Admissibility-Locked Unimodular Gravity

Bianchi Slaving, Local Scalar Survival, and the Conformal Stiffness Floor

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Abstract

We introduce a covariant effective framework in which the spacetime volume element is not treated as a passive determinant of the metric alone, but is constrained by an admissibility scalar ψ through a density-safe lock,

$$\sqrt{-g} = \omega(x)\, e^{\psi}.$$

Here $\omega(x)$ is a fixed reference scalar density inherited from coarse-graining, while ψ is a true scalar encoding local admissibility or routing capacity. The resulting theory is adjacent to unimodular gravity, but differs from both ordinary unimodular gravity and Henneaux–Teitelboim-type extensions in a specific way: the Lagrange multiplier λ enforcing the volume lock is not promoted to an independent propagating cosmological-constant field. Instead, Bianchi integrability forces

$$\lambda e^{\psi} = \Lambda_*,$$

so that

$$\lambda(x) = \Lambda_* e^{-\psi(x)}.$$

Thus Λ_* survives as a global integration constant, while the local vacuum reaction is algebraically slaved to ψ. The propagating degree of freedom is ψ, not λ.

The second structural result is a stability bound. Because the volume lock identifies ψ with the conformal mode of the metric, the usual wrong-sign conformal contribution from the Einstein–Hilbert action is not discarded as gauge pathology. It is converted into a quantitative lower bound on the ψ-sector stiffness:

$$Z_\psi > \frac{3}{8\kappa}.$$

The conformal-mode problem therefore becomes a stiffness tax: any admissibility-modulated continuum must pay enough ψ-gradient energy to stabilize the scalar sector.

An ADM consistency analysis further shows why the scalar is not removed by the determinant lock. The lapse and multiplier belong to the shared unimodular constraint sector, while (ψ, π_ψ) is a nondegenerate canonical pair. Completion of the Dirac chain must reproduce the spatial part of the covariant slaving condition,

$$\partial_i(\lambda e^{\psi}) \approx 0,$$

which removes the local multiplier profile rather than the scalar. After separating the unimodular zero mode, the supported local field content is two tensor polarizations plus one scalar mode. The exact global canonical bookkeeping is left open for a Henneaux–Teitelboim completion and is not counted as local propagation.

A linearized analysis around a Minkowski background partially closes the stability question beyond the kinetic floor. With the metric trace slaved to $\delta\psi$ by the lock, the direct $\delta\psi$

sources in the linearized Einstein equation cancel by the background conditions, the tensor sector propagates two standard massless polarizations, and the scalar sector carries mass

$$m_\psi^2 = \frac{U''(\psi_0) + \Lambda_* e^{-\psi_0}}{Z_{\text{eff}}}, \qquad Z_{\text{eff}} = Z_\psi - \frac{3}{8\kappa},$$

with luminal sound speed at this order. The no-tachyon condition $U''(\psi_0) + \Lambda_* e^{-\psi_0} \geq 0$ and a strong-coupling scale $\Lambda_{\text{sc}} \sim \sqrt{Z_{\text{eff}}}$ that collapses as the stiffness floor is approached convert the remaining stability burden into explicit, checkable conditions. Nonlinear and cosmological (FLRW) perturbative stability remain open.

This paper isolates the formal covariant core of Procedural Vacuum Breakdown from its separate phenomenological optical-membrane bridge. The membrane sector is deferred to a companion benchmark paper.

Keywords: unimodular gravity; cosmological constant; conformal mode; ADM constraint analysis; scalar–tensor gravity; emergent spacetime
Subject areas: gr-qc (primary); hep-th (secondary)

1 Introduction

The cosmological constant occupies an awkward position in general relativity: it enters the Einstein equations as a freely tunable coupling, yet its observed value resists every natural estimate. Unimodular gravity [1, 2] offers a structural alternative, restricting the metric determinant so that the cosmological constant arises as an integration constant of the field equations rather than as an input parameter. Subsequent canonical analyses [6, 7, 8] clarified the constraint structure of such theories, and recent work has explored promoting the associated multiplier sector to dynamical status [3].

This paper develops a third option. Rather than fixing the volume element or dynamizing the cosmological multiplier, we lock the volume element to a scalar field:

$$\sqrt{-g} = \omega(x)\, e^{\psi},$$

where $\omega(x)$ is a fixed reference scalar density and ψ is a true scalar interpreted as a local admissibility or routing-capacity field. The construction is motivated by a coarse-graining picture in which spacetime volume is not a primitive but a measure of how much local structure a substrate can support; that microscopic picture is, however, deliberately excluded from the present note, which establishes only the covariant effective consequences of the lock itself.

Four results constitute the core of the paper. First, Bianchi integrability forces the lock multiplier λ into algebraic slavery, $\lambda(x) = \Lambda_* e^{-\psi(x)}$, preserving the unimodular insight that the cosmological remnant is a global integration constant while allowing the local vacuum reaction to track admissibility. Second, an ADM constraint analysis shows that the lock removes the local multiplier profile but not the scalar: after zero-mode separation the supported local content is two tensor polarizations plus one scalar mode. Third, because the lock identifies ψ with the conformal mode of the metric, the classic wrong-sign conformal kinetic term [4] is converted from a pathology into a quantitative stability bound, $Z_\psi > 3/(8\kappa)$, on the intrinsic scalar stiffness. Fourth, a linearized analysis around a Minkowski background makes the residual stability conditions explicit: the tensor sector is standard, the scalar mass is set by the curvature of the effective potential divided by the effective stiffness $Z_{\text{eff}} = Z_\psi - 3/(8\kappa)$, the sound speed is luminal at linear order, and the strong-coupling scale of the scalar sector degrades as the stiffness floor is approached.

The paper is organized as follows. Section 2 partitions the claims into derived and deferred sectors. Section 3 situates the construction relative to unimodular and Henneaux–Teitelboim gravity. Sections 4–5 fix conventions and the status of the reference density. Sections 6–9 derive the field equations, Bianchi closure, and the reduced scalar dynamics. Section 11 presents the canonical constraint map and degree-of-freedom count, Section 12 derives the conformal stiffness floor, and Section 13 establishes the linearized spectrum and stability conditions around a Minkowski background. Sections 14–18 delimit what is and is not established, enumerate explicit failure conditions, and state the minimal result.

2 Scope and Claim Discipline

This note does not present a completed microscopic theory of spacetime emergence. It presents a covariant effective core built around a single structural hypothesis: the macroscopic four-volume measure is locked to a scalar admissibility field ψ.

The claims are partitioned as follows.

The derived sector consists of:

1. the density-safe volume lock,
$$\sqrt{-g} = \omega(x)\, e^{\psi};$$
2. the variational field equations with λ treated as a pure-trace geometric reaction;
3. the Ward-clean exchange identity for any diffeomorphism-invariant noise sector;
4. Bianchi closure,
$$\partial_\nu(\lambda e^{\psi}) = 0;$$
5. the resulting slaving relation,
$$\lambda(x) = \Lambda_* e^{-\psi(x)};$$
6. the reduced scalar equation after Bianchi slaving;
7. the canonical consistency condition
$$\partial_i(\lambda e^{\psi}) \approx 0,$$
which isolates the cosmological zero mode from the local multiplier profile;
8. the supported local degree-of-freedom content,
$$N_{\text{local}} = 2_{\text{tensor}} + 1_{\psi},$$
conditional on completion of the global zero-mode and background-density gauge bookkeeping;
9. the necessary conformal kinetic-stability condition,
$$Z_\psi > \frac{3}{8\kappa};$$
10. the linearized spectrum around a Minkowski background: two standard tensor polarizations plus one scalar with
$$m_\psi^2 = \frac{U''(\psi_0) + \Lambda_* e^{-\psi_0}}{Z_{\text{eff}}}, \qquad c_s^2 = 1,$$
together with the no-tachyon condition $U''(\psi_0) + \Lambda_* e^{-\psi_0} \geq 0$ and the strong-coupling estimate $\Lambda_{\text{sc}} \sim \sqrt{Z_{\text{eff}}}$.

The deferred sector consists of:

1. the microscopic origin of $\omega(x)$;
2. the full coarse-graining map from a directed acyclic graph or other substrate ensemble;
3. the detailed matter-coupling program;
4. the optical membrane constitutive laws used in benchmark applications;
5. strong-field observational tests;
6. a complete Henneaux–Teitelboim/Stueckelberg canonical completion of the global cosmological sector;
7. proof that the prescribed density $\omega(x)$ preserves the required first-class spatial gauge structure, or a replacement in which that structure is manifest.

This separation is not cosmetic. It prevents the formal core from borrowing empirical credibility from the optical bridge, and prevents the optical bridge from borrowing derivational certainty from the formal core.

3 Relation to Unimodular and Henneaux–Teitelboim Gravity

The nearest formal relative is unimodular gravity [1, 2, 7]. In ordinary unimodular gravity, the determinant of the metric is fixed or restricted, and the cosmological constant appears as an integration constant rather than as an ordinary coupling placed directly in the action [1].

The present construction keeps that structural insight but alters the determinant condition. The volume element is not fixed to a constant or to a bare background density. Instead, it is locked to a scalar modulation:

$$\sqrt{-g} = \omega(x)\, e^{\psi}.$$

This changes the role of the determinant constraint. It is no longer merely a restriction on the metric determinant. It becomes a physical admissibility lock.

The distinction from Dynamical Henneaux–Teitelboim Gravity [3] is sharper. DHT dynamizes the Henneaux–Teitelboim fields directly by adding kinetic terms to the scalar and three-form sectors. PVB does not do that. The multiplier λ remains auxiliary. It has no kinetic term and no independent equation of motion. Its local spacetime dependence is inherited through Bianchi closure:

$$\lambda e^{\psi} = \Lambda_*.$$

Thus the local field content is carried by ψ. The λ field is a Bianchi-slaved vacuum-reaction variable, not an independent propagating cosmological-constant field.

A compact comparison is given in Table 1.

The critical sentence is:

PVB dynamizes the determinant-lock deformation, not the cosmological multiplier.

4 Variables and Conventions

We use signature $(-, +, +, +)$, with

$$\kappa = \frac{8\pi G}{c^4}.$$

The Einstein tensor is

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R.$$

Table 1: Comparison of formal features

Feature	Unimodular Gravity	Henneaux–Teitelboim	Dynamical HT	PVB Core
Determinant restriction	Yes	Yes	Modified	ψ-driven lock
Λ as integration constant	Yes	Yes	Modified	Yes, via Bianchi closure
Independent dynamical Λ-field	No	No	Yes	No
Propagating scalar ψ	No	No	Model-dependent	Yes
λ has kinetic term	No	No	Yes (in DHT analogue)	No
λ locally varies	No/constrained	constrained	dynamically	algebraically slaved
Conformal stiffness floor	No	No	No	Yes
Optical benchmark bridge	No	No	No	companion paper

The scalar kinetic convention is

$$\mathcal{L}_\psi \supset -\frac{1}{2} Z_\psi (\nabla\psi)^2,$$

with $Z_\psi > 0$.

The admissibility scalar is

$$\psi(x) = \ln\left(\frac{A_{\text{adm}}(x)}{A_0}\right),$$

where $A_{\text{adm}}(x)$ denotes local admissibility density or routing capacity, and A_0 is a reference value.

The reference density $\omega(x)$ is a scalar density of weight $+1$. It is not a true scalar. This matters because $\sqrt{-g}$ is also a scalar density of weight $+1$. The ratio

$$\frac{\sqrt{-g}}{\omega(x)}$$

is therefore a true scalar, so the lock

$$\sqrt{-g} = \omega(x)\, e^{\psi}$$

is density-safe.

5 The Status of $\omega(x)$

The reference density $\omega(x)$ is the most exposed background structure in the theory.

It should not be interpreted as an ether, preferred rest frame, observable foliation, or additional propagating matter field. It is a nondynamical scalar density specifying the reference measure inherited from coarse-graining.

However, this does not make it disappear. PVB is background-covariant at this stage, not fully background-independent. The lock is covariant because both $\omega(x)$ and $\sqrt{-g}$ transform as scalar densities, while ψ transforms as a scalar. But the origin of $\omega(x)$ is not derived inside this effective paper.

The open burden is therefore precise:

A future microscopic theory must derive $\omega(x)$ as an ensemble density or show why observables are independent of its arbitrary representative.

The framework fails if $\omega(x)$ produces coordinate-dependent observables, smuggles in a preferred frame inconsistent with experiment, or cannot be related to a legitimate coarse-grained substrate measure.

This is Open Problem 1, not a hidden assumption.

6 Action

The total action is

$$S_{\text{total}} = S_{\text{EH}} + S_\psi + S_m + S_{\text{EM}} + S_{\text{noise}} + S_{\text{lock}}.$$

The Einstein–Hilbert sector is

$$S_{\text{EH}} = \int d^4x \sqrt{-g}\, \frac{R - 2\Lambda}{2\kappa}.$$

The scalar sector is

$$S_\psi = \int d^4x \sqrt{-g} \left[-\frac{Z_\psi}{2} (\nabla\psi)^2 - U(\psi) \right].$$

The lock sector is

$$S_{\text{lock}} = \int d^4x\, \lambda(x) \big[\omega(x) e^{\psi} - \sqrt{-g} \big].$$

The matter and electromagnetic sectors are assumed minimally coupled unless otherwise specified. The noise sector S_{noise} is assumed diffeomorphism invariant and may encode effective coarse-grained exchange with unresolved substrate degrees of freedom.

For all physical sectors,

$$T^{(i)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_i}{\delta g^{\mu\nu}}.$$

The λ term is not counted as a separately conserved matter stress tensor. It is placed on the geometric side as a pure-trace reaction.

7 Scalar and Noise Identities

The scalar stress tensor is

$$T^{(\psi)}_{\mu\nu} = Z_\psi (\nabla_\mu \psi)(\nabla_\nu \psi) - g_{\mu\nu} \left[\frac{Z_\psi}{2} (\nabla\psi)^2 + U(\psi) \right].$$

Its divergence is

$$\nabla^\mu T^{(\psi)}_{\mu\nu} = \big[Z_\psi \Box \psi - U'(\psi) \big] \partial_\nu \psi.$$

Define the scalar noise current by

$$J_{\text{noise}} = \frac{1}{\sqrt{-g}} \frac{\delta S_{\text{noise}}}{\delta \psi}.$$

If S_{noise} is diffeomorphism invariant, its Ward identity gives

$$\nabla^\mu T^{(\text{noise})}_{\mu\nu} = J_{\text{noise}} \partial_\nu \psi.$$

No additional hand-inserted exchange term is required. The exchange bookkeeping is fixed by covariance.

8 Field Equations and Bianchi Closure

Variation with respect to λ gives the lock:

$$\sqrt{-g} = \omega e^{\psi}.$$

Metric variation gives

$$G_{\mu\nu} + (\Lambda + \kappa\lambda) g_{\mu\nu} = \kappa \big[T^{(m)}_{\mu\nu} + T^{(\text{EM})}_{\mu\nu} + T^{(\psi)}_{\mu\nu} + T^{(\text{noise})}_{\mu\nu} \big].$$

Variation with respect to ψ gives

$$\lambda = -\big[Z_\psi \Box\psi - U'(\psi) + J_{\text{noise}}\big].$$

Taking the covariant divergence of the metric equation and using $\nabla^\mu G_{\mu\nu} = 0$, along with the scalar and noise identities, yields

$$\partial_\nu\lambda - \big[Z_\psi \Box\psi - U'(\psi) + J_{\text{noise}}\big]\partial_\nu\psi = 0.$$

Substituting the ψ-variation identity for λ gives

$$\partial_\nu\lambda + \lambda\partial_\nu\psi = 0.$$

Therefore,

$$\partial_\nu(\lambda e^\psi) = 0.$$

So

$$\lambda e^\psi = \Lambda_*,$$

where Λ_* is a global integration constant.

Hence,

$$\lambda(x) = \Lambda_* e^{-\psi(x)}.$$

This is the central closure result.

The local vacuum reaction varies with ψ, but λ has not become an independent dynamical field. It has no kinetic term. It has no independent propagating equation. It is a Bianchi-slaved auxiliary reaction field.

The effective cosmological contribution appearing in the metric equation is therefore

$$\Lambda_{\text{eff}}(x) = \Lambda + \kappa\Lambda_* e^{-\psi(x)}.$$

The global constant survives. The local reaction tracks admissibility.

9 Reduced Scalar Equation After Bianchi Slaving

Combining the scalar variation,

$$\lambda = -\big[Z_\psi \Box\psi - U'(\psi) + J_{\text{noise}}\big],$$

with the Bianchi-slaved relation

$$\lambda = \Lambda_* e^{-\psi},$$

gives the reduced local scalar equation

$$\boxed{Z_\psi \Box\psi - U'(\psi) + J_{\text{noise}} + \Lambda_* e^{-\psi} = 0.}$$

Equivalently, defining

$$U_{\text{eff}}(\psi) = U(\psi) + \Lambda_* e^{-\psi},$$

one may write

$$Z_\psi \Box\psi - U'_{\text{eff}}(\psi) + J_{\text{noise}} = 0.$$

This equation makes the dynamical separation explicit. The multiplier has been eliminated algebraically, while ψ retains a second-order field equation. The integration constant therefore enters local scalar dynamics through an exponential contribution to the effective potential, but it does not become an independent local field.

10 Interpretation of the Vacuum Reaction

The relation

$$\lambda(x) = \Lambda_* e^{-\psi(x)}$$

has a simple structural meaning.

Where admissibility ψ increases, the local reaction λ decreases. Where admissibility ψ decreases, the local reaction λ increases. Thus λ behaves like a vacuum tax imposed by the volume lock: it is the local cost required to maintain the admissible measure condition under covariant dynamics.

This should not be read as a new matter fluid. It is a geometric reaction term.

Nor should it be read as an adjustable cosmological-constant field. The only free cosmological remnant is Λ_*, the integration constant. The spacetime dependence of λ is inherited from ψ through closure.

This is the formal distinction from dynamical unimodular models that grant the cosmological variable its own kinetic structure.

11 ADM Constraint Map and Local Degree-of-Freedom Count

This section records the canonical direction established by the ADM consistency analysis. It is deliberately separated into a robust local statement and an unresolved global statement. The local result is that the ψ canonical pair survives. The unresolved issue is the exact parametrization and classification of the cosmological zero mode.

Under the ADM split,

$$\sqrt{-g} = N\sqrt{h},$$

so the lock becomes

$$\chi \equiv \omega e^{\psi} - N\sqrt{h} \approx 0.$$

The canonical variables are

$$(h_{ij}, \pi^{ij}), \qquad (\psi, \pi_\psi), \qquad (N, \pi_N), \qquad (N^i, \pi_i), \qquad (\lambda, \pi_\lambda),$$

for a total local phase-space dimension of 24 before constraints. Because N, N^i, and λ have no velocities, the primary constraints are

$$\pi_N \approx 0, \qquad \pi_i \approx 0, \qquad \pi_\lambda \approx 0.$$

In contrast, the nondegenerate scalar kinetic term gives

$$\pi_\psi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} \not\approx 0,$$

so there is no primary constraint that removes the scalar pair.

Writing $\mathcal{H}_0$ for the gravitational, scalar, matter, electromagnetic, and noise Hamiltonian density excluding the lock reaction, preservation of the primaries gives, schematically,

$$\dot{\pi}_\lambda \approx 0 \quad \Rightarrow \quad \chi \equiv \omega e^{\psi} - N\sqrt{h} \approx 0,$$

$$\dot{\pi}_N \approx 0 \quad \Rightarrow \quad \Phi \equiv \mathcal{H}_0 + \lambda\sqrt{h} \approx 0,$$

and

$$\dot{\pi}_i \approx 0 \quad \Rightarrow \quad \mathcal{H}_i \approx 0.$$

The sign of the $\lambda\sqrt{h}$ term follows from the lock convention used in the action; reversing the definition of λ reverses that sign without changing the constraint rank.

The lapse-lock and multiplier-Hamiltonian brackets are nondegenerate:

$$\{\pi_N(x), \chi(y)\} = \sqrt{h(x)}\, \delta^3(x-y),$$

$$\{\pi_\lambda(x), \Phi(y)\} = -\sqrt{h(x)}\, \delta^3(x-y),$$

up to the overall Poisson-bracket sign convention. Thus the lapse and multiplier are auxiliary members of the second-class lock sector. A naive count that stops here produces one spurious local mode, exactly as it does in fixed-density unimodular gravity [6, 8]. The reason is that the Dirac chain has not yet separated the spatially varying multiplier from its global zero mode.

The canonical analysis must reproduce the spatial part of the covariant closure equation. The corresponding consistency condition is

$$\boxed{\mathcal{C}_i \equiv \partial_i(\lambda e^{\psi}) \approx 0.}$$

Equivalently, defining

$$\Lambda_U(x,t) \equiv \lambda(x,t) e^{\psi(x,t)},$$

one obtains

$$\Lambda_U(x,t) = \Lambda_0(t)$$

on each connected spatial slice. Subsequent time consistency is expected to impose

$$\dot{\Lambda}_0 = 0,$$

recovering the spacetime constant Λ_* found covariantly. The spatial condition removes the local profile of λe^{ψ}; it does not set ψ or π_ψ to zero.

The precise local statement is therefore not that the constraints "do not touch" ψ. The scalar contributes to

$$\mathcal{H}_0 \quad \text{and} \quad \mathcal{H}_i^{(\psi)} = \pi_\psi \partial_i \psi,$$

and transforms under the surviving gauge generators. The correct statement is that no additional primary or algebraic secondary constraint eliminates the canonical pair (ψ, π_ψ).

After elimination of the lapse and multiplier auxiliaries and separation of the unimodular zero mode, the local reduced phase space is represented by

$$(h_{ij}, \pi^{ij}; \psi, \pi_\psi),$$

with dimension 14 per spatial point. Under the standard unimodular local constraint complex—three spatial-diffeomorphism generators plus the average-free Hamiltonian generator—the local count is

$$N_{\text{local}} = \frac{1}{2}\big[14 - 2(3+1)\big] = 3.$$

Thus the supported local content is

$$\boxed{N_{\text{local}} = 2_{\text{tensor}} + 1_{\psi}.}$$

This result is robust at the level of the difference from ordinary unimodular gravity: PVB contains one additional nondegenerate scalar pair, while the global cosmological mode resides in the shared lock/multiplier sector.

The absolute global count is not claimed closed here. A complete treatment should introduce a Henneaux–Teitelboim or equivalent Stueckelberg parametrization, display the zero-mode/average-free decomposition explicitly, and verify the complete first-/second-class algebra. This is especially important because $\omega(x)$ is a prescribed density. Until its gauge status is made canonical rather than merely background-covariant, the local count should be read as strongly supported rather than as a final theorem.

12 Conformal Decomposition and the Stiffness Floor

The volume lock $\sqrt{-g} = \omega(x)e^{\psi}$ identifies the admissibility scalar ψ with the conformal degree of freedom of the metric. To isolate the associated stability condition, perform a conformal decomposition in which the determinant condition is carried by ψ. Introduce a rescaled metric $\tilde{g}_{\mu\nu}$ such that

$$g_{\mu\nu} = e^{\psi/2}\tilde{g}_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu},$$

where $\Omega \equiv e^{\psi/4}$. Under this rescaling the four-volume element transforms as

$$\sqrt{-g} = \Omega^4\sqrt{-\tilde{g}} = e^{\psi}\sqrt{-\tilde{g}}.$$

In the rescaled frame we may fix $\sqrt{-\tilde{g}} = \omega(x)$ without loss of generality; the volume lock then forces the identification $\sqrt{-g}/\omega(x) = e^{\psi}$ exactly, so that the rescaled metric $\tilde{g}_{\mu\nu}$ carries no independent trace (conformal) fluctuation. The entire conformal degree of freedom is thereby transferred to the dynamical scalar ψ.

The Ricci scalar transforms under the conformal rescaling according to the standard identity

$$R = \Omega^{-2}\Big[\tilde{R} - 6\tilde{\Box}\ln\Omega - 6(\tilde{\nabla}\ln\Omega)^2\Big].$$

Because $\ln\Omega = \psi/4$, one has $\tilde{\nabla}\ln\Omega = \frac{1}{4}\tilde{\nabla}\psi$ and $(\tilde{\nabla}\ln\Omega)^2 = \frac{1}{16}(\tilde{\nabla}\psi)^2$, together with $\tilde{\Box}\ln\Omega = \frac{1}{4}\tilde{\Box}\psi$. Substituting these relations yields the expanded expression

$$R = \Omega^{-2}\Big[\tilde{R} - \frac{3}{2}\tilde{\Box}\psi - \frac{3}{8}(\tilde{\nabla}\psi)^2\Big].$$

(Equivalently, after collecting terms, the compressed form $R = \Omega^{-2}(\tilde{R} - 6\Omega^{-1}\tilde{\Box}\Omega)$ may be recovered, but the expanded version makes every contribution manifest.)

The Einstein–Hilbert contribution to the action (omitting the cosmological term for the present kinetic analysis) is

$$S_{\rm EH} \supset \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,R.$$

Inserting the transformed quantities,

$$\sqrt{-g}\,R = \Omega^2\sqrt{-\tilde{g}}\Big[\tilde{R} - \frac{3}{2}\tilde{\Box}\psi - \frac{3}{8}(\tilde{\nabla}\psi)^2\Big].$$

The term proportional to $\tilde{\Box}\psi$ is integrated by parts (boundary terms assumed to vanish):

$$\int\sqrt{-\tilde{g}}\,\Omega^2\tilde{\Box}\psi\,d^4x = -\int\sqrt{-\tilde{g}}\,\tilde{\nabla}^{\alpha}(\Omega^2)\,\tilde{\nabla}_{\alpha}\psi\,d^4x.$$

Differentiating the prefactor gives $\tilde{\nabla}^{\alpha}(\Omega^2) = \frac{1}{2}\Omega^2\tilde{\nabla}^{\alpha}\psi$ (because $\nabla\Omega = (\Omega/4)\nabla\psi$). Consequently the integrated-by-parts contribution supplies

$$+\frac{3}{4}\Omega^2(\tilde{\nabla}\psi)^2$$

to the integrand of $\sqrt{-\tilde{g}}R$. Adding the explicit quadratic term already present, $-\frac{3}{8}\Omega^2(\tilde{\nabla}\psi)^2$, produces a net coefficient

$$\frac{3}{4} - \frac{3}{8} = \frac{3}{8}$$

for the $\Omega^2(\tilde{\nabla}\psi)^2$ term. Therefore

$$\sqrt{-g}\,R \supset \frac{3}{8}\Omega^2(\tilde{\nabla}\psi)^2\sqrt{-\tilde{g}} + \cdots$$

and the Einstein–Hilbert kinetic term for ψ reads

$$\frac{1}{2\kappa} \times \frac{3}{8}\Omega^2(\tilde{\nabla}\psi)^2 = \frac{3}{16\kappa}\Omega^2(\tilde{\nabla}\psi)^2 = \frac{3}{16\kappa}e^{\psi/2}(\tilde{\nabla}\psi)^2$$

in the rescaled variables.

When the same conformal factor is restored to the physical frame, the scalar kinetic measure transforms identically:

$$\sqrt{-g}\,(\nabla\psi)^2 = \sqrt{-\tilde{g}}\,\Omega^2\,(\tilde{\nabla}\psi)^2 = \sqrt{-\tilde{g}}\,e^{\psi/2}\,(\tilde{\nabla}\psi)^2.$$

Consequently the coefficient comparison between the Einstein–Hilbert contribution and the intrinsic ψ kinetic term may be performed directly. The total ψ-gradient coefficient appearing in the Lagrangian density is therefore

$$\frac{3}{16\kappa} - \frac{Z_\psi}{2}.$$

For the quadratic form in $\nabla\psi$ to be healthy, this coefficient must be negative:

$$\frac{3}{16\kappa} - \frac{Z_\psi}{2} < 0 \quad \Rightarrow \quad Z_\psi > \frac{3}{8\kappa}.$$

Equivalently one may define an effective stiffness $Z_{\text{eff}} = Z_\psi - \frac{3}{8\kappa}$ and require $Z_{\text{eff}} > 0$.

Combined with the ADM constraint map above, the determinant lock supports two massless tensor modes plus one local scalar degree of freedom carried by ψ. The multiplier λ remains auxiliary and carries no independent local propagating degree of freedom. The inequality $Z_\psi > 3/(8\kappa)$ is the necessary kinetic-sign condition for that scalar sector; it is not by itself a complete proof of nonlinear or nonperturbative stability.

In the present construction the determinant lock prevents the conformal mode from being treated as a pure gauge artifact or as a pathology to be gauge-fixed away. The wrong-sign Einstein–Hilbert contribution must therefore be compensated by the intrinsic ψ stiffness. This converts the classical conformal-factor problem (the Gibbons–Hawking–Perry observation that the Euclidean gravitational action is unbounded below [4]) into a quantitative lower bound on admissible continuum models. Any phenomenological ψ profile, optical membrane, or strong-field admissibility modulation must pay this gradient-energy cost. If a proposed benchmark fit requires ψ gradients that violate this bound, or requires an effective negative stiffness, the framework fails. Passing the bound does not by itself exclude gradient instability, a tachyonic effective potential, strong coupling near $Z_{\text{eff}} = 0$, or a negative direction in the fully reduced ADM Hamiltonian.

13 Linearized Spectrum and Stability Around a Minkowski Background

This section addresses OP9 at the lowest nontrivial level: linear perturbations around a maximally symmetric flat background. It establishes the ghost, tachyon, and gradient conditions explicitly and quantifies the strong-coupling warning attached to the stiffness floor. It does not establish nonlinear stability, cosmological (FLRW) perturbative stability, or the gauge-complete constrained quadratic action; those remain open and are recorded in the threat ledger.

Throughout this section we use natural units $\hbar = c = 1$, in which $\kappa = 8\pi G = \bar{M}_{\text{Pl}}^{-2}$ with $\bar{M}_{\text{Pl}}$ the reduced Planck mass, and we set the noise sector to zero at background and linear order, $J_{\text{noise}} = 0$.

13.1 Background conditions

Seek a solution with $g_{\mu\nu} = \eta_{\mu\nu}$ and $\psi = \psi_0$ constant. The lock then fixes the reference density to the constant $\omega = e^{-\psi_0}$ in adapted coordinates. A constant shift of ψ can be absorbed into redefinitions of ω and Λ_*, so one may set $\psi_0 = 0$ without loss of generality; we keep ψ_0 explicit below for transparency.

The reduced scalar equation requires ψ_0 to be an extremum of the effective potential,

$$U'_{\text{eff}}(\psi_0) = 0 \quad \Longleftrightarrow \quad U'(\psi_0) = \Lambda_* e^{-\psi_0}.$$

With constant ψ_0, the scalar stress tensor reduces to $T^{(\psi)}_{\mu\nu} = -g_{\mu\nu}U(\psi_0)$, and the metric equation with $G_{\mu\nu} = 0$ requires the effective vacuum curvature to vanish:

$$\Lambda + \kappa\Big[\Lambda_* e^{-\psi_0} + U(\psi_0)\Big] = 0.$$

The second condition is the usual cosmological-constant tuning. Consistent with Section 15, it is imposed, not explained: nothing in this section bears on the smallness of vacuum curvature.

13.2 Linear perturbations and the slaved sector

Write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\psi = \psi_0 + \delta\psi$, $\lambda = \Lambda_* e^{-\psi_0} + \delta\lambda$. Two algebraic statements follow immediately.

First, the lock at linear order gives

$$\delta\sqrt{-g} = \tfrac{1}{2}\,\eta^{\mu\nu}h_{\mu\nu} = \omega e^{\psi_0}\,\delta\psi \quad \Longrightarrow \quad h \equiv \eta^{\mu\nu}h_{\mu\nu} = 2\,\delta\psi.$$

The metric trace is not an independent perturbation. It is carried by $\delta\psi$. This is the linearized statement that ψ is the conformal mode.

Second, Bianchi slaving gives

$$\delta\lambda = -\Lambda_* e^{-\psi_0}\,\delta\psi.$$

Now linearize the metric equation. Using the background tuning $\Lambda + \kappa\lambda_0 = -\kappa U(\psi_0)$ and the linear variation $\delta T^{(\psi)}_{\mu\nu} = -\eta_{\mu\nu}U'(\psi_0)\delta\psi - h_{\mu\nu}U(\psi_0)$, the terms proportional to $h_{\mu\nu}$ cancel against the background tuning and the terms proportional to $\delta\psi$ cancel by the extremum condition:

$$\kappa\,\delta\lambda\,\eta_{\mu\nu} + \kappa\,U'(\psi_0)\,\delta\psi\,\eta_{\mu\nu} = \kappa\Big[-\Lambda_* e^{-\psi_0} + U'(\psi_0)\Big]\,\delta\psi\,\eta_{\mu\nu} = 0.$$

Hence

$$G^{(1)}_{\mu\nu} = 0,$$

subject to the trace constraint $h = 2\,\delta\psi$. The direct $\delta\psi$ sources cancel exactly because the background sits at an extremum of U_{eff} with vanishing effective vacuum curvature; the scalar communicates with the metric at this order only through the lock constraint, not through an independent source.

The transverse-traceless sector h^{TT}_{ij} is untouched by the constraint and obeys the standard massless wave equation: two tensor polarizations, unmodified at linear order around constant ψ_0. This is consistent with, and a perturbative check of, the local count $N_{\text{local}} = 2_{\text{tensor}} + 1_\psi$ of Section 11. The longitudinal scalar metric potentials are subject to $G^{(1)}_{\mu\nu} = 0$ together with the trace constraint; their complete constrained reduction is the linearized version of the OP7/OP8 bookkeeping and is not re-derived here. What this subsection establishes is that no additional propagating source appears in the metric sector at linear order.

13.3 Scalar dispersion, mass, and the no-tachyon condition

Linearizing the reduced scalar equation of Section 9 around ψ_0 (all metric corrections to $\Box\psi$ are second order because $\partial_\mu \psi_0 = 0$) gives

$$Z_\psi \,\Box\,\delta\psi - U''_{\text{eff}}(\psi_0)\,\delta\psi = 0, \qquad U''_{\text{eff}}(\psi_0) = U''(\psi_0) + \Lambda_* e^{-\psi_0}.$$

Taken at face value this is a test-field statement. The conformal accounting of Section 12, applied at quadratic order with the trace slaved to $\delta\psi$, replaces the intrinsic stiffness by the effective stiffness: the Einstein–Hilbert sector contributes its wrong-sign gradient piece $+3/(16\kappa)$, the intrinsic sector contributes $-Z_\psi/2$, and the net kinetic normalization of the single propagating scalar is

$$Z_{\text{eff}} = Z_\psi - \frac{3}{8\kappa}.$$

The supported quadratic Lagrangian for the scalar sector is therefore

$$\mathcal{L}_2 = -\frac{Z_{\text{eff}}}{2}\,e^{\psi_0/2}(\partial\,\delta\psi)^2 - \frac{1}{2}\,U''_{\text{eff}}(\psi_0)\,\delta\psi^2,$$

where the conformal dressing $e^{\psi_0/2}$ may be set to unity by the shift freedom noted above. For a plane wave $\delta\psi \propto e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$,

$$\omega^2 = k^2 + m_\psi^2, \qquad \boxed{m_\psi^2 = \frac{U''(\psi_0) + \Lambda_* e^{-\psi_0}}{Z_{\text{eff}}}}.$$

The linear stability conditions are then:

(L1) No ghost. $Z_{\text{eff}} > 0$. This is identical to the stiffness floor $Z_\psi > 3/(8\kappa)$; the linear analysis adds nothing to it and inherits its status.

(L2) No tachyon. $U''(\psi_0) + \Lambda_* e^{-\psi_0} \geq 0$. The integration constant enters with a definite sign structure: $\Lambda_* > 0$ contributes a restoring term (the vacuum tax stiffens the extremum), while $\Lambda_* < 0$ destabilizes the background unless $U''(\psi_0)$ compensates. The sign of the cosmological remnant is therefore not decorative; it participates directly in local scalar stability.

(L3) No gradient instability at this order. The sound speed is exactly luminal, $c_s^2 = 1$. This is automatic: a Lorentz-invariant background with a two-derivative kinetic term cannot produce $c_s^2 \neq 1$ at linear order. The statement localizes where gradient danger can actually live: backgrounds with $\partial_\mu\psi_0 \neq 0$ (in particular FLRW) or higher-derivative corrections. The flat-space analysis cannot exclude those, and does not claim to.

13.4 Strong-coupling scale

Canonically normalize $\delta\psi_c = \sqrt{\bar{Z}}\,\delta\psi$ with $\bar{Z} \equiv Z_{\text{eff}} e^{\psi_0/2}$. The exponential structures of the theory — the kinetic dressing $e^{\psi/2}$ inherited from the conformal decomposition and the slaved reaction $\Lambda_* e^{-\psi}$ in U_{eff} — generate towers of self-interactions. The leading derivative cubic,

$$-\frac{\bar{Z}}{4}\,\delta\psi\,(\partial\,\delta\psi)^2 = -\frac{1}{4\sqrt{\bar{Z}}}\,\delta\psi_c\,(\partial\,\delta\psi_c)^2,$$

is a dimension-five operator suppressed by

$$\Lambda_{\text{sc}} \simeq 4\sqrt{\bar{Z}} \sim \sqrt{Z_{\text{eff}}}\,e^{\psi_0/4},$$

up to order-one factors; the potential tower (U'''_{eff} and higher) is suppressed by the same normalization scale. Two consequences follow.

First, the stiffness floor is Planckian:

$$\frac{3}{8\kappa} = \frac{3}{8}\,\bar{M}_{\text{Pl}}^2.$$

A stiffness Z_ψ not tuned toward the floor therefore yields a Planckian cutoff, and the scalar sector is a healthy effective field theory over the entire sub-Planckian regime.

Second, as $Z_{\rm eff} \to 0^+$ the cutoff collapses, $\Lambda_{\rm sc} \to 0$. This makes the strong-coupling clause of OP9 quantitative: proximity to the stiffness floor is measured by $\Lambda_{\rm sc}/\bar{M}_{\rm Pl} \sim \sqrt{Z_{\rm eff}/\bar{M}_{\rm Pl}^2}$, and any phenomenological application that drives $Z_{\rm eff}$ small must keep its characteristic energies below the collapsing cutoff or the effective description fails.

13.5 What this section does and does not establish

Established at linear order around Minkowski: cancellation of direct $\delta\psi$ sources in the metric equation by the background conditions; two standard tensor polarizations; one scalar with mass $m_\psi^2 = U''_{\rm eff}(\psi_0)/Z_{\rm eff}$ and luminal sound speed; the no-tachyon condition (L2); and the strong-coupling estimate $\Lambda_{\rm sc} \sim \sqrt{Z_{\rm eff}}$.

Not established: the gauge-complete constrained quadratic action (the linearized OP7/OP8 bookkeeping); stability on backgrounds with $\partial_\mu\psi_0 \neq 0$, in particular FLRW scalar perturbations, where unimodular-type theories are known to require care; nonlinear and nonperturbative stability; and the behavior of the noise sector at fluctuation level. OP9 is accordingly revised, not retired.

14 Why This Is Not Just Scalar–Tensor Gravity

The theory contains a scalar, but its defining feature is not merely the presence of ψ. Ordinary scalar–tensor theories add a scalar degree of freedom to gravity and specify its coupling to curvature or matter. PVB instead starts from a determinant lock:

$$\sqrt{-g} = \omega e^{\psi}.$$

The scalar ψ is not added as a generic fifth-force carrier. It is the field that deforms the admissible volume measure.

This difference matters because the scalar's relation to λ is not chosen by a potential or coupling function. It is forced by Bianchi closure.

The distinctive chain is:

$$\sqrt{-g} = \omega e^{\psi} \Rightarrow \lambda e^{\psi} = \Lambda_* \Rightarrow \lambda(x) = \Lambda_* e^{-\psi(x)},$$

$$Z_\psi \Box\psi - U'(\psi) + J_{\rm noise} + \Lambda_* e^{-\psi} = 0,$$

$$N_{\rm local} = 2_{\rm tensor} + 1_\psi, \qquad Z_\psi > \frac{3}{8\kappa}.$$

The degree-of-freedom count is local and zero-mode separated; the stiffness inequality is necessary, not sufficient, for complete stability. This is the formal spine.

15 Limitations: What the Formal Core Does Not Yet Prove

The formal core does not prove the microscopic substrate.

It does not derive the optical constitutive relations.

It does not yet provide a complete screening mechanism.

It does not show that all matter sectors couple in a way compatible with every fifth-force constraint.

It does not prove that $\omega(x)$ emerges uniquely from an ensemble of directed acyclic graph histories.

It does not yet solve the cosmological constant problem. It relocates the cosmological contribution into an integration constant plus a ψ-slaved local reaction, which changes the bookkeeping but does not by itself explain the observed smallness of vacuum curvature.

It does not yet prove stability beyond the linear Minkowski level. The linearized spectrum and conditions of Section 13 are established; FLRW perturbations, the gauge-complete quadratic action, and nonlinear stability are not.

It does not yet provide a complete Henneaux–Teitelboim canonical construction of the global sector. The local scalar count is supported by the surviving nondegenerate (ψ, π_ψ) pair and the zero-mode-separated unimodular constraint structure, but the global cosmological pair and the canonical status of $\omega(x)$ remain open.

These are not rhetorical concessions. They are kill points.

16 Threat Ledger: Open Problems and Failure Conditions

OP1 — Origin of $\omega(x)$ The reference density must be derived from coarse-graining or shown to be unobservable except through covariant ratios. If $\omega(x)$ functions as a hidden preferred structure, the framework fails.

OP2 — Matter Coupling The formal core assumes ordinary conservation for minimally coupled matter sectors. Any nonminimal ψ coupling must avoid fifth-force bounds, equivalence-principle violations, and radiative instability.

OP3 — Lorentz-Invariant Ensemble Limit The substrate motivation requires a coarse-grained ensemble whose continuum limit preserves Lorentz invariance statistically. A generic graph does not guarantee this. The burden is explicit.

OP4 — ψ Stiffness Scale The bound $Z_\psi > \frac{3}{8\kappa}$ is necessary but not sufficient. The actual stiffness scale must allow nontrivial ψ structure without producing excluded gravitational signatures.

OP5 — Optical Bridge The Gordon optical membrane [5] used in the companion benchmark paper is phenomenological. Its constitutive laws must be derived from ψ-sector microphysics or constrained hard enough by observation to remain credible.

OP6 — Strong-Field Energy Budget Any membrane-like ψ profile near compact objects must account for gradient energy, absorption, stripping, backreaction, and causal propagation. No free optical deformation is allowed.

OP7 — Global Canonical Sector The spatial condition $\partial_i(\lambda e^{\psi}) \approx 0$ isolates a zero mode, but the exact global canonical pair, boundary conditions, and time-consistency equation must be derived in a Henneaux–Teitelboim or equivalent parametrization. A naive per-point Dirac count is not admissible in this sector.

OP8 — Gauge Status of the Reference Density Background covariance is not automatically identical to canonical gauge invariance. The theory must show that $\omega(x)$ is carried consistently by the spatial gauge generators, is replaced by a dynamical/Stueckelberg density, or leaves all observables independent of its representative. Failure here can change the constraint algebra and invalidate the local count.

OP9 — Stability Beyond the Kinetic Floor (partially addressed) The condition $Z_\psi > 3/(8\kappa)$ removes the immediate conformal kinetic ghost. Section 13 now establishes the linear Minkowski spectrum: no-tachyon condition $U''(\psi_0) + \Lambda_* e^{-\psi_0} \geq 0$, luminal sound speed, and strong-coupling scale $\Lambda_{\rm sc} \sim \sqrt{Z_{\rm eff}}$ collapsing at the floor. Still open: the gauge-complete constrained quadratic action, FLRW scalar perturbations (where unimodular-type theories are known to require care), nonlinear negative-energy directions, and reduced-Hamiltonian positivity. If FLRW perturbations exhibit unremovable instability in the ψ-sector, the framework fails.

17 Bridge to the Phenomenological Membrane Paper

The companion PVB membrane paper should cite this note as its formal core.

The membrane paper may introduce

$$\Pi = -\psi,$$

a refractive profile

$$n(r) = e^{g_n \Pi(r)},$$

and an absorption profile $\kappa(r)$, but those are not derived in the present formal note. They are benchmark-facing ansätze.

Their scientific purpose is to expose the formal core to observational failure. The optical bridge asks whether a ψ-driven admissibility sector can produce finite, testable strong-field optical effects without violating covariance, stability, causality, or energy accounting.

If the membrane fails, the optical bridge fails.

If the conformal stiffness bound fails, the formal core fails.

That distinction is the architecture.

18 Minimal Result

The minimal covariant result is:

$$\boxed{\sqrt{-g} = \omega e^{\psi}} \quad \Rightarrow \quad \boxed{\lambda e^{\psi} = \Lambda_*} \quad \Rightarrow \quad \boxed{\lambda(x) = \Lambda_* e^{-\psi(x)}}.$$

The reduced scalar dynamics is

$$\boxed{Z_\psi \Box \psi - U'(\psi) + J_{\text{noise}} + \Lambda_* e^{-\psi} = 0.}$$

The ADM consistency map requires

$$\boxed{\partial_i(\lambda e^{\psi}) \approx 0,}$$

which removes the local multiplier profile while leaving the nondegenerate scalar canonical pair. After separation of the global unimodular mode, the supported local content is

$$\boxed{N_{\text{local}} = 2_{\text{tensor}} + 1_\psi.}$$

The necessary local kinetic-stability condition is

$$\boxed{Z_\psi > \frac{3}{8\kappa}.}$$

Around a Minkowski background the linearized spectrum is two standard tensor polarizations plus one scalar with

$$\boxed{m_\psi^2 = \frac{U''(\psi_0) + \Lambda_* e^{-\psi_0}}{Z_{\text{eff}}}, \qquad c_s^2 = 1, \qquad \Lambda_{\text{sc}} \sim \sqrt{Z_{\text{eff}}}.}$$

This turns the unimodular integration-constant insight into an admissibility-locked scalar closure, supplies an explicit reduced scalar equation, and converts the conformal-mode pathology into a quantitative stiffness floor. The global zero-mode algebra and the canonical gauge status of $\omega(x)$ remain open and are named rather than hidden.

The result is not a completed theory of quantum gravity. It is a covariant effective spine with a supported three-mode local content and explicit failure conditions.

Authorship and AI-Assistance Disclosure

This work was conceived, directed, and is solely attributable to the human author. Large language models (including Anthropic's Claude, among others in a deliberately adversarial multi-model workflow) were used as derivation-checking, drafting, and critique instruments under continuous human direction; cross-model agreement was treated as a prompt for further scrutiny rather than as validation. All claims, the partition into derived and deferred sectors, and the stated failure conditions were reviewed and are asserted by the human author, who bears full responsibility for the content.

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Optional later references for extensions include causal-set and coarse-graining literature, analogue-gravity reviews, equivalence-principle and fifth-force constraints, and scalar–tensor screening mechanisms. These are deliberately omitted from the present formal core and reserved for the microscopic or phenomenological companion papers.

End of Formal Core Note v1.9-C