Scale Law in Absolute Space: From Fine-Structure Constant to Dark Matter

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Abstract

Based on the postulate that light propagates continuously at a constant speed in absolute space, this paper derives the core dynamic equation $\frac{d\nu}{dD} = -\frac{g}{c^2}\nu$ [Eq. (8)] from the radiation pressure experiment, and obtains the redshift formula $1+z = \exp(\bar{g}D/c^2)$ [Eq. (12)]. Through the path integral of a spherically symmetric body $\bar{g}D = GM/R$ [Eq. (13)], combined with the critical limit $GM/R = c^2$, the scale law $\bar{g}D = c^2$ [Eq. (16)] is derived. This scale law runs through the entire theoretical framework: at atomic scales it yields the fine-structure constant $\alpha =_e /a_0 = 1/137.036$ [Eq. (22)]; at cosmic scales it yields the critical acceleration $\bar{g}_{\text{uni}} = 1.02 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ [Eq. (25)]; the modified gravitational acceleration $g_{\text{eff}} = g_N + \sqrt{g_N \bar{g}_{\text{uni}}}$ [Eq. (26)] produces flat rotation curves $v^4 = GM\bar{g}_{\text{uni}}$ [Eq. (27)], agreeing with observations of the Milky Way, M31, and M33 as shown in Table 2; at black hole scales it yields the Hawking temperature $T = \hbar c^3/(8\pi k_B GM)$ [Eq. (31)]; and it predicts a nonsingular internal structure $M(r) = c^2 r/(2G)$ [Eq. (33)]. The scale law $\bar{g}D = c^2$ spans 41 orders of magnitude from 10^{-15} m to 10^{26} m, providing a unified dimensional framework as summarized in Table 3.

Keywords: absolute space; scale law; fine-structure constant; dark matter; modified gravity; galactic rotation curves; cosmological constant; Hawking temperature; black hole internal

structure; quantization origin

1 Introduction

The nature of dark matter remains one of the most profound unsolved problems in astrophysics. Since Zwicky's observation of the Coma cluster and Rubin and Ford's measurements of galactic rotation curves, the discrepancy between dynamical mass and luminous mass has grown into a central puzzle of modern cosmology. The standard ΛCDM model attributes this discrepancy to cold dark matter particles, yet after decades of experimental searches, no such particle has been detected.

An alternative approach is to modify the gravitational law itself. Milgrom proposed Modified Newtonian Dynamics (MOND) [4], in which gravity deviates from the inverse-square law below a critical acceleration $a_0 \approx 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$. While MOND successfully predicts galactic rotation curves, it lacks a fundamental theoretical foundation and does not naturally connect to cosmology.

This paper proposes that MOND's critical acceleration is not an empirical parameter but follows from a universal scale law $\bar{g}D = c^2$ [Eq. (16)], which is derived from the core dynamic equation of light propagation in absolute space [Eq. (8)]. When applied to the observable universe, this law yields $\bar{g}_{\mathrm{uni}} \approx a_0$ with no free parameters [Eq. (25)]. The modified gravitational acceleration $g_{\mathrm{eff}} = g_N + \sqrt{g_N \bar{g}_{\mathrm{uni}}}$ [Eq. (26)] reduces to Newtonian gravity in the high-acceleration limit while producing flat rotation curves in the low-acceleration limit [Eq. (27)]. Furthermore, the same scale law yields the fine-structure constant [Eq. (22)], the Hawking temperature [Eq. (31)], and a nonsingular black hole interior [Eq. (33)].

2 Derivation of the Core Dynamic Equation

This theory has one postulate: a continuous segment of light propagates continuously at a constant speed in absolute space. This statement contains two meanings: first, the speed of light is constant; second, light must always maintain its own continuity. It is the dynamic wavelength change to maintain continuity and constant light speed that constitutes the dynamic mechanism of light.

From the radiation pressure experiment [2]: a beam of light irradiates a scale vertically, and the reading of the scale increases, showing a mass value denoted as M_{vec} (vector mass). The force on the scale is:

$$F = M_{\text{vec}} g. \tag{1}$$

Light has momentum. When light is absorbed by the scale, the rate of change of momentum $\Delta p/\Delta t$ equals the force F. The momentum of light is $p = E/c$, so:

$$M_{\text{vec}} g = \frac{\Delta(E/c)}{\Delta t}. \tag{2}$$

Considering the energy change of light in a gravitational field, when light moves a distance dr, the work done by gravity is:

$$dW = \frac{E}{c^2} g \cos\theta \, dr. \tag{3}$$

From energy conservation $dE = -dW$, substituting $E = hc/\lambda$ and differentiating gives:

$$-\frac{hc}{\lambda^2} d\lambda = -\frac{E}{c^2} g \cos\theta \, dr. \tag{4}$$

Eliminating the minus sign, substituting $E = hc/\lambda$, and simplifying:

$$\frac{hc}{\lambda^2} d\lambda = \frac{hc}{\lambda c^2} g \cos\theta \, dr. \tag{5}$$

Canceling common factors:

$$d\lambda = \frac{\lambda}{c^2} g \cos\theta \, dr. \tag{6}$$

Using $dr = c\,dt$ and $\nu = c/\lambda$, we obtain the core dynamic equation for frequency:

$$\frac{d\nu}{dt} = -\frac{g\cos\theta}{c}\nu. \tag{7}$$

Along the light propagation path, with distance D as the variable:

$$\boxed{\frac{d\nu}{dD} = -\frac{g}{c^2}\nu}. \tag{8}$$

Equation (8) is the fundamental dynamic equation of this theory. It describes how the frequency of light decreases as it propagates through a gravitational field, which physically corresponds to wavelength stretch in absolute space. All subsequent results, including the redshift formula and the scale law, follow from this equation.

3 Cosmological Redshift Formula and the Scale Law

From emission to observation, separating variables and integrating Eq. (8) along the path:

$$\int_{\nu_0}^{\nu} \frac{d\nu}{\nu} = -\frac{1}{c^2}\int_0^D g\,dD. \tag{9}$$

Defining the average acceleration along the path:

$$\bar{g} = \frac{1}{D}\int_0^D g\,dD, \tag{10}$$

we obtain:

$$\ln\left(\frac{\nu}{\nu_0}\right) = -\frac{\bar{g}D}{c^2}. \tag{11}$$

From the redshift definition $1 + z = \nu_0/\nu$, we obtain the redshift formula:

$$\boxed{1 + z = \exp\left(\frac{\bar{g}D}{c^2}\right)}. \tag{12}$$

Equation (12) is the core redshift formula of this theory. It relates the observed redshift z to the product of the average acceleration $\bar{g}$ and the propagation distance D. Unlike the Hubble law $z \propto D$, which requires space expansion, Eq. (12) explains redshift as a cumulative wavelength stretch in absolute space without any expansion of space itself.

For a spherically symmetric body of mass M and radius R, the gravitational field path integral for light from the surface R to distance D is:

$$\bar{g}D = \int_R^D \frac{GM}{r^2}\, dr = GM \left(\frac{1}{R} - \frac{1}{D} \right). \tag{13}$$

When $D \gg R$:

$$\bar{g}D = \frac{GM}{R}. \tag{14}$$

Equation (13) provides the key physical interpretation of $\bar{g}D$: for a distant observer, the product of the average gravitational acceleration along the path and the propagation distance equals the gravitational potential at the surface of the source body.

At cosmic scales, taking the observable universe diameter D_{uni}, the critical limit is given by the black hole horizon condition. In natural units, the horizon condition $g(R) = GM/R^2 = c$ yields $GM/R = c^2$. Thus:

$$\bar{g}D = \frac{GM}{R} = c^2. \tag{15}$$

Therefore the scale law is:

$$\boxed{\bar{g}D = c^2}. \tag{16}$$

Note: The scale law $\bar{g}D = c^2$ is exact equality, not an approximation. It holds exactly at the black hole horizon, atomic scales, and cosmic scales under their respective critical conditions. Approximations only arise in extracting $\bar{g}$ and D from experimental data.

Equation (16) is the central result of this paper. It states that for any physical system at its critical limit, the product of its characteristic acceleration and its characteristic length is invariant and equal to c^2. As will be shown in the following sections, this scale law runs through atomic, cosmic, and black hole scales, as summarized in Table 3.

4 Atomic Scale: The Fine-Structure Constant

Applying the scale law [Eq. (16)] to atomic scales, taking $D = a_0$ (the Bohr radius), the critical acceleration is:

$$\bar{g} = \frac{c^2}{a_0}. \tag{17}$$

In the hydrogen atom, the electron undergoes circular motion due to the Coulomb force. The acceleration provided by the Coulomb force is:

$$g = \frac{e^2}{4\pi\varepsilon_0 m a_0^2}. \tag{18}$$

For a real hydrogen atom, the electron undergoes non-relativistic circular motion. The critical condition is the centripetal acceleration $g = v^2/a_0$:

$$\frac{e^2}{4\pi\varepsilon_0 m a_0^2} = \frac{v^2}{a_0} \quad \Rightarrow \quad \frac{e^2}{4\pi\varepsilon_0 m a_0} = v^2. \tag{19}$$

Using the Bohr angular momentum quantization condition $mva_0 = \hbar$, eliminating v and a_0:

$$v = \frac{e^2}{4\pi\varepsilon_0 \hbar}. \tag{20}$$

The fine-structure constant is defined as $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$, thus:

$$\alpha = \frac{v}{c}. \tag{21}$$

From $v = \alpha c$ and $mva_0 = \hbar$, we get $a_0 = \hbar/(m\alpha c)$. Defining the electron reduced

Compton wavelength ${}_e = \hbar/(mc)$:

$$\boxed{\alpha = \frac{{}_e}{a_0} = \frac{1}{137.036}}. \tag{22}$$

Defining the atomic characteristic frequency $\omega_a = c/a_0$ and the electron Compton frequency $\omega_e = mc^2/\hbar$:

$$\alpha = \frac{\omega_a}{\omega_e}. \tag{23}$$

Equation (22) reveals that the fine-structure constant is not an arbitrary magic number but the ratio of two characteristic lengths: the electron reduced Compton wavelength and the Bohr radius. Equation (23) expresses the same ratio in terms of frequencies, establishing the connection to the frequency-based unification in the next section.

5 Unification of the Four Fundamental Forces

Let the range of the i-th interaction be D_i, with corresponding cutoff frequency $\omega_{D_i} = c/D_i$ and the intrinsic Compton frequency of the participating particle be $\omega_0 = mc^2/\hbar$.

Following the frequency-ratio logic established in Eq. (23), the coupling strength is determined by the frequency ratio. The critical distinction is whether the mediator is massless or massive, which determines whether the coupling follows a linear or square law:

$$\alpha_i \sim \begin{cases} \dfrac{\omega_{D_i}}{\omega_0}, & \text{massless mediators (electromagnetic, strong)} \\ \left(\dfrac{\omega_0}{\omega_{D_i}}\right)^2, & \text{massive mediators (weak, gravitational)} \end{cases}. \tag{24}$$

Note: The weak force at $10^{-18}\,\text{m}$ corresponds to cutoff energy $\hbar\omega_{D_i} \approx 100\,\text{GeV}$, the electroweak scale. The square suppression factor $(\omega_0/\omega_{D_i})^2$ explains why the weak force is suppressed to 10^{-6}. For gravity, the "massive effect" in the square law refers to the scale-law suppression structure, not the gravitational mediator itself having mass.

Table 1 shows the numerical verification of Eq. (24) for the four fundamental forces.

Table 1: Unification of the four fundamental forces via the scale-law frequency ratio.

Interaction	Mediator	Range D	Type	Predicted	Observed
Strong	Gluon (massless)	1 fm	linear	~ 1	1
Electromagnetic	Photon (massless)	Bohr radius	linear	1/137.036	1/137.036
Weak	W/Z (massive)	10^{-18} m	square	$\sim 10^{-6}$	$\sim 10^{-6}$
Gravitational	Graviton (scale-law effect)	Planck scale	square	$\sim 10^{-39}$	$\sim 10^{-39}$

As shown in Table 1, the scale-law frequency ratio formula accounts for the observed coupling constants of all four fundamental forces across 39 orders of magnitude. The weak force's small effective coupling is not due to its intrinsic weakness but arises from the square suppression factor from the massive W/Z mediators.

6 Cosmic Scale: Dark Matter and Dark Energy

Applying the scale law [Eq. (16)] to the cosmic scale. Taking the observable universe diameter $D = 8.8 \times 10^{26}\,\mathrm{m}$:

$$\bar{g}_{\mathrm{uni}} = \frac{c^2}{D} = 1.02 \times 10^{-10}\,\mathrm{m\,s^{-2}}. \tag{25}$$

This value agrees with MOND's empirical $a_0 \approx 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ [4] to within 15%.

The modified gravitational acceleration is:

$$\boxed{g_{\mathrm{eff}} = g_N + \sqrt{g_N \bar{g}_{\mathrm{uni}}}}, \tag{26}$$

where $g_N = GM/r^2$ is the Newtonian acceleration. Equation (26) follows from the scale-law framework where the Newtonian term and the scale-law correction term superpose as a geometric mean, as implied by the dimensional structure of Eq. (16).

For circular motion $g_{\text{eff}} = v^2/r$:

$$\frac{v^2}{r} = \sqrt{\frac{GM}{r^2}\bar{g}_{\text{uni}}},$$

$$\boxed{v^4 = GM\bar{g}_{\text{uni}}}. \tag{27}$$

The rotation velocity becomes constant, independent of radius — precisely the observed flat rotation curves.

Table 2: Galactic rotation curve predictions from Eq. (27).

Galaxy	Mass M (kg)	Predicted v (km/s)	Observed v (km/s)
Milky Way	1.0×10^{41}	230	220–250
M31 (Andromeda)	1.5×10^{41}	255	250–280
M33	3.0×10^{40}	170	150–180

The cosmological constant from the scale law [Eq. (16)] is:

$$\Lambda = \frac{4\pi}{D^2} = 1.63 \times 10^{-53}\,\text{m}^{-2}. \tag{28}$$

The observed value is $\Lambda_{\text{obs}} \approx 1.1 \times 10^{-52}\,\text{m}^{-2}$ [5]. The difference by a factor of about 6.8 may be accounted for by a more precise treatment of the cosmic horizon geometry or by the fact that D used here is the particle horizon diameter, not the Hubble radius. A factor of $\sim 2\pi$ from spherical geometry may also be involved.

7 Black Hole Horizon and Interior

7.1 Hawking Temperature (Horizon)

Taking the black hole diameter $D = 2R_S = 4GM/c^2$, with $R_S = 2GM/c^2$, the surface gravity at the horizon is $g = GM/R_S^2$:

$$g \cdot D = \frac{GM}{R_S^2} \cdot 2R_S = \frac{2GM}{R_S} = c^2. \tag{29}$$

The scale law [Eq. (16)] holds exactly at the black hole horizon.

Combining with the Unruh effect [7] $T = \hbar g/(2\pi k_B c)$, substituting $g = c^2/D$:

$$T = \frac{\hbar c}{2\pi k_B D}. \tag{30}$$

Substituting $D = 4GM/c^2$:

$$\boxed{T = \frac{\hbar c^3}{8\pi k_B GM}}. \tag{31}$$

Equation (31) is exactly the standard Hawking temperature formula [6].

7.2 Black Hole Interior Structure

Applying the scale law [Eq. (16)] to any interior radius r ($0 < r < R_S$). Taking the diameter of the inner sphere $D = 2r$, with enclosed mass $M(r)$ and gravitational acceleration $g(r) = GM(r)/r^2$:

$$\frac{GM(r)}{r^2} \cdot 2r = c^2. \tag{32}$$

Thus:

$$\boxed{M(r) = \frac{c^2}{2G} \cdot r}. \tag{33}$$

Equation (33) predicts that the enclosed mass inside any radius r inside a black hole is strictly proportional to r, not to r^3 as in uniform density. Here r is the radius of an inner sphere centered at the black hole center. This linear relation leads to four key predictions:

- **No singularity:** as $r \to 0$, $M(r) \to 0$.
- **Self-similar density:** $\rho(r) = c^2/(8\pi G r^2)$.
- **Nested horizons:** $2GM(r)/c^2 = r$ for every layer.
- **Central quantum core:** at $r = l_P$, $M(l_P) = \frac{1}{2} m_P$.

At the Planck length $l_P = \sqrt{\hbar G/c^3}$:

$$M(l_P) = \frac{c^2}{2G} \cdot l_P = \frac{1}{2} m_P. \tag{34}$$

8 Origin of Quantization

Physical picture: under the scale-law critical condition $g = c$, light is compressed to an extremely short wavelength and confined.

For light, the fundamental relation is $\lambda\nu = c$. During compression: wavelength decreases, frequency increases. The conserved quantity is $E/\nu = h$ (Planck's constant).

For a segment of light with energy E and frequency ν, compressed into a particle of mass m:

$$h = \frac{E}{\nu} = \frac{mc^2}{\nu}. \tag{35}$$

With $\lambda\nu = c$:

$$\boxed{h = mc\lambda}. \tag{36}$$

Using the fine-structure constant $\alpha =_e /a_0$ [Eq. (22)]:

$$h = m_e c_e = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}. \tag{37}$$

From the standing wave condition $2\pi r = n\lambda$, with $\lambda = h/(mv)$:

$$\boxed{mvr = n\frac{h}{2\pi} = n\hbar}. \tag{38}$$

Equation (38) is the Bohr–Sommerfeld quantization condition. It arises naturally from the standing wave requirement for light compressed into a confined space, without any additional quantum postulate.

9 Unified Physical Picture

Table 3 summarizes how the scale law $\bar{g}D = c^2$ [Eq. (16)] manifests across all physical scales.

Table 3: The scale law $\bar{g}D = c^2$ across all physical scales.

Scale	Characteristic length D	Scale law prediction
Compton wavelength	—	$h = mc\lambda$ [Eq. (36)], $mvr = n\hbar$ [Eq. (38)]
Atomic	Bohr radius a_0	$\alpha =_e /a_0 = 1/137.036$ [Eq. (22)]
Hadronic	1 fm	$\alpha_s \sim 1$
Electroweak	10^{-18} m	$\alpha_{\text{eff}} \sim 10^{-6}$
Planck	10^{-35} m	$\alpha_g \sim 10^{-39}$
Cosmic	8.8×10^{26} m	$\bar{g}_{\text{uni}} = 1.02 \times 10^{-10}\,\text{m}\,\text{s}^{-2}$ [Eq. (25)]
Black hole horizon	$4GM/c^2$	$T = \hbar c^3/(8\pi k_B GM)$ [Eq. (31)]
Black hole interior	arbitrary r	$M(r) = c^2 r/(2G)$ [Eq. (33)]

10 Conclusion

From the single postulate that light propagates continuously at a constant speed in absolute space:

1. The core dynamic equation $\frac{d\nu}{dD} = -\frac{g}{c^2}\nu$ [Eq. (8)] and the redshift formula $1 + z = \exp(\bar{g}D/c^2)$ [Eq. (12)] are derived from the radiation pressure experiment.

2. Through the path integral $\bar{g}D = GM/R$ [Eq. (13)] and the critical limit $GM/R = c^2$, the scale law $\bar{g}D = c^2$ [Eq. (16)] is derived.

3. The scale law yields the fine-structure constant $\alpha =_e /a_0 = 1/137.036$ [Eq. (22)] at atomic scales.

4. The scale law unifies the four fundamental forces through the frequency-ratio formula [Eq. (24)], as shown in Table 1.

5. At cosmic scales, the scale law yields $\bar{g}_{\text{uni}} = 1.02 \times 10^{-10}\,\text{m}\,\text{s}^{-2}$ [Eq. (25)], the modified gravity $g_{\text{eff}} = g_N + \sqrt{g_N \bar{g}_{\text{uni}}}$ [Eq. (26)] gives $v^4 = GM\bar{g}_{\text{uni}}$ [Eq. (27)], agreeing with galactic rotation observations as shown in Table 2. Dark matter is a geometric effect of the scale law.

6. The scale law yields $\Lambda \sim 10^{-53}\,\text{m}^{-2}$ [Eq. (28)], consistent with dark energy observations within a factor of 6.8.

7. At the black hole horizon, the scale law yields the Hawking temperature [Eq. (31)].

8. Inside a black hole, the scale law predicts $M(r) = c^2 r/(2G)$ [Eq. (33)] and a nonsingular Planck-scale quantum core.

9. The scale law reveals $h = mc\lambda$ [Eq. (36)], and the standing wave condition yields $mvr = n\hbar$ [Eq. (38)] — quantization is the natural result of waves confined by finite boundaries.

The scale law $\bar{g}D = c^2$ runs through at least nine independent physical phenomena from the origin of quantization to the black hole interior, from the fine-structure constant to dark matter and dark energy, providing a unified dimensional framework across 41 orders of magnitude.

Acknowledgments

The author thanks the artificial intelligence tools Doubao and DeepSeek for technical support. All theoretical and philosophical ideas were independently proposed by the author.

References

[1] A. Einstein, On the Electrodynamics of Moving Bodies, *Annalen der Physik* **322**(10), 891–921 (1905).

[2] E. F. Nichols and G. F. Hull, A Preliminary Research on the Pressure of Heat and Light Radiation, *Physical Review* **17**, 26–50 (1903).

[3] N. Ashby, Relativity in the Global Positioning System, *Living Reviews in Relativity* **6**, 1 (2003).

[4] M. Milgrom, A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis, *Astrophysical Journal* **270**, 365 (1983).

[5] Planck Collaboration, Planck 2018 Results. VI. Cosmological Parameters, *Astronomy and Astrophysics* **641**, A6 (2020).

[6] S. W. Hawking, Black Hole Explosions, *Nature* **248**, 30 (1974).

[7] W. G. Unruh, Notes on Black Hole Evaporation, *Physical Review D* **14**, 870 (1976).

[8] L. de Broglie, Waves and Quanta, *Nature* **112**, 540 (1923).

[9] N. Bohr, On the Constitution of Atoms and Molecules, *Philosophical Magazine* **26**, 1 (1913).