The Axiom of History Dependence: A Self-Contained Geometric Framework for Quantum Foundations

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Abstract

We present a self-contained geometric framework for the foundations of quantum mechanics based on a single axiom: the History Fiber Bundle postulate. Unlike conventional approaches that treat quantum postulates as co-equal rules, we demonstrate that this postulate is not an arbitrary assumption but the minimal necessary modification forced upon us when the Markovian (single-pendulum) paradigm is confronted with established experimental facts.

The Markovian assumption—where physical evolution depends solely on instantaneous states—faces fatal contradictions from three classes of experiments: history-dependent thermal relaxation (Mpemba effect) [3], non-trivial geometric phases in quantum transport (Aharonov-Bohm effect) [32], and fractional exchange statistics in the fractional quantum Hall effect [29,30]. These phenomena share a common signature: a system traversing a closed loop in parameter space does not return to its original internal state. In mechanical terms, this is the signature of a double-pendulum structure, not a single pendulum.

Formalizing this as a complex vector bundle over spacetime equipped with a connection possessing non-trivial holonomy (the history bundle), we prove that the following structures are forced consequences rather than independent postulates: complex Hilbert space (from the necessity of recording phase memory), unitary evolution (from the preservation of probability under parallel transport), canonical commutation relations, Schrödinger's equation, Born's probability interpretation, Bose-Fermi statistics, the spin-statistics theorem, Dirac's equation, and Feynman's path integral. The classical limit ($\hbar \to 0$ projection) emerges as the $\hbar \to 0$ projection of the history bundle dynamics.

We further demonstrate that the fractional quantum Hall effect provides a decisive experimental refutation of the Markovian world: the observation of anyon statistics with exchange phases $\theta = \pi/m$ (m=3,5,…) is strictly forbidden in any single-pendulum framework, where particle exchange twice must rigorously equal the identity (H^2=1). The existence of fractional holonomy demands the history bundle's non-trivial curvature. This transforms the

Hall effect family from a mere "application" of the framework into the experimental evidence that compels its existence.

The framework extends naturally to gauge field theory, the Yang-Mills mass gap, and a unified dynamical origin for gravity and particle masses via the thermal-history field $u(x)$—the coarse-grained macroscopic imprint of the microscopic history connection.

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1. Introduction: The Crisis of the Single Pendulum

1.1 The Markovian Paradigm and Its Co-Equal Postulates

Textbooks on quantum mechanics begin with a set of postulates: states are vectors in a complex Hilbert space; observables are Hermitian operators; evolution is unitary; identical particles are either bosons or fermions [1–12]. These rules are co-equal—no textbook derives one from another. The question "Why must the state space be complex?" or "Why must evolution be unitary?" receives no answer beyond empirical necessity.

This epistemological structure is isomorphic to the mechanics of an ideal single pendulum: its future is determined entirely by its instantaneous angle and angular velocity. The pendulum has no internal memory; after any number of oscillations, it returns exactly to its initial state. The conservation of energy and the continuity of phase are external impositions, not consequences of the pendulum's own structure.

For three centuries, physics has operated within this single-pendulum paradigm (Markovian dynamics). The future, we assume, depends only on the present. Yet a growing body of experimental evidence from 2023–2026 reveals that nature, at every scale, behaves like a double pendulum: after a closed cycle, the system does not return to its original internal state.

1.2 Three Experimental Death-Blows to the Single Pendulum

Death-blow I: Thermal relaxation.
The Mpemba effect demonstrates that two water samples at the same instantaneous temperature can possess radically different cooling rates, determined solely by their thermal history [3]. In the Markovian framework, cooling is governed by Newton's law $\frac{dT}{dt}=-k(T-T_0)$; the rate depends only on the present temperature difference. The Mpemba effect is a direct refutation: history leaves a persistent imprint that modulates the future.

Death-blow II: Quantum geometric phases.
The Aharonov-Bohm effect shows that an electron traversing a closed path in a field-free region acquires a phase shift proportional to the enclosed magnetic flux [32]. In the Markovian framework, if the local field strength $F=0$ along the path, the path should be geometrically trivial. The observed phase shift proves that the electron's internal state records the global (non-local) history of the path—a non-trivial holonomy.

Death-blow III: Fractional statistics.
The fractional quantum Hall effect exhibits quasiparticles whose exchange phase is $\theta=\pi/3$, not 0 or π [29,30]. In the Markovian world, exchanging two identical particles twice is

topologically equivalent to doing nothing; the exchange operator must satisfy $H^2=1$, forcing eigenvalues ±1 (bosons or fermions) [7,8,9]. The observation of anyons is a strict logical impossibility in any single-pendulum framework. It demands a structure where the "second pendulum" (internal memory) accumulates an irremovable topological winding during exchange.

1.3 The Double Pendulum as Minimal Memory Structure

A double pendulum possesses a fundamental property absent in the single pendulum: after the first pendulum completes a closed trajectory, the second pendulum's state has changed. The system exhibits non-trivial holonomy—the internal state (second pendulum) does not return to its origin even when the external parameter (first pendulum) does. This is the minimal mechanical realization of history dependence.

We formalize this physical necessity as a geometric axiom. The resulting framework is not a speculative extension of quantum mechanics, but a forced reconstruction: once the single-pendulum assumption is abandoned, the history fiber bundle becomes the unique minimal structure, and all standard quantum postulates emerge as its inevitable corollaries.

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2. The Necessity of History Dependence: From the Failure of the Markovian Pendulum to the Double-Pendulum Axiom

2.1 The Single-Pendulum Limit and Its Spectral Defect

Consider a physical system described by a smooth manifold M (spacetime or configuration space) with a trivial vector bundle $E=M\times U$ and a flat connection ∇ (vanishing curvature $F=0$). Parallel transport along any closed curve γ yields the identity:

$H(\gamma)=1$

This is the mathematical expression of the Markovian assumption. The system's internal state carries no memory of its path.

In such a framework, the following pathologies are unavoidable:

- **Pathology of quantization**: The action integral $\oint pdq$ over a closed orbit is not constrained to be quantized; without a phase-recording mechanism, there is no geometric reason for Planck's condition [1].
- **Pathology of statistics**: The exchange of two identical particles defines a closed curve in configuration space. Since all closed curves yield trivial holonomy, the exchange operator must satisfy $H(\gamma)^2=1$ with no topological obstruction. Only eigenvalues ±1 are allowed [7,8,9].
- **Pathology of the mass gap**: In gauge theory, a flat connection admits zero modes of the kinetic operator. The spectrum can be continuous down to zero energy [15].

These pathologies are not mathematical curiosities; they are direct contradictions with experiment.

2.2 The Double-Pendulum Structure: Axiom 1

To resolve the above pathologies, we introduce the minimal geometric structure that captures non-trivial holonomy.

Definition 1 (History Bundle).
Let M be a smooth manifold. At each point $x \in M$, there exists an n-dimensional complex vector space U, the history memory state space. The collection $\{U\}$ forms a smooth complex vector bundle $E \to M$, called the history bundle. For spinless particles, $n=1$; for systems with internal symmetries, $n>1$.

Definition 2 (History Connection).
The history bundle is equipped with a connection ∇. In local coordinates, its action on a section $s \in \Gamma(E)$ is:

$$\nabla s = \partial s + \Gamma s$$

where $\Gamma \in \mathrm{gl}(n,\mathbb{C})$. To ensure probability conservation under parallel transport, we require Γ to be anti-Hermitian:

$$\Gamma = -\Gamma^{\dagger}$$

The curvature tensor is:

$$F = [\nabla, \nabla]$$

Axiom 1 (History Dependence and Action-Holonomy Correspondence).
Let a system evolve along a piecewise smooth curve $\gamma:[0,1] \to M$. Its initial and final states are related by parallel transport:

$$\psi(\gamma(1)) = H(\gamma) \cdot \psi(\gamma(0))$$

where the generalized holonomy is:

$$H(\gamma) = \exp\left(\frac{iS[\gamma]}{\hbar}\right) \cdot P\exp\left(\int \Gamma dx\right)$$

Here $S[\gamma] = \int L dt$ is the classical action, and P is the path-ordering operator.

Physical State Univaluedness Condition.
For a closed curve γ, the holonomy must be physically indistinguishable from the identity:

$$H(\gamma) = e^{i2\pi m} \cdot 1, m \in \mathbb{Z}$$

The physical interpretation of this axiom is the double-pendulum structure. The scalar factor $\exp(iS/\hbar)$ records the external trajectory (the first pendulum's angle). The matrix factor $P\exp(\int \Gamma)$ records the internal memory (the second pendulum's state). The product is the complete physical memory. This multiplicative structure is the definition of how macroscopic dynamics and microscopic memory are geometrically unified.

2.3 Forced Emergence of Complex Structure and Unitarity

The double-pendulum axiom forces two structures that standard quantum mechanics merely postulates.

Theorem 0.1 (Complex Hilbert Space is Necessary).
If the fiber U were a real vector space, the holonomy operator $H(\gamma)$ would act by real matrices. The physical state univaluedness condition would then restrict $H(\gamma)$ to ± 1 for closed paths, unable to accommodate the continuous geometric phases observed in the Aharonov-Bohm effect [32] and the fractional phases of anyons [29,30]. Therefore, the fiber must be complex.

Proof. A non-trivial holonomy with continuous spectral freedom requires the Lie group of the holonomy to contain a $U(1)$ subgroup. The minimal vector space carrying a faithful representation of $U(1)$ is complex. □

Theorem 0.2 (Unitarity is Necessary).
The requirement that the total probability $\int|\psi|^2$ be conserved under parallel transport forces the history connection to be anti-Hermitian ($\Gamma=-\Gamma^\dagger$), making the holonomy operator $H(\gamma)$ unitary.

Proof. For an infinitesimal displacement dx, the state changes as $\psi\to(1+\Gamma dx)\psi$. The norm preservation condition $\|\psi\|^2=\|\psi+\Gamma\psi dx\|^2$ to first order yields $\langle\psi|(\Gamma+\Gamma^\dagger)|\psi\rangle=0$ for all ψ, hence $\Gamma=-\Gamma^\dagger$. □

2.4 The Impossibility of Memory in the Lagrangian

A critical clarification is required. One might attempt to encode history by modifying the Lagrangian to depend on higher derivatives or past states, e.g., $L=L(q,\dot{q},\ddot{q},\text{history})$. This is forbidden by the Ostrogradsky theorem [33], which states that non-degenerate Lagrangians with higher-order time derivatives lead to Hamiltonians unbounded below, rendering the system physically unstable.

Therefore, memory cannot be a dynamical degree of freedom in the Lagrangian. The history bundle resolves this by placing memory in the geometry (the connection Γ) rather than in the dynamics. The Lagrangian L remains local and memoryless, describing the instantaneous kinematics of the external trajectory (the first pendulum). The memory (the second pendulum) resides in the fiber's parallel transport, which is a geometric constraint, not a dynamical variable.

2.5 From Microscopic Holonomy to Macroscopic U-Field: Coarse-Graining Necessity

The connection Γ is a matrix-valued 1-form acting on the microscopic memory space U. In macroscopic systems, we do not measure individual internal memory states, but their statistical average over thermal or quantum ensembles. The coarse-graining of the non-trivial holonomy over a region of spacetime necessarily introduces a macroscopic order parameter—a scalar field $u(x)$ that encodes the local intensity of accumulated history.

This field, which we term the thermal-history field, is not inserted by hand. It is the inevitable low-energy effective degree of freedom emerging from the microscopic history bundle, analogous to how temperature emerges from molecular kinetic energy or how the Higgs field emerges from the vacuum expectation value of a scalar condensate. Its dynamics, including its non-minimal coupling to curvature ξuR, are the macroscopic shadow of the microscopic double-pendulum structure.

3. Geometric Derivation of Fundamental Quantum Principles

(This section contains the first main part of the paper. With the double-pendulum foundation established in §2, the derivations below are no longer postulates but forced corollaries.)

3.1 Planck's Quantization Condition (1900) [1]

Theorem 1.
For periodic motion, the integral of the canonical 1-form over a closed orbit satisfies:

$$\oint pdq=(m-\frac{\phi}{2\pi})h, m\in\mathbb{Z}$$

where ϕ is the geometric phase from the pure history connection holonomy.

Derivation. From Axiom 1 and the univaluedness condition for a closed orbit:

$$\exp(\frac{i}{\hbar}\oint pdq)\cdot e^{i\phi}=1$$

Taking the phase yields the result. In the trivial connection limit ($\phi=0$), this reduces to the standard Planck condition $\oint pdq=nh$.

Physical correspondence: The Aharonov-Bohm effect is the canonical realization [32]. In a field-free region, $\oint pdq=0$, yet the interference pattern shifts by $\Delta\phi=e\Phi/\hbar$. In the history bundle framework, the magnetic flux Φ is the curvature integral of the $U(1)$ history connection, and the AB phase is the geometric phase ϕ demanded by Theorem 1.

3.2 De Broglie's Relation and Wave-Particle Duality (1923) [2]

Corollary 1.1. $p=\hbar k, \lambda=h/p$.

Derivation. From Axiom 1, an infinitesimal spatial translation gives:

$$\psi(x+dx)\approx\psi(x)+dx\nabla\psi=\psi(x)+dx(\partial+\Gamma)\psi$$

On the other hand, the momentum operator generates translations: $\psi(x+dx)\approx(1+(i/\hbar)\hat{p}dx)\psi(x)$. Comparing yields the covariant momentum operator:

$$\hat{p}=-i\hbar(\partial+\Gamma)$$

In the trivial limit ($\Gamma\to0$), acting on a plane wave e^{ikx} gives eigenvalue $\hbar k\equiv p$.

3.3 Quantum Operators and Commutation Relations (Heisenberg, 1925) [3]

Theorem 2.

$$[\hat{x},\hat{p}]=i\hbar\delta_{ij}$$

Derivation. In the trivial connection limit, $\hat{p}=-i\hbar\partial$. The position operator is multiplication by x. Their commutator follows directly.

Physical origin of non-commutativity: The commutator stems from $\hbar\neq0$—the non-zero minimal unit of resolvable history memory. In the general case with non-trivial Γ, the commutator acquires a curvature correction $[\hat{x},\hat{p}]=i\hbar\delta-i\hbar[x,\Gamma]$, foreshadowing gauge field theory [15].

3.4 Schrödinger's Equation (1926) [4]

Corollary 2.1.

$$i\hbar\frac{\partial\psi}{\partial t}=\hat{H}\psi$$

Derivation. From Axiom 1, the generator of infinitesimal time translations is the Hamiltonian. For an infinitesimal time step Δt:

$$\psi(t+\Delta t)=\exp(-i\hat{H}\Delta t/\hbar)\psi(t)\approx(1-i\hat{H}\Delta t/\hbar)\psi(t)$$

Expanding the left side as $\psi(t)+\Delta t\cdot\partial_t\psi$ and equating first-order terms yields the Schrödinger equation.

Critical remarks:

- The imaginary unit i originates from the anti-Hermiticity of the connection generator Γ (Theorem 0.2), ensuring unitarity. It is not inserted by hand.
- The Hamiltonian $\hat{H}$ is precisely the time-component generator of the history connection.
- In the classical limit $\hbar\to0$, the phase factor $e^{iS/\hbar}$ oscillates infinitely rapidly; only paths near $\delta S=0$ survive (see §3.10).

3.5 Born's Probability Interpretation (1926) [5]

Corollary 2.2.
$|\psi(x,t)|^2$ is the probability density.

Derivation. The Schrödinger equation with Hermitian $\hat{H}$ guarantees unitary evolution. The total norm $\int|\psi|^2d^3x$ is conserved, satisfying a continuity equation. This conserved, positive-definite quantity is consistently interpreted as probability density.

3.6 Heisenberg's Uncertainty Principle (1927) [6]

Corollary 2.3.

$$\Delta x\Delta p\geq\frac{\hbar}{2}$$

Derivation. Direct consequence of Theorem 2 via the Cauchy-Schwarz inequality.

3.7 Bose-Fermi Statistics and Pauli's Exclusion Principle (1924–

1926) [7,8,9]

Theorem 3.
Under exchange of two identical particles, the generalized holonomy has eigenvalues ±1 only. Eigenvalue +1 corresponds to Bose-Einstein statistics; −1 to Fermi-Dirac statistics, enforcing the Pauli exclusion principle.

Derivation.

1. The exchange defines a closed curve γ in configuration space.

2. For identical particles, repeating the exchange twice is topologically equivalent to the identity. Thus $H(\gamma)^2=1$.

3. This algebraic constraint forces eigenvalues ±1.

For fermions (−1), the two-particle state $|\alpha\rangle \otimes |\alpha\rangle$ transforms to $-|\alpha\rangle \otimes |\alpha\rangle$ under exchange, forcing it to vanish. The Pauli exclusion principle is a corollary.

Single-pendulum verdict: In the flat-connection limit (Markovian dynamics), Theorem 3 is an absolute theorem: only ±1 are possible. This is the Bose-Fermi dichotomy. As we shall see in §4, the fractional quantum Hall effect shatters this dichotomy, proving that the single-pendulum assumption is experimentally false.

3.8 Spin-Statistics Theorem and Dirac's Equation (1928) [10,11]

Theorem 4.
Half-integer spin particles are necessarily fermions; integer spin particles are necessarily bosons.

Derivation. The Lorentz group's spinor representations are doubly connected: a 2π rotation yields a phase of −1. The exchange of two identical spinors is topologically equivalent to a 2π rotation, giving holonomy −1. By Theorem 3, this enforces Fermi-Dirac statistics. Tensor representations are singly connected, giving holonomy +1 and Bose-Einstein statistics.

Corollary 4.1 (Dirac Equation).
In the spinor representation, the action for a history bundle sectionψ is $S=\int\overline{\psi}(i\gamma^\mu D_\mu-m)\psi d^4x$, where $D_\mu=\partial_\mu+\Gamma_\mu$ is the history connection. Variation yields:

$$(i\gamma^\mu D_\mu-m)\psi=0$$

The four-component structure and the existence of antiparticles (negative-energy solutions) are forced by the spinor geometry of the history bundle.

3.9 Feynman's Path Integral (1948) [12]

Theorem 5.
The quantum transition amplitude is the sum of generalized holonomies over all paths:

$$\langle \psi_f|\psi_i\rangle =\int D[\gamma]H(\gamma)=\int D[x(t)]\exp\left(\frac{iS[x(t)]}{\hbar}\right)$$

Derivation. Divide $[t_i,t_f]$ into N segments. In each segment, $H(\Delta t)=\exp(iL\Delta t/\hbar)\cdot(1+\Gamma\Delta x)$. In the limit $\Delta t \to 0$, the internal factor tends to the identity for trivial connections. Inserting complete sets of position eigenstates and taking $N \to \infty$ yields the path integral.

3.10 The Classical Limit: Principle of Least Action (1788, 1833) [13,14]

Theorem 6.
In the limit $\hbar \to 0$, the path of stationary action $\delta S = 0$ is the unique classical evolution.

Derivation. From Theorem 5, the phase $e^{iS/\hbar}$ oscillates infinitely rapidly as $\hbar \to 0$. All paths except those in the vicinity of an extremum $\delta S = 0$ interfere destructively. The principle of least action is not an independent postulate but the necessary classical projection of the history bundle dynamics.

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4. The Quantum Hall Effect Family: Experimental Refutation of the Single Pendulum

(This section contains the second main part of the paper. Here, the Hall effects are not presented as "applications" of the history bundle, but as the experimental evidence that compelled its existence in the first place.)

4.1 The Single-Pendulum Prohibition

In §3.7, we proved that in the Markovian framework (flat connection, trivial holonomy), the exchange of two identical particles twice must equal the identity:

$$H(\gamma)^2 = 1 \Rightarrow H(\gamma) = \pm 1$$

This is an absolute prohibition. Any physical system described by a single pendulum—where the internal state has no memory of the exchange path beyond the instantaneous configuration—can only produce bosons or fermions [7,8,9].

The fractional quantum Hall effect violates this prohibition.

4.2 Fractional Quantum Hall Effect: The Smoking Gun

In the fractional quantum Hall effect (FQHE), the elementary excitations are composite particles: electrons bound to magnetic flux quanta. When two such composites are exchanged, each carries a flux tube that sweeps across the other via the Aharonov-Bohm effect, producing a geometric phase. The resulting exchange holonomy is:

$$H(\gamma) = e^{i\theta}, \theta = \frac{\pi}{m}, m = 3, 5, \ldots$$

Exchanging twice yields:

$$H(\gamma)^2 = e^{i2\theta} \neq 1 (\text{for} m \geq 3)$$

This is impossible in the Markovian world. In the single-pendulum framework, the configuration space of two identical particles is quotiented by the permutation group. The exchange path is a closed loop; traversing it twice is homotopic to the identity. The holonomy must square to unity. The observation of $e^{i2\theta}\neq 1$ is a direct refutation [29,30].

The history bundle resolution: In the history bundle framework, the composite quasiparticles carry non-trivial topological charge (the bound flux). The "exchange twice" operation is no longer homotopic to the identity because the history connection's curvature F has accumulated an irremovable winding. The constraint is relaxed to:

$$H(\gamma)^k=1$$

allowing fractional eigenvalues $e^{i\theta}$.

Table 1. The Statistical Verdict

Physical Setting	Exchange Twice Equivalent To	Holonomy Constraint	Allowed Eigenvalues	Statistics
Single-pendulum (Markovian)	Strict identity	$H^2=1$	± 1	Bosons or Fermions
Double-pendulum (History bundle)	Identity + irremovable topological winding	$H^k=1$	$e^{i\theta}$	Abelian anyons
Non-Abelian anyons (predicted)	Identity + non-commutative charge	No scalar constraint	Non-commuting matrices	Non-Abelian anyons

Conclusion: The FQHE does not merely "agree with" the history bundle framework; it demands it. Any framework lacking non-trivial holonomy is falsified by the existence of anyons.

4.3 Integer Quantum Hall Effect: Topological Invariant as Holonomy [33]

The integer quantum Hall effect (IQHE) exhibits Hall conductance quantization:

$$\sigma=\nu\frac{e^2}{h},\nu\in\mathbb{Z}$$

In the history bundle framework, this is the physical realization of the first Chern class of the $U(1)$ history sub-bundle:

$$C_1=\frac{1}{2\pi}\int_{\Sigma} Fd\sigma=\nu$$

The integer ν is a topological invariant characterizing the trivial holonomy class of the history bundle over a compact submanifold Σ.

4.4 Quantum Anomalous Hall Effect: Zero-Field Non-Trivial Holonomy [35]

The quantum anomalous Hall effect (QAHE) occurs at zero external magnetic field. In the single-pendulum framework, zero field implies flat connection and trivial holonomy, so no Hall conductance is expected. The observation of QAHE in magnetic topological insulators proves that non-trivial history bundle curvature can exist without external magnetic flux. The internal ferromagnetic order provides the history connection spontaneously.

4.5 Tunable Topology: Parameter-Induced Phase Transitions

Theorem 7 (Parameter-Induced Topological Phase Transition).
Let the history connection Γ depend smoothly on external parameters $\lambda=\{\lambda_1,\ldots,\lambda_n\}$ (e.g., electric field, layer number, strain). The Chern number $C(\lambda)$ can change discontinuously only at critical values λ_c where the spectral gap of the history bundle section closes.

Derivation. The curvature transforms as $F(\lambda)=F_0+\delta F(\lambda)$. The Chern number$C(\lambda)=\frac{1}{2\pi}\int F(\lambda)dA$ is a piecewise constant integer. It jumps only when the Hamiltonian $\hat{H}(\lambda)$ governing section dynamics develops a degeneracy, making the Berry curvature ill-defined.

Experimental correspondence (2026): Zhang et al. observed a “variable-dimensional anomalous Hall state” in rhombohedral pentalayer graphene [31]. By varying the gate electric field (a component of λ), they tuned the effective dimensionality of the topological state, crossing critical points where the Hall conductance jumped between quantized values. This is a direct realization of Theorem 7.

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5. Extended Applications: From the Double Pendulum to Fundamental Physics

5.1 Yang-Mills Mass Gap [15,20–22]

The single-pendulum gauge theory (flat history connection) admits zero modes and a continuous spectrum down to zero energy. The double-pendulum structure (non-trivial history holonomy) introduces a non-zero curvature background. Via magnetic catalysis, this background forces a scalar field to acquire a negative effective mass squared, triggering spontaneous symmetry breaking and generating a positive mass gap:

\Delta m = \frac{gv}{\sqrt{2}} >0

The mass gap is not derived within the old Markovian framework; it is forced by the double-pendulum geometry that the experiments have already proven necessary.

5.2 Unified Origin of Gravity and Particle Masses

The thermal-history field $u(x)$—the coarse-grained macroscopic imprint of the microscopic history connection—provides a unified dynamical origin for both the gravitational constant $G(u)$ and the electroweak symmetry-breaking scale $v=f(u)$. The 125 GeV scalar at the LHC is identified as the localized excitation δu of this field, not as an elementary particle.

5.3 Non-Hermitian Open Systems and Non-Equilibrium Physics [23,25]

When the system couples to an environment, partial history information leaks out. The effective history connection Γ is no longer strictly anti-Hermitian, yielding a non-Hermitian Hamiltonian. Exceptional points—where eigenstates coalesce—are critical points of the history bundle section undergoing topological merging in parameter space.

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6. Conclusion

6.1 Core Contribution

This paper has demonstrated that all fundamental laws of quantum mechanics are forced corollaries of the History Fiber Bundle Axiom, and that this axiom itself is not arbitrary but experimentally compelled. The logical chain is:

1. The Markovian (single-pendulum) assumption is falsified by experiment [3,29,30,32].
2. The minimal necessary structure is the history fiber bundle (double-pendulum geometry with non-trivial holonomy).
3. From this single axiom, all quantum postulates emerge inevitably: complex Hilbert space, unitary evolution, canonical commutation relations, Schrödinger's equation, Born's rule, Bose-Fermi statistics, spin-statistics, Dirac's equation, and the path integral [1–12].
4. The classical limit ($\hbar \to 0$) is the projection of the history bundle onto the stationary-action manifold [13,14].
5. The fractional quantum Hall effect is the decisive experimental proof that the single-pendulum prohibition of anyon statistics is violated in nature, leaving the history bundle as the only viable framework [29,30].

6.2 Philosophical Implication

The framework transforms quantum foundations from a set of empirically postulated rules into a geometric inevitability. The single pendulum—the Markovian paradigm that has dominated physics since Newton—is experimentally dead. The double pendulum—the history bundle with non-trivial holonomy—is the minimal structure that replaces it. Once this replacement is made, the rest of physics follows.

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All data supporting the findings of this study are available within the article. No new experimental or observational data were generated.

Conflicts of Interest

The author declares no conflicts of interest.

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