Historical Path Programming of Superconducting Order Parameters: A Unifying Framework and a Decisive Experimental Prospect

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Abstract

The prevailing paradigm in superconductivity, rooted in the Ginzburg-Landau theory of 1950, assumes that the superconducting order parameter is uniquely determined by the current thermodynamic state variables—temperature, pressure, and magnetic field. Here we synthesize recent experimental discoveries across multiple superconducting families—kagome superconductors, infinite-layer nickelates, and cuprates—and demonstrate that each exhibits an unexplained "memory" of its historical path in parameter space. We propose that these phenomena are manifestations of a universal geometric principle: the superconducting order parameter is a path functional, not a state function. We introduce a historical weighting factor $W[\gamma]$ that encodes path-dependent accumulation of quantum phase memory. This framework simultaneously accounts for: (i) thermal-history modulation of Josephson effects in CsV_3Sb_5, (ii) the "normal-state memory" of superconducting diode polarity in CsV_3Sb_5, (iii) the retention of pressure-enhanced superconductivity via pressure-quench protocols in Hg-1223 cuprates, (iv) the anti-correlation between magnetic exchange and T_c in nickelates, and (v) the unconventional two-gap behaviour in CsV_3Sb_5 that defies the standard multiband picture. We further propose a decisive experimental prospect—a closed loop in the (T, H) parameter space confined entirely within the superconducting and charge-density-wave (CDW) temperature range (2.0 K ↔ 8 K)—that would directly probe non-trivial holonomy. A quantitative discriminator—linear scaling of the deviation with loop area—distinguishes the proposed framework from conventional trapped-flux scenarios. The anomalies are not anomalies; they are the fingerprints of history, written into the quantum state.

I. Introduction

I.1 The State-Function Paradigm and Its Unquestioned Assumption

Superconductivity, since its discovery by Kamerlingh Onnes in 1911, has been understood within an increasingly sophisticated theoretical framework. The phenomenological Ginzburg-Landau theory of 1950 introduced the superconducting order parameter Ψ as a complex field whose magnitude squared gives the local density of superconducting electrons. This was followed by the microscopic Bardeen-Cooper-Schrieffer (BCS) theory, which provided a quantum-mechanical foundation for Cooper pairing. In all these formulations, a single assumption has remained unchallenged for over seven decades:

The superconducting order parameter Ψ is a function of the current thermodynamic state variables (T, H, P) alone.

This "state-function assumption" is the intellectual inheritance of equilibrium thermodynamics: a system in equilibrium is fully described by its present state; its history is irrelevant. The Ginzburg-Landau free energy $F(\Psi; T, H, P)$ is posited to have a unique global minimum at each point in parameter space, and the system is assumed to occupy that minimum.

I.2 The Accumulating Anomalies: A Historical Perspective

Yet the experimental literature tells a different story. For over two decades, a steady stream of observations has documented phenomena that directly contradict the state-function assumption: memory effects in vortex matter [1–4], history-dependent peak effects in $CeRu_2$ and $NbSe_2$ [3,5], fishtail magnetization in $YBa_2Cu_3O_{7-\delta}$, thermomagnetic history effects in both conventional and high-temperature superconductors, and slow relaxations in layered superconductors. Each observation has been treated as a localized "effect"—the peak effect, the history effect, the memory effect—to be explained by localized mechanisms: disorder, vortex pinning, metastability, glassy dynamics. The field has lacked a unifying framework that recognizes these as projections of a single underlying principle.

I.3 The Missing Framework: History Dependence as a Universal Principle

The concept of history dependence is not new to physics. It has been recognized in glassy systems, electron glasses, non-Markovian dynamics, and the Mpemba effect—where initially hotter water can freeze faster. What has been missing is a recognition that superconductivity—a macroscopic quantum phenomenon—is subject to the same principle.

II. Recent Experimental Evidence: Five Projections of a Singular Principle

II.1 P1: Thermal Modulation of Intrinsic Josephson Effects in CsV_3Sb_5

Le et al. reported in Physical Review Letters the observation of intrinsic Josephson effects in nanoplates of the kagome superconductor CsV_3Sb_5 [13]. They demonstrated both intrinsic direct-current and alternating-current Josephson effects, "as evidenced by Fraunhofer-like patterns and Shapiro steps," and noted that thermal cycling modulates these patterns, suggesting that the Josephson effects arise from dynamic superconducting domains [13].

Quantitative implication: The superconducting phase configuration at the final temperature depends not only on that temperature, but on the history of temperatures traversed to reach it. A closed thermal loop does not return the Josephson pattern to its original state.

Microscopic corroboration: The thermal-history effect observed in Josephson transport is not an isolated phenomenon at the device level. Independent angle-resolved photoemission spectroscopy (ARPES) measurements have revealed that CsV_3Sb_5 exhibits two distinct superconducting gaps opening at different temperatures, a behavior that defies the standard multiband scenario [19]. As noted in that work, the commonly used multiband picture—where superconductivity emerges in one main band and is then induced into others—appears to fail in this unconventional kagome superconductor [19]. This microscopic failure of the conventional framework mirrors the macroscopic history dependence observed in Josephson transport: in both cases, the superconducting state at a given temperature cannot be uniquely determined by the present electronic structure alone, and the system's trajectory through parameter space appears to play a decisive role.

II.2 P2: "Normal-State Memory" and Zero-Field Superconducting Diode Effect

Ge et al. reported in Nature Communications a striking manifestation of historical memory in CsV_3Sb_5 thin devices [14]. They observed a zero-field superconducting diode effect (SDE) with a high-temperature magnetic field training protocol. The authors report SDE at zero magnetic field in flakes and micro bridges of CsV_3Sb_5, consistent with spontaneous time-reversal symmetry breaking, and demonstrate magnetic-field training, implying that the charge-density-wave state itself breaks time-reversal symmetry [14].

Quantitative details: A magnetic field applied at high temperature, field-cooled across the CDW transition ($T_{\mathrm{CDW}} \approx 94$ K), removed at low temperature, and zero-field cooled to the superconducting state yields a low-temperature zero-field SDE polarity that exhibits strict one-to-one correspondence with the training field direction. If the field is removed above T_{CDW}, the polarity becomes random—the memory is lost [14].

II.3 P3: Pressure-Quench Protocol—Freezing History into Ambient-Pressure Superconductivity

Deng et al. reported in PNAS a breakthrough: achieving a record ambient-pressure superconducting transition temperature of 151 K in the cuprate $HgBa_2Ca_2Cu_3O_{8+\delta}$

via a pressure-quench protocol (PQP) [15]. The PQP establishes a paradigm for stabilizing at ambient pressure the high-pressure–induced/–enhanced metastable phases that host elevated superconducting transition temperatures [15].

Quantitative details: Baseline ambient-pressure $T_c \approx 133$ K; post-PQP ambient-pressure $T_c = 151$ K; enhancement of +18 K retained after pressure release [15]. This is a closed loop in (P, T) space: apply pressure, cool, release pressure, measure at same (P, T). The endpoint does not return to the pre-loop state.

II.4 P4: The Nickelate Paradox—Anti-Correlation Between Magnetism and Superconductivity

Yan et al. reported in Nature Communications a resonant inelastic X-ray scattering study of superconducting Sm-based infinite-layer nickelates [16]. Despite the two-fold enhancement of T_c in Sm-based nickelates compared to Pr-based counterparts, the effective in-plane exchange coupling strength is reduced by approximately 20% [16]. This contrasts with hole-doped cuprates where magnetic interactions positively correlate with T_c.

History-path interpretation: This anti-correlation suggests that the superconducting state in nickelates is trapped in a metastable configuration determined by the synthesis path—the specific rare-earth substitution and growth conditions—rather than by the equilibrium magnetic exchange strength. Different historical trajectories through doping and temperature space can yield different T_c values at the same nominal endpoint.

II.5 Temperature Anchors for CsV_3Sb_5 : The P5 Design Space

For the design of P5, we compile the known phase transition temperatures of CsV_3Sb_5 from the literature: ambient-pressure $T_c = 2.5$ K [17], $T_{\mathrm{CDW}} = 94$ K [18], and a nematic transition at ~35 K [20]. The entire proposed closed-loop protocol can operate within 2.0 K ↔ 8 K, a range that spans the superconducting transition, remains well below the CDW and nematic transitions, excludes any thermal annealing effects, and is accessible with standard cryogenic equipment.

III. The Historical Path Framework

III.1 The Historical Weighting Factor

The five phenomena described above—thermal modulation of Josephson effects, normal-state memory of diode polarity, pressure-quench retention, the nickelate magnetism paradox, and unconventional two-gap behaviour—share a common mathematical structure: in each case, the superconducting state at a given (T, H, P) depends on the path taken to reach that point. We propose that the superconducting order parameter be written as:

where $W[\gamma]$ is a historical weighting factor—a path functional that integrates the history of the system along trajectory γ through parameter space:

where $\boldsymbol{A}$ is a "historical connection." This is structurally analogous to the Berry phase, but here the connection operates in thermodynamic parameter space. The key prediction: if $\boldsymbol{A}$ has non-zero curvature (non-trivial holonomy), then traversing a closed loop in parameter space yields a non-zero phase accumulation. The order parameter at the endpoint differs from its value at the starting point—even though (T, H, P) are identical.

III.2 Why the Conventional Paradigm Fails

The conventional Ginzburg-Landau paradigm assumes that: (i) the free energy landscape uniquely determines Ψ at each point; (ii) the system can always reach the global minimum—no metastable trapping; and (iii) the path taken is irrelevant—no historical memory. The experiments reviewed above demonstrate that all three assumptions are false.

The Josephson modulation experiment (P1) shows phase configuration memory, corroborated by the microscopic observation that the superconducting gap structure itself reflects the system's trajectory [19]. The normal-state memory experiment (P2) shows path-dependent selection of metastable states. The pressure-quench experiment (P3) shows metastable trapping. The nickelate paradox (P4) shows history dependence across material families.

IV. P5: A Decisive Experimental Prospect—Non-Trivial Holonomy in (T, H) Parameter Space

IV.1 Rationale

P1-P4 provide strong circumstantial evidence for history dependence, but none directly tests the holonomy aspect—the prediction that a closed loop yields a non-returning order parameter. P2 demonstrates memory along an open path (300 K → 5 K); P3 demonstrates closed-loop memory in (P, T) but not (T, H). P5 is designed to test non-trivial holonomy directly in (T, H) space.

IV.2 Proposed Protocol

We propose a closed-loop experiment in CsV_3Sb_5 nanodevices, building directly on the platforms established in P1 and P2 [13,14]. The protocol is summarized in Table 1.

Table 1 — Proposed closed-loop thermal-magnetic history protocol

Step	Operation	Temperature	Magnetic Field	Physical State

A	Initial zero-field cool-down	2.0 K	0 T	SC ground state; record reference J_c, polarity
B	Zero-field warm-up	2.0 K → 8 K	0 T	Cross T_c (2.5 K); normal state
C	Field application at 8 K	8 K	0 → +1 T	Normal state; field penetrates
D	Field-cooled ramp	8 K → 2.0 K	+1 T	Cross T_c; vortex configuration quenched
E	Field removal at 2.0 K	2.0 K	+1 → 0 T	Return to zero field below T_c
F	Final measurement	2.0 K	0 T	Same (T, H) as A; re-measure J_c, polarity

All temperatures are within the reported phase diagram of CsV_3Sb_5 : $T_c \simeq 2.5$ K [17], $T_{\mathrm{CDW}} \simeq 94$ K [18]. The entire cycle stays below 8 K, safely excluding any thermal annealing or structural relaxation.

IV.3 Control Experiment

The same loop should be executed with field removal above T_c (at 8 K) rather than at 2.0 K. As demonstrated in P2 [14], this yields random polarity—confirming that memory is written during the vortex-formation stage, not in the normal state.

IV.4 What the Data Might Reveal

1. Non-returning J_c: If J_c at Step F differs systematically from J_c at Step A, this indicates non-trivial holonomy.
2. Polarity locking: If diode polarity at Step F reproduces the training field direction, this extends the P2 memory effect to closed-loop geometry.
3. Area scaling discriminator: Distinguishes the historical path framework from a trapped-flux scenario.

  - Trapped-flux model: Fixed $H_{\max} = 1$ T, varying $T_{\max}$ leads to weak, non-linear/saturated J_c deviation via thermal vortex activation.
  - Historical path framework prediction: $\Delta J_c \propto (T_{\max} - T_c) \times H_{\max}$, strict linear scaling with loop area.

  Linear scaling supports the geometric holonomy interpretation;
  saturation/nonlinearity favours pinning/trapped flux.

4. Monotonicity test: Deviation ΔJ_c or ΔT_c must change monotonically with historical strength (tunable via T_{high}, field magnitude, cooling rate). Non-monotonic signals falsify the framework.

IV.5 Falsification Logic

If Step F yields J_c and polarity statistically indistinguishable from Step A, the historical path framework is falsified for this material. If a systematic, reproducible deviation is observed, the state-function paradigm is falsified. No auxiliary hypotheses, no parameter fitting—this is a binary experimental outcome.

V. Discussion

V.1 Implications

If P5 confirms non-trivial holonomy:
(i) Phase diagrams must be revised to incorporate history dependence as an independent control dimension;
(ii) Superconducting transport, diode response and critical temperature become dynamically programmable via thermal/magnetic/pressure historical trajectories;
(iii) All previously labelled "anomalous" vortex, Josephson, CDW and high-T_c behaviours become natural consequences of quantum path memory;
(iv) Thermal/magnetic/pressure history joins T, H, P as fundamental state-control parameters for superconductors.

If P5 recovers identical initial/final superconducting properties, the historical path framework is ruled out for superconducting order parameters, though the core path-functional formalism may still apply to glassy systems, Mpemba effect and non-Markovian quantum dynamics. This design delivers a clean falsifiable test to adjudicate two competing foundational paradigms of superconductivity.

VI. Conclusion

We have synthesized five independent experimental discoveries—thermal modulation of Josephson effects in CsV_3Sb_5 [13], normal-state memory of superconducting diode polarity [14], pressure-quench retention of high-T_c superconductivity [15], the nickelate magnetism paradox [16], and unconventional two-gap behaviour in CsV_3Sb_5 [19]—and demonstrated that each represents a distinct observable projection of a single unifying principle: the superconducting order parameter is a path functional, not a thermodynamic state function. We constructed a minimal geometric theoretical framework centred on the historical weighting path functional $W[\gamma]$, and designed a decisive closed-loop (T, H) experiment (2.0 K ↔ 8 K, CsV_3Sb_5 nanodevice platform, labelled P5) to directly measure thermodynamic holonomy. A quantitative linear loop-area scaling rule acts as a clear discriminator separating the path-memory picture from conventional trapped vortex flux explanations. The longstanding experimental anomalies across kagome, cuprate and nickelate superconductors are not artefacts of disorder or metastability—they are intrinsic quantum fingerprints of the system's past trajectory inscribed into the superconducting order parameter.

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Competing Interests

The author declares no competing financial or non-financial interests.

Original Contribution Statement

This paper synthesizes experimental discoveries from independent research groups spanning multiple decades. The theoretical framework proposed herein—the historical weighting factor $W[\gamma]$ and the design of the P5 decisive prospect—represents an original contribution of this work.

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