A Completed-Cosh Thawing-Scalar Diagnostic
with a Candidate Two-Measure Parent Mechanism

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Status: exploratory theory-and-numerics draft; not a likelihood analysis; not a compact-object proof; not yet a complete matter-coupled parent theory.

Abstract

Purpose. This manuscript studies a canonical thawing-scalar diagnostic generated by the completed-cosh potential

$$V_{\rm sym}(\Phi) = \Lambda_0\left(\frac{4}{\Phi^2} + \frac{\Phi^2}{4} - 2\right) = \Lambda_0\left(\frac{\Phi}{2} - \frac{2}{\Phi}\right)^2 .$$

Method. The diagnostic branch is evolved as a flat-FLRW canonical scalar initialized above its finite zero-energy endpoint at $\Phi = 2$, with the present matter normalization fixed by $\Omega_\Phi(0) = 0.685$. **Results.** For the fiducial initial condition $\Phi_i = 4$ at $N = \ln a = -8$, the background solution gives $\Phi_0 \simeq 3.734$, $Q_0 \simeq 3.485$, $\mathrm{d}\Phi/\mathrm{d}N|_0 \simeq -0.509$, and $(w_0, w_a) \simeq (-0.874, -0.214)$. This places the branch in the non-phantom thawing quadrant and makes it a concrete target for dynamical-dark-energy comparison, although not yet a likelihood-level fit. **Parent mechanism and limitations.** The manuscript also identifies a candidate two-measure parent construction in which the same completed potential arises from the effective form $V_{\rm eff} = (V_1 + M)^2/(4V_2)$, rather than being inserted directly. In the minimal Palatini version, the Einstein-frame kinetic map preserves the Fisher-Rao kinetic coefficient, so the same canonical scalar background equations are recovered. This parent construction remains conditional. The decisive unresolved issues are safe matter coupling, fifth-force avoidance, continuation through the endpoint measure-degeneracy surface, measure-sector radiative stability, displacement selection, Boltzmann-level observables, and direct likelihood comparison.

Keywords: thawing dark energy; quintessence; Full-Fisher CAL; completed-cosh potential; Fisher-Rao metric; two-measure theory; scalar cosmology; DESI diagnostic.

1 Scope and status

This manuscript separates the thawing dark-energy diagnostic from the compact-object branch of the broader project. The diagnostic potential may be treated phenomenologically, but the candidate two-measure parent mechanism below can generate the reciprocal term, additive zero, and completed-square structure through an algebraic measure constraint.

This parent mechanism does not make the model complete. The main unresolved issues are the assignment of massive matter without producing an excluded fifth force, the continuation of the parent variables through the endpoint where the independent measure degenerates, and the stability of the measure-sector structure under quantum or effective-field-theory corrections.

The compact-object construction remains outside the scope of this manuscript. The present manuscript does not claim a DESI likelihood fit, a compact-object solution, or a completed matter-coupled theory of dark energy.

2 Introduction

The compact-object branch of the broader program is not part of the main claim here. It requires fixed-load continuation, physical matter normalization, metric matching, stability analysis, cutoff-sensitivity checks, and independent reproduction before any physical-object interpretation is defensible. The cosmology sector addresses the narrower question of whether the completed-cosh scalar branch can produce a reproducible thawing-dark-energy trajectory while also retaining a finite zero-energy endpoint.

The label “Full-Fisher CAL” denotes the scalar branch studied in this manuscript. It should not be read as a standard external formalism. Operationally, the branch is specified by the Fisher-Rao kinetic normalization, the canonical variable $\Phi = 2\sqrt{Q}$, and the completed-cosh potential written below. The acronym is less important than the explicit equations.

This manuscript presents the cosmology sector as a standalone thawing-dark-energy note. The claim is deliberately narrow: a reproducible scalar-background diagnostic plus a candidate two-measure parent mechanism for the completed potential. The comparison with DESI-style CPL parameters is used only as a qualitative orientation tool, not as a statistical likelihood analysis. The value of the note is that it turns the rough statement “initial displacement near $\Phi_i = 4$ gives thawing behavior” into a reproducible one-parameter background solve with a clear displacement parameter and explicit failure conditions.

In standard language, thawing quintessence refers to a scalar field initially frozen by Hubble friction with $w \approx -1$, which later begins to roll as dark energy becomes dynamically important [1, 2]. The branch studied here is of that type: the scalar begins frozen at early times, remains above its zero-energy endpoint, and rolls downward by the present epoch. Because the diagnostic model is a canonical scalar, the branch remains non-phantom.

3 Methods and model assumptions: Fisher-Rao kinetic normalization

The canonical-frame normalization issue is whether the Φ-canonical solve is merely a convenient field choice or follows from the proposed kinetic geometry. Within the assumptions stated below, it is not arbitrary.

Here an admissibility weight means a positive local scalar weight assigned to the allowed vacuum/configuration sector. Let

$$w = e^{\psi}, \qquad Q = \eta e^{\psi}.$$

The Fisher-Rao line element on positive weights has the local form

$$\mathrm{d}s^2 \propto \frac{\mathrm{d}w^2}{w}.$$

Pulling this metric back through $w = e^{\psi}$ gives

$$\frac{\mathrm{d}w^2}{w} = e^{\psi}\mathrm{d}\psi^2.$$

With the branch normalization,

$$Z_{\psi}(\psi) = \frac{\eta e^{\psi}}{\kappa}.$$

The canonical coordinate is therefore

$$\mathrm{d}\Phi = \sqrt{Z_\psi}\,\mathrm{d}\psi, \qquad \Phi = 2\sqrt{\frac{\eta}{\kappa}}e^{\psi/2}.$$

In the diagnostic units $\kappa = 1$, this is

$$\Phi = 2\sqrt{\eta e^{\psi}} = 2\sqrt{Q}, \qquad Q = \eta e^{\psi} = \frac{\Phi^2}{4}.$$

This also fixes the normalization error in the older parent draft: the expression $\Phi = 2\sqrt{\eta e^{\psi}/2}$ is inconsistent with $Q = \Phi^2/4$. The "/2" belongs nowhere inside the square root. All numerical results below use the corrected relation $\Phi = 2\sqrt{Q}$.

The theorem-level claim must remain limited. Chentsov uniqueness applies directly to normalized probability simplices under Markov morphisms, not automatically to a single homogeneous cosmological weight. For unnormalized positive measures, extended information-metric characterizations are less unique unless additional locality/extensivity assumptions are imposed [3, 4, 5]. Therefore the defensible claim is:

> The Full-Fisher kinetic choice is conditionally justified if admissibility weights are treated as a local positive measure and the scalar kinetic metric is identified with the local Fisher-Rao metric. This explains $Z_\psi \propto e^{\psi}$ and $\Phi = 2\sqrt{Q}$. It does not determine the potential.

A second limitation is important: the reflection $w \to 1/w$ is not an isometry of the one-dimensional Fisher-flat coordinate. Thus Fisher geometry does not protect the reciprocal term, the coefficient equality, or the additive zero. Those must come from another mechanism.

4 Model definition: diagnostic completed-cosh branch

In the diagnostic normalization $\eta = 1$, the completed branch is

$$V_{\mathrm{sym}}(\psi) = \Lambda_0(e^{-\psi} + e^{\psi} - 2) = 2\Lambda_0(\cosh\psi - 1),$$

which becomes

$$V_{\mathrm{sym}}(\Phi) = \Lambda_0\left(\frac{4}{\Phi^2} + \frac{\Phi^2}{4} - 2\right) = \Lambda_0\left(\frac{\Phi}{2} - \frac{2}{\Phi}\right)^2.$$

The endpoint is

$$\Phi_{\mathrm{vac}} = 2, \qquad Q_{\mathrm{vac}} = 1, \qquad V(2) = 0, \qquad V_{,\Phi}(2) = 0.$$

The curvature is

$$V_{,\Phi\Phi} = \Lambda_0\left(\frac{24}{\Phi^4} + \frac{1}{2}\right) > 0$$

for $\Phi > 0$, so the canonical positive-field branch has no tachyonic interval.

The endpoint is not the present cosmological state. If the field already sits at $\Phi = 2$, it provides no thawing dark energy. The cosmological branch must be displaced above the endpoint today. That displacement is an open selection problem, not a solved feature.

5 Parent mechanism: candidate two-measure construction

The diagnostic potential can be treated phenomenologically, but the candidate two-measure construction below gives a possible parent-level mechanism for generating the same completed form. The completed potential arises through a constraint algebra.

Introduce a metric-independent measure density built from four measure scalars φ^a:

$$\Omega = \frac{1}{4!}\epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}\partial_\mu\varphi^a\partial_\nu\varphi^b\partial_\rho\varphi^c\partial_\sigma\varphi^d.$$

Consider the minimal Palatini action

$$S = \int \mathrm{d}^4x\,\Omega\left[-\frac{1}{\kappa}R(\Gamma,g) + \frac{1}{2}Z_\psi(\psi)g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - V_1(\psi)\right] + \int \mathrm{d}^4x\sqrt{-g}\,[-V_2(\psi)]\,,$$

with

$$V_1(\psi) = ce^\psi, \qquad V_2(\psi) = \frac{c^2}{4\Lambda_0}e^\psi, \qquad Z_\psi(\psi) = \frac{\eta e^\psi}{\kappa}.$$

The measure-scalar variation gives

$$L_1 = M,$$

where M is an integration constant. Defining

$$\zeta \equiv \frac{\Omega}{\sqrt{-g}},$$

the metric and trace equations give the usual two-measure algebraic constraint, with the sign conventions used in this draft,

$$\zeta = \frac{2V_2}{V_1 + M}.$$

The Einstein-frame effective potential is then

$$V_{\text{eff}} = \frac{(V_1 + M)^2}{4V_2}.$$

Substituting the weight-linear sector potentials gives

$$V_{\text{eff}}(\psi; M) = \Lambda_0 e^\psi + 2\Lambda_0\frac{M}{c} + \Lambda_0\left(\frac{M}{c}\right)^2 e^{-\psi}.$$

For the zero-energy branch $M = -c$,

$$V_{\text{eff}}(\psi) = \Lambda_0(e^\psi - 2 + e^{-\psi}),$$

and therefore

$$V_{\text{eff}}(\Phi) = \Lambda_0\left(\frac{\Phi^2}{4} + \frac{4}{\Phi^2} - 2\right) = \Lambda_0\left(\frac{\Phi}{2} - \frac{2}{\Phi}\right)^2.$$

This is the central parent-candidate result. The reciprocal term is not written into the action. It is generated by division through V_2 after the measure constraint is solved. The additive zero is not an arbitrary Lagrangian constant. It is the cross term from the integration constant M. In general,

$$C_0 = 2\Lambda_0\frac{M}{c}, \qquad \Lambda_* = \Lambda_0\left(\frac{M}{c}\right)^2, \qquad C_0^2 = 4\Lambda_0\Lambda_*.$$

On $M = -c$, this becomes $C_0 = -2\Lambda_0$ and $\Lambda_* = \Lambda_0$. The coefficient equality is therefore not an independent matching condition in the same way it was in the single-measure diagnostic. It is controlled by the integration-constant branch and the normalization of the admissibility origin.

This differs from common scale-invariant TMT charge assignments. Standard examples often use different exponential weights, such as $V_1 \propto e^{\alpha\phi}$ and $V_2 \propto e^{2\alpha\phi}$, producing a different effective potential [6]. The present construction is a CAL-specific weight-linear assignment. Its status is a candidate parent mechanism, not a generic theorem of two-measure theory.

6 Einstein-frame kinetic map

The two-measure parent must reproduce the same canonical background numerics. This requires the Fisher kinetic coefficient to survive the Einstein-frame map.

Let

$$\bar{g}_{\mu\nu} = \zeta g_{\mu\nu}.$$

In four dimensions,

$$\sqrt{-\bar{g}} = \zeta^2\sqrt{-g}, \qquad \bar{g}^{\mu\nu} = \zeta^{-1}g^{\mu\nu}.$$

Therefore

$$\Omega g^{\mu\nu} = \zeta\sqrt{-g}\, g^{\mu\nu} = \sqrt{-\bar{g}}\,\bar{g}^{\mu\nu}.$$

This weight-neutral identity applies to both the Palatini curvature term and the scalar kinetic term. The scalar sector becomes

$$\int \mathrm{d}^4x\, \Omega \frac{1}{2} Z_\psi g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi = \int \mathrm{d}^4x \sqrt{-\bar{g}}\frac{1}{2} Z_\psi \bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi.$$

Thus the Einstein-frame kinetic coefficient is unchanged:

$$Z_{\mathrm{eff}}(\psi) = Z_\psi(\psi) = \frac{\eta e^\psi}{\kappa}.$$

The parent candidate therefore preserves the canonical coordinate $\Phi = 2\sqrt{Q}$ and transfers the completed-cosh thawing numerics without changing the background equations. This is the main technical improvement supplied by the parent-mechanism analysis.

7 Endpoint behavior and measure-degeneracy surface

On the $M = -c$ branch,

$$\zeta(\psi) = \frac{c}{2\Lambda_0}\frac{e^\psi}{e^\psi - 1}.$$

At the endpoint $\psi = 0$, equivalently $\Phi = 2$, the Einstein-frame system is smooth:

$$V_{\mathrm{eff}} = 0, \qquad V_{\mathrm{eff},\psi} = 0, \qquad V_{\mathrm{eff},\psi\psi} = 2\Lambda_0, \qquad Z_\psi = \frac{\eta}{\kappa}.$$

The phenomenological scalar can therefore cross the endpoint and oscillate, giving the same scalar-dust end state as the completed-cosh diagnostic, provided the usual coherent-oscillation averaging applies.

The parent variables are not equally smooth. As $\psi \to 0$, $\zeta \to \pm\infty$, while

$$g_{\mu\nu} = \zeta^{-1}\bar{g}_{\mu\nu} \to 0, \qquad \Omega = \zeta^{-1}\sqrt{-\bar{g}} \to 0.$$

The endpoint is therefore a measure-degeneracy or orientation-change surface in the parent description. On $\Omega = 0$, the usual measure-scalar argument enforcing $L_1 = M$ loses rank. Continuing the parent solution through repeated post-endpoint oscillations requires an explicit junction rule preserving M across $\Omega = 0$. The observable pre-endpoint thawing branch is parent-derived. The post-endpoint scalar-dust phase is Einstein-frame regular, but parent continuation remains conditional on this junction rule.

8 Numerical method: background equations and prescription

The completed-cosh diagnostic is a homogeneous flat-FLRW background calculation with $M_{\rm Pl} = 1$ and $\kappa = 1$. The independent variable is $N = \ln a$, with today at $N = 0$. The scalar is initialized at $N_i = -8$ with

$$\Phi(N_i) = \Phi_i, \qquad \frac{\mathrm{d}\Phi}{\mathrm{d}N}(N_i) = 0.$$

For each input Φ_i, the present-day matter normalization is chosen by shooting so that

$$\Omega_\Phi(0) = 0.685.$$

The background system is the standard flat-FLRW canonical-scalar system written in N-time. Let

$$u \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}N}, \qquad \rho_m(N) = \rho_{m0} e^{-3N},$$

where ρ_{m0} is the shooting parameter. With V denoting the scaled completed-cosh potential used in the numerical solve, the algebraic Friedmann closure is

$$H^2 = \frac{\rho_m + V(\Phi)}{3 - u^2/2}.$$

The evolution equations are

$$\frac{\mathrm{d}\Phi}{\mathrm{d}N} = u,$$

$$\frac{\mathrm{d}u}{\mathrm{d}N} = -\left(3 + \frac{\mathrm{d}\ln H}{\mathrm{d}N}\right) u - \frac{V_{,\Phi}}{H^2},$$

with

$$\frac{\mathrm{d}\ln H}{\mathrm{d}N} = -\frac{1}{2}\left(\frac{\rho_m}{H^2} + u^2\right).$$

Equivalently, these equations follow from

$$3H^2 = \rho_m + \frac{1}{2}H^2u^2 + V(\Phi), \qquad \Phi'' + \left(3 + \frac{H'}{H}\right)\Phi' + \frac{V_{,\Phi}}{H^2} = 0,$$

where primes denote $\mathrm{d}/\mathrm{d}N$. The scalar density fraction is

$$\Omega_\Phi = \frac{\frac{1}{2}H^2u^2 + V}{3H^2} = \frac{u^2}{6} + \frac{V}{3H^2}.$$

In the $\eta = 1$ completed branch, the dimensionless potential and derivative are

$$V(\Phi) = \frac{4}{\Phi^2} + \frac{\Phi^2}{4} - 2, \qquad V_{,\Phi} = -\frac{8}{\Phi^3} + \frac{\Phi}{2}.$$

The scalar equation of state is computed from the kinetic energy

$$K = \frac{1}{2}H^2\left(\frac{\mathrm{d}\Phi}{\mathrm{d}N}\right)^2$$

as

$$w(N) = \frac{K - V}{K + V}.$$

The CPL summary parameters are reported as $w_0 = w(0)$ and $w_a = -\mathrm{d}w/\mathrm{d}N|_0$. This matches $w(a) = w_0 + w_a(1 - a)$ at $a = 1$, because $N = \ln a$. The original numerical script used `solve_ivp` with RK45, `rtol=1e-9`, `atol=1e-12`, `max_step=0.02`, dense output, and Brent shooting for the matter-density normalization.

9 Results: one-parameter thawing locus

The relevant control parameter is the initial displacement Φ_i, not the present value Φ_0. The phrase "Φ about 4 today" should be read only as a rounded high-field branch label. The direct solve gives Φ_0 as an output. The resulting numerical scan is summarized in Table 1.

Φ_i	Φ_0	Q_0	$\mathrm{d}\Phi/\mathrm{d}N\|_0$	w_0	w_a	$V(\Phi_0)/\Lambda_0$
3.6	3.230	2.608	-0.699	-0.762	-0.454	0.991
3.8	3.492	3.048	-0.586	-0.833	-0.297	1.376
4.0	3.734	3.485	-0.509	-0.874	-0.214	1.772
4.2	3.964	3.929	-0.451	-0.901	-0.163	2.184
4.6	4.407	4.855	-0.371	-0.933	-0.106	3.061

Table 1: Completed-cosh thawing diagnostic outputs. The fiducial run is $\Phi_i = 4.0$, giving $\Phi_0 \simeq 3.734$ and $(w_0, w_a) \simeq (-0.874, -0.214)$.

The scan separates into three qualitative regions. At lower displacement, around $\Phi_i \simeq 3.5$–3.6, the field rolls too rapidly and produces w_0 far from -1. Around $\Phi_i = 4$, the branch enters a phenomenologically relevant thawing window: the present field remains above the endpoint, rolls downward, and yields non-phantom dynamics with $w_0 \simeq -0.87$ and $w_a \simeq -0.21$. At higher displacement, around $\Phi_i \simeq 4.6$–5, the field becomes more frozen and approaches Λ-like behavior.

The fiducial point is therefore not special because it solves a selection problem. It is useful because it is a coherent diagnostic target: it keeps the scalar displaced from the endpoint, lands in the thawing quadrant, and provides a concrete present-day field value $\Phi_0 \simeq 3.734$.

10 Discussion: matter coupling and fifth-force constraint

The parent mechanism improves the potential story but sharpens the matter problem. A Hubble-scale scalar with ordinary universal conformal coupling is constrained by solar-system tests. In scalar-tensor notation, a light scalar with coupling β typically gives

$$|\gamma - 1| \simeq 2\beta^2$$

up to convention. The Cassini measurement of γ implies an order-of-magnitude constraint $\beta \lesssim \text{few}\times 10^{-3}$ [7].

The naive matter assignment is not safe. If massive Standard Model matter is placed under the original $\sqrt{-g}$ measure, then in Einstein frame it sees

$$g_{\mu\nu} = \zeta^{-1}\bar{g}_{\mu\nu}.$$

On the $M = -c$ branch,

$$\zeta \propto \frac{\Phi^2}{\Phi^2 - 4}.$$

The induced conformal coupling is

$$\beta(\Phi) = -\frac{1}{2}\frac{\mathrm{d}\ln\zeta}{\mathrm{d}\Phi} = \frac{4}{\Phi(\Phi^2 - 4)}.$$

At the fiducial present value $\Phi_0 \simeq 3.734$, this gives

$$\beta_0 \simeq 0.108,$$

which is far above the Cassini-scale bound if unscreened. Worse, β diverges as $\Phi \to 2$. Therefore the earlier possible resolution "put the Standard Model under $\sqrt{-g}$" does not solve both vacuum-energy routing and fifth-force safety. Those two requirements pull apart.

The surviving parent options are narrower:

1. massive matter couples directly to the Einstein-frame metric $\bar{g}_{\mu\nu}$, with the admissibility scalar confined to the gravity/vacuum sector;
2. matter couples only derivatively to $\mathrm{d}\ln w$, protected by global rescaling $w \to \lambda w$;
3. mass generation is sequestered into a sector that avoids the dangerous conformal factor;
4. the parent interpretation is abandoned while the phenomenological quintessence diagnostic is retained.

This manuscript does not solve this. Matter-sector assignment is therefore the leading viability condition for the parent-theory interpretation.

11 Discussion: radiative and structural protection status

The two-measure mechanism protects more than the rigid square did, but it does not complete the quantum theory.

The protected part is algebraic. The endpoint zero arises from the numerator square $(V_1 + M)^2$. A denominator-side vacuum shift can deform the shape without lifting the zero at $V_1 + M = 0$. The reciprocal term and additive cross term are generated by the measure constraint, not inserted directly.

The unprotected part is also clear. Corrections to V_1, V_2, or the measure sector can deform the observational thawing locus. Mixed-measure operators, such as schematic terms of the form $\Omega^2/\sqrt{-g}$, would spoil the minimal constraint structure unless forbidden by a deeper principle. Matter loops are harmless only after a safe matter-sector assignment is specified. Thus the revised status is:

> The completed square is conditionally generated by a candidate two-measure parent. The endpoint zero is better protected than in the rigid diagnostic. Full radiative stability remains open.

12 Discussion: relation to DESI-style CPL diagnostics

The completed-cosh locus lies in the non-phantom thawing quadrant $w_0 > -1$, $w_a < 0$. This sign pattern is consistent with the thawing-quintessence orientation used in the surrounding literature [1, 9]. It is also qualitatively relevant because DESI-era discussions have increased interest in time-varying dark energy and CPL summaries of the form $w(a) = w_0 + w_a(1-a)$ [8]. However, the present calculation is not a DESI likelihood analysis and must not be described as a statistical fit.

The earlier source manuscript used approximate DESI-style CPL overlays for orientation. Those overlays are not official covariance surfaces, exclusion statistics, or parameter constraints. The correct claim is weaker and cleaner: the completed-cosh branch generates a reproducible one-parameter thawing trajectory in a qualitative region relevant to dynamical-dark-energy model building. Direct likelihood comparison remains required.

13 Open problem: displacement selection

The two-measure parent candidate explains the potential shape. It does not yet explain why the early field displacement should land near $\Phi_i \simeq 4$. The diagnostic scan shows that this displacement is not knife-edge fine tuning, but it is still genuine selection. Lower values roll too strongly or approach the endpoint too early; larger values become more Λ-like.

The working prediction of this branch is that the $\Phi_i \sim 4$ displacement is not a numerical accident. This manuscript cannot prove that prediction. The defensible statement is narrower: the fiducial displacement identifies a finite, reproducible thawing target that a parent preparation mechanism could plausibly select. Possible routes include a boundary condition, a measure prior, an attractor in a larger parent phase space, or a pre-thawing dynamical preparation mechanism. None has been derived here.

14 Updated claim-status table

Claim	**Status in this manuscript**
The completed branch has a finite zero-energy endpoint.	Yes in the diagnostic branch; conditionally generated in the two-measure parent on the $M = -c$ branch.
The branch admits a thawing high-field trajectory.	Yes in the completed-cosh background diagnostic scan.
The fiducial $\Phi_i = 4$ run yields $w_0 \simeq -0.874$, $w_a \simeq -0.214$.	Yes, reproduced from the completed-cosh table.
The Fisher kinetic coefficient is arbitrary.	No. It is conditionally justified by the Fisher-Rao metric on positive admissibility weights, with explicit postulates.
$\Phi = 2\sqrt{Q}$ is the correct canonical coordinate.	Yes, given $Z_\psi \propto e^\psi$. The older /2 inside the square root is a normalization error.
The completed-cosh potential is merely hand-selected.	No longer the best status. A two-measure parent candidate generates it through $V_{\text{eff}} = (V_1+M)^2/(4V_2)$.
The rigid Hamilton-Jacobi square protects the potential under gravity.	No. The rigid square remains a diagnostic identity, not the gravitating protection mechanism.

Claim	Status in this manuscript
The two-measure parent is a completed theory.	No. It is conditional on matter-sector assignment, endpoint junction, and measure-sector radiative stability.
Naive Standard Model matter under $\sqrt{-g}$ is safe.	No. It gives $\beta_0 \simeq 0.108$, excluded if unscreened.
The result is a DESI likelihood fit.	No. A direct model-level likelihood remains open.
The compact-object branch is established.	No. Compact-object claims remain outside this paper.
The displacement Φ_i is dynamically selected.	No. This remains a main open consistency condition.

15 Limitations and remaining consistency requirements

The parent-theory interpretation requires the following additional results before it can be regarded as a complete dark-energy model.

1. **Matter-sector assignment.** Determine whether massive matter can couple safely without generating an excluded fifth force.
2. **Two-measure parent write-up.** Re-derive the parent action, constraint, Einstein-frame map, and potential using primary TMT sources.
3. **Endpoint junction rule.** Decide whether the integration constant M is preserved across $\Omega = 0$ during post-endpoint oscillations.
4. **Measure-sector radiative stability.** Identify what forbids mixed-measure operators and dangerous corrections to V_1 and V_2.
5. **Direct likelihood implementation.** Replace CPL overlays with a direct CLASS/Cobaya or equivalent model-level likelihood fit.
6. **Boltzmann observables.** Replace smooth-background/growth estimates with CMB and matter-power calculations.
7. **Displacement selection.** Derive or explicitly fail to derive a preparation mechanism for $\Phi_i \sim 4$.

16 Conclusion

The completed-cosh cosmology sector has a controlled positive result. The completed-cosh diagnostic gives a reproducible non-phantom thawing trajectory: the fiducial displacement $\Phi_i = 4$ evolves to $\Phi_0 \simeq 3.734$ and produces $(w_0, w_a) \simeq (-0.874, -0.214)$. The compact-object branch remains outside the scope of this paper.

The parent-theory status has also improved. The completed-cosh potential is not merely a selected square in the revised construction. A candidate two-measure model with weight-linear sectors generates the same potential through the algebraic constraint $V_{\rm eff} = (V_1 + M)^2/(4V_2)$, while the Einstein-frame kinetic map preserves the Fisher coefficient and canonical coordinate. That is a substantive upstream mechanism, not merely a retuning of the phenomenological potential.

The controlled prediction is that the completed-cosh thawing locus is pointing at a real parent-sector selection principle rather than an accidental numerical fit. That prediction is not established here. It becomes credible only if a safe matter assignment, endpoint junction rule, and measure-sector stability principle can be supplied. More seriously, the naive massive-matter assignment produces a conformal coupling far above fifth-force bounds. Therefore the revised status is: reproducible thawing diagnostic advanced; candidate two-measure parent mechanism identified; matter-sector viability and measure-sector closure remain decisive.

Ethics statement

Ethics approval was not required for this theoretical and numerical study because it did not involve human participants, human data, human tissue, animals, or clinical material.

Data and code availability

This manuscript is based on the completed-cosh diagnostic and its companion numerical script, which generate background tables, scan outputs, finite-difference checks, and CPL diagnostic overlays. A publication-ready version should deposit the executable script, environment file, CSV outputs, and plotting scripts in a public archive. The two-measure parent derivation is algebraic and should be independently reproduced from the parent action before submission.

AI assistance statement

The author used AI tools for drafting, mathematical exposition, symbolic consistency-check prompting, code prototyping, numerical workflow organization, and manuscript editing. The author supervised the project direction and accepts responsibility for all claims, calculations, errors, and interpretations.

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Conflicts of interest

The author declares no conflicts of interest.

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