Procedural Vacuum Breakdown: A Three-Form Completion of the Gravitational Core

Canonical Closure, Scalar Dynamics, an Exact Planar Wall, and a Spherical-Gravity Audit

Chris Clifton
Independent Researcher, Delaware, USA

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Abstract

Procedural Vacuum Breakdown (PVB) is an effective gravitational program in which a scalar admissibility field ψ modulates the local vacuum reaction through the spacetime measure. The original realization imposed $\sqrt{-g} = \omega(x)e^{\psi}$ with a prescribed density ω. Although passively covariant, that realization fails to preserve the lock under the unrestricted canonical spatial-diffeomorphism generator for generic ω:

$$\delta_\xi \chi\big|_{\chi=0} = -e^{\psi}\mathcal{L}_\xi \omega.$$

This paper replaces the external density by the metric-independent four-form density of a nonpropagating three-form A and completes the lock as

$$\sqrt{-g} = e^{\psi}\Omega[A], \qquad \Omega[A] = \frac{1}{4!}\tilde{\epsilon}^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}.$$

Variation of A gives $d(\lambda e^{\psi}) = 0$, so $\lambda = \Lambda_* e^{-\psi}$ on each connected component. The complete Dirac–Bergmann analysis shows that the multiplier pair is second class, the three-form contributes no local propagating mode, and the reduced local constraints close as those of Einstein gravity plus one scalar with sector potential $U(\psi) + \Lambda_* e^{-\psi}$. The local physical content is therefore 2 tensor modes plus one admissibility scalar, together with a global topological pair.

Around constant-ψ_0 Minkowski space, eliminating the nondynamical metric and measure response gives physical scalar normalization Z_ψ, not the determinant-adapted diagonal entry $Z_\psi - 3/(8\kappa)$. The scalar is locally ghost-free for $Z_\psi > 0$, has luminal principal propagation, and is nontachyonic when $U''(\psi_0) + \Lambda_* e^{-\psi_0} \geq 0$. A fixed bare potential also admits an exact selected-sector planar tanh wall with asymptotically AdS exteriors. Its fixed-sector scalar master action has positive time-kinetic term and a nonnegative Friedrichs spatial operator; the formal zero mode is non-normalizable, and no resonance pole is claimed.

The completed core is then tested in static spherical symmetry. The full backreacted Einstein–scalar equations and a local three-form reconstruction are derived. Under a nonnegative sector potential and positive effective radial friction, a pressure-monotonicity identity obstructs the intended regular, horizonless, asymptotically flat scalar-only bubble. Adding a conserved two-dimensional surface load changes the result: the de Sitter–Schwarzschild junction system has a closed-form equilibrium family containing radially stable, standard-orientation, horizonless ultracompact points. An explicit benchmark occurs at $R_0 = 2.8M$, but approximately 86% of the summed proper component-energy estimate is carried by the added load. The spherical equilibrium is therefore a matter-assisted extension, not an intrinsic stabilization mechanism of the bare PVB scalar. Finite thickness, formation, leakage, nonspherical stability, the optical constitutive law, and the global topological sector remain open.

Public revision summary. This edition presents the completed three-form gravitational core and the spherical-gravity audit as one claim-disciplined preprint. It removes release-candidate language, makes the novelty boundary explicit, and separates three results that must not be conflated: local canonical closure of the PVB core, exact planar Einstein–scalar wall dynamics in a selected sector, and a matter-assisted thin-shell equilibrium in spherical symmetry. The distributional loaded shell is a radial existence benchmark, not a finite-thickness compact-object theory.
Keywords: Procedural Vacuum Breakdown; PVB; unimodular gravity; three-form measure; ADM constraint analysis; scalar–tensor gravity; domain walls; spherical gravity; thin shells; ultracompact objects
Subject areas: gr-qc (primary); hep-th (secondary)

1 Introduction

Procedural Vacuum Breakdown (PVB) is a layered effective-theory program built around one organizing possibility: a scalar admissibility variable ψ may alter the local vacuum reaction while remaining part of a generally covariant gravitational system. The program has three logically distinct layers. The first is the gravitational core: the measure lock, field equations, constraint algebra, and propagating degrees of freedom. The second is phenomenology: wall profiles, optical response, and possible compact configurations. The third is microscopic interpretation: what admissibility represents and why matter or an underlying substrate would couple to it. This paper addresses the first layer and uses a planar wall only as a controlled application.

The original PVB core encoded the coupling through

$$\sqrt{-g} = \omega(x)e^{\psi},$$

where ω was a prescribed weight-+1 density. This made the intended relation transparent: the admissibility scalar deformed the metric volume relative to a reference measure, and the multiplier response became locally slaved to ψ. However, the same fixed reference measure created a load-bearing canonical problem. The standard spatial-diffeomorphism generator acts on the metric, lapse, and ψ, but a prescribed external density has no canonical momentum and therefore does not transform as part of the phase space. For generic ω, the lock is not preserved by unrestricted active spatial diffeomorphisms.

The present paper keeps the PVB mechanism that survives this test and replaces the part that does not. The fixed density is replaced by the metric-independent four-form density of a three-form A,

$$\Omega[A] = \frac{1}{4!}\tilde{\epsilon}^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}, \qquad F = dA,$$

so that the completed PVB lock is

$$\sqrt{-g} = e^{\psi}\Omega[A].$$

The three-form is varied, transforms under diffeomorphisms, and has the gauge redundancy $A \mapsto A + dB$. It has no kinetic term and adds no local particle. Its role is to carry the measure inside the theory rather than leave it as an external background.

This completion changes the local interpretation in a precise way. The completed lock no longer forces the metric determinant to follow ψ relative to a rigid prescribed profile; the nonpropagating measure can adjust locally. What remains physically specific to PVB is the weighted relation between admissibility and the vacuum multiplier. Variation of the three-form makes λe^{ψ} a spacetime constant, so the local reaction is

$$\lambda = \Lambda_* e^{-\psi}.$$

Thus the original rigid-measure realization is not retained, but the central PVB slaving structure survives in a fully diffeomorphism-invariant theory.

This paper establishes the local gravitational core of PVB: the prescribed-density realization is obstructed, while the scalar-weighted three-form realization has a closing constraint algebra with two tensor modes and one admissibility scalar.

Seven results organize the paper. First, the fixed-density obstruction is derived explicitly. Second, the three-form completion restores the full local diffeomorphism constraint algebra and gives the local count $2_{\text{tensor}} + 1_{\psi}$, with only a global topological pair added. Third, the physical scalar normalization around constant-background Minkowski space is Z_{ψ}, not the unreduced diagonal coefficient $Z_{\text{diag}} = Z_{\psi} - 3/(8\kappa)$. Fourth, a fixed bare potential admits an exact selected-sector planar tanh wall. Fifth, the fixed-sector wall perturbation problem is standard Einstein–scalar theory, with a positive scalar time-kinetic term and a nonnegative Friedrichs spatial operator. Sixth, the static spherical reduction yields a conditional obstruction to the intended positive-energy scalar-only compact bubble. Seventh, an explicitly added conserved tangential load produces a closed-form family of radially stable thin-shell equilibria, while leaving finite thickness and nonspherical stability open.

Section 2 places these results inside the wider PVB program and partitions the claims by status. Section 3 compares the completion with standard and dynamical Henneaux–Teitelboim gravity. Sections 4–5 define the three-form measure and derive the prescribed-density obstruction. Sections 6–9 derive the completed field equations and sector dynamics. Section 11 states the local canonical result, and Appendix A displays the complete Dirac chain. Sections 12 and 13 treat the diagonal conformal block and physical Minkowski scalar. Section 14 constructs and audits the exact wall as a worked application. Section 15 derives the spherical scalar obstruction and the loaded-shell extension. The final sections state what this completion means for PVB, what remains open, and how the separate optical program may use the results without borrowing claims that have not been derived.

2 The PVB Program, Scope, and Claim Discipline

The completed theory should be read as the gravitational core of PVB, not as the whole PVB program. Its relation to the wider program is summarized in Table 1. The formal core is derived here. The optical layer remains a separate phenomenological construction, and the microscopic layer remains open. No result in one layer is used as proof of another.

Table 1: Logical layers of the PVB program

Layer	Question	Status in this paper
Gravitational core	Can the admissibility-weighted measure, vacuum slaving relation, and local degrees of freedom be made canonically consistent?	Closed locally by the three-form completion.
Phenomenology	Can admissibility support walls, optical response, compact shells, or observable propagation effects?	An exact planar wall and a spherical existence audit are derived. The stable spherical benchmark requires an added conserved load; optical constitutive physics remains separate.
Microscopic interpretation	What is admissibility, what underlying system produces it, and how does matter couple to it?	Not derived.

Claims below are separated by logical status.

Directly derived results:

1. the fixed-density obstruction
$$\delta_\xi \chi\big|_{\chi=0} = -e^{\psi}\mathcal{L}_\xi \omega;$$

2. the fully diffeomorphism-invariant lock $\sqrt{-g} = e^{\psi}\Omega[A]$ and three-form gauge symmetry $A \mapsto A + dB$;

3. topological slaving $\partial_\mu(\lambda e^{\psi}) = 0$, hence $\lambda = \Lambda_* e^{-\psi}$;

4. the local first-class constraints of the completed theory and the count
$$N_{\text{local}} = 2_{\text{tensor}} + 1_{\psi};$$

5. the reduced sector equation
$$Z_\psi \Box\psi - U'(\psi) + J_{\text{noise}} + \Lambda_* e^{-\psi} = 0;$$

6. the determinant-adapted diagonal coefficient $Z_{\text{diag}} = Z_\psi - 3/(8\kappa)$, explicitly not promoted to a physical eigenvalue;

7. the local nonzero-mode Minkowski scalar action
$$S^{(2)}_{\text{phys}} = -\frac{1}{2}\int d^4x \left[Z_\psi(\partial\varphi)^2 + \big(U''(\psi_0) + \Lambda_* e^{-\psi_0}\big)\varphi^2\right];$$

8. the exact selected-sector planar wall, its AdS asymptotics, and the flat-bulk obstruction;

9. in a fixed Λ_* sector, the standard Einstein–scalar master action with positive time-kinetic term for $Z_\psi > 0$;

10. the limit-point center, exclusion of the formal zero mode, positive-semidefinite factorization, and the numerical finite-box approach to the $m^2 = 0$ continuum threshold.

11. the full static spherical Einstein–scalar reduction and local reconstruction of the nonpropagating three-form measure;

12. a conditional scalar-only obstruction for nonnegative sector potential and positive effective radial friction;

13. the closed-form loaded-shell equilibrium family and the explicit radially stable, standard-orientation benchmark at $R_0 = 2.8M$, within the stated thin-shell model.

Derived diagnostics without a resonance claim:

1. the regular partner potential has threshold $m^2 = 0$ for every $\epsilon_g > 0$;

2. the flat-space partner state at $m^2\delta^2 = 3$ is embedded in that continuum when gravity is turned on and its intertwining image is singular at the center;

3. the fixed trial profile gives
$$\mathcal{R}_{\epsilon_g}[g_0] = 3 + \left(\ln 4 - \frac{3}{2}\right)\epsilon_g + O(\epsilon_g^2).$$

Deferred problems:

1. the microscopic interpretation of the three-form measure and its coupling, if any, to unresolved substrate variables;
2. global cohomology, flux quantization, boundary terms, and the global canonical pair;
3. a cubic reduction and physical strong-coupling scale;
4. FLRW and other time-dependent backgrounds;
5. an outgoing-wave or analytically continued scattering calculation for quasi-localized wall poles;
6. a covariant finite-thickness spherical scalar–matter solution, its formation channel, leakage control, and full nonspherical perturbation spectrum;
7. a microscopic electromagnetic constitutive law and compatibility with fifth-force and equivalence-principle bounds.

This partition preserves the original breadth of the PVB program without pretending that every layer has been derived. It prevents a local gravitational completion from being misrepresented as a microscopic theory, and it prevents a finite-box or partner-potential diagnostic from being mislabeled a resonance.

3 Relation to Unimodular and Henneaux–Teitelboim Gravity

Standard fixed-density unimodular gravity restricts the spacetime volume element and retains only density-preserving diffeomorphisms. The Henneaux–Teitelboim formulation replaces the background density by the exterior derivative of a three-form, restoring unrestricted diffeomorphism invariance while leaving the cosmological variable and the three-form nonpropagating [1, 6, 7]. Dynamical Henneaux–Teitelboim gravity adds kinetic terms to those fields and therefore changes their local dynamical status [3].

The PVB completion uses the same topological mechanism for the measure but changes the lock:

$$\text{HT: } \sqrt{-g} = \Omega[A], \qquad \text{PVB: } \sqrt{-g} = e^{\psi}\Omega[A].$$

The exponential factor makes the determinant deformation a genuine scalar mode. Variation of the three-form gives

$$d(\lambda e^{\psi}) = 0,$$

not $d\lambda = 0$. Thus the global quantity is $\Lambda_* = \lambda e^{\psi}$, while the local multiplier response is $\lambda = \Lambda_* e^{-\psi}$. The local sector is Einstein gravity plus a scalar with an integration-sector contribution $\Lambda_* e^{-\psi}$ to its potential.

The completion remains distinct from dynamical HT: neither A nor λ has a kinetic term, and neither adds a local propagating degree of freedom. The only additional object is the global pair associated with the integrated three-form coordinate and its constant momentum.

Novelty boundary. The use of a three-form to restore unrestricted diffeomorphism invariance is the Henneaux–Teitelboim mechanism and is not claimed as new. The result specific to PVB is the e^{ψ}-weighted lock, the conserved combination $\Lambda_* = \lambda e^{\psi}$, the resulting sector potential $\Lambda_* e^{-\psi}$, and the explicit local constraint reduction retaining one propagating admissibility scalar.

Interpretive boundary. The completion does not preserve the old picture of a scalar fixing the metric volume relative to an immutable local reference density. Because $\Omega[A]$ is varied and nonpropagating, it can carry the local measure adjustment required by the lock. The completed local theory is therefore best understood as a scalar-weighted topological measure theory whose fixed-sector dynamics are Einstein gravity plus one scalar. The PVB content lies in the status of

Table 2: Comparison of formal features

Feature	Fixed-density UG	Henneaux–Teitelboim	Dynamical HT	Completed PVB
Full diffeomorphism invariance	No	Yes	Yes	Yes
Measure carrier	prescribed density	three-form	dynamical scalar/three-form	three-form with e^{ψ} lock
Integration constant	Yes	Yes	modified/dynamical	$\Lambda_* = \lambda e^{\psi}$
Propagating admissibility scalar	No	No	model-dependent	Yes, one local scalar
Local three-form mode	No field	No	Yes when kinetically activated	No
λ kinetic term	No	No	Yes in DHT	No
Local multiplier profile	constrained	constant	dynamical	$\lambda = \Lambda_* e^{-\psi}$
Local gravitational count	two tensor	two tensor	model-dependent	two tensor plus one scalar
Optical benchmark	No	No	No	separate companion paper

ψ as the admissibility deformation and in the topological origin of the exponential sector term, not in a surviving rigid background measure.

The defining sentence is:

The viable PVB gravitational core is a scalar-weighted topological measure theory: admissibility remains dynamical, the vacuum reaction remains slaved, and no rigid external density remains.

4 Variables and Conventions

We use signature $(-,+,+,+)$ and

$$\kappa = \frac{8\pi G}{c^4}.$$

The scalar convention is

$$\mathcal{L}_\psi \supset -\frac{1}{2} Z_\psi (\nabla\psi)^2, \qquad Z_\psi > 0.$$

The admissibility scalar may be parametrized as

$$\psi = \ln\left(\frac{A_{\mathrm{adm}}}{A_0}\right).$$

Let $A_{\mu\nu\rho}$ be a three-form and

$$F_{\mu\nu\rho\sigma} = 4\partial_{[\mu} A_{\nu\rho\sigma]}.$$

The alternating symbol $\widetilde{\epsilon}^{\mu\nu\rho\sigma}$ is a contravariant density, and

$$\Omega[A] \equiv \frac{1}{4!}\widetilde{\epsilon}^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} = \frac{1}{3!}\widetilde{\epsilon}^{\mu\nu\rho\sigma}\partial_\mu A_{\nu\rho\sigma}$$

is a scalar density of weight $+1$. We work in an oriented sector with $\Omega[A] > 0$, so it can equal the positive metric volume density. The completed lock

$$\sqrt{-g} = e^{\psi}\Omega[A]$$

is therefore density-safe. The action is invariant under

$$A \longmapsto A + dB$$

for any two-form B. Locally, any smooth nonvanishing four-form density can be written as dA; global existence and flux are boundary/cohomology questions treated separately. In particular, on a compact boundaryless manifold a globally exact positive volume form requires a patched potential or a nontrivial higher-form bundle rather than a single globally defined A.

5 Fixed-Density Obstruction and Topological Measure Completion

The earlier realization used

$$\chi \equiv \omega e^{\psi} - N\sqrt{h} \approx 0$$

with a prescribed spatial density ω. Let $D[\xi]$ be the standard spatial-diffeomorphism generator acting on the canonical metric, lapse, and scalar variables. If ω has no canonical momentum, then $D[\xi]$ does not generate its density Lie derivative. On the lock surface,

$$\delta_\xi \chi = \omega e^{\psi} \mathcal{L}_\xi \psi - \mathcal{L}_\xi (N\sqrt{h}) \tag{1}$$

$$= \omega e^{\psi} \mathcal{L}_\xi \psi - \mathcal{L}_\xi (\omega e^{\psi}) \tag{2}$$

$$= -e^{\psi} \mathcal{L}_\xi \omega. \tag{3}$$

For a weight-+1 density,

$$\mathcal{L}_\xi \omega = \partial_i (\xi^i \omega).$$

Thus a generic fixed density preserves only its stabilizer subgroup. Passive coordinate covariance does not repair the missing active gauge transformation. The prescribed-density shortcut therefore fails the stated goal of a full first-class diffeomorphism complex.

The completion replaces ω by the dynamical topological density $\Omega[A]$. The lock is

$$\boxed{\sqrt{-g} = e^{\psi} \Omega[A].}$$

Both sides now transform as dynamical scalar densities. The three-form gauge symmetry removes local redundancy in A, while variation of A imposes the global slaving equation. No preferred density profile is inserted by hand.

This changes the status of the old open problem. The local gauge obstruction is closed. What remains is global: the allowed four-form flux, boundary data for A, possible quantization conditions in a microscopic completion, and the interpretation of the global conjugate variable. Those questions do not alter the local degree-of-freedom count.

6 Action

The completed action is

$$S_{\text{total}} = S_{\text{EH}} + S_\psi + S_m + S_{\text{EM}} + S_{\text{noise}} + S^{(A)}_{\text{lock}},$$

with

$$S_{\text{EH}} = \int d^4x \sqrt{-g}\, \frac{R - 2\Lambda}{2\kappa},$$

$$S_\psi = \int d^4x \sqrt{-g} \left[-\frac{Z_\psi}{2} (\nabla\psi)^2 - U(\psi) \right],$$

and

$$\boxed{S^{(A)}_{\text{lock}} = \int d^4x\, \lambda \left[e^{\psi} \Omega[A] - \sqrt{-g} \right].}$$

The multiplier λ and three-form A have no kinetic terms. Matter, electromagnetism, and the effective noise sector are assumed independent of A in the minimal completion. A boundary term or boundary condition fixing the pullback of A is required for a well-posed global variational problem; local field equations are unaffected.

For all metric-coupled physical sectors,

$$T^{(i)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_i}{\delta g^{\mu\nu}}.$$

The lock term is kept on the geometric side as an auxiliary pure-trace reaction.

7 Scalar and Noise Identities

The scalar stress tensor is

$$T^{(\psi)}_{\mu\nu} = Z_\psi \nabla_\mu \psi \nabla_\nu \psi - g_{\mu\nu} \left[\frac{Z_\psi}{2} (\nabla\psi)^2 + U(\psi) \right],$$

with

$$\nabla^\mu T^{(\psi)}_{\mu\nu} = \left[Z_\psi \Box\psi - U'(\psi) \right] \partial_\nu \psi.$$

Define

$$J_{\text{noise}} = \frac{1}{\sqrt{-g}} \frac{\delta S_{\text{noise}}}{\delta\psi}.$$

If S_{noise} is diffeomorphism invariant and its additional variables satisfy their equations of motion,

$$\nabla^\mu T^{(\text{noise})}_{\mu\nu} = J_{\text{noise}} \partial_\nu \psi.$$

Unlike the prescribed-density realization, no external-measure functional term appears: the measure carrier is a varied field. If a future matter or noise model couples directly to A, its Euler–Lagrange term must be included in the Ward identity and in the three-form equation.

8 Field Equations, Topological Slaving, and Bianchi Consistency

Variation with respect to λ gives

$$\boxed{\sqrt{-g} = e^\psi \Omega[A].}$$

Metric variation gives

$$G_{\mu\nu} + (\Lambda + \kappa\lambda) g_{\mu\nu} = \kappa \left[T^{(m)}_{\mu\nu} + T^{(\text{EM})}_{\mu\nu} + T^{(\psi)}_{\mu\nu} + T^{(\text{noise})}_{\mu\nu} \right].$$

Scalar variation gives

$$\lambda = - \left[Z_\psi \Box\psi - U'(\psi) + J_{\text{noise}} \right].$$

Variation of the three-form is most transparent in differential-form notation:

$$\delta_A S^{(A)}_{\text{lock}} = \int \lambda e^\psi d(\delta A) = - \int d(\lambda e^\psi) \wedge \delta A + \text{boundary}.$$

Therefore

$$\boxed{d(\lambda e^\psi) = 0.}$$

On each connected spacetime component,

$$\boxed{\lambda e^\psi = \Lambda_*, \qquad \lambda = \Lambda_* e^{-\psi}.}$$

Taking the covariant divergence of the metric equation and using the scalar/noise Ward identities yields the same equation,

$$\partial_\nu \lambda + \lambda \partial_\nu \psi = 0.$$

The Bianchi identity is therefore a consistency check rather than the only source of slaving. The global integration constant survives, while the local multiplier profile follows algebraically from ψ.

9 Reduced Scalar Equation in a Fixed Topological Sector

Using $\lambda = \Lambda_* e^{-\psi}$ in the scalar equation gives

$$\boxed{Z_\psi \Box\psi - U'(\psi) + J_{\text{noise}} + \Lambda_* e^{-\psi} = 0.}$$

Define

$$U_{\text{eff}}(\psi; \Lambda_*) = U(\psi) + \Lambda_* e^{-\psi}.$$

Then

$$Z_\psi \Box\psi - U'_{\text{eff}}(\psi; \Lambda_*) + J_{\text{noise}} = 0.$$

For local calculations at fixed Λ_*, the metric and scalar equations are generated by the sector action

$$S^{\text{loc}}_{\Lambda_*} = \int d^4x\sqrt{-g}\left[\frac{R - 2\Lambda}{2\kappa} - \frac{Z_\psi}{2}(\nabla\psi)^2 - U(\psi) - \Lambda_* e^{-\psi}\right] + S_m + S_{\text{EM}} + S_{\text{noise}},$$

with the understanding that Λ_* is a global momentum fixed by boundary/sector data rather than a Lagrangian coupling. This local sector action is a Routhian representation of the completed topological theory, not a replacement for its global three-form sector.

10 Interpretation of the Vacuum Reaction

The relation

$$\lambda = \Lambda_* e^{-\psi}$$

separates a global conserved quantity from a local reaction. Increasing admissibility lowers the local multiplier response; decreasing admissibility raises it. The multiplier is neither a matter fluid nor an independently adjustable cosmological field. It is an auxiliary reaction whose profile is slaved to ψ, while Λ_* is the momentum of the global topological coordinate.

This is the completed distinction from dynamical Henneaux–Teitelboim models: PVB varies the measure potential to restore gauge invariance but does not assign a kinetic term to the measure or multiplier sectors.

11 Topological ADM Completion and Local Degree-of-Freedom Count

Dualize the three-form to a vector density

$$\tau^\mu \equiv \frac{1}{3!}\tilde{\epsilon}^{\mu\nu\rho\sigma} A_{\nu\rho\sigma}, \qquad \Omega[A] = \partial_\mu \tau^\mu,$$

and write $T \equiv \tau^0$. For a shift-covariant spatial split define

$$\bar{\tau}^i \equiv \tau^i + N^i T, \qquad \tau^i = \bar{\tau}^i - N^i T.$$

The topological term is then

$$\lambda e^\psi \left[\dot{T} + \partial_i \bar{\tau}^i - \partial_i(N^i T)\right] - \lambda N\sqrt{h}.$$

Its primary constraints are

$$\pi_N \approx 0, \qquad \pi_i^{(N)} \approx 0, \qquad p_\lambda \approx 0, \qquad P_i \approx 0, \qquad \Gamma \equiv P_T - \lambda e^\psi \approx 0.$$

The only nonzero bracket inside the multiplier pair is

$$\{p_\lambda(x), \Gamma(y)\} = e^{\psi(y)}\delta^3(x-y),$$

so (p_λ, Γ) is second class. Its preservation fixes multipliers rather than generating a new constraint. Preservation of P_i gives

$$\mathcal{C}_i^A \equiv \partial_i P_T \approx 0,$$

and preservation of $\mathcal{C}_i^A$ gives no further local condition. The Hamilton equation for T reproduces the lock itself. Appendix A displays every step, the unreduced spatial-diffeomorphism generator, and the complete weak algebra.

After eliminating (p_λ, Γ), the Dirac bracket on

$$(h_{ij}, \pi^{ij}; \psi, \pi_\psi; T, P_T)$$

remains the canonical Poisson bracket. The three-form sector has no local mode: for each nonzero Fourier mode, the three constraints $\partial_i P_T$ have rank one, while the three primary constraints P_i remove the three spatial measure variables. The zero mode survives as a global pair.

On a connected slice,

$$P_T = \Lambda_*.$$

The remaining local phase space is

$$(h_{ij}, \pi^{ij}; \psi, \pi_\psi),$$

with dimension 14, and the local constraints are

$$\mathcal{C}_\perp = \mathcal{H}_\perp^{g\psi} + \sqrt{h}\,\Lambda_* e^{-\psi} \approx 0,$$

$$\mathcal{C}_i = -2h_{ik}D_j\pi^{jk} + \pi_\psi \partial_i \psi \approx 0.$$

Because Λ_* is spatially constant and $\sqrt{h}e^{-\psi}$ is a scalar density, these constraints obey the standard hypersurface-deformation algebra. Hence

$$\boxed{N_{\text{local}} = \frac{1}{2}\left[14 - 2(4)\right] = 3 = 2_{\text{tensor}} + 1_\psi.}$$

The completed statement is therefore local and exact: the fixed-density version is obstructed, while the three-form version closes the local first-class complex. Global flux sectors, compact topology, boundary charges, and the interpretation or quantization of the conjugate integrated three-form coordinate remain open.

12 Measure-Adapted Conformal Decomposition and the Diagonal Hessian Coefficient

The completed lock permits the exact decomposition

$$g_{\mu\nu} = e^{\psi/2}\tilde{g}_{\mu\nu}, \qquad \sqrt{-\tilde{g}} = \Omega[A].$$

The Ricci scalar transforms as

$$R = e^{-\psi/2}\left[\tilde{R} - \frac{3}{2}\tilde{\Box}\psi - \frac{3}{8}(\tilde{\nabla}\psi)^2\right].$$

After integration by parts,

$$\sqrt{-g}R = \Omega[A]e^{\psi/2}\left[\tilde{R} + \frac{3}{8}(\tilde{\nabla}\psi)^2\right] + \text{boundary}.$$

On the lock surface, the determinant-adapted action therefore contains

$$S_{\text{det-adapted}} = \int d^4x\, \Omega[A] \left[\frac{e^{\psi/2}}{2\kappa} \tilde{R} - \frac{1}{2} e^{\psi/2} \left(Z_\psi - \frac{3}{8\kappa} \right) (\tilde{\nabla}\psi)^2 - e^{\psi} \left(\frac{\Lambda}{\kappa} + U(\psi) \right) \right].$$

The explicit diagonal entry is

$$\boxed{Z_{\text{diag}} = Z_\psi - \frac{3}{8\kappa}.}$$

It is obtained by holding the remaining measure-adapted metric and three-form variables fixed. It is not the Schur complement of the full constrained Hessian and is not the coefficient of a physical scalar eigenmode.

The completed canonical analysis makes the distinction sharper. In a fixed Λ_* sector, the local theory is ordinary Einstein gravity plus a scalar with potential $U + \Lambda_* e^{-\psi}$. Lapse, shift, scalar metric response, and the nonpropagating measure variables must be eliminated before reading a kinetic residue. The physical constant-background coefficient is Z_ψ; no universal lower bound follows from the sign of Z_{diag}.

13 Constraint-Reduced Scalar Dynamics Around Minkowski

13.1 Background and fixed-sector equations

Let $g_{\mu\nu} = \eta_{\mu\nu}$ and $\psi = \psi_0$ be constant in a sector Λ_*. The background conditions are

$$U'(\psi_0) = \Lambda_* e^{-\psi_0},$$

$$\Lambda + \kappa \left[U(\psi_0) + \Lambda_* e^{-\psi_0} \right] = 0.$$

The measure density is fixed dynamically by the lock,

$$\Omega_0 = e^{-\psi_0},$$

in adapted Minkowski coordinates.

The fixed-sector local action is Einstein gravity plus a minimally coupled scalar with

$$U_{\text{eff}}(\psi) = U(\psi) + \Lambda_* e^{-\psi}.$$

Because $U'_{\text{eff}}(\psi_0) = 0$, the scalar perturbation has no linear stress source. The linear metric equation is the vacuum equation

$$G^{(1)}_{\mu\nu}[h] = 0,$$

while the scalar obeys

$$Z_\psi \Box \varphi - \left[U''(\psi_0) + \Lambda_* e^{-\psi_0} \right] \varphi = 0.$$

13.2 Role of the measure perturbation

Let

$$\sigma \equiv \frac{\delta\Omega}{\Omega_0}.$$

The linearized lock gives

$$\frac{1}{2} h = \varphi + \sigma.$$

Unlike the prescribed-density theory, this relation does not force a physical metric trace: the nonpropagating three-form measure adjusts so that the lock is satisfied. In a measure-adapted gauge $\sigma = 0$, one recovers $h = 2\varphi$ and the explicit Einstein–Hilbert conformal cancellation found in the earlier calculation. In a metric-trace gauge $h = 0$, the same lock is carried by $\sigma = -\varphi$. The two descriptions are gauge allocations of the same local scalar mode.

13.3 Physical quadratic action

After the standard lapse, shift, and scalar metric constraints are eliminated, the local nonzero-mode action is

$$\boxed{S^{(2)}_{\text{phys}} = -\frac{1}{2}\int d^4x \left[Z_\psi \partial_\mu\varphi\partial^\mu\varphi + M^2\varphi^2\right],}$$

where

$$M^2 = U''(\psi_0) + \Lambda_* e^{-\psi_0}.$$

Thus

$$\omega^2 = k^2 + m_\psi^2, \qquad m_\psi^2 = \frac{M^2}{Z_\psi}, \qquad c_s^2 = 1.$$

The local conditions are

$$\boxed{Z_\psi > 0}$$

for absence of a scalar ghost and

$$\boxed{M^2 \geq 0}$$

for absence of a tachyon.

13.4 Status of strong coupling and general backgrounds

The topological completion closes the local gauge count but does not determine the nonlinear cutoff. A cubic and higher-order constrained reduction is still required. On nonconstant backgrounds, the local fixed-sector system is standard Einstein–scalar theory, so its gauge-invariant perturbation action can be used; nevertheless, background-dependent kinetic degeneracies, boundary modes, and singular field redefinitions must be checked for each solution. The exact wall is treated next.

14 Selected-Sector Planar AdS Wall and Scalar Spectrum

This section constructs an exact nonconstant background in a selected global sector, applies the fixed-sector Einstein–scalar perturbation reduction, states the singular operator domain, and audits the spatial spectrum numerically. The positive time-kinetic term follows from the completed fixed-sector master action; the spatial plots are not used as a substitute for that kinetic statement. No outgoing-wave resonance pole is claimed.

14.1 High-frequency characteristic on a smooth background

In a fixed Λ_* sector, the local scalar equation is that of a minimally coupled scalar with normalization Z_ψ. For WKB perturbations $\varphi = \mathcal{A}e^{i\theta/\epsilon}$, the principal scalar term is

$$-Z_\psi \bar{g}^{\mu\nu}k_\mu k_\nu \mathcal{A}, \qquad k_\mu = \bar{\nabla}_\mu\theta.$$

The metric-scalar background-gradient mixing contains fewer derivatives at principal order. Therefore

$$\boxed{\bar{g}^{\mu\nu}k_\mu k_\nu = 0.}$$

The characteristic cone is the metric null cone. The sign of the physical time-kinetic term is not inferred from this equation alone; it is fixed by the sector quadratic action, which is the standard Einstein–scalar action and is positive for $Z_\psi > 0$. A future noise or matter sector with its own second-order derivative coupling would have to be included in the principal matrix.

14.2 Local Einstein–scalar equivalence in a fixed integration sector

For fixed global momentum Λ_*, define

$$\mathcal{U}_{\Lambda_*}(\psi) = U_{\text{bare}}(\psi) + \Lambda_* e^{-\psi} + \frac{\Lambda}{\kappa}.$$

The local equations are exactly

$$G_{\mu\nu} = \kappa \left[Z_\psi \nabla_\mu \psi \nabla_\nu \psi - g_{\mu\nu} \left(\frac{Z_\psi}{2} (\nabla\psi)^2 + \mathcal{U}_{\Lambda_*} \right) \right],$$

$$Z_\psi \Box \psi - \mathcal{U}'_{\Lambda_*}(\psi) = 0.$$

With $\Phi = \sqrt{Z_\psi}(\psi - \psi_c)$, these are the Euler–Lagrange equations of minimally coupled Einstein–scalar theory. The three-form field supplies the measure required by the lock and carries no local perturbative mode. Consequently the standard local gauge-invariant scalar reduction applies in each fixed Λ_* sector. Global fluctuations that change Λ_* are excluded from this local normal-mode problem and belong to the topological zero-mode sector.

14.3 Static planar equations and the flat-bulk obstruction

Use proper normal coordinate y:

$$ds^2 = e^{2A(y)} \eta_{ab} dx^a dx^b + dy^2, \qquad a, b = 0, 1, 2, \qquad \Phi = \Phi(y).$$

The independent equations are

$$3A_y^2 = \kappa \left(\frac{1}{2} \Phi_y^2 - V_{\Lambda_*} \right),$$

$$2A_{yy} + 3A_y^2 = -\kappa \left(\frac{1}{2} \Phi_y^2 + V_{\Lambda_*} \right),$$

$$\Phi_{yy} + 3A_y \Phi_y - \partial_\Phi V_{\Lambda_*} = 0.$$

Subtracting the Einstein equations gives

$$\boxed{A_{yy} = -\frac{\kappa}{2} \Phi_y^2 \leq 0.}$$

For a nontrivial wall,

$$A_y(+\infty) - A_y(-\infty) = -\frac{\kappa}{2} \int_{-\infty}^{+\infty} \Phi_y^2 \, dy < 0.$$

Therefore a static flat-sliced wall cannot have $A_y \to 0$ at both infinities. Within this branch, no nontrivial static wall is Minkowski on both sides.

In the thin-wall flat-bulk limit, an unsupported wall instead has expanding worldvolume [14, 15],

$$H = \frac{\kappa\sigma}{4}.$$

For the gravity-off tanh profile,

$$\sigma = \frac{4 Z_\psi a^2}{3\delta}, \qquad \epsilon_g \equiv \kappa Z_\psi a^2, \qquad H\delta = \frac{\epsilon_g}{3}.$$

The prescribed flat wall used as an optical toy profile is therefore quasi-static or externally supported, not the exact static solution below.

14.4 Fixed bare potential and selected-sector exact wall

The Lagrangian potential must be fixed independently of the global momentum produced by the three-form equation. Introduce a fixed parameter Λ_0 and choose the fake superpotential [16]

$$\mathscr{W}(\Phi) = \frac{v}{\delta}\left(\Phi - \frac{\Phi^3}{3v^2}\right), \qquad v = \sqrt{Z_\psi}\, a, \qquad \epsilon_g = \kappa v^2.$$

Define the target Einstein–scalar potential

$$V_{\text{target}}(\Phi) = \frac{1}{2}\mathscr{W}_{,\Phi}^2 - \frac{3\kappa}{4}\mathscr{W}^2$$

and the fixed bare PVB potential

$$\boxed{U_{\text{bare}}(\psi;\Lambda_0) = V_{\text{target}}\left(\sqrt{Z_\psi}(\psi-\psi_c)\right) - \Lambda_0 e^{-\psi} - \frac{\Lambda}{\kappa}.}$$

In a general integration sector,

$$\mathcal{U}_{\Lambda_*}(\psi) = V_{\text{target}}(\Phi) + (\Lambda_* - \Lambda_0)e^{-\psi}.$$

The exact symmetric wall exists in the selected branch

$$\boxed{\Lambda_* = \Lambda_0.}$$

In that branch, the first-order system

$$\Phi_y = \mathscr{W}_{,\Phi}, \qquad A_y = -\frac{\kappa}{2}\mathscr{W}$$

solves the full background equations and gives

$$\boxed{\Phi(y) = v\tanh x, \qquad x = \frac{y}{\delta},}$$

$$\boxed{A(y) = -\epsilon_g\left[\frac{1}{3}\ln\cosh x + \frac{1}{12}\tanh^2 x\right], \qquad A(0) = 0.}$$

At either vacuum,

$$V_{\text{target,vac}} = -\frac{3\kappa}{4}\mathscr{W}_{\text{vac}}^2 = -\frac{\kappa v^4}{3\delta^2} < 0,$$

and

$$A(y) \sim -\frac{\epsilon_g}{3\delta}|y| \qquad (|y| \to \infty).$$

The selected-sector static wall is therefore asymptotically AdS on both sides. Sector selection $\Lambda_* = \Lambda_0$ is an explicit tuning condition, not an identity built into the action. It selects the branch on which this particular reconstructed wall is exact; it is not presented as a solution of the cosmological-constant problem or as a dynamical selection principle.

14.5 Master action and singular physical domain

Introduce conformal normal coordinate r by

$$dr = e^{-A(y)}dy,$$

and define

$$\mathcal{H}_A = \frac{dA}{dr}, \qquad z(r) = e^A\frac{d\Phi/dr}{\mathcal{H}_A} = e^A\frac{\Phi_y}{A_y}.$$

The fixed-sector Einstein–scalar reduction gives the canonical master action [17]

$$\boxed{S_{\rm sc}^{(2)} = \frac{1}{2}\int dr\, d^3x \left[(\partial_t G)^2 - (\nabla_\parallel G)^2 - (\partial_r G)^2 - \frac{z''}{z}G^2\right].}$$

The time-kinetic coefficient is positive. Normal modes obey

$$\mathcal{H}_+ f = m^2 f, \qquad \mathcal{H}_+ = -\frac{d^2}{dr^2} + \frac{z''}{z}.$$

With

$$Q = \frac{d}{dr} + \frac{z'}{z}, \qquad Q^\dagger = -\frac{d}{dr} + \frac{z'}{z},$$
$$\mathcal{H}_+ = QQ^\dagger.$$

For the exact wall,

$$z(r) \sim \frac{C}{r}, \qquad \mathcal{H}_+ \sim -\frac{d^2}{dr^2} + \frac{2}{r^2} \qquad (r \to 0).$$

The local homogeneous behaviors are r^2 and r^{-1}; only r^2 is square integrable. Since $2 > 3/4$, the center is limit-point on each half-line and introduces no self-adjoint extension parameter. The maximal half-line domain is

$$\mathcal{D}_{\rm max}^{\pm} = \{f \in L^2(\mathbb{R}_\pm) : f, f' \in AC_{\rm loc},\ \mathcal{H}_+ f \in L^2\}.$$

The semibounded normal operator is the Friedrichs realization of the quadratic form

$$\mathfrak{q}[f] = \|Q^\dagger f\|^2.$$

Hence

$$\langle f, \mathcal{H}_+ f\rangle = \|Q^\dagger f\|^2 \geq 0.$$

The formal zero solution $f_0 \propto z \sim 1/r$ is not normalizable. The remaining domain issue is not positivity but the precise left–right matching relation induced by regular gauge-invariant metric–scalar perturbations across the wall center.

14.6 Regular partner potential and failure of automatic isospectrality

The algebraically regular partner expression is

$$\mathcal{H}_- = Q^\dagger Q = -\frac{d^2}{dr^2} + V_-(r).$$

Writing

$$t = \tanh x, \qquad x = \frac{y}{\delta},$$

one obtains

$$\boxed{V_-(r(x)) = \frac{e^{2A(x)}}{\delta^2}\left[2(1+t^2) + \frac{\epsilon_g}{3}(2t^4 - 3t^2 + 3)\right].}$$

The bracket is positive for $0 \leq t^2 \leq 1$, while

$$V_-(|r| \to \infty) \to 0$$

for every $\epsilon_g > 0$. The partner continuum therefore begins at $m^2 = 0$.

The regular full-line partner is not automatically isospectral to the physical singular operator. Near the center,

$$Q \sim \frac{d}{dr} - \frac{1}{r}.$$

For a regular even partner function

$$g(r) = g_0 + O(r^2),$$

one finds

$$Qg \sim -\frac{g_0}{r},$$

which is not in the L^2 domain of $\mathcal{H}_+$. Parity and center-domain conditions therefore matter.

At exactly $\epsilon_g = 0$,

$$V_-\delta^2 = 4 - 2\,\mathrm{sech}^2\, x$$

has the normalized bound state

$$g_0(x) = \frac{1}{\sqrt{2}}\,\mathrm{sech}\, x, \qquad m^2\delta^2 = 3.$$

Because $g_0(0) \neq 0$, its image under Q is singular at the center. The present analysis therefore does not identify this partner state with an admissible physical master mode when gravity is nonzero.

14.7 Inner-region Rayleigh diagnostic

Although no physical resonance is claimed, the regular partner potential retains a localized inner well continuously connected to the $\epsilon_g = 0$ partner problem. Write

$$A(\xi) = \epsilon_g \alpha(\xi), \qquad \xi = \frac{y}{\delta}, \qquad \alpha(\xi) = -\frac{1}{3}\ln\cosh\xi - \frac{1}{12}\tanh^2\xi.$$

Let

$$C_0 = 2(1+\tanh^2\xi), \qquad C_1 = \frac{1}{3}\left(2\tanh^4\xi - 3\tanh^2\xi + 3\right).$$

For a fixed trial function $g(\xi)$, the dimensionless Rayleigh functional of the full partner expression is

$$\mathcal{R}_{\epsilon_g}[g] = \frac{\displaystyle\int_{-\infty}^{\infty} d\xi\, e^{A}\left[(g')^2 + (C_0 + \epsilon_g C_1)g^2\right]}{\displaystyle\int_{-\infty}^{\infty} d\xi\, e^{-A} g^2}.$$

At $\epsilon_g = 0$, use

$$g_0(\xi) = \frac{1}{\sqrt{2}}\,\mathrm{sech}\,\xi, \qquad \mathcal{R}_0[g_0] = 3.$$

Differentiating at fixed g_0 gives

$$\left.\frac{d\mathcal{R}_{\epsilon_g}[g_0]}{d\epsilon_g}\right|_{\epsilon_g=0} = \int_{-\infty}^{\infty} d\xi\left[\alpha\left((g_0')^2 + C_0 g_0^2\right) + C_1 g_0^2 + 3\alpha g_0^2\right].$$

With $u = \tanh\xi$, this becomes

$$\int_{-1}^{1} du\left[\frac{1}{2} - \frac{17}{24}u^2 + \frac{5}{24}u^4 + \left(\frac{5}{12} + \frac{1}{4}u^2\right)\ln(1-u^2)\right] = \ln 4 - \frac{3}{2}.$$

Therefore

$$\boxed{\mathcal{R}_{\epsilon_g}[g_0] = 3 + \left(\ln 4 - \frac{3}{2}\right)\epsilon_g + O(\epsilon_g^2),}$$

where

$$\ln 4 - \frac{3}{2} = -0.113705638880109\ldots.$$

The negative coefficient shows that this localized trial functional decreases at first order. Because the continuum threshold is zero for every $\epsilon_g > 0$, this is not an ordinary discrete-eigenvalue perturbation. It does not establish a physical excitation or resonance pole. Such a claim requires the physical domain map and an outgoing-wave or analytically continued scattering calculation.

14.8 Numerical background and spectral audit

The accompanying reproducibility script evaluates the exact background at $v = \delta = \kappa = 1$ ($\epsilon_g = 1$). Substitution into the two independent Einstein equations and the scalar equation gives a maximum double-precision residual

$$2.8 \times 10^{-16}.$$

Figure 1 shows the analytic wall and warp factor.

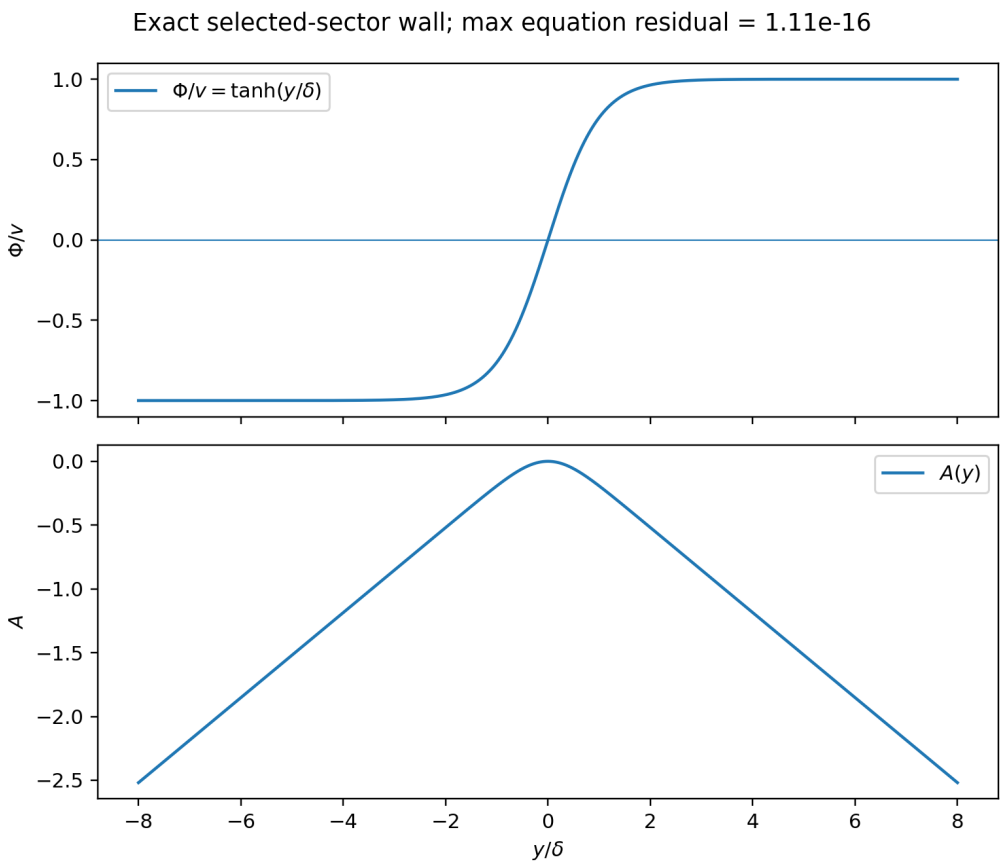

Figure 1: Exact selected-sector wall and warp factor. The displayed residual is the maximum absolute residual of the three background equations on the plotted grid.

Figure 2 constructs $V_+ = z''/z$ in conformal radius and discretizes $\mathcal{H}_+$ on half-line boxes with the regular center condition. For box radii $R/\delta = 10, 20, 40, 80$ and spacing 0.02δ, the lowest levels are

$$m^2_{\min}\delta^2 = 0.863,\ 0.268,\ 0.0751,\ 0.0200.$$

They approach the continuum threshold from above. At fixed $R/\delta = 40$, refining the spacing $h/\delta = 0.08, 0.04, 0.02, 0.01$ gives

$$0.075119,\ 0.075107,\ 0.075110,\ 0.075109,$$

showing that the quoted infrared trend is not a grid artifact. This is a finite-volume audit, not the proof of positivity; positivity follows from the master action and Friedrichs factorization.

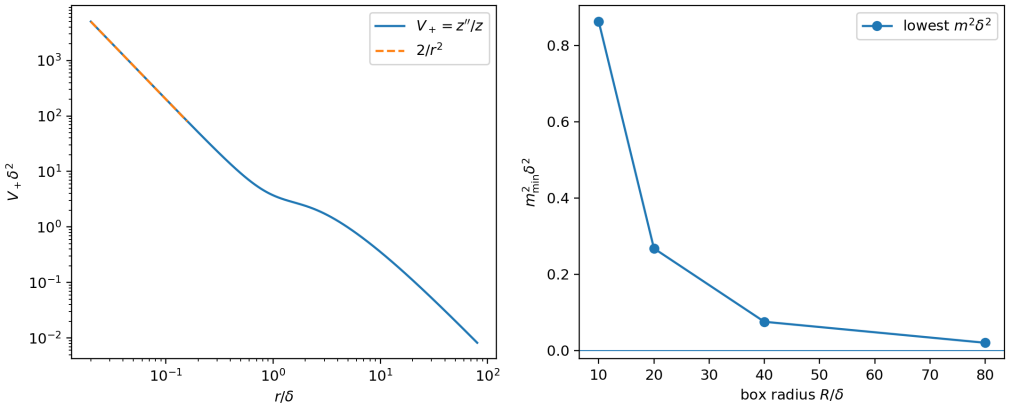

Figure 2: Singular master potential and finite-box approach to the $m^2 = 0$ threshold. No negative spatial mode is observed. Ghost freedom is a separate statement supplied by the positive time-kinetic term in the fixed-sector master action.

The partner-potential and intertwining audits are shown in Figures 3 and 4. They make the domain mismatch visible: the flat-space regular partner state is not carried into an admissible $\mathcal{H}_+$ state.

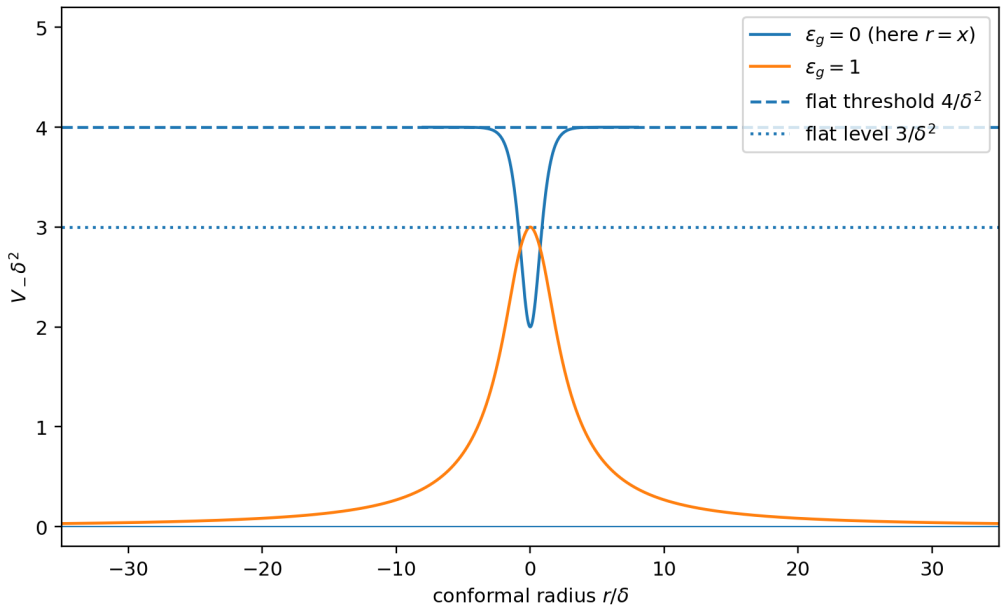

Figure 3: For $\epsilon_g = 0$, the regular partner has threshold $4/\delta^2$ and a discrete level at $3/\delta^2$. For $\epsilon_g > 0$, the asymptotic threshold is 0; the former level is embedded in the continuum rather than remaining a discrete bound state.

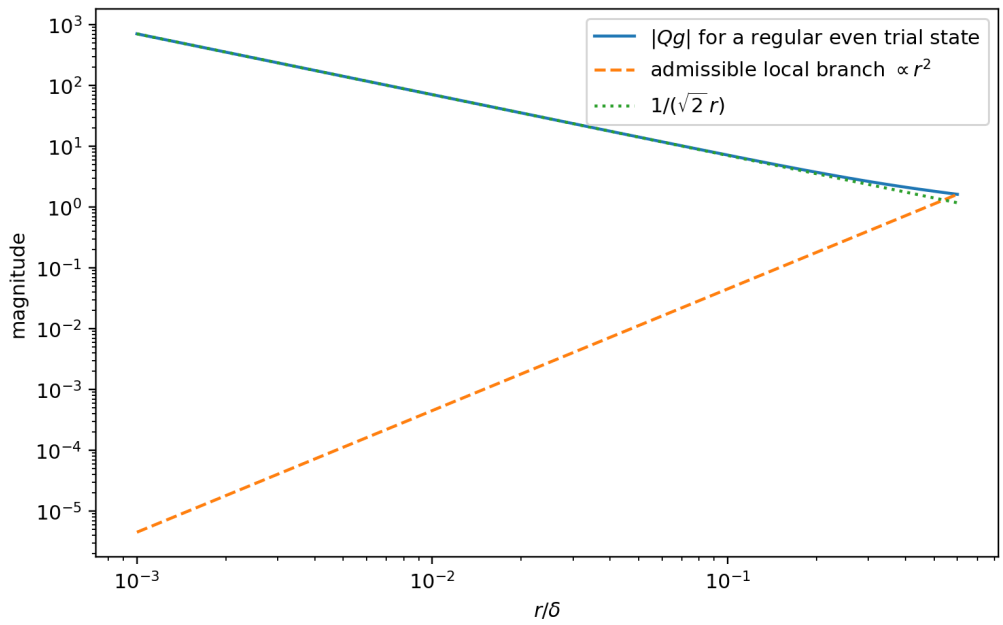

Figure 4: The intertwining image Qg of a regular even partner trial state diverges as $-g(0)/r$, whereas the square-integrable local branch of $\mathcal{H}_+$ behaves as r^2.

14.9 Status and relation to the optical benchmark

Statement	Status
Three-form completion and local $2+1$ count	Derived, subject to standard boundary regularity; a global topological pair remains.
Selected-sector tanh background and warp factor	Derived after fixing $U_{\text{bare}}(\psi;\Lambda_0)$ and imposing $\Lambda_* = \Lambda_0$.
Flat-bulk obstruction and AdS asymptotics	Derived within the static flat-sliced ansatz.
Metric-null high-frequency characteristic	Derived in the fixed-sector Einstein–scalar system.
Positive scalar master time-kinetic term	Derived for $Z_\psi > 0$ in the fixed Λ_* sector.
Nonnegative $\mathcal{H}_+$	Derived for the Friedrichs realization; physical left–right matching remains to be stated in original variables.
Formal zero solution	Excluded because $z \sim 1/r$ is not square integrable.
Finite-box spectrum	Reproducible numerical audit; lowest level approaches 0^+ as box size grows.
Partner bound state	Not a physical wall mode; threshold becomes zero and the intertwining image violates the center domain.
Inner Rayleigh coefficient	Derived diagnostic; no resonance pole claimed.
Strong-coupling scale	Open pending cubic reduction.

The exact wall background is planar and asymptotically AdS, so it cannot inherit a spherical light-ring equation whose areal-radius factor is essential. Section 15 shows that the bare scalar does not provide the desired positive-energy spherical floor on the benchmark branch, while an added conserved tangential load can produce a radially stable junction equilibrium. That result still does not derive a finite-thickness optical shell or a Gordon constitutive law.

15 Static Spherical Audit and Loaded-Shell Extension

This section tests the completed local core in spherical symmetry. The scalar-only result and the loaded-shell result are deliberately separated: the former follows from the fixed-sector Einstein–

scalar core, while the latter introduces an additional conserved matter stress. Neither result is used to establish the canonical closure derived earlier.

15.1 What spherical completion means

The completed PVB core uses the scalar-weighted topological lock

$$\sqrt{-g} = e^{\psi}\Omega[A], \qquad d(\lambda e^{\psi}) = 0, \qquad \lambda e^{\psi} = \Lambda_*.$$

In a fixed global sector Λ_*, the local equations are generated by

$$S_{\Lambda_*} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}Z_\psi(\nabla\psi)^2 - \mathcal{U}_{\Lambda_*}(\psi)\right] + S_{\text{matter}},$$

where

$$\mathcal{U}_{\Lambda_*}(\psi) = U_{\text{bare}}(\psi) + \Lambda_* e^{-\psi} + \frac{\Lambda}{8\pi G}.$$

Define the canonical scalar

$$\Phi = \sqrt{Z_\psi}\,(\psi - \psi_{\text{out}}), \qquad V(\Phi) = \mathcal{U}_{\Lambda_*}(\psi(\Phi)).$$

The local spherical problem is therefore an Einstein–scalar problem. The specifically PVB information is the interpretation of Φ and the topological origin of the exponential sector term.

A spherical calculation must answer three separate questions:

1. Can the core scalar and gravity alone support a regular static compact phase region?
2. If not, what additional stress can produce a self-consistent spherical equilibrium?
3. Does the resulting equilibrium survive finite thickness, leakage, formation, and nonspherical perturbations?

This section completes the first question under stated conditions and gives a radially stable thin-shell answer to the second. The third remains open.

15.2 Full static spherical reduction

Use areal radius and write

$$ds^2 = -e^{2\delta(r)}N(r)dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega_2^2, \qquad N(r) = 1 - \frac{2Gm(r)}{r}.$$

For a static canonical scalar $\Phi(r)$, the independent equations are

$$m' = 4\pi r^2\left[\frac{1}{2}N\Phi'^2 + V(\Phi)\right], \tag{4}$$

$$\delta' = 4\pi G r\,\Phi'^2, \tag{5}$$

$$\left(e^{\delta}r^2 N\Phi'\right)' = e^{\delta}r^2 V_{,\Phi}. \tag{6}$$

The regular-center and asymptotically flat conditions are

$$m(0) = 0, \qquad \Phi'(0) = 0, \qquad \Phi(r) \to 0, \qquad V(0) = V_{,\Phi}(0) = 0, \qquad \delta(\infty) = 0.$$

The ADM mass is $M = m(\infty)$. A horizonless solution also requires $N(r) > 0$ everywhere.

Near the center,

$$\Phi(r) = \Phi_c + \frac{V_{,\Phi}(\Phi_c)}{6}r^2 + O(r^4), \qquad m(r) = \frac{4\pi}{3}V(\Phi_c)r^3 + O(r^5).$$

These equations include full spherical backreaction. They are not the scalar equation on a prescribed metric.

15.2.1 Local reconstruction of the three-form measure

For the metric above,

$$\sqrt{-g} = e^{\delta} r^2 \sin\theta.$$

The PVB lock requires

$$\Omega[A] = e^{-\psi(r)} e^{\delta(r)} r^2 \sin\theta.$$

Dualize the three-form to a vector density τ^μ with $\Omega[A] = \partial_\mu \tau^\mu$. On a local spherical patch one may choose

$$\tau^r(r,\theta) = \sin\theta \int_0^r ds\, e^{\delta(s)-\psi(s)} s^2, \qquad \tau^t = \tau^\theta = \tau^\phi = 0.$$

Then $\partial_r \tau^r = \Omega[A]$. Thus any regular local Einstein–scalar solution in the selected sector admits a local three-form reconstruction. The measure field enforces the lock but does not add an independent local radial force.

15.3 Scalar-only static obstruction

The scalar stress tensor has

$$\rho = \frac{1}{2} N \Phi'^2 + V, \qquad p_r = \frac{1}{2} N \Phi'^2 - V, \qquad p_t = -\frac{1}{2} N \Phi'^2 - V.$$

Using Eq. (6), the radial pressure satisfies the exact identity

$$p_r' = -\Phi'^2\, \mathcal{F}(r), \qquad \mathcal{F}(r) = \frac{2N}{r} + N\delta' + \frac{1}{2} N'. \tag{7}$$

Equivalently, after using the Einstein equations,

$$\mathcal{F}(r) = \frac{1+3N}{2r} + 4\pi G r\, p_r.$$

Conditional no-soliton statement. Assume

1. $N(r) > 0$ and the solution is regular and asymptotically flat;
2. $V(\Phi) \geq 0$ along the profile;
3. $\mathcal{F}(r) > 0$ wherever $\Phi' \neq 0$.

At the center, $p_r(0) = -V(\Phi_c) \leq 0$. At infinity, $p_r(\infty) = 0$. Equation (7) says that p_r is nonincreasing outward. It cannot begin at a nonpositive value and approach the larger value zero unless $p_r = 0$ and $\Phi' = 0$ everywhere. Therefore the only solution under these conditions is the trivial vacuum.

This is not a theorem against every possible real-scalar soliton. It identifies the precise escape routes: the physical potential must become negative somewhere, the geometry must make $\mathcal{F}$ change sign, a horizon or singular branch must intervene, the scalar must be time dependent or charged, or an additional matter stress must be present.

For an approximately de Sitter core with $V = \rho_v > 0$ and

$$N \simeq 1 - H^2 r^2, \qquad H^2 = \frac{8\pi G \rho_v}{3},$$

one finds

$$\mathcal{F} \simeq \frac{2}{r} - 3H^2 r.$$

It is positive for $Hr < \sqrt{2/3}$. The compact benchmark constructed below has $HR_0 \simeq 0.340$, well inside this positive-friction region. The intended scalar-only positive-energy core is therefore on the obstructed branch.

The thin-wall energy gives the same physical diagnosis. For a spherical phase wall with tension σ and positive interior-exterior vacuum difference $\Delta V > 0$,

$$E(R) \simeq 4\pi\sigma R^2 + \frac{4\pi}{3}\Delta V R^3.$$

Both terms decrease when the sphere contracts. Neither provides a static radius.

15.4 Minimal extended model: a conserved surface load

The previous obstruction says that the completed PVB core does not secretly contain the required stabilizer. Introduce a minimal additional source: a conserved two-dimensional ultrarelativistic population confined to the phase boundary.

Let

$$f_-(R) = 1 - H^2R^2, \qquad f_+(R) = 1 - \frac{2M}{R}.$$

The interior is de Sitter-like and the exterior is Schwarzschild. The shell carries scalar tension σ and a conserved load with energy $E_L = \Xi/R$. Its surface density and tangential pressure are

$$\Sigma(R) = \sigma + \frac{\Xi}{4\pi R^3}, \qquad P(R) = -\sigma + \frac{\Xi}{8\pi R^3}.$$

The standard-orientation Israel junction equation is [18, 19]

$$\sqrt{\dot{R}^2 + f_-} - \sqrt{\dot{R}^2 + f_+} = 4\pi G\Sigma R.$$

Use geometrized units $G = 1$ and define

$$x = \frac{R}{M}, \qquad h = HM, \qquad q = 4\pi\sigma M, \qquad \zeta = \frac{\Xi}{M^2}, \qquad \mathcal{S}(x) = qx + \frac{\zeta}{x^2}.$$

Then

$$\dot{x}^2 + V_{\rm sh}(x) = 0,$$

with dimensionless proper time τ/M and

$$V_{\rm sh}(x) = 1 - \frac{2}{x} - \frac{\left[\frac{2}{x} - h^2x^2 - \mathcal{S}^2(x)\right]^2}{4\mathcal{S}^2(x)}. \tag{8}$$

The ordinary de Sitter-interior/Schwarzschild-exterior branch requires both orientation functions to be positive.

15.5 Closed-form static equilibrium family

At a static standard-orientation shell,

$$\mathcal{S}(x) = \Delta(x; h) \equiv \sqrt{1 - h^2x^2} - \sqrt{1 - \frac{2}{x}}.$$

A true equilibrium is a double root of the radial potential. Equivalently,

$$\mathcal{S}'(x) = \Delta'(x; h),$$

where

$$\Delta' = -\frac{h^2x}{\sqrt{1 - h^2x^2}} - \frac{1}{x^2\sqrt{1 - 2/x}}.$$

Because $\mathcal{S} = qx + \zeta/x^2$, the two equilibrium conditions are linear in the two component loads. Solving them gives

$$q_{\rm eq}(x,h) = \frac{2\Delta + x\Delta'}{3x}, \tag{9}$$

$$\zeta_{\rm eq}(x,h) = \frac{x^2}{3}\left(\Delta - x\Delta'\right). \tag{10}$$

These formulas parameterize the entire static radial family before finite-thickness or nonspherical tests. A physical point requires

$$x > 2, \qquad hx < 1, \qquad \Delta > 0, \qquad q_{\rm eq} \geq 0, \qquad \zeta_{\rm eq} \geq 0.$$

At such a point the orientation functions reduce to

$$\beta_- = \sqrt{1 - h^2x^2} > 0, \qquad \beta_+ = \sqrt{1 - 2/x} > 0.$$

Radial stability at fixed h, q, and ζ requires

$$V''_{\rm sh}(x_0) > 0.$$

The linear radial mode then obeys

$$\delta\ddot{x} + \frac{1}{2}V''_{\rm sh}(x_0)\delta x = 0.$$

Figure 5 shows the physical equilibrium family found from Eqs. (9)–(10). Stable radial equilibria are not isolated; a continuous stable branch extends into the ultracompact region $2 < x < 3$.

15.6 Ultracompact benchmark

To connect directly with the earlier loaded-shell calculation, retain

$$q = 4\pi\sigma M = 0.0025$$

and require a static shell at

$$x_0 = R_0/M = 2.8.$$

Solving Eq. (9) for h, followed by Eq. (10), gives

$$\boxed{h = 0.1213639119, \qquad \zeta = 3.127912158.}$$

The numerical audit gives the values in Table 3.

The shell is outside the Schwarzschild radius $2M$ and inside the exterior photon radius $3M$. The exterior Schwarzschild photon sphere therefore remains part of the physical exterior geometry, while no event horizon is present in the reduced construction.

The component-energy diagnostic is

$$\frac{E_L}{M} = \frac{\zeta}{x} = 1.11711, \qquad \frac{E_{\rm wall}}{M} = qx^2 = 0.01960, \qquad \frac{E_{\rm dS}}{M} = \frac{1}{2}h^2x^3 = 0.16167.$$

Thus

$$\frac{E_L}{E_L + E_{\rm wall} + E_{\rm dS}} = 0.8604.$$

This is not an ADM-mass decomposition. It is a component comparison showing that the stabilizing stress is load-dominated. The surface load density is about 57 times the scalar tension density at the shell.

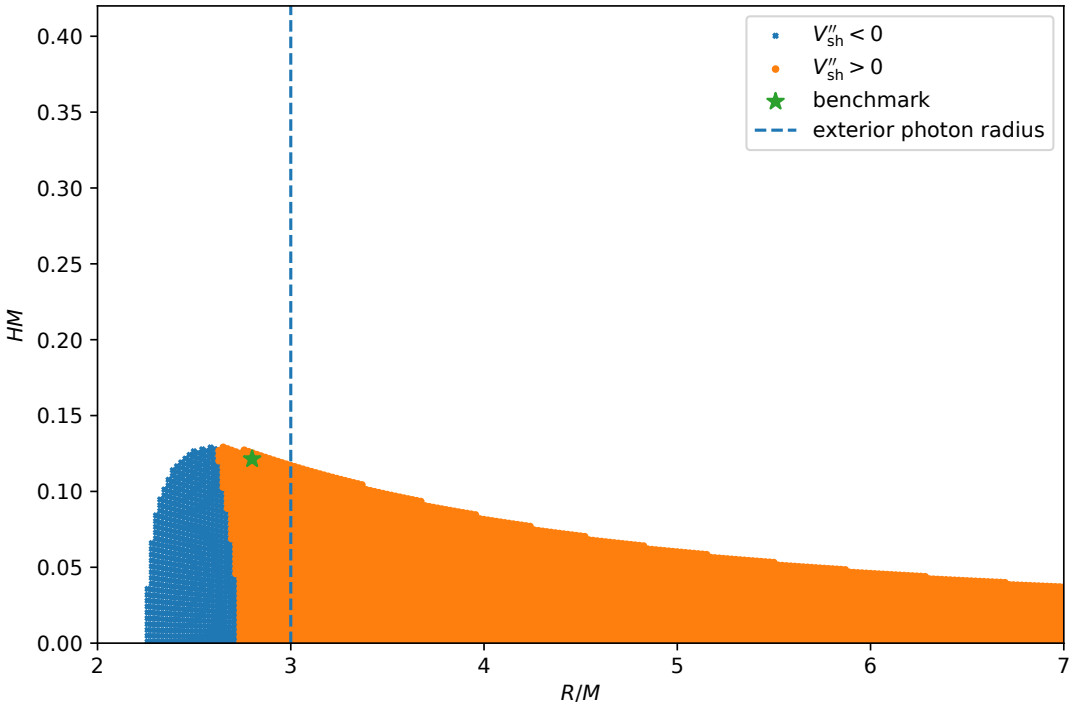

Figure 5: Physical standard-orientation loaded-shell equilibria with nonnegative tension and load. The plotted classification is only radial: circles have $V''_{\rm sh} > 0$ and crosses have $V''_{\rm sh} < 0$. The vertical line is the exterior Schwarzschild photon radius.

Table 3: Radially stable ultracompact equilibrium benchmark.

Quantity	Value
R_0/M	2.8000000
HM	0.1213639
$4\pi\sigma M$	0.0025000
Ξ/M^2	3.1279122
$V_{\rm sh}(R_0)$	-1.1×10^{-16}
$V'_{\rm sh}(R_0)$	0
$V''_{\rm sh}(R_0)$	0.1140778
$\beta_-(R_0)$	0.9404909
$\beta_+(R_0)$	0.5345225
HR_0	0.3398189
radial frequency ωM	0.2388282
radial period T/M	26.3084

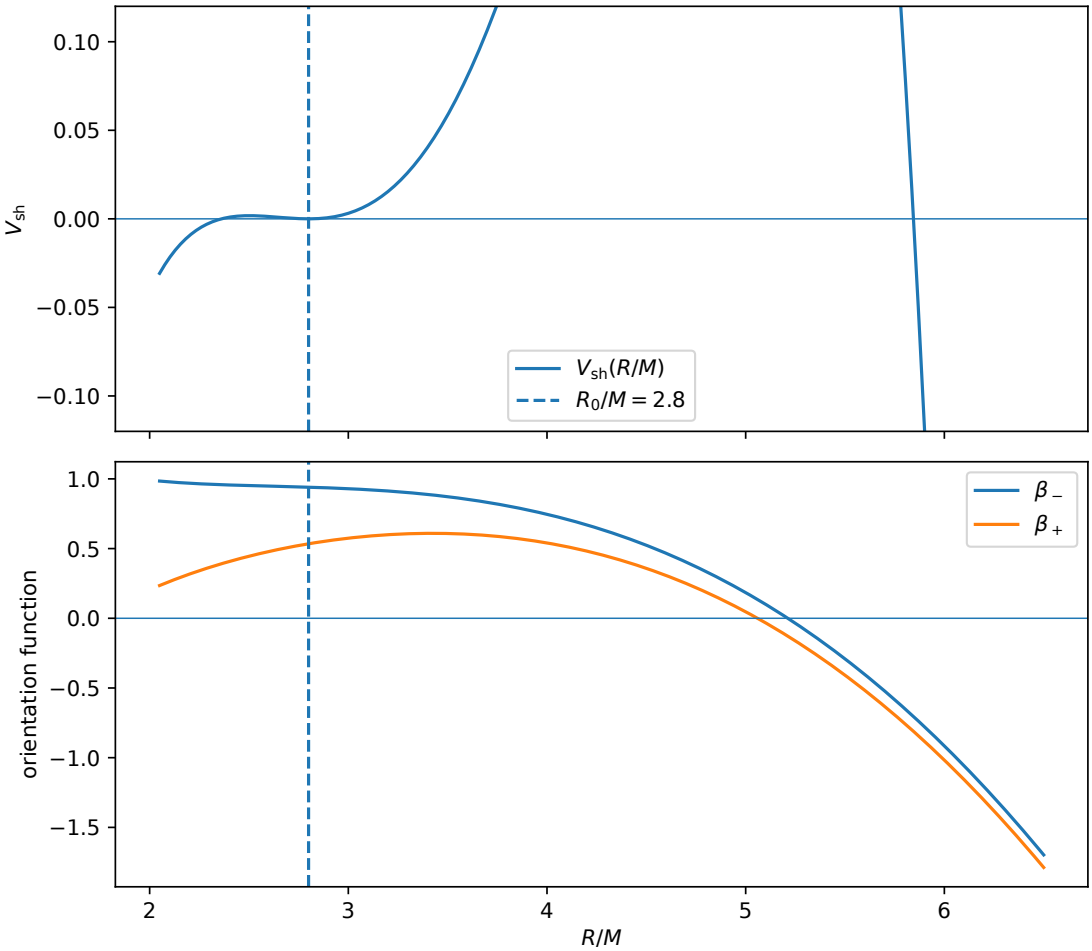

Figure 6: Effective radial potential and orientation functions for the benchmark. The shell sits at a double root with positive curvature. Both orientation functions are positive at the equilibrium. Other algebraic branches outside the local equilibrium region must not be confused with the selected standard-orientation shell.

15.7 What has been solved and what has not

Established in this section

1. The completed PVB core has a well-defined full static spherical reduction.
2. The three-form measure can be reconstructed locally and supplies no hidden local stabilizing pressure.
3. A nonnegative scalar-only positive-energy core is obstructed under the explicit positive-friction condition.
4. The conserved-load thin-shell model has a closed-form static equilibrium family.
5. That family contains standard-orientation, horizonless, radially stable ultracompact points, including the explicit $R_0 = 2.8M$ benchmark.

Not established

1. The scalar field and load have not been solved as a single finite-thickness matter system.
2. Radial junction stability is not nonspherical stability.
3. The conserved load has no controlled microscopic production and confinement mechanism in the present effective model.
4. Leakage, radiation, charge transfer, and formation before trapped-surface creation remain open.
5. The optical constitutive response is not derived by this calculation.
6. The global three-form flux and boundary sector have not been fixed.

The calculation therefore changes the spherical question. The issue is no longer whether the reduced gravitational junction equations can admit a horizonless radial equilibrium. They can. The decisive issue is whether PVB can supply a controlled finite-thickness phase boundary and a conserved tangential stress that survives all perturbations and formation constraints.

15.8 Next decisive calculation

A legitimate continuation must replace the distributional shell by a covariant finite-thickness action. The minimum required model contains

1. the completed PVB Einstein–scalar core;
2. a matter field whose mass or gap changes across the scalar phase boundary;
3. a conserved occupation or charge producing tangential pressure;
4. the full spherical metric and matter equations;
5. boundary data yielding a regular center and asymptotically flat exterior.

The numerical task is then a coupled boundary-value problem, followed by radial and nonspherical perturbation calculations. A smooth solution must converge toward the junction benchmark as the thickness decreases. Failure of that convergence would reject the loaded-shell mechanism even though the idealized junction equilibrium is correct.

15.9 Spherical result and remaining burden

The spherical calculation returns a split verdict.

The completed PVB scalar and gravity do not, by themselves, provide the desired static positive-energy spherical floor in the physically relevant branch. The topological measure fixes the consistency of the theory but does not create a new local pressure. This is a genuine negative result for an intrinsic scalar-only PVB compact object.

A conserved tangential load changes the result. The de Sitter–Schwarzschild junction problem has an analytically parameterized family of static equilibria, and that family includes a radially stable horizonless shell at $R = 2.8M$. This is a real self-consistent spherical gravity benchmark rather than a prescribed scalar profile. It is also explicitly load-driven.

The PVB compact-object program therefore survives in a narrower form: the scalar can define the phase boundary, while an additional conserved wall sector must supply most of the stabilizing stress. Whether that extended construction is physically controlled remains the next decisive falsification test.

16 What the Completion Means for PVB

In each fixed topological sector, the local equations are those of minimally coupled Einstein–scalar gravity with

$$\mathcal{U}_{\Lambda_*}(\psi) = U_{\text{bare}}(\psi) + \Lambda_* e^{-\psi} + \frac{\Lambda}{\kappa}.$$

This local equivalence is a result, not a loss of identity. It tells us exactly which part of PVB is new and which part uses established gravitational dynamics.

The local wall equations and perturbation equations are standard Einstein–scalar equations once Λ_* is fixed. The PVB distinction is how that sector term enters and what the scalar represents:

$$\sqrt{-g} = e^{\psi}\Omega[A] \quad \Longrightarrow \quad P_T = \lambda e^{\psi} = \Lambda_*.$$

The coefficient of the exponential scalar term is not inserted only as an ordinary local coupling; it is the conserved momentum of a nonpropagating global measure sector. The field ψ is the surviving admissibility degree of freedom.

The completion therefore preserves the central PVB structure in a more specific form. It preserves a dynamical admissibility scalar, a vacuum reaction slaved exponentially to that scalar, and a measure-based origin for the global sector parameter. It does not preserve the earlier rigid reference-density interpretation. The three-form can absorb local measure variation, so the lock is no longer an extra local restriction on the metric determinant relative to a fixed background profile.

PVB is consequently broader than this paper, but it is no longer undefined at its gravitational foundation. The present work establishes the locally consistent gravitational layer. The microscopic meaning of admissibility, its matter coupling, the optical constitutive response, and a covariant finite-thickness spherical realization remain independent burdens.

17 Limitations: What the Formal Core Does Not Yet Prove

The completion does not derive a microscopic substrate, an electromagnetic constitutive law, a screening mechanism, or compatibility with all fifth-force and equivalence-principle constraints.

The local canonical obstruction of a generic prescribed density is closed by the three-form measure, and the local count is $2 + 1$. The remaining canonical questions are global: boundary generators, flux sectors, compact topology, possible quantization, and the interpretation of the global conjugate coordinate.

The selected-sector wall requires the fixed bare parameter Λ_0 and the branch condition $\Lambda_* = \Lambda_0$. It is an exact existence result, not a dynamical explanation of sector selection.

The fixed-sector wall master action has positive time-kinetic term for $Z_\psi > 0$ and a nonnegative Friedrichs spatial operator. The precise matching map across the singular center should still be written directly in regular gauge-invariant metric–scalar variables. No outgoing-wave resonance pole has been computed.

The exact wall is planar and asymptotically AdS. The spherical audit establishes a radially stable distributional shell only after adding a conserved tangential load; it does not establish a finite-thickness optical object, a formation channel, leakage control, or nonspherical stability. FLRW stability, nonlinear reduced-Hamiltonian positivity beyond the quadratic fixed-sector problem, fluctuation-level noise dynamics, and the cubic strong-coupling scale also remain open.

18 Open Problems and Failure Conditions

OP1 — Microscopic Meaning of the Topological Measure The three-form completion removes the preferred fixed density but does not explain why admissibility should couple to the spacetime measure through $e^\psi dA$. A microscopic or coarse-grained derivation remains required.

OP2 — Matter Coupling Any nonminimal ψ or A coupling must avoid fifth-force bounds, equivalence-principle violations, radiative instability, and unwanted energy exchange.

OP3 — Lorentz-Invariant Ensemble Limit The substrate motivation requires a continuum limit that preserves Lorentz invariance statistically.

OP4 — Strong Coupling The local quadratic scalar is healthy for $Z_\psi > 0$. A physical cutoff requires the cubic and higher constrained action.

OP5 — Optical Bridge The Gordon response remains phenomenological. Its constitutive law and medium four-velocity must be derived or constrained.

OP6 — Finite-Thickness Loaded Shell The distributional spherical equilibrium is load-dominated. A covariant scalar–matter boundary-value problem must reproduce the thin-shell limit while controlling gradient energy, leakage, radiation, charge transfer, backreaction, and causal formation.

OP7 — Global Topological Sector Flux, boundary terms, global canonical variables, compact topology, and possible sector transitions remain open. A failure to define a consistent global variational principle would invalidate the completion beyond local patches.

OP8 — Closed: Generic Prescribed-Density Gauge Obstruction The fixed-density realization fails for generic ω because $\delta_\xi \chi = -e^\psi \mathcal{L}_\xi \omega$. The three-form completion replaces, rather than repairs, that realization and restores the local first-class diffeomorphism algebra. The complete local Dirac chain is given in Appendix A; OP8 is no longer used as a conditional assumption.

OP9 — Nonconstant-Background and Cosmological Stability The exact wall is controlled at quadratic order in a fixed sector. FLRW backgrounds, time-dependent walls, nonlinear stability, and boundary sectors remain open.

OP10 — Optical-Background Compatibility The exact planar AdS wall, the loaded spherical junction equilibrium, and the prescribed Gordon optical benchmark are different models. Results cannot be transferred by profile shape alone, and the electromagnetic constitutive response remains underived.

OP11 — Nonspherical Compact-Object Stability Radial thin-shell stability is not full stability. Polar, axial, shape, matter, and leakage modes must be computed for a finite-thickness solution.

OP12 — Center Matching and Quasi-Localized Spectrum The Friedrichs half-line operators are nonnegative, but the physical left–right matching rule should be derived from regular gauge-invariant perturbations. A resonance claim requires an outgoing-wave or analytically continued scattering calculation.

19 Position Within the Wider PVB Program

The completed PVB program now has a clean division of labor.

The gravitational core, established here. The locally consistent statements are

$$\sqrt{-g} = e^{\psi}\Omega[A], \qquad \lambda e^{\psi} = \Lambda_*, \qquad Z_{\text{physical}}^{(\text{Mink})} = Z_\psi, \qquad N_{\text{local}} = 2_{\text{tensor}} + 1_\psi.$$

These are the results that later PVB constructions may inherit.

The phenomenological membrane program. The three-form measure does not by itself produce an electromagnetic refractive law. The exact branch derived here is planar and asymptotically AdS. The spherical section supplies a radially stable, horizonless junction benchmark only after an added conserved load is introduced. The Gordon constitutive response remains a separate matter-sector ansatz, and no calculation here identifies that loaded shell with the prescribed optical benchmark.

The microscopic PVB program. This paper does not identify an underlying substrate, derive the admissibility field from microscopic variables, or explain why matter should develop a Gordon response. Those questions remain the wider theory-building objective.

The program therefore remains partitioned by a strict boundary. The formal core supplies a consistent gravitational scaffold; the spherical audit supplies a scalar-only obstruction and a matter-assisted radial existence benchmark; the optical paper explores a phenomenological response on prescribed backgrounds. None of these results proves the missing microscopic bridge, and the optical benchmark cannot be treated as the constitutive completion of either gravitational solution.

20 Core PVB Result

The gravitational core of PVB survives in a more specific form: the fixed-density realization is obstructed, while the scalar-weighted three-form realization is locally complete.

The generic fixed-density realization has the canonical obstruction

$$\boxed{\delta_\xi \chi\big|_{\chi=0} = -e^{\psi}\mathcal{L}_\xi \omega.}$$

The completed theory replaces it by

$$\boxed{\sqrt{-g} = e^{\psi}\Omega[A]}, \qquad \Omega[A] = \frac{1}{4!}\tilde{\epsilon}^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}.$$

Three-form variation gives

$$\boxed{d(\lambda e^{\psi}) = 0} \quad \Longrightarrow \quad \boxed{\lambda = \Lambda_* e^{-\psi}}.$$

The topological sector has no local propagating mode. Its momentum is spatially and temporally constant, and the local first-class constraints close. Therefore

$$\boxed{N_{\text{local}} = 2_{\text{tensor}} + 1_\psi.}$$

In a fixed sector,

$$\boxed{Z_\psi \Box\psi - U'(\psi) + J_{\text{noise}} + \Lambda_* e^{-\psi} = 0.}$$

Around constant-ψ_0 Minkowski space,

$$\boxed{Z_{\text{physical}} = Z_\psi, \qquad m_\psi^2 = \frac{U''(\psi_0) + \Lambda_* e^{-\psi_0}}{Z_\psi}, \qquad c_s^2 = 1.}$$

The determinant-adapted diagonal entry $Z_{\rm diag} = Z_\psi - 3/(8\kappa)$ is not the physical normalization.

For the fixed bare potential $U_{\rm bare}(\psi;\Lambda_0)$, the selected sector $\Lambda_* = \Lambda_0$ admits

$$\Phi = v\tanh(y/\delta), \qquad A(y) = -\epsilon_g\left[\frac{1}{3}\ln\cosh(y/\delta) + \frac{1}{12}\tanh^2(y/\delta)\right].$$

The wall is asymptotically AdS. Its fixed-sector master action has positive time-kinetic term and

$$\mathcal{H}_+ = QQ^\dagger \geq 0$$

for the Friedrichs realization. The formal zero solution is non-normalizable; the lowest finite-box level approaches the $m^2 = 0$ continuum threshold from above. The regular partner's flat-space level does not map into the physical center domain. No resonance pole is claimed.

The static spherical reduction adds a sharp status result. Under a nonnegative potential and positive radial friction, the intended scalar-only positive-energy compact bubble is obstructed. An added conserved surface load admits a radially stable standard-orientation equilibrium at $R_0 = 2.8M$, but the load supplies most of the component-energy estimate and the construction remains distributional.

This is the completed local gravitational core of PVB together with its present strongest planar and spherical existence tests. The remaining burdens belong either to its global completion or to the wider PVB program: global topological data, center matching in regular variables, cubic strong coupling, cosmological backgrounds, microscopic admissibility and matter coupling, outgoing-wave poles, optical constitutive physics, a finite-thickness loaded shell, formation, leakage control, and nonspherical stability.

A Complete Dirac–Bergmann Chain for the Three-Form Completion

This appendix supplies the canonical chain underlying Section 11. Matter and noise variables may be appended in the standard way; they do not change the topological classification provided they do not couple directly to A.

A.1 Canonical variables and primary constraints

For the gravity–scalar sector,

$$\mathcal{H}_\perp^{g\psi} = \frac{2\kappa}{\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2\right) - \frac{\sqrt{h}}{2\kappa}\,{}^{(3)}R + \frac{\sqrt{h}\Lambda}{\kappa} + \frac{\pi_\psi^2}{2Z_\psi\sqrt{h}} + \frac{Z_\psi\sqrt{h}}{2}D_i\psi D^i\psi + \sqrt{h}\,U(\psi),$$

$$\mathcal{H}_i^{g\psi} = -2h_{ik}D_j\pi^{jk} + \pi_\psi\partial_i\psi.$$

The topological variables are (T, P_T), $(\bar\tau^i, P_i)$, and (λ, p_λ). Together with lapse and shift, the complete primary set is

$$\boxed{\pi_N, \quad \pi_i^{(N)}, \quad p_\lambda, \quad P_i, \quad \Gamma = P_T - \lambda e^\psi.}$$

Here $\pi_i^{(N)}$ denotes the momentum conjugate to N^i. The nonzero primary-pair bracket is

$$\boxed{\{p_\lambda(x), \Gamma(y)\} = e^{\psi(y)}\delta^3(x-y).}$$

Thus (p_λ, Γ) is an invertible second-class pair wherever $e^\psi \neq 0$.

Modulo a redefinition of the multiplier of Γ, the total Hamiltonian can be written

$$H_T = \int d^3x\big[N\mathcal{C}_\perp + N^i\mathcal{H}_i^{\rm tot} + \bar\tau^i\mathcal{C}_i^A + u_N\pi_N + u_N^i\pi_i^{(N)} + u_\lambda p_\lambda + u_\Gamma\Gamma + u_A^i P_i\big], \tag{11}$$

where

$$\mathcal{C}_\perp = \mathcal{H}_\perp^{g\psi} + \sqrt{h}\,P_T e^{-\psi}, \qquad \mathcal{C}_i^A = \partial_i P_T.$$

A.2 Unreduced spatial-diffeomorphism generator

The generator must Lie-drag every unreduced canonical field. A local representative is

$$\begin{aligned}\mathcal{H}_i^{\rm tot} &= \mathcal{H}_i^{g\psi} + \pi_N \partial_i N + \pi_j^{(N)} \partial_i N^j + \partial_j \Big(\pi_i^{(N)} N^j\Big) + p_\lambda \partial_i \lambda - T \partial_i P_T \\ &\quad + P_j \partial_i \bar{\tau}^j + \partial_j \big(P_i \bar{\tau}^j\big) - \partial_i \big(P_j \bar{\tau}^j\big)\,. \end{aligned} \tag{12}$$

With $D[\xi] = \int d^3x\, \xi^i \mathcal{H}_i^{\rm tot}$,

$$\{T, D[\xi]\} = \mathcal{L}_\xi T = \partial_i(\xi^i T), \qquad \{P_T, D[\xi]\} = \mathcal{L}_\xi P_T = \xi^i \partial_i P_T,$$

and similarly for $\bar{\tau}^i$, λ, N, N^i, h_{ij}, and ψ. This is the canonical term absent in the prescribed-density realization.

A.3 Preservation equations

Define the normal scalar velocity

$$v_\psi \equiv N \frac{\pi_\psi}{Z_\psi \sqrt{h}}, \qquad \dot{\psi} = v_\psi + \mathcal{L}_{\vec{N}} \psi.$$

The primary constraints evolve as follows, up to terms proportional to primaries and standard boundary generators:

$$\dot{\pi}_N = -\mathcal{C}_\perp \approx 0, \qquad \dot{\pi}_i^{(N)} = -\mathcal{H}_i^{\rm tot} \approx 0.$$

The second-class pair does not generate additional constraints:

$$\dot{p}_\lambda = e^\psi u_\Gamma + \mathcal{L}_{\vec{N}} p_\lambda \approx 0 \quad \Longrightarrow \quad \boxed{u_\Gamma = 0},$$

$$\dot{\Gamma} = \mathcal{L}_{\vec{N}} \Gamma - e^\psi \left(u_\lambda + \lambda v_\psi\right) \approx 0 \quad \Longrightarrow \quad \boxed{u_\lambda = -\lambda v_\psi}.$$

Preservation of the spatial three-form momenta produces

$$\dot{P}_i = -\mathcal{C}_i^A \approx 0 \quad \Longrightarrow \quad \boxed{\mathcal{C}_i^A = \partial_i P_T \approx 0}.$$

Since the Hamiltonian is independent of T except through the diffeomorphism generator,

$$\dot{P}_T = \mathcal{L}_{\vec{N}} P_T = N^i \partial_i P_T,$$

and hence

$$\dot{\mathcal{C}}_i^A = \partial_i (N^j \mathcal{C}_j^A) \approx 0.$$

No tertiary local constraint is generated.

Finally, the Hamilton equation for T is

$$\dot{T} = N\sqrt{h}\, e^{-\psi} + \partial_i (N^i T) - \partial_i \bar{\tau}^i + u_\Gamma.$$

Using $u_\Gamma = 0$ and $\tau^i = \bar{\tau}^i - N^i T$ gives

$$\boxed{N\sqrt{h} = e^\psi \left(\dot{T} + \partial_i \tau^i\right) = e^\psi \Omega[A].}$$

Thus the determinant lock reappears as the Hamilton equation for the topological coordinate; it is not an extra secondary constraint imposed after the Legendre transform.

A.4 Reducibility and local count of the three-form sector

The secondary constraints obey the off-shell identities

$$\partial_{[i}\mathcal{C}^A_{j]} \equiv 0.$$

For a nonzero Fourier mode k_i,

$$\mathcal{C}^A_i(k) = ik_i P_T(k),$$

so the three displayed equations have rank one: only the longitudinal scalar condition is independent. Before eliminating (p_λ, Γ), the topological–multiplier sector has ten phase-space dimensions,

$$(T, P_T), \qquad (\bar{\tau}^i, P_i), \qquad (\lambda, p_\lambda).$$

It contains two second-class constraints and four independent local first-class constraints: the three P_i plus the single independent nonzero-mode condition represented by $\mathcal{C}^A_i$. Therefore

$$N_{\text{top,local}} = \frac{1}{2}\,[10 - 2 - 2(4)] = 0.$$

At $k = 0$, $\mathcal{C}^A_i$ vanishes identically. On a connected slice the surviving momentum is spatially constant, $P_T = \Lambda_*$, and its conjugate is the global coordinate

$$Q_T \equiv \int_\Sigma d^3x\, T,$$

subject to boundary and large-gauge identifications.

A.5 Dirac bracket after eliminating the multiplier pair

Let $\chi_1 = p_\lambda$ and $\chi_2 = \Gamma$. Their constraint matrix is

$$\Delta_{ab}(x, y) = \begin{pmatrix} 0 & e^\psi \\ -e^\psi & 0 \end{pmatrix} \delta^3(x - y).$$

For functionals F, G independent of λ and p_λ, every Dirac correction contains either $\{F, p_\lambda\}$ or $\{p_\lambda, G\}$ and therefore vanishes. Consequently

$$\boxed{\{F, G\}_D = \{F, G\} \quad \text{on} \quad (h_{ij}, \pi^{ij}; \psi, \pi_\psi; T, P_T; \bar{\tau}^i, P_i).}$$

One may set

$$p_\lambda = 0, \qquad \lambda = P_T e^{-\psi},$$

without deforming the canonical brackets of the surviving variables.

A.6 Constraint algebra before and after local reduction

Introduce

$$H[N] = \int d^3x\, N\mathcal{C}_\perp, \quad D[\xi] = \int d^3x\, \xi^i \mathcal{H}^{\text{tot}}_i, \quad C[\alpha] = \int d^3x\, \alpha^i \mathcal{C}^A_i, \quad P[\rho] = \int d^3x\, \rho^i P_i.$$

With smearings of the appropriate tensor density weights, the unreduced weak algebra is

$$\{D[\xi], D[\eta]\} = D[[\xi, \eta]], \qquad \{D[\xi], H[N]\} = H[\mathcal{L}_\xi N],$$

$$\{D[\xi], C[\alpha]\} = C[\mathcal{L}_\xi \alpha], \qquad \{D[\xi], P[\rho]\} = P[\mathcal{L}_\xi \rho],$$

$$\{H[N], C[\alpha]\} = 0, \qquad \{H[N], P[\rho]\} = 0, \qquad \{C[\alpha], C[\beta]\} = 0, \qquad \{P[\rho], C[\alpha]\} = 0.$$

For

$$v^i = h^{ij}(N\partial_j M - M\partial_j N),$$

the Hamiltonian bracket is

$$\boxed{\{H[N], H[M]\} = D[v] + C[Tv] + \mathcal{P}[v].}$$

Here $\mathcal{P}[v]$ is a linear combination of the primary constraints π_N, $\pi_i^{(N)}$, p_λ, P_i, and Γ required by the fully improved generator. The term $C[Tv]$ is the difference between the gravity–scalar momentum constraint produced by the bracket and the total momentum generator, whose topological contribution is $-T\partial_i P_T$. Thus the algebra closes weakly before reduction, including the three-form constraints explicitly.

After imposing $P_i = 0$, $\partial_i P_T = 0$, and the second-class pair, one has $P_T = \Lambda_*$ locally and

$$\mathcal{C}_\perp = \mathcal{H}_\perp^{g\psi} + \sqrt{h}\,\Lambda_* e^{-\psi}, \qquad \mathcal{C}_i = \mathcal{H}_i^{g\psi}.$$

The algebra reduces to

$$\{D[\xi], D[\eta]\} = D[[\xi, \eta]],$$
$$\{D[\xi], H[N]\} = H[\mathcal{L}_\xi N],$$
$$\boxed{\{H[N], H[M]\} = D\big[h^{ij}(N\partial_j M - M\partial_j N)\big]\,.}$$

This establishes the local first-class complex used in the $2+1$ count. The theorem is local: global flux, boundary generators, large gauge transformations, compact topology, and the precise domain of Q_T remain separate questions.

Reproducibility Package

The public release contains this manuscript source and PDF, the script `pvb_public_preprint_audit.py`, six generated figures, JSON audit metadata, and two CSV tables for the finite-box wall spectrum. The script reconstructs the exact wall, verifies the background equations, computes the half-line finite-box levels, displays the partner-domain mismatch, derives the spherical benchmark parameters, checks the shell potential and orientation functions, and maps the radial equilibrium family.

From the release directory, run

```
python -m pip install -r requirements.txt
python pvb_public_preprint_audit.py --out-dir results
pdflatex PVB_aiXiv_Public_Preprint.tex
pdflatex PVB_aiXiv_Public_Preprint.tex
```

The manuscript values must agree with `results/pvb_public_preprint_audit.json` and the accompanying CSV files to the displayed precision.

Authorship and AI-Assistance Disclosure

This work was conceived, directed, and is solely attributable to the human author. Large language models (including Anthropic's Claude, among others in a deliberately adversarial multi-model workflow) were used as derivation-checking, drafting, and critique instruments under continuous human direction; cross-model agreement was treated as a prompt for further scrutiny rather than as validation. All claims, the partition into derived and deferred sectors, and the stated failure conditions were reviewed and are asserted by the human author, who bears full responsibility for the content.

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