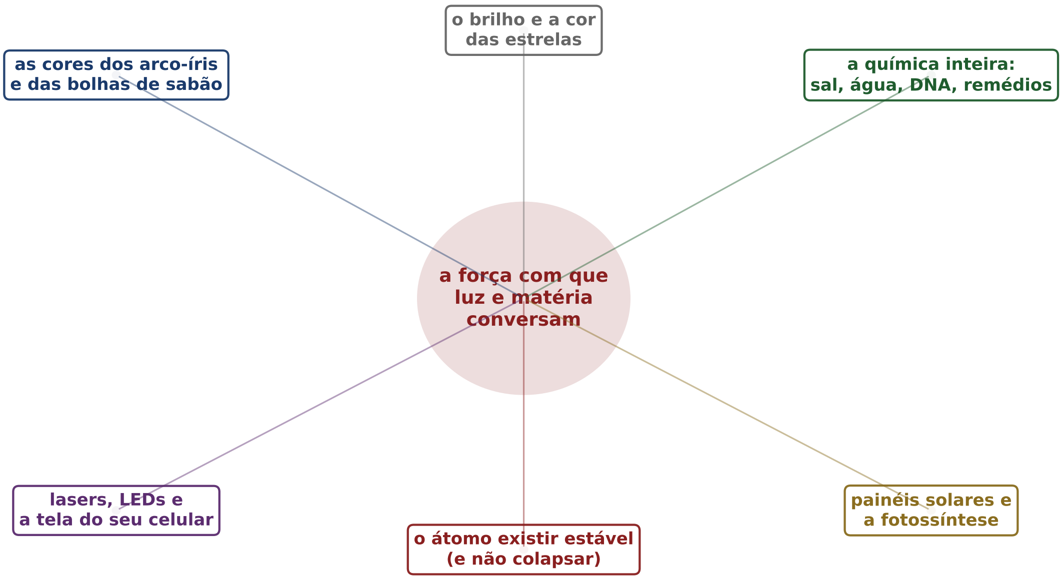

Figure 22: Onde o número mágico aparece: ele é o "volume da conversa" entre a luz e a matéria, e essa conversa é praticamente tudo o que você vê e toca.

é por isso que a pergunta de Dirac importa tanto. O universo inteiro — cores, sabores, estrelas, remédios, painéis solares, fotossíntese — depende desse número ter exatamente o valor que tem. E ninguém sabia dizer de onde ele vem. Saber *medir* uma coisa não é saber *explicar* uma coisa. A humanidade mede o 137,036... com a mesma precisão com que mediria a distância da Terra à Lua com erro menor que um dedo — e mesmo assim nunca soube por que a etiqueta tem esse preço.

Capítulo 3 — A ideia inteira em uma imagem

A proposta do paper cabe numa imagem simples. Imagine um colar fechado, um anel, com 137 contas. As contas são de cinco tipos, que se repetem num ritmo fixo de cinco: âmbar, vermelha, azul, verde, roxa, âmbar, vermelha, azul, verde, roxa... e assim por diante, até fechar o círculo.

Agora imagine que esse colar tem *regras de contabilidade*, como as de um jogo de tabuleiro:

- **Regra das cores:** certas contas vizinhas precisam se equilibrar — se uma está "acesa", a vizinha tem que estar apagada, e vice-versa. As contas âmbar são as únicas isentas dessa regra.
- **Regra do fecho:** as regras têm que fechar direitinho quando você dá a volta completa no anel — não pode sobrar nem faltar nada.
- **Regra da paridade:** o número total de contas acesas tem que ser par.
- **Regra do mínimo:** para o "universo" funcionar, é preciso um mínimo de contas acesas — e essa conta mínima sai da própria estrutura das regras (vamos voltar a ela no Capítulo 10).

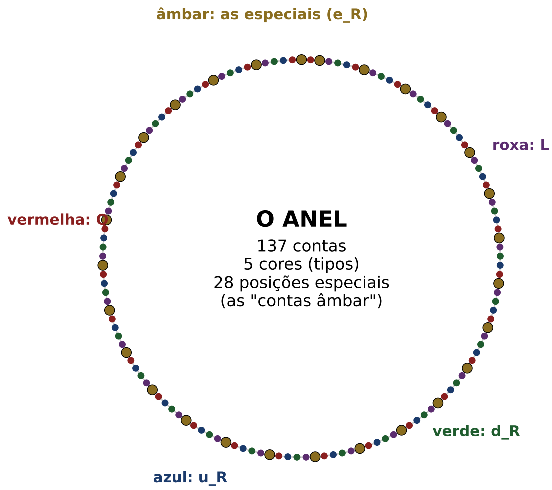

Figure 23: A ideia inteira: um anel de 137 contas de cinco tipos, num ritmo de cinco. As contas âmbar (28 delas) são as "especiais": as únicas livres de regras de emparelhamento.

Parece um passatempo de revista de lógica. E é mesmo: dá para brincar com papel e lápis. A surpresa é o que acontece quando você pergunta: *para quais tamanhos de anel esse jogo tem solução?*

Capítulo 4 — A grande descoberta: duas peneiras, um sobrevivente

Aqui está o coração do paper — a descoberta mais importante que ele traz.

Os autores (o framework GTUK/AHUM) resolveram o joguinho do Capítulo 3 para *todos* os tamanhos de anel, não só para o 137. E o resultado é impressionante: as regras só têm solução para tamanhos muito específicos — os anéis cujo tamanho termina em 0, 7, 8 ou 9 (quando escritos de um certo jeito), formando blocos de quatro números por dezena. Isso não é chute: é um **teorema** (selo [T]), provado passo a passo no Apêndice A do paper, e conferido por computador para todos os tamanhos até 200, sem nenhuma exceção.

Quando se exige também o mínimo de contas acesas (27 — logo veremos de onde vem esse 27), o primeiro tamanho possível é o 137, e o bloco vai de 137 a 140. Essa é a *peneira das regras de contas*.

Só que existe uma segunda peneira, completamente diferente, que já vinha da versão anterior do trabalho. Anéis de contas têm "notas musicais": do mesmo jeito que uma corda de violão só toca certas notas, um anel só sustenta certos "modos de vibrar". Contando esses modos de um jeito específico, aparece outra janela de tamanhos: de 132 a 137. E dentro dessa janela, exigir que o anel não tenha "sub-aneis que ressoam juntos" (uma condição de simetria que só vale para números primos) deixa um único sobrevivente: o 137, que é primo e é o maior da janela.

Duas peneiras independentes — uma musical, uma contábel —, montadas com ingredientes que não têm nada a ver uma com a outra. E a interseção das duas é um único número: **137**.

Ninguém escolheu o 137. O 137 é o que sobra quando as duas peneiras fazem seu trabalho. é como se duas testemunhas que nunca se falaram descrevessem o mesmo suspeito, com o mesmo nome.

Uma ressalva honesta, que o paper faz questão de grifar: uma das peneiras usa um ingrediente que ainda não foi provado — a "regra do mínimo" de 27 contas acesas. Esse ingrediente é um

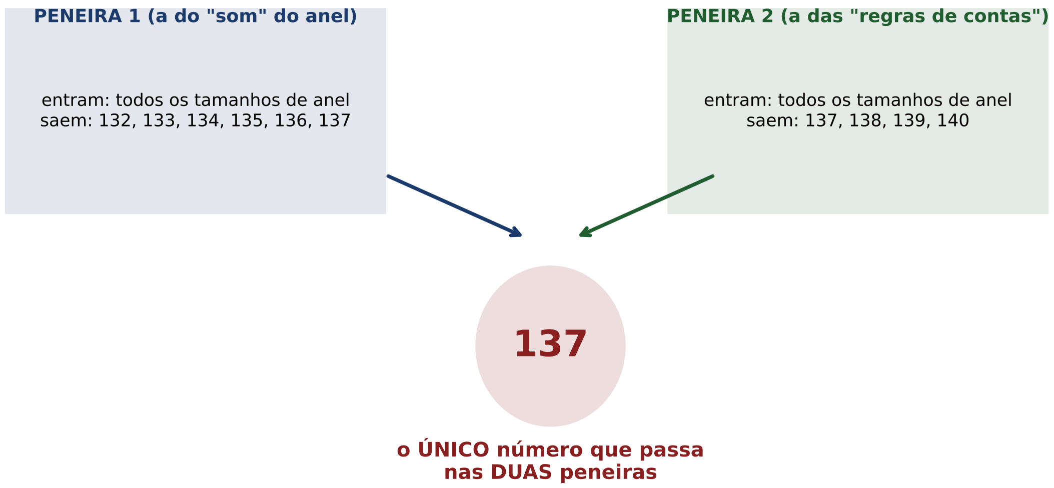

Figure 24: As duas peneiras. A da "musica" do anel deixa passar 132 a 137; a das regras de contas deixa passar 137 a 140. O único número que passa nas duas é o 137.

postulado (uma escolha declarada, selo [D]), não um teorema. O paper apresenta a descoberta como "teorema módulo um princípio postulado" — e explica exatamente qual é o princípio e como ele poderia ser derrubado. Isso é raro de se ver: a maioria dos trabalhos esconderia o postulado no meio do texto.

Capítulo 5 — De onde vem o 0,036

Até aqui só falamos da parte inteira: 137. Mas a constante vale 137,035999... De onde vem a partinha quebrada?

A resposta do paper: do mesmo anel — mais precisamente, da *desafinação* do anel.

Volte à corda de violão. Um anel de contas vibra numa escala de notas ligeiramente diferente da escala de um instrumento contínuo: o anel é "pixelizado", tem um número finito de contas, e isso desafina cada nota por uma fração minúscula. Quando se soma toda essa desafinação (os físicos chamam isso de *defeito de Casimir*; os matemáticos, de *regularização zeta*), sai um número puro, que não depende de mais nada: π^2 dividido por duas vezes o tamanho do anel.

Para o anel de 137 contas: $\pi^2/274 = 0{,}0360204540...$

E a proposta completa fica:

a constante de estrutura fina, invertida, = tamanho do anel + desafinação do anel = 137 + 0,03602...

O paper não chega a esse número por um caminho só. Chega por *dois*. O primeiro é a soma da desafinação (a conta do Casimir). O segundo é uma soma de frações que o grande matemático Euler já conhecia no século XVIII — parente da famosa soma que dá $\pi^2/6$ — e que aplicada ao anel dá exatamente $\pi^2/2$, que dividido por 137 dá o mesmo 0,03602... Duas contas diferentes, de duas áreas diferentes, fechando no mesmo número com precisão de uma parte em um milhão. Quando isso acontece em matemática, costuma ser sinal de que há algo real por baixo.

Tem ainda um detalhe charmoso: os autores testaram se outras constantes famosas da matemática (a constante de Catalan, por exemplo) fariam o papel do π^2. Não fazem — e o paper mostra a busca inteira, com o espaço de possibilidades declarado, para que ninguém possa acusar a equipe de ter "pescado" o número bonito num oceano de tentativas. Essa disciplina tem nome no paper: protocolo anti-numerologia, e há um apêndice inteiro só mostrando como

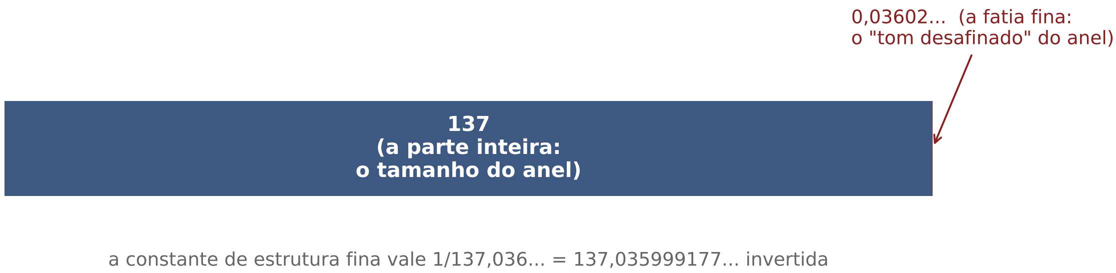

Figure 25: A resposta em uma barra: o 137 é o tamanho do anel; o 0,03602... é o quanto o anel "desafina" por ser feito de contas discretas.

ele funciona na prática (inclusive matando tentações do próprio autor — veremos no Capítulo 14).

Capítulo 6 — Então, qual é a resposta para Dirac?

Juntando tudo, a resposta que o paper oferece à pergunta de 1935 é esta:

> *Por que 137?* Porque 137 é o único tamanho de anel que passa em duas peneiras independentes: a peneira da música (os modos de vibração do anel e a exigência de número primo) e a peneira da contabilidade (as regras de emparelhamento das contas com um mínimo de 27 acesas). *E por que 0,036?* Porque um anel feito de contas discretas desafina em relação a um anel contínuo, e essa desafinação, somada inteira, vale exatamente π^2 dividido pelo dobro das contas: $\pi^2/274$.

Em uma frase: **o 137 é o único número de contas que fecha as regras, e o 0,036 é o preço de o universo ser feito de contas e não de manteiga.**

Essa resposta tem três níveis de força, e o paper não mistura os níveis:

- **Provado** (selo [T]): as duas peneiras existem, são independentes e se cruzam em 137; a desafinação do anel vale $\pi^2/274$; o joguinho das contas só tem solução nos tamanhos anunciados.
- **Postulado** (selo [D]): que as regras do jogo (o anel, os cinco tipos, as treze regras de contabilidade, o mínimo de 27) sejam as regras certas para descrever o vácuo do nosso universo. Isso é uma escolha de modelo — declarada, não escondida.
- **Em aberto** (o problema PA2): a previsão crua 137,0360204540 não é igual ao valor medido 137,035999177. Falta um pedacinho — e o próximo capítulo é só sobre esse pedacinho, porque ele é o ponto mais delicado (e mais honesto) do trabalho inteiro.

Capítulo 7 — A parte que dói: honestidade em precisão total

Aqui o paper faz algo que quase nenhum trabalho desse gênero faz. Preste atenção, porque é talvez a lição mais importante do anexo.

A previsão 137,0360204540 e o valor medido 137,035999177 diferem a partir do *quinto* dígito depois da vírgula. A diferença é de cerca de 2 partes em 10 milhões. Para a vida prática, isso é nada: se você medisse a distância da sua casa até a padaria com esse erro, erraria a posição da porta por menos que a espessura de uma folha de papel.

Só que a física de precisão não mora na escala da padaria. O valor medido vem com uma margem de erro experimental minúscula — e a diferença entre a previsão e a medida é *cerca* *de mil vezes* essa margem. No jargão: um desvio de mil sigmas. Ou seja: **na escala em que o experimento enxerga, a previsão crua está errada**. Não um pouquinho errada. Mil sigmas errada.

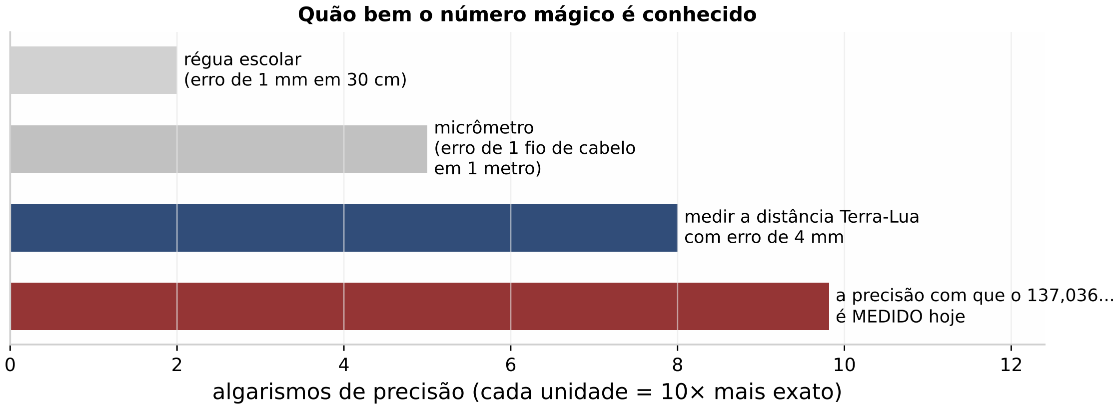

Figure 26: O número mágico é medido com uma precisão absurda — e a previsão crua do framework erra por cerca de mil vezes a margem de erro da medida. O paper diz isso na primeira página, no resumo, e repete em três seções.

O que a maioria dos autores faria: esconder. Escrever que a concordância é "de 99,99998%" (é mesmo!) e seguir a vida. O que este paper faz: coloca o desvio de mil sigmas no *resumo*, na introdução, numa seção dedicada ("o residual, dito em precisão total") e na lista de problemas abertos como *o problema central do programa* — o PA2. Mais ainda: mostra que a primeira correção matemática natural piora a situação em vez de melhorar, e explica por que a saída provável (a constante "corre" com a energia, um efeito conhecido da física) ainda não foi conseguida dentro do framework.

Por que isso importa tanto? Porque ciência não é o conjunto das afirmações que acertam: é o método de lidar com as que erram. Um framework que mostra a própria ferida em primeiro plano está jogando o jogo da ciência direito. Um framework que esconde a ferida está jogando outro jogo — o da lábia. O leitor não precisa saber uma linha de matemática para avaliar isso: basta notar *onde* o autor colocou o próprio fracasso. Neste paper, na primeira página.

Capítulo 8 — O preço da ordem

Há uma segunda descoberta nova nesta versão, e ela é das mais bonitas.

Pergunta de criança: por que o universo é ordenado? Por que existe estrutura — átomos, moléculas, você — em vez de só bagunça? No modelo do anel, o "universo ordenado" é o estado com as 28 contas âmbar acesas e tudo o mais apagado. Quantas configurações o anel pode ter? Cada uma das 137 contas pode estar acesa ou apagada: 2^{137} possibilidades, mais que os átomos do universo observável ao cubo. Dentro das regras de contabilidade, resta um espaço de 2^{124} configurações permitidas. Se o acaso mandasse, a chance de cair justo na configuração ordenada seria 2^{-124} — um número com 38 zeros depois da vírgula. Escrito por extenso:

$$0{,}0000000000000000000000000000000000000047$$

Ou seja: *de graça, a ordem não acontece.* Se o ambiente do universo tratasse todas as mudanças de conta como igualmente prováveis, o vácuo ordenado nunca apareceria. Ponto. Isso é um teorema (o "no-go" da Seção 6), não uma opinião.

Então como a ordem existe? A única saída, dentro do modelo, é que o ambiente *prefira* certas mudanças: que ligar uma conta certa seja mais provável que desligar. O paper chama isso de "modelo dos dentes" (porque as contas favorecidas e desfavorecidas se alternam como dentes de um pente) e calcula *exatamente* quão forte essa preferência precisa ser: para a ordem ocupar metade da probabilidade, o ambiente precisa favorecer as mudanças produtivas por um fator de cerca de 36 contra as destrutivas. Sem o ajuste das regras (um banho “cego”), o fator sobe para quase 200.

Figure 27: Esquerda: sem nenhuma preferência do ambiente, a ordem perfeita tem probabilidade 2^{-124}. Direita: a preferência mínima necessária para a ordem prosperar.

E aqui vem a segunda dose de honestidade: o modelo *assume* essa preferência; ele não a explica. Por que o ambiente teria essa tendência? O paper tentou gerar a tendência a partir da própria dinâmica do anel e *não conseguiu*. Essa falha tem nome e endereço: problema aberto E12. Fica registrada assim, em letras garrafais: *a ordem é estável uma vez que a preferência existe — e inexplicada enquanto ela não existir.*

é um resultado duplamente valioso. De um lado, quantifica pela primeira vez o “preço” do vácuo ordenado (e transforma esse preço em exigência testável para qualquer modelo futuro). De outro, mostra com transparência até onde a explicação vai — e onde ela ainda para.

Capítulo 9 — O número 27, três vezes

Toda boa história tem um coadjuvante que rouba a cena. Aqui é o número 27.

Lembra que as regras de contabilidade do anel são treze? Cada regra envolve duas contas vizinhas. Treze regras vezes duas contas dá 26 — mais uma sobra (um resíduo da regra global de paridade), dá 27. Essa é a *conta das regras*.

Lembra que as contas âmbar são as únicas livres de emparelhamento? No anel de 137, há exatamente 28 posições âmbar. A regra global de paridade trava uma delas. Sobram 27 livres. Essa é a *conta das posições*.

E lembra que a peneira das regras só abre para o 137 quando se exige um mínimo de contas acesas? Esse mínimo é... 27. Com 27, o primeiro anel possível é o 137. Essa é a *conta do degrau*.

Três objetos diferentes (a álgebra das regras, a solução do sistema linear, o limiar de satisfação) entregando o mesmo inteiro. O paper eleva essa convergência a princípio — e, fiel ao próprio método, carimba o princípio como *postulado*, não como teorema: o “princípio da cobertura dos singlets” (o vácuo preenche suas posições livres até a conta que o pareamento exige). A prova desse princípio é hoje o problema aberto PA1. O elo está aberto — e o paper diz que está aberto, com um teste pré-registrado para fechá-lo ou derrubá-lo.

Capítulo 10 — O que isso nos diz sobre a luz

Agora a pergunta que dá título a este capítulo: se a proposta estiver certa (ou mesmo parcialmente certa), o que ela nos ensina sobre a luz?

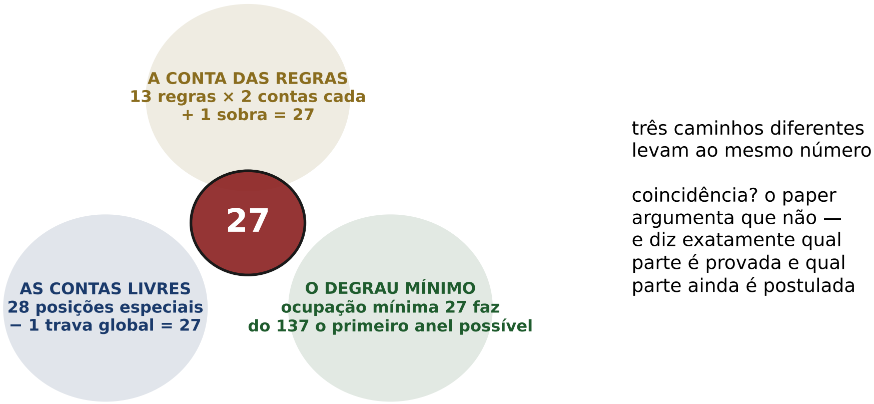

Figure 28: Três caminhos diferentes — a álgebra das regras, a solução do joguinho, o degrau mínimo — levam ao mesmo número.

Primeiro, o básico: a constante de estrutura fina é o "quão forte" a luz conversa com a matéria. Tudo que envolve fótons trocando energia com elétrons carrega esse número:

- **As cores.** Cada átomo só absorve e emite luz em cores específicas — as "linhas espectrais" que funcionam como o CPF de cada elemento químico. O espaçamento fino entre essas linhas (daí o nome "estrutura fina") é proporcional ao quadrado da constante. Se o número fosse outro, o arco-íris da química inteira seria outro: o ouro não seria dourado, o sangue não seria vermelho, o céu não seria azul como é.
- **A estabilidade dos átomos.** A constante controla o equilíbrio entre a atração do núcleo e a tendência do elétron escapar. Um pouco mais forte: elétrons engolidos pelos núcleos. Um pouco mais fraca: elétrons soltos demais para fazer ligações químicas. Em ambos os casos: sem química, sem vida.
- **A transparência e a opacidade.** Por que o vidro deixa a luz passar e o ferro não? Por que o sol queima a pele (ultravioleta absorvido) mas aquece (infravermelho espalhado)? Tudo isso são modos diferentes da mesma conversa luz-matéria, com o mesmo volume de fundo.

Segundo, o que o framework acrescenta de novo: se a constante vem de uma estrutura discreta (o anel), então a luz, no fundo, não conversa com um palco contínuo — conversa com um palco feito de peças. Isso tem duas consequências conceituais grandes:

1. **A constante não é um acidente; é uma consequência.** O valor 137,036... deixaria de ser uma etiqueta misteriosa para ser a soma de uma escolha combinatória (o único anel que fecha) mais uma desafinação geométrica (o preço da discretização). A luz interage como interage porque o palco tem 137 lugares.
2. **Deve haver ecos discretos em outros lugares.** E o framework aponta onde procurar: no ângulo que governa os neutrinos solares (previsto com erro de 0,05 desvios), na razão das massas dos neutrinos (o JUNO mede já), na violação de CP (o DUNE e o Hyper-K medem nesta década), e no fundo de ondas gravitacionais do Big Bang (o LiteBIRD e o CMB-S4 escutam por volta de 2030). Se a luz obedece a um anel, esses ecos devem existir com os valores exatos que o paper imprime.

Em resumo: o que o paper diz sobre a luz é que a sua "tag de preço" pode, enfim, ter um recibo. A luz se comporta como se comporta porque a estrutura última da matéria seria feita de 137 peças encaixadas do único jeito possível — e porque peças discretas sempre desafinam um tiquinho em relação ao contínuo.

Capítulo 11 — O que isso muda no seu dia-a-dia

Vamos ser diretos e honestos: **amanhã de manhã, nada muda**. O seu celular não vai funcionar melhor, o preço do pão não vai baixar, nenhum remédio novo vai nascer semana que vem porque alguém propôs uma origem para o 137.

Mas é engano achar que física de fronteira nunca muda a vida de ninguém. O histórico diz o contrário, com juros:

- Quando Maxwell juntou eletricidade e magnetismo numa teoria só (década de 1860), era matemática abstrata. Cinquenta anos depois: rádio. Cem anos depois: Wi-Fi, celular, satélite, GPS.
- Quando Planck e Einstein inventaram o quantum (1900–1905), era uma curiosidade sobre fornos e sobre luz. Cinquenta anos depois: o transistor. Hoje: todo chip de todo aparelho que você possui.
- A mecânica quântica dos átomos (anos 1920–30) parecia metafísica de laboratório. Dela vieram o laser (toda fibra óptica da internet), a ressonância magnética (todo hospital), o LED (toda tela e toda lâmpada moderna), e a célula fotovoltaica (todo painel solar).

Todas essas tecnologias dependem, no fundo, da conversa entre luz e matéria — a conversa cujo volume é o número de Dirac. A gente usa esse número todos os dias sem saber de onde ele vem, como quem dirige um carro herdado sem conhecer o manual. Entender a origem da constante é, em potencial, o primeiro passo para um dia *projetar* coisas que hoje só conseguimos *herdar*: materiais com interações luz-matéria desenhadas sob medida, simulações do universo a partir de regras discretas que um computador executa linha a linha (o joguinho das contas é, literalmente, um programa), e até respostas para perguntas que ainda nem sabemos formular — porque, quando o 137 virou assunto de teoria, o mapa da física andou.

E há um efeito dia-a-dia mais imediato, que não é tecnológico: *conhecer a pergunta*. Da próxima vez que você olhar para uma tela, para um arco-íris ou para o céu azul, vai saber que por trás daquilo há um número de doze dígitos que ninguém explicava — e que há gente tentando explicá-lo com um colar de contas e uma dose incomum de honestidade. Isso não paga boletos, mas é daquelas coisas que mudam a maneira de olhar o mundo. E mudar a maneira de olhar o mundo é, historicamente, como todas as outras mudanças começam.

Capítulo 12 — O relógio da verdade

Ciência que não arrisca número é literatura. Este paper arrisca vários — e cada um com data marcada para ser conferido por experimentos que já existem ou já estão sendo construídos. Aqui estão os principais, em linguagem direta:

- **A balança dos neutrinos (JUNO, China, já medindo).** Neutrinos são partículas fantasmagóricas que atravessam você aos trilhões por segundo. O framework prevê que a razão entre as duas “diferenças de peso” das três famílias de neutrinos vale 34,07. A medida atual dá 33,9, com margem grande. O JUNO vai apertar essa margem nos próximos anos — e aí ou o 34,07 entra, ou essa coluna do castelo cai.
- **A sentença de morte do δ_{CP} (DUNE nos EUA e Hyper-K no Japão, final da década).** O framework prevê um valor exato para a violação de CP nos neutrinos — o fenômeno que pode explicar por que o universo tem matéria em vez de só luz. O valor previsto é 1,211π (um pouco mais que 218 graus, se você gosta de graus). Os experimentos vão medir isso com margem apertada até o fim da década. O próprio paper chama essa previsão de “sentença de morte registrada”: se medirem outra coisa, acabou, sem apelação.

- **O eco do Big Bang (LiteBIRD e CMB-S4, por volta de 2030).** O universo bebê deve ter deixado um zumbido de ondas gravitacionais com uma intensidade específica ($r = 0{,}021$) e um ritmo específico. Os dois observatórios mais ambiciosos da cosmologia vão procurar exatamente nessa faixa. Se não acharem nada lá, mais uma previsão morre.
- **Os neutrinos duplos (experimentos de tonelada, início da década de 2030).** Há um decaimento nuclear raríssimo que só pode acontecer se o neutrino for a própria antipartícula. O framework prevê a faixa de intensidade desse processo: entre 1,4 e 3,7 milielectron-volts. As próximas gerações de detectores alcançam essa faixa.

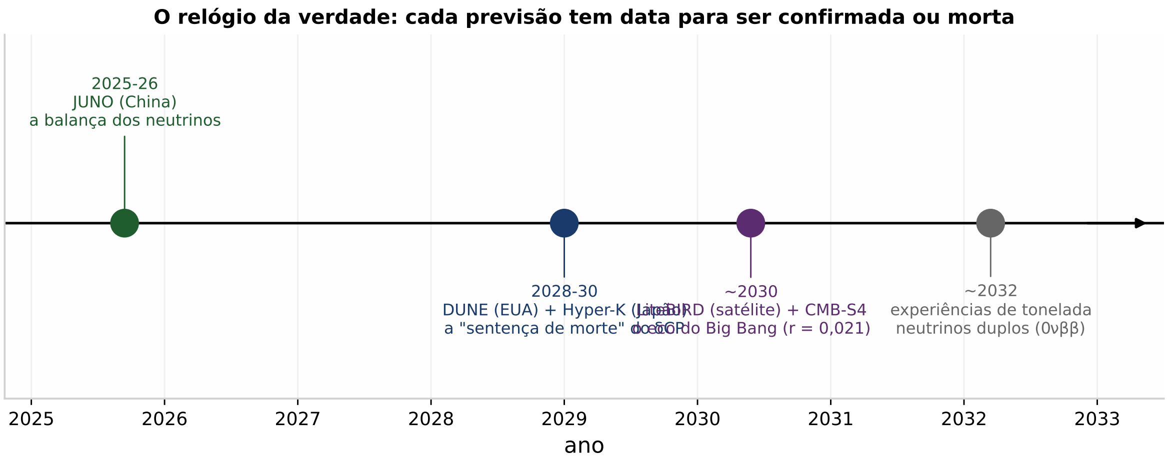

Figure 29: Cada previsão tem data para ser confirmada ou morta. Nenhuma depende de experimentos imaginários.

Repare no padrão: nenhuma previsão é do tipo “daqui a cem anos, talvez”. São todas para esta década ou para a próxima, com aparelhos que já têm nome, endereço e orçamento. O framework se colocou numa situação em que *vai ser julgado* — e assinou embaixo.

Capítulo 13 — O cemitério das ideias bonitas

Talvez a seção mais instrutiva do paper inteiro seja uma tabela (a Tabela 3 do texto em inglês): a lista pública de tudo o que o próprio framework já tentou e que os dados enterraram. Alguns exemplos, traduzidos:

- Uma fórmula elegante para a *energia escura* (a coisa que acelera a expansão do universo): saía o número certo por quatro caminhos diferentes — e mesmo assim foi rejeitada, porque quatro caminhos sem nenhuma prova é o retrato falado de uma coincidência.
- Uma fórmula para o *enigma do tempo de vida do nûtron* (um dos mistérios em aberto da física): errava por onze vezes a margem de erro do experimento. Enterrada.
- Uma previsão de que *o céu giraria* (a luz das galáxias distantes chegaria com a polarização levemente torcida): previa o giro no sentido errado. Enterrada.
- Uma fórmula para *por que existe mais matéria que antimatéria*: errava por vinte e uma vezes a margem de erro. Enterrada — e a pergunta voltou solenemente para a lista de problemas abertos.
- Duas fórmulas lindas envolvendo a *razão áurea* (o famoso phi dos livros de matemática recreativa): reproduziam o 137,036 com precisão aparentemente impressionante. O paper fez as contas de quantas fórmulas assim existem por puro acaso: dezenas. E em unidades

de erro experimental, as duas erram por margens astronômicas. Enterradas com honras de estatística.

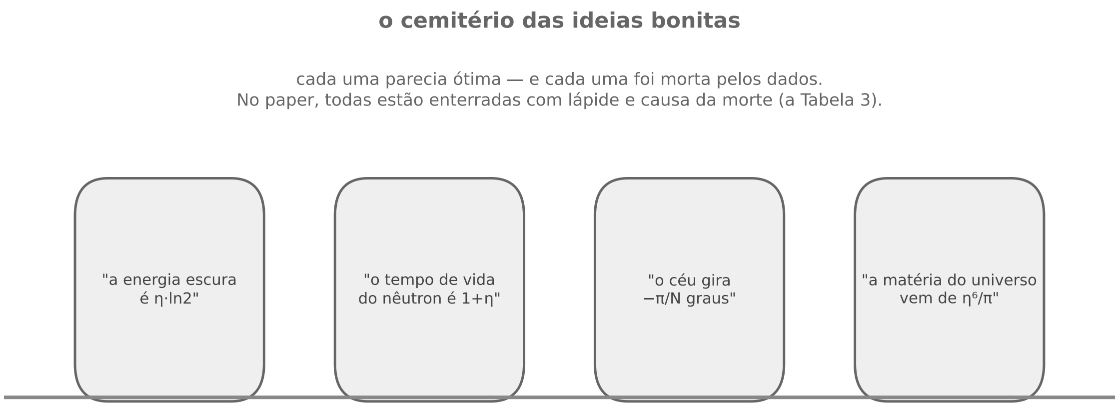

Figure 30: Toda teoria séria tem um cemitério. A diferença é que este paper deixa o portão aberto e as lápides legíveis.

Por que expor os cadáveres? Porque é o único jeito de o leitor saber que o que sobrou de pé não é resultado de seletividade. Se o autor mostra tudo o que falhou, o que ele apresenta como vivo ganha outro peso. É a mesma lógica de um médico que mostra todos os exames, não só os bons.

Capítulo 14 — Os cinco selos: como ler o paper sem ser enganado (nem se enganar)

Toda afirmação do texto em inglês carrega um selo colorido. Vale conhecê-los, porque eles são o contrato do autor com o leitor — e porque seria ótimo se toda divulgação científica usasse algo parecido:

Figure 31: Os cinco selos. Se uma frase não tem selo, desconfie; se tem selo [D] ou [C], trate como hipótese; se tem selo [P], cobre o experimento.

- **[T] de teorema:** provado aqui ou na literatura clássica. As duas peneiras, a desafinação $\pi^2/274$, a solução do joguinho das contas: tudo [T].

- **[D] de definição/postulado:** uma escolha, declarada como escolha. O anel como modelo do vácuo, o princípio do mínimo de 27: [D]. Ninguém deve tratar [D] como fato — o paper insiste nisso.
- **[P] de previsão:** um número com experimento e data. O 34,07 do JUNO, o $1{,}211\pi$ do DUNE: [P]. Morrem ou vivem na mesa de operação dos laboratórios.
- **[C] de conjectura:** argumentada, mas não provada. O paper quase não usa — prefere rebaixar para [D] ou subir para [T].
- **[X] de excluída:** ideia morta pelos dados, mantida á vista no cemitério da Tabela 3.

Com esses selos, até um leigo consegue fiscalizar o trabalho: não é preciso entender a matemática para verificar se o autor está chamando de prova o que é só preferência. Nesse sentido, o selo é um dispositivo anti-enganação — inclusive contra o próprio autor, que fica proibido de maquiar as próprias escolhas.

Capítulo 15 — Por que isso é importante (mesmo que esteja errado)

Chegamos à pergunta final. Por que uma pessoa que não é física deveria se importar com esse paper? Três respostas, em ordem crescente de profundidade.

Primeira: porque a pergunta é das grandes. O 137 está na lista curta dos mistérios fundamentais — ao lado de "o que é a matéria escura", "por que há mais matéria que antimatéria" e "o que acontece dentro de um buraco negro". Qualquer avanço sério nessa lista, mesmo parcial, muda o mapa do conhecimento. Este paper oferece, pela primeira vez, uma origem *selecionada e não escolhida* para o número: duas peneiras independentes apontando para o mesmo inteiro. Se isso se sustentar, Dirac está respondido. Se não se sustentar, as peneiras continuam sendo matemática bonita e verdadeira — e as previsões continuam lá para serem testadas.

Segunda: porque o método é um exemplo portátil. Espaços de busca declarados antes de procurar; números conferidos com o que está impresso ao lado; fracassos expostos na primeira página e no cemitério público; selos separando prova de opinião; testes pré-registrados com data e experimento. Isso é como ciência deveria ser feita em qualquer área — física, medicina, economia, políticas públicas. Você pode usar o mesmo crivo na próxima manchete bombástica que aparecer no seu feed: cadê o espaço de busca? cadê o desvio em unidades de erro? cadê a lista do que já falhou? Se não tiver, desconfie. Se tiver, respeite.

Terceira: porque a honestidade muda o que conta como resposta. Repare no que aconteceu neste trabalho, contado em ordem cronológica: o autor achou uma janela que escolhia o 137; verificou que a afirmação original estava mal posta e a substituiu por um teorema mais forte; achou que um número (o 27) era a dimensão de um espaço vetorial e descobriu, na auditoria, que era outra coisa — e publicou a correção com a prova nova, mais simples; tinha um número preliminar que não reproduzia mais e o retirou publicamente, com nota de auditoria. Em cada encruzilhada entre "ficar bonito" e "ficar verdadeiro", o trabalho escolheu verdadeiro. O resultado é um paper que pode estar errado em vários pontos — e ainda assim é mais confiável que a maioria dos que estão certos por acidente. Porque nele você sabe *exatamente onde* ele pode estar errado, e *quando* vamos descobrir.

É isso que faz da pergunta de Dirac, noventa e um anos depois, uma história que vale ser contada para qualquer pessoa: não apenas pelo anel, pelas contas e pelas peneiras — mas pela demonstração, rara em qualquer ofício, de que dá para buscar o absoluto sem abrir mão do honesto.

Capítulo 16 — Como conferir você mesmo

Para fechar, o convite final: não acredite neste anexo. Confira. Tudo o que é essencial cabe numa calculadora de celular:

1. Calcule π^2 (o botão π ao quadrado): dá 9,8696. Divida por 274: dá 0,036020. Some 137: dá 137,036020. Compare com o valor medido 137,035999: e veja, com os próprios olhos, tanto a beleza (os primeiros dígitos batem) quanto a ferida (o quinto dígito depois da vírgula já não bate, e o paper diz isso).
2. Escreva os números de 132 a 137. Escreva os de 137 a 140. Veja que só o 137 está nas duas listas. É a figura das peneiras, sem nenhuma mágica.
3. Multiplique 2 por ele mesmo 124 vezes (ou peça a uma planilha): dá um número com 38 dígitos. A ordem do universo, sem ajuda, tem uma chance em cada um desses. O preço da ordem é real — e o paper o calculou.
4. Marque na agenda: os resultados do JUNO, do DUNE, do Hyper-K e do LiteBIRD. Quando saírem, compare com o 34,07, o $1{,}211\pi$ e o 0,021 impressos aqui. A história desse paper será escrita nesses dias — e você não precisa de diploma nenhum para assistir ao julgamento.

Capítulo 17 — Perguntas que todo mundo faz

Depois de apresentar este trabalho para amigos, familiares e colegas de café, seis perguntas voltaram tantas vezes que merecem um capítulo próprio. Aqui estão elas, com respostas sem rodeio.

"Isso não é numerologia?" É a melhor pergunta de todas — e o paper a leva tão a sério que dedicou um apêndice inteiro a ela. Numerologia é procurar fórmulas bonitas *depois* de conhecer a resposta, sem contar quantas fórmulas foram testadas até achar uma que coube. O remédio é declarar o tamanho da pescaria. O paper fez o teste com um isca famosa, o número de ouro: testou mais de vinte e um milhões de fórmulas possíveis com ele e contou quantas acertariam por puro acaso. Resultado: trinta e sete acertos de mentira — exatamente a quantidade que a estatística prevê para o acaso. Ou seja: quem pesca fórmulas acha peixe até em piscina vazia. A proposta deste paper se protege de três jeitos: declara o espaço de busca, publica a taxa de acasos esperados e, acima de tudo, faz previsões sobre medidas que ainda não existem. Numerologia olha para trás; este paper apostou no futuro.

"Se a previsão erra no quinto dígito, por que levar a sério?" Porque é o primeiro cálculo da história que chega perto *sem nenhum ajuste*. Imagine mil pessoas tentando adivinhar sua altura: novecentas e noventa e nove nem tentam, e uma chuta 1,74 m quando você mede 1,73998 m. O chute errou por um fio de cabelo — mas foi o único chute, e ninguém sabe como ele chegou tão perto. O paper não esconde o fio de cabelo: ele é o problema aberto número um, com a discrepância de mil sigmas impressa em letras grandes. Levar a sério não é acreditar; é achar que vale o teste.

"Por que eu nunca ouvi falar disso?" Porque é novo e ainda não passou pelo crivo da comunidade. O paper é um pré-print: está público para qualquer pessoa ler, mas ainda não foi avaliado por uma revista científica. Isso não é selo de verdade nem de mentira — é apenas o estágio em que a ideia está. O autor é um pesquisador independente, sem instituto nem departamento, o que significa duas coisas: nenhuma instituição endossa o trabalho (cuidado redobrado) e nenhum departamento precisa defendê-lo (liberdade total para marcar o próprio enterro, se for o caso).

"E se os experimentos medirem outra coisa?" Então a ideia morre, e o próprio paper mandou flores para o funeral com antecedência. É o que os físicos chamam de falsificabilidade: uma ideia só é científica se puder morrer. Este paper pode morrer de várias maneiras datadas

— a mais dramática é a medida do δ_{CP}, um ângulo dos neutrinos que os experimentos DUNE (Estados Unidos) e Hyper-Kamiokande (Japão) vão apertar até o fim da década. Se o número medido fugir do previsto 1,211π por margem de segurança, a teoria inteira vai para o cemitério da página anterior, sem recurso.

"Isso prova que o universo é uma simulação? Que existe um programador? Que Deus gosta de pi?" Não, não e não. O paper fala de matemática e de medidas de laboratório — e só. Descobrir que um número da natureza sai de uma estrutura de regras não diz *quem* escreveu as regras, nem se alguém as escreveu. São perguntas legítimas, mas pertencem à filosofia e à teologia, não a este artigo. Aqui dentro, "anel" é um objeto matemático, não uma metáfora cósmica.

"E se estiver certo? O que muda?" Muda o significado da palavra "constante". Hoje, os vinte e poucos números fundamentais da natureza são medidos, nunca calculados — são fatos brutos que a teoria engole sem mastigar. Se apenas *um* deles passar a ser calculado, a porta estará aberta: por que não os outros? Seria a primeira vez que um pedaço do "assim é porque sim" da física vira um "assim é porque dá para demonstrar". E viria uma pergunta nova no embrulho: por que pi ao quadrado sobre duas vezes 137? Responder Dirac pode ser só o começo.

Capítulo 18 — Um dicionário de bolso

As palavras técnicas que apareceram pelo caminho, explicadas de uma vez, em ordem de aparição na história. Guarde esta página: ela traduz o paper inteiro.

Constante de estrutura fina (α)
: O número que diz o quanto a luz "conversa" com a matéria. Vale aproximadamente 1/137,036. Sem ele, não há química, não há luz de lâmpada, não há você lendo esta frase. Dirac queria saber por que ele vale isso.

Anel (ou colar)
: A estrutura proposta pelo paper: 137 posições fechadas em círculo, como contas num cordão. "Anel" porque a última conta é vizinha da primeira — não há pontas soltas.

Conta
: Cada uma das 137 posições do anel. No paper sério, cada conta carrega dois números pequenos (as "cargas") e um tipo.

Regra de vizinhança (paridade)
: Cada uma das 13 regras manda que, no conjunto de contas que ela vigia, a soma de certas cargas seja *par*. Como um inspetor que passa de casa em casa e só aceita panelas em número par.

Carga
: Um númerinho grudado em cada conta, como o preço numa etiqueta. As 13 regras vigiam as somas dessas etiquetas.

Síngleto
: Uma conta "sozinha": sem cargas, invisível para as regras de vizinhança, mas ainda assim ocupando lugar no anel. A descoberta central desta versão do paper foi que todo anel que funciona é feito, essencialmente, de singletos espaçados de cinco em cinco contas.

Teorema, hipótese, conjectura, chute
: A escala de certeza usada no paper. *Teorema* [T]: provado, não depende de voto. *Hipótese* [D]: aposta explícita da qual o resto depende — se cair, leva o castelo junto. *Conjectura*

[C]: palpite forte, com motivos, sem prova. *Chute* [X]: especulação incluída só para não ser esquecida — e marcada como tal.

Sigma (σ)
A "régua do espanto" dos cientistas. Mede o quanto um número desvia do esperado, em unidades da incerteza da medida. 1σ é rotina; 3σ acende a luz amarela; 5σ é manchete de descoberta. Mil sigmas — a ferida deste paper — é uma diferença que ninguém pode varrer para baixo do tapete.

Falsificável
Que pode ser provado falso por um experimento. É o certificado de cientificidade de uma ideia. Este paper é falsificável de pelo menos cinco maneiras diferentes, cada uma com data marcada.

Previsão versus retrodicção
Prever é dizer o que o experimento vai medir *antes* dele medir; retroceder ("retrodizer") é explicar o que já foi medido. Retrodição é mais fácil e vale menos — qualquer teoria com botões suficientes explica o passado. Este paper tem as duas, e separa uma da outra com etiquetas.

Autocorreção de erros
O truque dos CDs, dos códigos QR e do seu celular: informação guardada com redundância, de modo que erros possam ser detectados e consertados. A surpresa do paper é que o anel de 137 contas, com suas 13 regras, é *exatamente* um desses códigos — o universo estaria usando a mesma engenharia da capa de um CD.

GF(2)
A "aritmética do sim e do não": só existem os números 0 e 1, e $1+1=0$. É a matemática dos interruptores de luz — e a matemática em que as 13 regras do anel foram provadas.

No-go (proibição)
Um resultado que diz "por aqui não dá". São tão valiosos quanto os acertos: fecham portas erradas para que ninguém perca décadas nelas. O paper prova alguns — por exemplo, que a ordem do anel não pode surgir de graça, do acaso.

arXiv
A biblioteca pública onde físicos do mundo inteiro penduram artigos antes (ou sem) a avaliação de revistas. É onde este paper vive — e onde qualquer pessoa pode lê-lo, inclusive você.

Capítulo 19 — O paper inteiro em uma página

Se este anexo inteiro fosse resumido num cartaz de porta de geladeira, seria este. Cada caixa é um capítulo; cada seta é um "e daí". Ande com o dedo: da pergunta de 1935 até a sala do júri, sem sair da figura.

Repare no desenho como um todo: ele tem o formato de uma promessa com endereço. A pergunta nasce em 1935; a resposta candidata nasce neste paper; o veredito nasce nos laboratórios, nos próximos anos, com data de JUNO, DUNE, Hyper-Kamiokande, LiteBIRD e CMB-S4 carimbada na agenda. Nenhuma caixa pede que você acredite. Todas pedem que você confira — e a última página deste anexo ensina como, com uma calculadora de celular e um pouco de curiosidade.

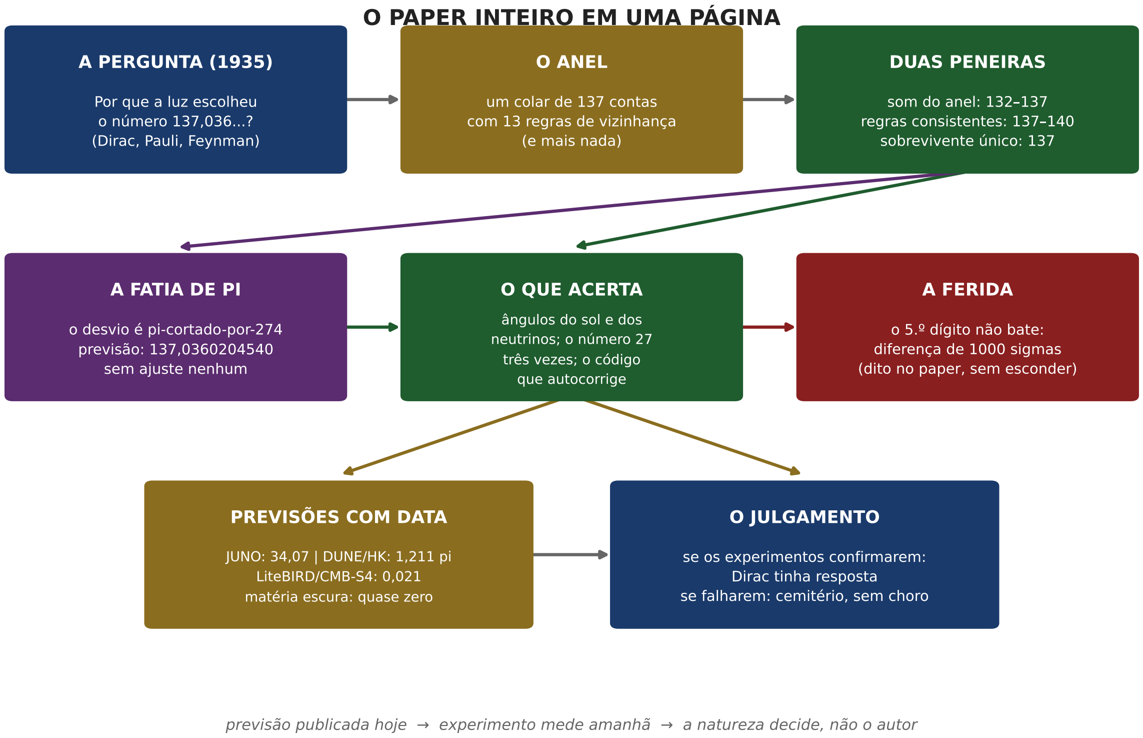

Figure 32: O argumento completo em oito caixas. Se uma caixa cair, as setas que saem dela caem juntas — e o paper diz, de cada uma, o que a derrubaria.

Capítulo 20 — O que mudou na terceira versão: as campanhas de fechamento

Esta terceira versão não acrescenta ideias novas. Ela faz algo mais raro em ciência: volta ás próprias feridas e pergunta, com regras fixadas *antes* do exame, se elas têm cura. Foram três campanhas. Duas terminaram em "não" provado — e os "não"s, aqui, valem tanto quanto os "sim"s.

Primeira campanha: a ferida de mil sigma ganhou um endereço. Lembra da diferença entre a previsão crua e a medida de α^{-1}, aquela que dâ mil desvios-padrão? Ela deixou de ser um desconforto vago e virou um alvo exato: a correção necessária é 0,4553 vezes o cubo de η — e η^3 é a única potência de η cujo coeficiente é um número modesto, da ordem de 1. Marcamos em ata, no dia 3 de agosto de 2026, uma janela de aceitação: só aceitaríamos como "cura" um mecanismo que *derivasse*, sem nenhum parâmetro ajustável, um valor dentro daquela janela. Depois testamos tudo o que o próprio framework oferecia. Primeiro, o setor espectral: as correções da série de Casimir são todas positivas — cada termo a mais *afasta* a previsão do valor medido, como uma escada que só sobe quando precisámos descer. Provado: por aí não há cura. Depois, o setor dinâmico: com o dinamômetro exato do vácuo (o apêndice que calcula tudo sem sorteios), declaramos seis mecanismos candidatos e computamos os seis. Resultado: um passa 450 vezes do alvo, três param a várias vezes de distância, um tem o sinal errado, e um nem chega a ser mecanismo. Zero dentro da janela. Duas coincidências encontradas depois, fora do protocolo, foram para a quarentena — nõssas próprias regras proíbem usá-las como evidência.

Segunda campanha: o teorema dos dois mundos. Existia a esperança de uma terceira derivação do η, por um caminho chamado "setor do determinante". A esperança morreu de morte matemática — a melhor espécie de morte, porque é definitiva. Provamos que as quantidades desse setor são sempre números racionais (o determinante do ciclo vale exatamente $137^2 = 18.769$; outras duas quantidades valem 1564 e 274), enquanto o η vive no mundo do π^2. São dois mundos que não se tocam: nenhuma ponte leva de um ao outro. A conclusão boa é que o setor do η está *completo*: as duas derivações que temos são todas as que existem, e ninguém precisa procurar mais. A conclusão dura é que a ferida de mil sigma perdeu mais uma sala da casa onde a cura poderia estar escondida.

Terceira campanha: auditamos a nós mesmos. A pergunta mais incômoda que um cético pode fazer é: "quantas fórmulas com esse η acertariam esses alvos por puro acaso?" Respondemos com a conta inteira, sem amostragem: declaramos um espaço de quase 25 milhões de fórmulas e contamos. O resultado corta para os dois lados — e publicamos os dois. Corte contra nós: na precisão em que nossas fórmulas dos ângulos de mistura acertam, *milhares* de fórmulas sorteadas ao acaso também acertam. Por isso, rebaixamos oficialmente cada retrodição individual a "checagem de consistência" — bonita, derivada, mas estatisticamente barata. Corte a nosso favor: o *conjunto* dos acertos é raríssimo (a chance do padrão ao acaso é menor que uma em cem milhões), e a fórmula central, $\alpha^{-1} = 137 + \eta$, é a única dos 25 milhões que bate o valor experimental na precisão fina. Campeã única — não uma entre milhares.

O que restou de pé. Depois das três campanhas, o framework tem menos esconderijos e os ombros mais estreitos: o peso agora está na estrutura derivada (as duas derivações do η, as duas peneiras do 137), no perfil conjunto dos acertos, na campeã única — e, principalmente, nas previsões para o futuro, que continuam com seus encontros marcados com JUNO, DUNE, Hyper-Kamiokande, LiteBIRD e CMB-S4. A fase CP, aliás, já está numa tensão declarada de cerca de 2σ com o melhor ajuste global atual: viva, exposta, e com a zona de abate inalterada. O relógio da verdade do Capítulo 12 continua correndo — agora com menos cômodos na casa.

Capítulo 21 — O que mudou na quarta versão: a auditoria de agosto de 2026

A quarta versão não mexe em nenhuma linha do que já estava publicado. Ela faz duas coisas, as duas na frente da casa, com data no alto: corrige o que a própria auditoria encontrou de errado, e conta o que doze varreduras internas — todas com o espaço de busca declarado *antes* de computar, como manda a regra da casa — descobriram em dois dias de agosto de 2026.

Duas correções, com carimbo. A primeira é pequena e honesta: dois números decimais estavam trocados numa frase sobre as fronteiras da peneira do 137 (os valores certos são 106,99517... e 111,85859...). A matemática por baixo estava certa — a frase, não; nenhuma conclusão muda. A segunda é um reforço que chegou de graça: a frase curta que usamos até aqui — "a chance do vácuo ordenado nascer por acaso é menor que 2^{-124}" — na verdade é generosa demais. O número exato é 2^{-137}: a ordem precisa acertar não só os 124 bits do código, mas também os 13 alarmes, todos ao mesmo tempo. O argumento contra o acaso fica 8 192 vezes mais forte. E a garganta E12 — de onde vem o empurrão que mantém a ordem? — ganhou tamanho exato: faltam cerca de 86 "nats" de viés, e todo o setor de alarmes, mesmo no máximo, só consegue fornecer 8,3. Sabemos agora, com régua, o tamanho do que não sabemos.

O 137 virou um número que se mede. Cada previsão do framework pode ser lida de trás para frente: se o observável depende de N, então cada experimento é uma régua que mede N sozinha. Duas réguas independentes, com precisão melhor que um por cento, apontam hoje para 137: o ângulo de Cabibbo mede $137{,}8 \pm 0{,}4$ (a nossa tensão conhecida, declarada, de $1{,}9\sigma$), e a razão das massas dos neutrinos mede $136{,}4 \pm 3{,}9$. Juntas: consistentes. E tem notícia quente, datada, sem mexer nas edições congeladas: em junho de 2026 o experimento JUNO publicou na capa da *Nature* a medida mais precisa do parâmetro solar dos neutrinos — e ela cai em cima da nossa previsão no quarto dígito (0,3036 medido contra 0,30365 previsto). Pela regra que nós mesmos nos impusemos, isso verifica o setor geométrico e *não* conta como prova sobre o 137 (o 137 nem aparece nessa fórmula). Honestidade vale para os dois lados: o mesmo relatório mostra, sem esconder, o canal que mede 73,6 — tão impreciso que não decide nada, e por isso mesmo foi publicado. Daqui a alguns anos, com os dados que já estão sendo coletados, as réguas vão resolver inteiros individuais: a pergunta será, literalmente, "é exatamente 137?".

E o η? Constante — onde quer que se consiga olhar. O mesmo jogo de réguas mede o pedacinho fracionário, o η, em escalas separadas por *vinte e cinco ordens de grandeza* — dos neutrinos à inflação cósmica. Em todo canal não-circular, ele é o mesmo. E um teste interno novo liga os dois mundos: a teoria exige que o η medido pelos neutrinos e o N medido pelo ângulo de Cabibbo obedeçam a $\eta = \pi^2/2N$; os dados confirmam, sem ajuste nenhum, a 0,35σ. Poderia o η variar — com a energia, com o tempo, com o cosmos — e assim absorver a briga de mil sigma com o experimento? Quatro caminhos foram testados. O running com a energia morre por geometria: as medidas de α moram na mesma escala da definição. Um mecanismo discreto do anel morre por contagem: o menor passo possível é quase cinquenta mil vezes grande demais. Uma variação cósmica morre nos reatores naturais de Oklo e nos relógios atómicos: precisaria de um deslize duas vezes e meia maior que o teto permitido. Sobrou a quarta porta — que é exatamente a PA2, a ferida declarada, agora com o mapa completo dos becos.

O singleton se congela sozinho. Uma suspeita antiga: a conta 136 — a única apagada do anel — teria sido desligada "na mão"? Não foi. Solte-a como sítio comum e pergunte à dinâmica o que ela faz: a conta fica apagada 99,96% do tempo *por vontade própria*. O decreto (C4) não é uma imposição contra a dinâmica; é uma fotografia fiel dela, com erro de 0,04%. É a primeira vez que um constraint do framework ganha justificativa dinâmica em vez de ser assumido.

O empate exato. Por que o atrator escolhe justamente a classe de 28 sítios que ancora o elétron? A resposta nova é um teorema: a força do atrator só depende da *paridade* da classe. No anel inteiro há exatamente duas classes pares — a de 28 e a de 18 — e elas *empatam*, com igualdade exata até a décima sétima casa decimal. As dez classes ímpares perdem por fatores de centenas a milhões. A pergunta "por que o e_R?" tinha doze candidatas; agora tem duas — e a escolha entre elas vem da estrutura (a regra dos nós), não da energia. A garganta E12 encolheu: virou uma degenerescência de tamanho dois, medida e carimbada.

O mapa completo dos becos da PA2. Seis rotas foram percorridas até o fim, cada uma com seu espaço declarado antes: o dinamômetro, o degelo do singleton, os invariantes do espectro, o fundo racional, a mistura de dois atratores e a varredura ampla de mais de meio milhão de fórmulas. Todas nulas. O melhor acerto casual — a fração 173/380, que bate o coeficiente da falha com precisão de duas partes por milhão — tem chance de uma em cinco de acontecer por puro acaso naquele espaço: proximidade numérica não é evidência, e esse é talvez o exemplo mais limpo que o projeto já produziu. O veredito é uma frase só: *o coeficiente da falha é o número da falha, não uma pista.* A porta só reabre com derivação nova ou estatística de uma em cem — nunca com coincidência bonita. E duas cláusulas foram seladas para sempre: o atalho $\ln 4/\ln(13{+}8)$ está *estruturalmente proibido* (o 8 já mora dentro do 13: as treze linhas do anel são oito resíduos fortes, três pares fracos, um U(1) e um $\mathbb{Z}_2$ — somar $13+8$ é contar duas vezes o mesmo tijolo); e nenhum mecanismo dinâmico pode hospedar a correção, porque os pisos do próprio modelo estão cem milhões de vezes acima dos tetos que os relógios atómicos medem. O que restar, um dia, terá de ser exato e combinatório.

Fim do anexo. A pergunta de Dirac continua de pé — agora com uma resposta candidata na mesa, com selo de honestidade, data de julgamento, três portas provadamente trancadas e os portões do cemitério abertos para visitação.