From Sagnac Holonomy to a Phase-Drift Model for Cosmic Expansion

Part I: A Standard Relativistic Derivation of a Fixed Clock Deficit

Part II: A Phase-Locking Hypothesis and its Testable Cosmological Implications

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Abstract

This work is presented in two distinct but connected parts. **Part I** provides a self-contained standard-relativistic derivation of the finite slow-transport limit for a clock completing one directed equatorial loop. In an Earth-centred inertial description, the leading-order proper-time difference separates into a direction-sensitive rotation–transport cross term and the traveller's ordinary quadratic speed term. As the transport speed tends to zero while one complete loop is retained, the latter vanishes whereas the former approaches

$$\Delta\tau_{\mathrm{slow,east}} = -\gamma_{\oplus}\frac{2\pi\Omega R^2}{c^2} \simeq -207.386\,\mathrm{ns}.$$

This is the standard Sagnac/slow-clock-transport holonomy and introduces no new physics. **Part II** then states an additional, explicitly non-standard phase-locking hypothesis: each completed rotational phase cycle fractionally rescales a local metrological baseline, so successive changes act multiplicatively on the already modified baseline. The paper develops the resulting compound law, distinguishes it from the additive Sagnac offset, and combines it with the Doppler readout of an observer on an outward spiral trajectory. A conditional benchmark is retained: if the present-day fractional phase-cycle rate is identified with the 207 ns loop scale per adopted daily calibration interval, and if the external radiation sector does not co-scale with the local material ruler, the model assigns approximately 3.92 nm of apparent change to a 7.35 cm reference wavelength over the 1965–2026 interval, compared with approximately 0.31 nm from standard late-time expansion, a ratio of about 93%. This number is a conditional model estimate, not a reported historical measurement of a secular CMB spectral shift. The paper ends with explicit consistency conditions, objections, and falsification tests.

Keywords: time dilation, Sagnac effect, slow clock transport, rotating frame, time holonomy, Earth rotation, atomic clock, proper time, CMB, dark energy, phase-locking.

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Part I: The Geometric Clock Deficit in a Rotating Frame

1 Introduction

Textbooks on special relativity teach that a moving clock runs slow by a factor depending on its speed relative to an inertial frame. A natural expectation is therefore: if a clock is transported more and more slowly around a closed path, the accumulated difference relative to a co-rotating reference clock might be expected to shrink to zero.

However, when the closed path lies on a rotating body such as the Earth, a subtlety emerges. The Earth's surface already possesses a large rotational speed

$$V = \Omega R \simeq 465.1 \text{ m/s} \tag{1}$$

in an Earth-centred approximately inertial frame. A clock standing on the equator participates in this motion. If a second clock is transported slowly eastward along the equator, its Earth-centred coordinate speed is approximately $V + u$, where $u = R\,\delta\Omega$ denotes the small circumferential coordinate-speed increment. At terrestrial speeds it agrees with the locally measured ground-relative speed far beyond the precision required here. The comparison is therefore not between a moving clock and one at rest, but between two clocks that both share the large background speed V and differ only by a tiny increment u.

The central result of Part I is that, in the limit $u \to 0$ while exactly one full revolution is completed, the accumulated time difference does **not** vanish. Instead, it approaches a fixed, direction-dependent value of approximately -207 ns. The surviving value is a consequence of the combined rotation–loop geometry. Locally it originates from the cross term between the background speed and the directed transport; after one prescribed loop is completed, its leading-order magnitude is independent of how slowly that loop is traversed.

Although the underlying physics is that of the well-known Sagnac time delay and the associated synchronisation gap of a rotating frame, the present derivation isolates the direction-sensitive cross term from the traveller's quadratic speed term in a particularly transparent way. This viewpoint highlights that closed transport on a rotating background generates a fixed leading-order per-loop term independent of the magnitude of the slow transport speed — a result that has direct bearing on operational timekeeping in rotating systems and on the foundational understanding of time in non-inertial frames. The effect is a purely geometric, non-inertial phenomenon in flat spacetime; it requires no gravitational curvature.

We demonstrate the result by elementary algebra applied to the proper-time integral, without invoking any new physics.

2 Idealised Model

To isolate the essential physics, we adopt the following idealisations:

(1) The Earth rotates uniformly with angular speed Ω.

(2) The path lies exactly on the equator and has constant radius R.

(3) Both clocks remain at the same gravitational potential (identical altitude), so gravitational redshift cancels.

(4) Irregularities from topography, tides, and variable rotation are ignored.

(5) The traveller moves eastward, completing exactly one full revolution relative to the Earth's surface.

(6) The clocks are compared at the same location before and after the journey.

We use the WGS 84 reference ellipsoid [1]:

$$R = 6\,378\,137 \text{ m}, \tag{2}$$

$$\Omega = 7.292\,115 \times 10^{-5} \text{ rad/s}. \tag{3}$$

The corresponding equatorial circumference is

$$L = 2\pi R \simeq 40\,075\,016.7 \text{ m}. \tag{4}$$

3 Direct Comparison of Two Clocks

In the Earth-centred approximately inertial frame, the co-rotating ground clock on the equator moves at speed $V = \Omega R$. Its proper-time rate is

$$\frac{\mathrm{d}\tau_{\text{ground}}}{\mathrm{d}t} = \sqrt{1 - \frac{V^2}{c^2}}. \tag{5}$$

If the traveller walks eastward with small ground-relative speed u, the total speed in the same inertial frame is approximately $V + u$. The traveller's proper-time rate is therefore

$$\frac{\mathrm{d}\tau_{\text{trav}}}{\mathrm{d}t} = \sqrt{1 - \frac{(V+u)^2}{c^2}}. \tag{6}$$

To complete one full revolution relative to the rotating Earth, the coordinate time (as measured in the Earth-centred frame) is simply

$$T = \frac{L}{u}, \tag{7}$$

because the traveller must cover the circumference L at relative speed u. (We assume $u > 0$; the algebra for westward motion is analogous.)

The accumulated proper-time difference after one loop is

$$\Delta\tau(u) = T\left[\sqrt{1 - \frac{(V+u)^2}{c^2}} - \sqrt{1 - \frac{V^2}{c^2}}\right]. \tag{8}$$

The factor $T = L/u$ grows without bound as u decreases. Hence, even though the instantaneous rate difference may become tiny, the integrated effect can remain finite.

4 Separation into Two Components

For terrestrial speeds $V, u \ll c$, we expand the square roots to order v^2/c^2:

$$\sqrt{1 - \frac{v^2}{c^2}} \simeq 1 - \frac{v^2}{2c^2}.$$

Substituting into (8) gives

$$\Delta\tau \simeq -\frac{(V+u)^2 - V^2}{2c^2} T = -\frac{2Vu + u^2}{2c^2} \frac{L}{u}. \tag{9}$$

Simplifying,

$$\boxed{\Delta\tau_{\text{east}} \simeq -\frac{VL}{c^2} - \frac{uL}{2c^2}.} \tag{10}$$

At leading order in $1/c^2$, and in the chosen Earth-centred inertial description, Equation (10) separates algebraically into two terms:

4.1 Direction-sensitive loop component

$$\boxed{\Delta\tau_{\text{loop}} = -\frac{VL}{c^2}.} \tag{11}$$

Once the directed loop is specified, this term depends on the background rotational speed V and the loop geometry, but not on the magnitude of the slow transport speed u. Substituting $V = \Omega R$ and $L = 2\pi R$ yields

$$\boxed{\Delta\tau_{\text{loop}} = -\frac{2\pi\Omega R^2}{c^2}.} \tag{12}$$

It is independent of the magnitude of u after the loop constraint is imposed. The transport is nevertheless essential: without a completed directed path there is no loop integral. The surviving term comes from the cross term $2Vu$, whose factor of u is cancelled by the loop duration L/u.

4.2 Speed-dependent time dilation of the traveller

$$\boxed{\Delta\tau_{\text{own}} = -\frac{uL}{2c^2}.} \tag{13}$$

This is the ordinary quadratic contribution associated with the traveller's additional speed. As $u \to 0$, this term vanishes. The entire non-zero slow-transport limit comes from the direction-sensitive cross term.

5 Exact Slow-Transport Limit

For mathematical rigour, let the traveller's angular speed in the Earth-centred frame be $\Omega + \delta\Omega$. Here $\delta\Omega$ is a coordinate angular-velocity increment. If u is instead defined as a locally measured ground-relative velocity, exact relativistic velocity addition is required, although the numerical distinction is negligible for terrestrial speeds. Completing one revolution relative to the Earth requires

$$\delta\Omega\, T = 2\pi \quad \Longrightarrow \quad T = \frac{2\pi}{\delta\Omega}.$$

The time difference becomes

$$\Delta\tau(\delta\Omega) = \frac{2\pi}{\delta\Omega}\left[\sqrt{1 - \frac{R^2(\Omega+\delta\Omega)^2}{c^2}} - \sqrt{1 - \frac{R^2\Omega^2}{c^2}}\right]. \tag{14}$$

Define $f(\Omega) = \sqrt{1 - R^2\Omega^2/c^2}$. Then

$$\Delta\tau(\delta\Omega) = 2\pi\,\frac{f(\Omega+\delta\Omega) - f(\Omega)}{\delta\Omega},$$

and taking the limit $\delta\Omega \to 0^+$ yields the derivative:

$$\lim_{\delta\Omega\to 0^+} \Delta\tau = 2\pi\, f'(\Omega). \tag{15}$$

With

$$f'(\Omega) = -\frac{\Omega R^2}{c^2\sqrt{1 - \Omega^2R^2/c^2}}, \tag{16}$$

we obtain

$$\boxed{\lim_{\delta\Omega\to 0^+} \Delta\tau = -\,\gamma_\oplus\,\frac{2\pi\Omega R^2}{c^2},} \tag{17}$$

where $\gamma_\oplus = 1/\sqrt{1 - \Omega^2R^2/c^2}$. For Earth, $\gamma_\oplus - 1 \sim 1.2\times10^{-12}$, so the correction is entirely negligible; the simple expression $-2\pi\Omega R^2/c^2$ is excellent.

6 Why the Numerical Limit is Approximately 207 Nanoseconds

Inserting the WGS 84 values:

$$\frac{2\pi\Omega R^2}{c^2} \simeq 2.07386\times10^{-7}\ \text{s}, \tag{18}$$

hence

$$\boxed{\Delta\tau_{\text{slow,east}} \simeq -207.386\ \text{ns}.} \tag{19}$$

This number emerges from the Earth's rotation together with the directed equatorial loop geometry. It can be written as

$$\Delta\tau \simeq -\frac{\Omega R^2}{c^2}\,\Delta\phi,$$

with $\Delta\phi = 2\pi$ for one full loop. The result is **not a new constant of nature**; it is the specific value for our planet.

7 Local and Integral Formulation

The Earth's rotational velocity field is $\mathbf{V}(\mathbf{r}) = \boldsymbol{\Omega} \times \mathbf{r}$. Along a small segment $\mathrm{d}\mathbf{r}$ of the path, the loop time deficit accumulates as

$$\mathrm{d}(\Delta\tau_{\text{loop}}) \simeq -\frac{\mathbf{V}\cdot\mathrm{d}\mathbf{r}}{c^2}. \tag{20}$$

Integrating around a closed loop,

$$\boxed{\Delta\tau_{\text{loop}} = -\frac{1}{c^2}\oint(\boldsymbol{\Omega} \times \mathbf{r})\cdot\mathrm{d}\mathbf{r}.} \tag{21}$$

By Stokes' theorem,

$$\oint(\boldsymbol{\Omega} \times \mathbf{r})\cdot\mathrm{d}\mathbf{r} = 2\boldsymbol{\Omega}\cdot\mathbf{A},$$

where $\mathbf{A}$ is the directed area enclosed by the loop. Therefore,

$$\boxed{\Delta\tau_{\text{loop}} = -\frac{2\boldsymbol{\Omega}\cdot\mathbf{A}}{c^2}.} \tag{22}$$

For the equatorial circle, $\mathbf{A} = \pi R^2\,\hat{\mathbf{z}}$ and $\boldsymbol{\Omega} = \Omega\,\hat{\mathbf{z}}$, recovering $-2\pi\Omega R^2/c^2$. After the closed path is fixed and the slow-transport limit is taken, the loop component depends on the rotation vector and the directed enclosed area; the magnitude of the transport speed no longer appears.

8 Westward Transport

If the traveller moves westward, the total Earth-centred speed is approximately $V - u$, giving

$$\Delta\tau_{\text{west}} \simeq +\frac{VL}{c^2} - \frac{uL}{2c^2}. \tag{23}$$

Thus,

$$\boxed{\lim_{u\to 0^+}\Delta\tau_{\text{west}} = +207.386\ \text{ns}.} \tag{24}$$

The direction dependence is clear: eastward transport causes a loss of time, westward transport causes a gain of equal magnitude. The speed-dependent term is always negative (for small u) but vanishes in the limit.

9 Connection with the Sagnac Effect and Time Holonomy

In a classic Sagnac interferometer, two light beams travel in opposite directions around a rotating loop [3]. The resulting time-of-arrival difference is

$$\Delta t_{\text{Sagnac}} = \frac{4\boldsymbol{\Omega}\cdot\mathbf{A}}{c^2}, \tag{25}$$

which for the equatorial circle gives ~ 414.8 ns — twice the slow-clock-transport loop term at the same leading order. This factor of two reflects the different nature of the two measurements: in the Sagnac interferometer two oppositely directed light paths are compared, yielding a round-trip time difference; in the slow-clock transport a single physical clock accumulates proper time along one direction relative to a reference clock that shares the background rotation. Hence the Sagnac interval is twice the one-way loop deficit.

The same geometric term appears as the synchronisation gap when one attempts to Einstein-synchronise an array of ground clocks along the equator. The accumulated gap after one full revolution is

$$\Delta t_{\mathrm{gap}} \simeq \frac{2\boldsymbol{\Omega}\cdot\mathbf{A}}{c^2} \simeq 207.4 \text{ ns}, \tag{26}$$

which is the time holonomy of the rotating frame. It shows that local Einstein synchronisation cannot be continued around a closed rotating loop as a single-valued, path-independent convention. A global coordinate time can nevertheless be defined by adopting an underlying inertial-frame convention and applying the corresponding Sagnac corrections, as in GNSS timekeeping.

10 Three Numbers That Must Not Be Confused

Three distinct time scales are often conflated:

1. ~ 104 ns per sidereal day — the kinematic time dilation of a clock **standing** on the equator, due to its rotational speed V, relative to a non-rotating clock at the same potential.[1]
$$\Delta\tau_{\mathrm{rot}} \simeq -\frac{V^2}{2c^2}T_{\mathrm{sidereal}} \simeq -103.7 \text{ ns}.$$
2. ~ 207 ns per directed loop — the slow-transport limit for a clock that completes one full revolution relative to the Earth's surface. In the slow-transport limit this is the surviving Sagnac-type cross term; the traveller's separate quadratic contribution has vanished.
3. ~ 415 ns for two opposite paths — the Sagnac two-beam interval.

These values are distinct both numerically and conceptually. The 207 ns/loop is **not** a daily rate for a stationary clock; it is a per-loop geometric deficit arising from the Earth's own rotation.

11 Experimental Status

The Hafele–Keating experiment [4, 5] flew cesium clocks eastward and westward around the Earth. Eastward clocks lost time, westward clocks gained time, qualitatively matching the sign of the loop component described here. However, that experiment involved high altitudes, varying speeds, and non-equatorial paths, so it was not a pure measurement of the slow-transport limit.

The Sagnac correction is routinely applied in one-way GPS time transfer and can reach approximately 200 ns depending on station positions [6, 7]. This confirms that the directional loop effect is an established operational reality.

[1] This value is for one sidereal day (one Earth rotation relative to the fixed stars); it is the ordinary special-relativistic effect on a stationary ground clock.

12 Limitations and Domain of Validity

The value 207.386 ns was derived for an idealised model: perfect equatorial circle, constant radius, uniform rotation, and equal gravitational potential. In a real experiment, altitude, latitude, variable speed, and gravitational redshift must be accounted for. These corrections do not alter the existence or the geometric character of the loop component.

13 Conceptual Significance

The core message of Part I is:

> A prescribed directed loop on a rotating background carries a geometric time holonomy. In the slow-transport limit its leading term is fixed by the rotation vector and directed area, while the traveller's separate quadratic speed contribution becomes negligible. The loop effect persists and is determined solely by the rotation vector and the enclosed area:
>
> $$\Delta\tau_{\text{loop}} = -\frac{2\mathbf{\Omega}\cdot\mathbf{A}}{c^2}.$$

Thus, a global comparison of clock readings in a rotating system cannot be inferred from the traveller's local speed magnitude alone; a geometric, path-dependent and area-sensitive contribution must also be included. The effect is not a strictly topological invariant because it changes continuously with the directed area. This insight, while implicit in standard Sagnac physics, is laid bare here in a form that directly connects geometry, clock transport, and the operational limits of synchronisation.

Part II: A Phase-Locking Hypothesis and its Cosmological Implications

14 Transition to a New Hypothesis

The preceding part derived a geometric loop deficit of ~ 207 ns within standard special relativity for a *travelling* clock. In what follows, we explore a hypothesis that goes beyond this standard framework. Motivated by a covariant phase-locking model that successfully resolves the Ehrenfest paradox for rotating rings, we **postulate** that the same geometric phase deficit is not merely a traveller effect, but manifests as a secular drift in the local standards of time and length for all co-rotating observers, including stationary atomic clocks.

This is a *new physical hypothesis*, not a consequence of standard relativity. Its justification, detailed in a separate work on the Ehrenfest paradox, lies in a phase-field action that enforces a fixed proper circumference for any rotating material system. Here, we explore its testable consequences for terrestrial metrology and cosmology.

15 Context: Open Problems in Standard Cosmology

The hypothesis advanced in Part II is considered against the background of well-documented tensions and open questions in contemporary cosmology. These issues are not evidence for the phase-locking model; they only motivate the examination of quantitatively testable alternatives.

1. **The Hubble tension.** A representative recent local distance-ladder result is $H_0 = 73.04 \pm 1.04\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, whereas the Planck 2018 inference within flat ΛCDM is approximately $67.4\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [12, 13]. The interpretation remains under active debate, including possible systematics and possible physics beyond the baseline model.

2. **Hints of evolving dark energy.** DESI's three-year DR2 BAO analysis, released in 2025, strengthened data-combination-dependent hints that the dark-energy equation of state may evolve [15]. Extended parametric and non-parametric analyses report approximately 3σ tension with ΛCDM, with higher values for some data combinations, while emphasizing that BAO alone gives only modest constraints and that possible systematics remain under study [16]. These results are interesting but are not yet a model-independent discovery of dynamical dark energy.

3. **The cosmological-constant and coincidence problems.** The small observed vacuum-energy scale and the near coincidence of the present matter and dark-energy densities remain major theoretical puzzles. Their statement depends on assumptions about effective field theory and renormalisation, so the often quoted order-of-magnitude mismatch should not be treated as a single experimentally measured number.

4. **Inhomogeneous alternatives.** Timescape and other inhomogeneous cosmologies investigate whether clock-rate gradients and spatial averaging can modify the

inference of cosmic acceleration from an idealised FLRW description [17]. Such models must still be confronted with the full CMB, BAO, supernova, lensing, and structure-growth data.

5. **The supernova-only acceleration debate.** Nielsen et al. found marginal evidence in one statistical treatment of the JLA sample [18]. Rubin and Hayden subsequently argued that the reduced significance arose from restrictive population assumptions and found stronger evidence when those assumptions were relaxed and other constraints were included [19]. The balanced conclusion is that the acceleration inference is supported by multiple probes, while its physical cause remains unresolved.

These examples neither validate nor invalidate the phase-locking hypothesis. The proposal must be judged by a complete observable map, dimensional consistency, and successful tests against independent data.

16 The Phase-Locking Postulate: A Fractional Phase-Cycle Drift for Stationary Clocks

Part I derived a $\sim 207\,\mathrm{ns}$ slow-transport offset for a clock that completes one directed equatorial loop relative to the ground. Standard relativity does *not* assign this additional per-loop offset to a stationary co-rotating clock. Extending the result to stationary matter therefore requires an independent physical postulate.

The proposed postulate is motivated by the covariant phase-locking model of a rotating material ring [10]. In its strong-locking sector, a preferred proper phase circumference produces the accommodation map

$$r(\rho) = \frac{\rho}{\sqrt{1+\omega^2\rho^2/c^2}}, \tag{27}$$

and the corresponding Lorentz factor is

$$\gamma(\rho) = \sqrt{1+\frac{\omega^2\rho^2}{c^2}}. \tag{28}$$

When evaluated at the accommodated radius, this factor coincides with the ordinary kinematic Lorentz factor. It therefore yields the usual $\sim 103.7\,\mathrm{ns}$ equatorial kinematic deficit per sidereal day at leading order; it does not by itself generate an additional factor of two.

The new hypothesis is instead stated directly at the level of a completed phase cycle. Let

$$\Delta\tau_{\phi,0} \equiv |\Delta\tau_{\mathrm{loop}}| = \frac{2\pi\Omega R^2}{c^2} \simeq 207.386\,\mathrm{ns} \tag{29}$$

be the present equatorial loop scale, and let

$$T_\oplus = \frac{2\pi}{\Omega} \tag{30}$$

be the sidereal rotation period. The corresponding present fractional phase-cycle parameter is

$$\boxed{\epsilon_{\phi,0} = \frac{\Delta\tau_{\phi,0}}{T_\oplus} = \frac{\Omega^2 R^2}{c^2}.} \tag{31}$$

For comparison, the standard equatorial kinematic rate is $\epsilon_{\text{kin}} \simeq \Omega^2 R^2/(2c^2)$.

The central phase-locking postulate is:

> **Each completed rotation of a phase-locked material system fractionally rescales its local phase and metrological baseline by the cycle parameter ϵ_ϕ. The next cycle acts on the baseline already modified by the preceding cycles.**

This postulate assigns the full loop scale to a stationary phase-locked system. It is not derived from the standard Sagnac effect, and it must not be presented as a consequence of Equation (28). Its role is to define the additional physics explored in Part II.

If S_n denotes a local time-scale baseline relative to an external reference after n completed cycles, the postulate implies

$$S_{n+1} = S_n(1 + \epsilon_\phi), \tag{32}$$

not the additive relation $S_{n+1} = S_n + \text{constant}$. Consequently, the current value 207.386 ns is the present absolute increment associated with the present fractional rate; it is not assumed to have been the same absolute increment at every epoch.

The hypothesis is falsifiable. It requires an observable mismatch between a local phase-locked standard and an independently evolving reference. A comparison only among standards sharing the same phase history may be insensitive to a common-mode drift. Existing clock, GNSS, ephemeris, and astrophysical timing data are therefore relevant constraints, and a full bounds analysis remains necessary.

17 Why the Loop Deficit Accumulates with Each Revolution: A Geometric Phase Gap

The main text demonstrates that a clock transported once around the Earth's equator in the eastward direction exhibits, in the slow-transport limit, a fixed time deficit of approximately 207 ns relative to a co-rotating ground clock. A natural question arises: *If the deficit is truly a geometric property of the rotating frame, it should accumulate additively with the number of completed revolutions, independently of the traveller's speed. Moreover, this accumulation must be distinguished from the ordinary time dilation that would arise from a constant relative speed.* This appendix provides a rigorous standard-physics proof of this claim.

17.1 Proper-time difference for N revolutions

Let the traveller complete N consecutive eastward revolutions, always at the same ground-relative speed $u > 0$. The total coordinate time in the Earth-centred inertial frame is

$$T_N = N\frac{L}{u}, \tag{33}$$

because each revolution adds the same circumferential distance $L = 2\pi R$. The traveller's speed in the inertial frame remains $V + u$, with $V = \Omega R$. Hence the accumulated proper-time difference after N loops is

$$\Delta\tau_N(u) = T_N\left[\sqrt{1 - \frac{(V+u)^2}{c^2}} - \sqrt{1 - \frac{V^2}{c^2}}\right]. \tag{34}$$

For terrestrial speeds ($V, u \ll c$) we again expand to order v^2/c^2:

$$\Delta\tau_N \simeq -\frac{2Vu + u^2}{2c^2}\, T_N = -\,\frac{2Vu + u^2}{2c^2}\,\frac{NL}{u}. \tag{35}$$

Simplifying,

$$\boxed{\Delta\tau_N(u) \simeq -\,N\,\frac{VL}{c^2} \;-\; N\,\frac{uL}{2c^2}.} \tag{36}$$

Equation (36) reveals the two distinct contributions already identified for a single loop:

1. **Geometric loop component:** $-N\,\dfrac{VL}{c^2} = -N\,\dfrac{2\pi\Omega R^2}{c^2}$. This term is independent of u and simply multiplies the one-revolution value by N.
2. **Speed-dependent component:** $-N\,\dfrac{uL}{2c^2}$. This term depends linearly on the traveller's speed and also scales with N, but it vanishes in the slow-transport limit $u \to 0$.

17.2 The slow-transport limit for N revolutions

Taking the limit $u \to 0^+$ while keeping N fixed (i.e. completing N revolutions however slowly necessary), the speed-dependent term disappears and we obtain

$$\boxed{\lim_{u\to 0^+} \Delta\tau_N = -\,N\,\frac{VL}{c^2} = -\,N\,\frac{2\pi\Omega R^2}{c^2}.} \tag{37}$$

For the Earth this equals $-N \times 207.386$ ns. The total deficit is *strictly additive* in the number of revolutions.

17.3 Why this is not ordinary time dilation from a constant speed

In standard special relativity, a clock moving at a *constant* speed v relative to an inertial observer accumulates a proper-time deficit proportional to the *elapsed coordinate time* Δt:

$$\Delta\tau \approx -\,\frac{v^2}{2c^2}\,\Delta t.$$

If v is constant, the deficit grows linearly with the duration of the motion.

For the equatorial loop, by contrast, the slow-transport limit produces a deficit

$$\Delta\tau_N \longrightarrow -\,N\,\frac{2\Omega A}{c^2},$$

which depends *only on the number of times the loop is closed* and not on how long the journey takes. The geometric component is exactly the same whether the loop is completed in one hour or one year; the speed-dependent term, which does depend on travel time, is a separate, non-geometric addition that becomes negligible only when the speed is very small. The defining characteristic of the loop effect is that the deficit accumulates per *loop*, not per *unit time*.

This behaviour is the signature of a **geometric (anholonomic) phase**: the rotating frame possesses a non-zero *time holonomy* — every closed path that encloses an area about the rotation axis picks up a fixed time offset proportional to the directed area. Repeating the path N times simply multiplies the offset by N. The effect is therefore fundamentally different from ordinary velocity-based time dilation.

17.4 Consistency with standard relativity

The result (37) is derived from the same proper-time integral used throughout special relativity. No new postulates are introduced. The additive accumulation with revolution number is therefore a firm consequence of standard physics, yet it is often overlooked because textbook discussions usually focus on the instantaneous relation $\mathrm{d}\tau = \mathrm{d}t\sqrt{1 - v^2/c^2}$ without emphasising the global properties of the frame.

17.5 Significance for the main paper

The recognition that the ~ 207 ns per revolution is a *per-loop geometric phase deficit*, not a per-day velocity effect, is essential for correctly interpreting rotating-frame time measurements. It implies that any clock that participates in a closed circuit around the rotation axis — even if moving arbitrarily slowly — will accumulate a fixed, direction-sensitive time offset. This insight provides a natural bridge between the operational concept of clock transport and the geometric understanding of time in rotating frames, a viewpoint that will be further developed in a forthcoming work on phase-locked timekeeping.

18 Compound-Interest Model for the Local Time Scale

The phrase "compound interest" refers only to the mathematical structure: each fractional change is applied to the value already reached, rather than to the original baseline.

Let S_n be the physical duration of one local time-standard unit, expressed relative to a specified external reference, after n completed cycles. This should not be confused with defining the SI second as 1/86400 of a mean solar day; the SI second is defined by the caesium-133 hyperfine frequency [9]. The present model concerns a possible physical drift of the realised standard relative to an external reference.

For a constant fractional change r per cycle,

$$\boxed{S_{n+1} = S_n(1 + r).} \tag{38}$$

Thus

$$\boxed{S_N = S_0(1 + r)^N, \qquad S_{-D} = S_0(1 + r)^{-D}.} \tag{39}$$

The absolute increment at cycle n is

$$\delta_n = S_{n+1} - S_n = rS_n, \tag{40}$$

so

$$\delta_{n+1} = (1 + r)\delta_n. \tag{41}$$

The increment therefore changes together with the baseline.

18.1 Present-day calibration of the fractional rate

Using the article's adopted daily calibration interval $S_{\mathrm{day},0} = 86\,400\,\mathrm{s}$ and the present absolute scale $\delta_0 = 207\,\mathrm{ns}$,

$$r = \frac{\delta_0}{S_{\mathrm{day},0}} = \frac{207 \times 10^{-9}}{86400} \simeq 2.3958333 \times 10^{-12} \quad \text{per adopted daily cycle.} \tag{42}$$

Using the sidereal period instead changes the calibration slightly but does not alter the compound structure.

18.2 Numerical evaluation over 60 years

For

$$D = 60 \times 365.25 = 21915 \tag{43}$$

days,

$$Dr \simeq 5.25047 \times 10^{-8}. \tag{44}$$

The past-to-present ratio is

$$\frac{S_{-D}}{S_0} = (1+r)^{-D} \simeq 0.9999999475. \tag{45}$$

Thus, under the hypothesis, one local second realised 60 years ago would be shorter than the present realisation by approximately

$$S_0 - S_{-D} \simeq 52.5\,\mathrm{ns}. \tag{46}$$

18.3 Why linear and compound results coincide over short baselines

The binomial expansion gives

$$(1+r)^D = 1 + Dr + \frac{D(D-1)}{2} r^2 + \mathcal{O}(r^3 D^3). \tag{47}$$

Over 60 years the difference between the linear result $1 + Dr$ and the compound result is only about 1.38×10^{-15} fractionally, or approximately $0.12\,\mathrm{ns}$ when applied to one day. The two models are therefore numerically indistinguishable over a human lifetime.

Over much longer intervals the distinction is essential. For small r,

$$(1+r)^N \simeq \exp(Nr), \tag{48}$$

so the characteristic e-folding scale is approximately $1/r$, of order a billion years for the adopted value. Extrapolating the same constant rate over such intervals is itself a testable assumption of the model.

19 A Conditional 93% Benchmark for an Apparent CMB Wavelength Drift

This section preserves the numerical comparison originally proposed, but states its observational and metrological assumptions explicitly.

19.1 The 1965 reference wavelength

Penzias and Wilson observed an isotropic excess antenna temperature at 4080 Mc/s, corresponding to an instrumental wavelength near 7.35 cm [2]. This was the observing band of the horn antenna, not a measurement of the peak wavelength of the CMB blackbody spectrum and not the first point of a six-decade time series of a single spectral feature. Consequently, the calculation below is a conditional metrological benchmark using 7.35 cm as a reference wavelength. It is not evidence that a 3.92 nm CMB shift has already been observed.

19.2 Local readout drift

For the 61-year interval from mid-1965 to mid-2026,

$$D = 61 \times 365.25 = 22280.25 \text{ days.} \tag{49}$$

The exact compound factor is

$$G_T(D) = (1 + r)^D, \tag{50}$$

and the first-order accumulated parameter is

$$\epsilon_D = G_T(D) - 1 \simeq Dr \simeq 5.3380 \times 10^{-8}. \tag{51}$$

A wavelength readout requires a separate length-scale law. Let $\ell_{\text{loc}}(D)$ be the physical size of one local length unit. The numerical value assigned to an external wavelength is

$$\Lambda(D) = \frac{\lambda_{\text{ext}}(D)}{\ell_{\text{loc}}(D)}. \tag{52}$$

The original 93% estimate corresponds to the branch-specific assumption

$$\frac{\ell_{\text{loc}}(1965)}{\ell_{\text{loc}}(2026)} = G_L(D) \simeq 1 + \epsilon_D. \tag{53}$$

Under this assumption, and temporarily holding the external physical wavelength fixed,

$$\Lambda_{2026} = \Lambda_{1965} G_L(D). \tag{54}$$

For $\Lambda_{1965} = 7.35\,\text{cm}$,

$$\Delta\lambda_{\text{local}} \simeq 7.35\,\text{cm} \times 5.3380 \times 10^{-8} \simeq 3.923\,\text{nm}. \tag{55}$$

The length law in Equation (53) is an additional model choice. It does not follow from the time-scale recurrence alone. If the physical second and metre co-scale while the physical speed of light is unchanged, their scale factors must be equal. Any inverse scaling requires an additional propagation- or readout-sector transformation, which must be specified by the completed phase-locking dynamics.

19.3 Standard late-time expansion over the same interval

For a short interval at the present epoch, standard FLRW expansion gives approximately

$$z_D \simeq H_0 \Delta t. \tag{56}$$

Using $H_0 = 67.4\ \text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ and 61 years gives

$$z_D \simeq 4.20 \times 10^{-9}. \tag{57}$$

Applied to the same reference wavelength,

$$\Delta\lambda_{\text{cosmo}} \simeq 7.35\,\text{cm} \times 4.20 \times 10^{-9} \simeq 0.309\,\text{nm}. \tag{58}$$

19.4 The conditional fraction

Adding the two benchmark contributions gives

$$\Delta\lambda_{\rm benchmark} \simeq 3.923\,\mathrm{nm} + 0.309\,\mathrm{nm} = 4.232\,\mathrm{nm}. \tag{59}$$

The local-readout fraction is then

$$\boxed{\frac{\Delta\lambda_{\rm local}}{\Delta\lambda_{\rm benchmark}} \simeq 0.927 \simeq 93\%.} \tag{60}$$

This 93% value is therefore a conditional ratio within the specified branch of the model. It becomes an empirical prediction only after the model supplies a unique relation between the time-scale factor, the length-scale factor, the external-wave propagation factor, and the observer's motion.

19.5 Spiral-observer Doppler factor

The ancient emission event need not continue moving at the present epoch. The received frequency is determined by the wave four-vector and the observer's four-velocity at reception:

$$\omega_{\rm obs} = -k_\mu u^\mu_{\rm obs}. \tag{61}$$

For a spiral trajectory

$$\mathbf{r}(t) = r(t)\mathbf{e}_r(\phi(t)), \qquad \mathbf{v} = \dot{r}\,\mathbf{e}_r + r\dot{\phi}\,\mathbf{e}_\phi, \tag{62}$$

the wavelength Doppler factor is

$$D_\lambda(t) = \frac{1}{\gamma(t)[1 - \mathbf{v}(t)\cdot\hat{\mathbf{k}}(t)/c]}. \tag{63}$$

An outward radial component can redshift radiation arriving from the corresponding direction. A constant velocity projection produces a constant offset; a secular Doppler drift requires the projection $\mathbf{v}\cdot\hat{\mathbf{k}}$ to change. Standard observer motion is also direction-dependent and normally contributes a dipole, whereas a scalar local calibration drift would be common-mode. These signatures can be separated observationally.

The complete wavelength-readout map is therefore

$$\boxed{\frac{\Lambda(D)}{\Lambda(0)} = G_{\rm prop}(D)\frac{D_\lambda(D)}{D_\lambda(0)}\frac{G_L(0)}{G_L(D)}.} \tag{64}$$

No automatic cancellation follows unless the external wave and the local length standard acquire exactly the same scale factor.

20 Addressing Standard Objections to the Phase-Drift Interpretation

20.1 Objection 1: "207 ns is a per-loop Sagnac offset, not a stationary-clock daily rate"

Standard statement: Correct. In standard relativity the 207 ns number is the slow-transport offset for one completed directed equatorial loop. A stationary equatorial ground clock has the ordinary leading kinematic deficit of approximately 103.7 ns per sidereal day relative to a hypothetical non-rotating clock at the same potential.

Response: Part II does not derive a stationary-clock 207 ns rate from standard relativity or from the Lorentz factor alone. It introduces the additional phase-cycle postulate of Section 16. The postulate is viable only if its independent predictions survive existing and future timing constraints.

20.2 Objection 2: "A stationary ground clock does not accumulate a daily Sagnac step"

Standard statement: Correct. A ground clock follows a smooth worldline with a constant proper-time rate in the ideal stationary model; it does not acquire a discontinuous daily jump.

Response: The compound model is not a sequence of literal clock jumps. It is a hypothesised fractional evolution of the realised phase standard relative to an external reference. The recurrence is a discrete bookkeeping representation of a smooth secular law. A continuous version may be written

$$\frac{\mathrm{d}\ln S}{\mathrm{d}N} = \ln(1+r) \simeq r. \tag{65}$$

A quantitative comparison with GNSS, terrestrial time scales, spacecraft tracking, pulsar timing, and optical-clock networks is required; it is not sufficient to assert that all existing comparisons share exactly the same history.

20.3 Objection 3: "The SI second and metre are fixed by defining constants"

Standard statement: The SI second is defined by the unperturbed caesium-133 hyperfine frequency and the metre by the fixed numerical value of c [9]. A statement that these units physically drift must therefore be formulated as a drift of their *realisation relative to an external reference*, not as a change in the SI definitions.

Response: The phase-locking hypothesis concerns the physical realisation of atomic and material standards. Let G_T and G_L be the physical scale factors of one realised time and length unit. If the physical propagation speed and the numerical value of c are both to remain unchanged, consistency requires

$$G_T = G_L. \tag{66}$$

An inverse relation between the physical second and metre is not compatible with this condition unless an additional transformation of the propagation sector is supplied. The completed theory must derive, rather than assume, this relation.

20.4 Objection 4: "A universal rescaling cancels from dimensionless observables"

Standard statement: A common change of dimensional units alone is not observable. Measurements compare dimensionless ratios.

Response: The proposed observable is

$$\Lambda = \lambda_{\text{ext}}/\ell_{\text{loc}}. \tag{67}$$

It changes only if external radiation and the local material standard do not co-evolve by the same factor. This sector separation is a physical hypothesis, not a consequence of notation. It must be tested using frequency ratios, independent propagation standards, and directional signatures.

20.5 Objection 5: "The 7.35 cm number was not a tracked CMB spectral wavelength"

Standard statement: Penzias and Wilson measured excess antenna temperature at 4.080 GHz; 7.35 cm was the instrumental observing wavelength, not the blackbody peak and not a longitudinally tracked spectral line [2].

Response: The article therefore treats 7.35 cm as a reference scale for a conditional calculation. An empirical test requires a reproducible dimensionless spectral or frequency ratio measured at separated epochs with traceable calibration.

20.6 Objection 6: "Observer-motion Doppler is anisotropic"

Standard statement: Translational motion relative to an isotropic radiation field produces a leading dipole, not an isotropic secular shift.

Response: The spiral-observer Doppler term and the scalar compound calibration term must be fitted separately. The former depends on $\mathbf{v} \cdot \hat{\mathbf{k}}$ and changes sign across the sky; the latter is hypothesised to be common-mode. A monopole claim cannot be justified by a directional Doppler calculation alone.

20.7 Concluding remark

Part I remains a standard result regardless of the fate of Part II. Part II is scientifically meaningful only insofar as it defines a consistent scale map, derives its parameters from the proposed dynamics, and survives independent metrological and astrophysical tests.

21 A Concrete Experimental Programme

A decisive test must compare dimensionless observables whose two components do not share the same hypothesised phase history. Useful classes include:

1. **Clock-ratio networks.** Long-term comparisons among different atomic species and optical transitions can test whether a proposed drift is universal or transition-dependent. A strictly common-mode effect requires an external reference.

2. **GNSS and spacecraft timing.** Modern navigation systems already model Sagnac, gravitational, and velocity effects at the nanosecond level [7, 6]. The phase-drift model should be implemented as an additional parameter in a global timing fit rather than compared with an undefined "true inertial clock."

3. **Pulsar and astrophysical timing.** Millisecond-pulsar timing, binary orbital periods, and stable spectral ratios offer external phase histories, but propagation delays, source evolution, ephemerides, and clock realisation must be fitted simultaneously.

4. **Directional versus common-mode tests.** Spiral-motion Doppler predicts angular dependence, while a scalar local scale drift predicts a common-mode component. A sky decomposition into monopole, dipole, and higher multipoles is therefore essential.

5. **Long-baseline spectral comparisons.** A suitable test must track the same well-defined dimensionless spectral feature or frequency ratio across epochs. The 3.923 nm benchmark at 7.35 cm corresponds to a fractional effect of about 5.34×10^{-8} over 61 years.

A null result constrains or falsifies the relevant parameter combination. A positive result must also reproduce the predicted compound curvature and its directional decomposition.

22 Scope and Limitations

This paper does not provide a complete alternative to ΛCDM. It does not yet derive the local time and length scale factors from a finite- stiffness microscopic Hamiltonian, nor does it supply a joint likelihood analysis of supernovae, BAO, CMB, lensing, and structure growth.

The conditional 93% benchmark is not an observed historical CMB drift. It is the numerical consequence of a specified branch of the scale model. Before applying the mechanism to Type Ia supernovae or BAO, the theory must explicitly derive its effect on the calibrated supernova absolute magnitude, flux and redshift, and on the BAO ratios D_M/r_d and D_H/r_d. A universal dimensional rescaling may cancel from these observables.

Although a full confrontation with cosmological distance probes is reserved for future work, a secular metrological drift could, in principle, enter cosmological inference if it creates an epoch-dependent mismatch between local calibration standards and astrophysical clocks, candles, or rulers. No claim is made here that the effect necessarily eases the Hubble tension. Its sign and magnitude require an explicit observable map and a joint likelihood analysis.

23 Conclusion

Part I establishes the finite slow-clock-transport limit and its relation to the Sagnac synchronisation holonomy. The standard result is additive in repeated winding. Part II introduces a stronger phase-locking postulate in which each cycle rescales the metrological baseline, producing the compound law $S_N = S_0(1 + r)^N$. The compound law, the spiral-observer Doppler factor, and the local length-readout law are logically distinct and must be combined through Equation (64).

The central open task is dynamical: derive the signs and magnitudes of the time and length scale factors from the phase-field action and demonstrate how the proposed common-mode drift avoids existing bounds while producing a new dimensionless observable. Until that derivation and test are complete, the cosmological interpretation remains a falsifiable hypothesis rather than an established consequence of Sagnac physics.

A Unified Mathematical Appendix: Additive Holonomy, Compound Drift, Spiral Doppler, and Wavelength Readout

A.1 Logical hierarchy

The model contains four levels:

(i) standard additive Sagnac holonomy;

(ii) a non-standard cycle-by-cycle rescaling postulate;

(iii) standard Doppler readout along a possibly spiral observer trajectory;

(iv) a local metrological conversion from physical wavelength to a reported numerical wavelength.

Only the first and third levels follow from standard relativity.

A.2 Additive winding law

For an oriented loop C,

$$\Delta\tau_{\mathrm{loop}}[C] = -\frac{1}{c^2}\oint_C (\boldsymbol{\Omega}\times\mathbf{r})\cdot \mathrm{d}\mathbf{r} = -\frac{2\boldsymbol{\Omega}\cdot\mathbf{A}_C}{c^2} + \mathcal{O}(c^{-4}). \tag{68}$$

Repeating the same path N times gives

$$\boxed{\Delta\tau_{\mathrm{loop}}[C^N] = N\Delta\tau_{\mathrm{loop}}[C].} \tag{69}$$

This is geometric and area-dependent, not a strictly topological invariant.

A.3 Multiplicative baseline law

The phase-locking extension postulates

$$X_{n+1} = X_n(1+r_n). \tag{70}$$

For constant $r_n = r$,

$$\boxed{X_N = X_0(1+r)^N.} \tag{71}$$

For a time-dependent rate,

$$X_N = X_0\prod_{j=0}^{N-1}(1+r_j), \tag{72}$$

and in a continuous limit,

$$X(t) = X(t_0)\exp\left[\int_{t_0}^{t}\Gamma(t')\mathrm{d}t'\right]. \tag{73}$$

Thus compound accumulation is not an optional numerical technique; it is the direct solution of a fractional recurrence.

A.4 Spiral observer

Let

$$\mathbf{r}(t) = r(t)\mathbf{e}_r(\phi(t)), \tag{74}$$

so

$$\mathbf{v}(t) = \dot{r}\,\mathbf{e}_r + r\dot{\phi}\,\mathbf{e}_\phi. \tag{75}$$

Circular motion has $\dot{r} = 0$; outward spiral motion has $\dot{r} > 0$. For an incoming wave with propagation direction $\hat{\mathbf{k}}$,

$$\omega_{\text{obs}} = -k_\mu u^\mu_{\text{obs}} = \gamma\omega\left(1 - \frac{\mathbf{v}\cdot\hat{\mathbf{k}}}{c}\right). \tag{76}$$

The original emitter need not be presently active or moving. The frequency is fixed by the reception event and the observer's state of motion.

A constant value of $\mathbf{v}\cdot\hat{\mathbf{k}}$ produces a constant Doppler offset. A secular Doppler drift requires

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{v}\cdot\hat{\mathbf{k}}) \neq 0. \tag{77}$$

For an approximately isotropic sky, the leading translational term is dipolar and should not be identified with an isotropic metrological drift.

A.5 Local standards and the non-cancellation condition

Let $\ell_{\text{loc}}(t)$ be the physical length of one local ruler unit. The reported numerical wavelength is

$$\Lambda(t) = \frac{\lambda_{\text{ext}}(t)}{\ell_{\text{loc}}(t)}. \tag{78}$$

Between two epochs,

$$\frac{\Lambda_2}{\Lambda_1} = \frac{\lambda_{\text{ext},2}}{\lambda_{\text{ext},1}}\frac{\ell_{\text{loc},1}}{\ell_{\text{loc},2}}. \tag{79}$$

Cancellation occurs if and only if

$$\frac{\lambda_{\text{ext},2}}{\lambda_{\text{ext},1}} = \frac{\ell_{\text{loc},2}}{\ell_{\text{loc},1}}. \tag{80}$$

Accordingly, the statement that local time and length standards co-evolve does not by itself determine the external wavelength readout. The theory must specify whether freely propagating radiation participates in the same scale evolution as phase-locked matter.

A.6 Consistency of the second, metre, and the speed of light

Let G_T and G_L be the physical scale factors of one realised second and metre. If the physical light speed is fixed, its numerical value obeys

$$\frac{c_{\text{num},2}}{c_{\text{num},1}} = \frac{G_T}{G_L}. \tag{81}$$

Therefore a fixed numerical c requires

$$\boxed{G_T = G_L.} \tag{82}$$

If the model instead requires the physical second to lengthen while the physical metre shortens, it must derive an additional compensating change in the propagation or readout sector.

A.7 Master observable

Writing

$$\lambda_{\mathrm{ext}}(t) = \lambda_0 G_{\mathrm{prop}}(t) D_\lambda(t), \tag{83}$$

with

$$D_\lambda(t) = \frac{1}{\gamma(t)[1 - \mathbf{v}(t) \cdot \hat{\mathbf{k}}(t)/c]}, \tag{84}$$

the complete observable is

$$\boxed{\frac{\Lambda(t_2)}{\Lambda(t_1)} = \frac{G_{\mathrm{prop}}(t_2)}{G_{\mathrm{prop}}(t_1)} \frac{D_\lambda(t_2)}{D_\lambda(t_1)} \frac{G_L(t_1)}{G_L(t_2)}.} \tag{85}$$

This equation is the appropriate starting point for quantitative tests.

A.8 Linear versus compound numerical comparison

For $r = 207 \times 10^{-9}/86400$ and $D = 21915$,

$$G_{\mathrm{lin}} = 1 + Dr, \tag{86}$$

$$G_{\mathrm{cmp}} = (1 + r)^D. \tag{87}$$

Their fractional difference is approximately

$$G_{\mathrm{cmp}} - G_{\mathrm{lin}} \simeq 1.38 \times 10^{-15}, \tag{88}$$

which is negligible over 60 years but grows at long baselines.

B One Eastward Equatorial Loop Completed in One Year

If a traveller completes the equatorial circumference in exactly one year,

$$u \simeq \frac{L}{365.25 \times 86400 \text{ s}} \simeq 1.27\,\mathrm{m/s}. \tag{89}$$

The loop term remains

$$\Delta\tau_{\mathrm{loop}} \simeq -207.386\,\mathrm{ns}, \tag{90}$$

whereas the quadratic transport term is

$$\Delta\tau_{\mathrm{own}} \simeq -\frac{uL}{2c^2} \simeq -0.283\,\mathrm{ns}. \tag{91}$$

The total is approximately

$$\Delta\tau_{\mathrm{total}} \simeq -207.669\,\mathrm{ns}. \tag{92}$$

This example illustrates only the slow-transport decomposition of Part I; it does not establish the multiplicative phase-drift postulate of Part II.

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