The Uncircumscribability Theorem: Why Straight Lines Vanish in Curved Spaces and How Algebra Recovers Them

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Abstract

We prove that in any space of non-vanishing Gaussian curvature, Euclidean straight lines do not exist—not because space "bends" them, but because π's transcendence erects an ontological barrier to finite geometric construction [1]. This barrier, which we formalize as the Uncircumscribability Theorem, states that any finite algebraic or piecewise-geodesic attempt to "draw" a straight line in a curved 2-manifold necessarily incurs an irremovable π-twist [2,3,4,5,7]. Consequently, Riemannian geodesics are not "straight lines in curved space"; they are the residual traces left after the construction of a straight line fails. The statement "parallel lines intersect in curved space" is therefore doubly ill-posed: its premise (the existence of lines) is false, and its object (geodesics) are not lines.

We further show that the recovery of true, non-intersecting isometric lines requires abandoning geometric construction altogether. The recent solution of the Erdős unit-distance problem by an OpenAI reasoning model [14], verified by Alon et al. [15], exemplifies this paradigm: by defining isometry purely within high-dimensional algebraic number fields—without ever "drawing" a circle or a line—the construction bypasses π-twist entirely and recovers exact, non-intersecting isometric structures. Euclid's fifth postulate is thus not an axiom about space, but a theorem about constructibility under the jurisdiction of π [1].

As a corollary, we prove that the standard interpretation of geodesic motion in General Relativity—"test particles follow the straightest possible path in curved spacetime" [16,17,18]—is mathematically undefined in any region where the Riemann curvature tensor $R \neq 0$. The referent "straight line" does not exist in such regions; therefore the geodesic postulate, as commonly verbally formulated, commits a category error. The Einstein field equations remain mathematically self-consistent; only their semantic overlay is falsified.

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1. Introduction

Euclid's fifth postulate asserts that through a point not on a given line, there exists exactly one line parallel to the given line, and that parallel lines never meet [4]. This postulate holds in flat space (zero Gaussian curvature).

Riemann's geometry (1854) demonstrated that in spaces of positive curvature, geodesics that are initially "parallel" can converge and intersect [6]. The standard example is two meridians on a sphere: they appear parallel at the equator yet meet at the poles.

However, this standard formulation contains an unexamined presupposition: that straight lines exist in curved space. In this paper, we demonstrate that they do not—not due to the curvature itself, but due to the transcendence of π acting as an ontological barrier to finite construction [1]. We formalize this as the Uncircumscribability Theorem (Theorem 6.2): any finite attempt to "draw" a straight line in a space with $K \neq 0$ necessarily fails, because the discrete, history-dependent recalibration process (the method of exhaustion [2,3]) can never terminate, and its irremovable gap is amplified by the Gauss–Bonnet theorem [5,7] into a geometric obstruction.

The geodesics of a curved manifold are therefore not straight lines. They are curves whose apparent straightness is the macroscopic limit of a discrete, history-dependent recalibration process—but this limit is a compromise, not a realization. Riemannian geometry does not generalize Euclid; it sidesteps Euclid by substituting a constructible local optimum (the geodesic) for an unconstructible global ideal (the straight line).

A crucial corollary follows. If straight lines do not exist in curved 2-space, neither do parallel lines, and the question of their intersection is ill-posed. Yet true isometric lines can be recovered if one abandons the very act of "drawing" and instead defines isometry algebraically. The recent resolution of the Erdős unit-distance problem by an OpenAI reasoning model [14], verified by Alon et al. [15], provides the paradigm: by constructing high-dimensional algebraic integer lattices in which "unit distance" is declared via the field norm N_{K/Q}(x)=1—never approximating a circle—the model bypasses π-twist entirely and recovers exact, non-intersecting isometric structures.

Finally, we address the geodesic postulate of General Relativity. We prove that in any pseudo-Riemannian manifold with $R \neq 0$, the statement "geodesics are the straightest possible lines" [16,17,18] is mathematically undefined, because the set of straight lines in such a manifold is empty. This is not a philosophical critique but a strict mathematical consequence of Theorem 3.1 [1,5,7,8] applied to spacelike and timelike 2-sections. The Einstein field equations remain intact; only their verbal interpretation is falsified.

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2. Preliminaries

2.1 Transcendence of π

Theorem 2.1 [1]. π is transcendental; that is, there exists no nonzero polynomial $f(x)$ with integer coefficients such that $f(\pi) = 0$.

Corollary. π cannot be obtained from the rationals by any finite sequence of algebraic operations (addition, subtraction, multiplication, division, and root extraction). In particular, no finite polygon can exactly represent a circle.

2.2 The Method of Exhaustion and Monotonicity

Proposition 2.2 [2,3]. Let S be a circle of radius r in the Euclidean plane, and let P_n denote the perimeter of the inscribed regular n-gon. Then the sequence $\{P_n\}_{n \geq 3}$ satisfies
$P_3 < P_4 < P_5 < \cdots < 2\pi r$
and
$\lim_{n\to\infty} P_n = 2\pi r$.

Proof [2,3]. Let AB be a side of the inscribed regular n-gon, subtending arc $\hat{AB}$. Let C be the midpoint of the arc. By the triangle inequality, AC + CB > AB. Applying this operation to every side yields the inscribed regular 2n-gon, whose perimeter is strictly greater. The upper bound follows from the circumscribed polygon.

Corollary [1,2,3]. By Theorem 2.1 [1], for every finite $n \in \mathbb{N}$, $\delta_n := 2\pi r - P_n > 0$. That is, the perimeter gap between the circle and any inscribed polygon is irremovable.

Proposition 2.3. Let A_n denote the area of the inscribed regular n-gon, where
$A_n = (1/2)nr^2 \sin(2\pi/n)$.
Then $\Delta A_n := \pi r^2 - A_n > 0$ for all finite n, and
$\lim_{n\to\infty} \Delta A_n = 0$.

Proof [2,3]. Since $\sin x < x$ for $x > 0$, we have $A_n < \pi r^2$. Monotonicity follows from properties of sin on $(0,\pi)$. □

2.3 The Gauss–Bonnet Theorem

Theorem 2.4 [5,7,9]. Let D be a simply connected region with piecewise smooth boundary ∂D on a two-dimensional Riemannian manifold (M,g). Then
$\iint_D K\, dA + \oint_{\partial D} k_g\, ds + \Sigma_{i=1}^m \alpha_i = 2\pi$
where K is the Gaussian curvature, k_g is the geodesic curvature of ∂D, and α_i are the exterior angles at the corners.

2.4 Asymptotic Expansion of Geodesic Disks

Proposition 2.5 [11]. Let (M,g) be a two-dimensional Riemannian manifold, $p \in M$, and K(p) the Gaussian curvature at p. For the geodesic disk D_r of geodesic radius r centered at p:
$L(r) = 2\pi r - (\pi/3)K(p)r^3 + O(r^5)$
$A(r) = \pi r^2 - (\pi/12)K(p)r^4 + O(r^6)$

Corollary [11]. When $K(p) > 0$, for sufficiently small $r > 0$: $L(r) < 2\pi r$ and $A(r) < \pi r^2$. When $K(p) < 0$: $L(r) > 2\pi r$ and $A(r) > \pi r^2$. In either case, if $K(p) \neq 0$, the geodesic circle deviates from the Euclidean circle of the same radius.

2.5 Holonomy and Curvature

Definition [12]. Let (M,g) be a Riemannian manifold with Levi-Civita connection ∇. For a closed curve $\gamma: [0,1] \to M$, the holonomy transformation H\γ *is the orthogonal transformation on T*\{γ(0)}M induced by parallel transport along γ. The holonomy group is $\mathrm{Hol}(\nabla) = \{ H_\gamma \mid \gamma \text{ closed} \}$.

Theorem 2.6 [8]. The holonomy algebra equals the algebra generated by the curvature tensor:

$(\nabla) = \mathrm{span}\{ R(X,Y) \mid X,Y \in T_p M, p \in M \}$

Corollary [8]. $\mathrm{Hol}(\nabla) \neq \{e\}$ if and only if $R \neq 0$.

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3. Main Theorem

Definition (Euclidean straight line). A path ℓ in a Riemannian manifold (M,g) is a Euclidean straight line if and only if both of the following hold:

- **Flatness**: The geodesic curvature of ℓ vanishes identically ($k_g \equiv 0$) and parallel transport along any segment of ℓ preserves vector directions (trivial holonomy along ℓ).
- **Global minimality**: For any two points $A,B \in \ell$, the length of the segment of ℓ between A and B equals the infimum of lengths of all piecewise smooth paths from A to B in M.

Remark [11]. In a flat manifold ($K \equiv 0$), geodesics satisfy both conditions. In a curved manifold, geodesics satisfy condition 2 (local minimality) but, as we shall prove, cannot simultaneously satisfy condition 1 globally.

Theorem 3.1 (Impossibility of Straight Lines in Curved Spaces). Let (M,g) be a complete, two-dimensional Riemannian manifold with $K \neq 0$ at some point $p \in M$. Then no Euclidean straight line exists in M.

Proof
Step 1 [1,2,3]. π's transcendence guarantees an irremovable gap between circles and polygons. By Propositions 2.2 [2,3] and 2.3, and Theorem 2.1 [1], the perimeter gap $\delta_n = 2\pi r - P_n > 0$ and the area gap $\Delta A_n = \pi r^2 - A_n > 0$ are strictly positive for all finite n. These gaps cannot be eliminated by any finite process.

Step 2 [5,7,9,11]. The Gauss–Bonnet theorem identifies this gap with nonzero curvature.
By Proposition 2.5 [11], the circumference of a geodesic circle of radius r in (M,g) satisfies
$L(r) = 2\pi r - (\pi/3)K(p)r^3 + O(r^5)$.
If $K(p) \neq 0$, then $L(r) \neq 2\pi r$ for all sufficiently small $r > 0$. The geodesic circle deviates from the Euclidean circle by an amount proportional to K(p).

Applying the Gauss–Bonnet theorem (Theorem 2.4 [5,7,9]) to the geodesic disk D_r:
$\iint*\{D_r\} K\, dA + \oint*\{\partial D_r\} k_g\, ds = 2\pi$.
When $K(p) \neq 0$, the integral $\iint_{D_r} K\, dA \neq 0$, and the boundary geodesic curvature integral is correspondingly adjusted. The space itself distorts the shape of every circle: no inscribed polygon can approximate a geodesic circle exactly, because the geodesic curvature of the circle's boundary is affected by the ambient curvature. The gap between straight segments (the polygon's sides) and curved arcs (the circle's boundary) is thus an intrinsic geometric quantity determined by K.

Therefore, $K \neq 0$ implies that the geometric mismatch between straight paths and geodesic paths is irremovable. □

Step 3 [8]. Nontrivial holonomy.
Since $K(p) \neq 0$ at some point, the curvature tensor $R \neq 0$ (in two dimensions, R is entirely determined by K). By Theorem 2.6 [8]:
$R \neq 0 \implies \quad (\nabla) \neq \{0\} \implies \mathrm{Hol}(\nabla) \neq \{e\}$.
The holonomy group is nontrivial: parallel-transporting a vector along a sufficiently large closed curve and returning to the starting point yields a vector pointing in a different direction.
□

Step 4 [8]. Straight lines do not exist.
By Step 3, $\mathrm{Hol}(\nabla) \neq \{e\}$. We now show that this violates the flatness condition of a Euclidean straight line.

Suppose, for contradiction, that a Euclidean straight line ℓ exists in M. By the flatness condition, parallel transport along ℓ preserves vector directions. Consider a closed curve γ composed of:
(a) a segment of ℓ from point A to point B,
(b) a geodesic arc from B back to A (not lying on ℓ).

Since ℓ is a straight line, parallel transport along the ℓ-segment from A to B preserves all vector directions. Since the return arc is a geodesic, parallel transport along it preserves the tangent direction. However, the composition of these two parallel transports around γ must equal the holonomy transformation H_γ.

By Step 3, for γ enclosing a region where $K \neq 0$, $H_\gamma \neq \mathrm{id}$. This means that after traversing the full closed curve γ, a vector returns with a different orientation. But this is a contradiction: if ℓ were a Euclidean straight line, parallel transport along ℓ is trivial, and parallel transport along the geodesic return arc is also trivial (geodesics preserve their own tangent), so H_γ should be the identity.

The contradiction arises because, in a curved space, the flatness condition and the global minimality condition cannot be simultaneously satisfied. A geodesic satisfies minimality but acquires nontrivial holonomy when combined with other geodesics into closed curves. A locally flat path (if it could exist) would not be globally minimal.

Therefore, no Euclidean straight line exists in (M,g) when $K \neq 0$.

Conclusion: $K \neq 0 \rightarrow \mathrm{Hol}(\nabla) \neq \{e\} \implies$ Euclidean straight lines do not exist.

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4. The Recalibration Principle: The Method of Exhaustion as Mathematical Prototype [2,3]

Theorem 3.1 [1,5,7,8,11] proves the impossibility of straight lines in curved spaces. A natural question arises: what are geodesics, if not straight lines?

We answer this by identifying geodesics as the macroscopic limit of a discrete, history-dependent recalibration process, whose rigorous mathematical prototype is the method of exhaustion [2,3].

4.1 The Method of Exhaustion as Recalibration [2,3]

In the method of exhaustion, each step consists of:

1. Locate the current path: A side AB of the inscribed n-gon.
2. Detect deviation: The arc $\hat{AB}$ does not coincide with the segment AB; the gap $\delta_n > 0$ persists.
3. Recalibrate: Insert a new vertex C on the arc, replacing AB with AC + CB.
4. Update history: The new perimeter $P_{n+1} > P_n$; the history is irreversibly altered.
5. Return to step 1.

This process has four essential features [2,3]:

- **Discreteness**: Each step is a finite operation.
- **History dependence**: Each result P_{n+1} depends strictly on the previous step P_n. History cannot be skipped, reversed, or erased.
- **Non-termination**: By the transcendence of π (Theorem 2.1 [1]), $P_n < 2\pi r$ for all finite n. The process never terminates.
- **Monotonicity**: $P_3 < P_4 < P_5 < \cdots$; the direction of approach never reverses.

4.2 From Exhaustion to Curved Spaces

Method of Exhaustion [2,3]	Geodesic in Curved Space
Approximate arc $\hat{AB}$ with segment AB	Approximate curved path with local Euclidean segment
Detect gap $\delta_n > 0$ [1,2,3]	Detect deviation due to curvature [5,7,9,11]
Insert new vertex C, recalibrate	At the next point, re-execute local minimization
$P_{n+1} > P_n$; history updated	Path length changes; holonomy accumulates [8]
Never terminates (transcendence of π [1])	Never terminates (curvature is irremovable [5,7,9])

A geodesic in curved space is the continuous limit of the method of exhaustion applied to curved arcs. It is not a "straight line" but an infinite sequence of local Euclidean approximations, each based on the history of all previous approximations, each producing an irremovable deviation.

4.3 The Gauss–Bonnet Theorem as the Bridge [5,7,9]

The Gauss–Bonnet theorem (Theorem 2.4 [5,7,9]) serves as the bridge between the discrete recalibration process and the continuous geometry:

In the method of exhaustion, the gap δ_n is a numerical quantity arising from the transcendence of π [1].

In a curved space, the corresponding gap is a geometric quantity: $\iint_D K \, dA \neq 0$ [5,7,9].

The Gauss–Bonnet theorem [5,7,9] establishes that the numerical gap (irremovable by π's transcendence [1]) and the geometric gap (irremovable by nonzero curvature [5,7,9]) are two manifestations of the same phenomenon.

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5. Consequences

5.1 Euclid's Fifth Postulate Is Exact [4]

Euclid's fifth postulate asserts: through a point not on a given line, there exists exactly one line parallel to the given line, and parallel lines never meet [4].

If straight lines exist—that is, if $K \equiv 0$—this postulate is exact. It is not an axiom in need of proof; it is a theorem of flat geometry.

5.2 Riemann's Geometry Describes Spaces Without Straight Lines [6]

Riemann's geometry [6] describes the structure of spaces where $K \neq 0$ and straight lines do not exist. The geodesics of such spaces are not straight lines; they are curves produced by the infinite recalibration process described in Section 4 [2,3].

The standard formulation—"parallel lines intersect in curved space"—is therefore imprecise. A more precise formulation is:

In curved space, straight lines do not exist. The objects that appear to be "straight" (geodesics) are curves whose apparent straightness is the macroscopic limit of a discrete, history-dependent recalibration process [2,3]. Since straight lines do not exist in such spaces, neither do parallel lines, and the question of their intersection is ill-posed.

5.3 The Reconciliation

	Euclid [4]	Riemann [6]	This Paper
Space	Flat ($K = 0$)	Curved ($K \neq 0$)	Curved ($K \neq 0$)
Straight lines	Exist	Do not exist	Do not exist [1,5,7,8,11]
Parallel lines	Exist; never intersect [4]	Do not exist	Do not exist
Fifth postulate	Exact [4]	Not applicable	Exact (under premise $K = 0$ [4])

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6. The Uncircumscribability Theorem

6.1 Existence vs. Constructibility

Mathematics distinguishes two ontological registers:

- **Existence**: An object is well-defined in the continuous limit of a complete metric space.
- **Constructibility**: An object can be produced by a finite terminating procedure from finitely many given data (e.g., rational numbers, initial points, algebraic operations, piecewise geodesic segments).

Proposition 6.1 (Continuous existence of the circle). The Euclidean circle $S^1(r)$ and the line $\mathbb{R}$ exist as continuous geometric objects. Specifically, $\lim_{n\to\infty} P_n = 2\pi r$ [2,3] and the geodesics of $\mathbb{R}^2$ satisfy both conditions of the Euclidean straight line.

Proof. Immediate from the completeness of $\mathbb{R}$ and Proposition 2.2 [2,3]. □

Theorem 6.2 (The Uncircumscribability Theorem). Let denote any finite sequence of algebraic operations (addition, subtraction, multiplication, division, root extraction), finite point-set selections, and piecewise-smooth curve concatenations. Then:
(i) **Algebraic barrier**: No execution of from rational data yields π (Theorem 2.1 [1]).
(ii) **Geometric barrier**: No execution of yields an exact Euclidean circle or an infinite Euclidean straight line.
(iii) **Curved-space barrier**: In any 2-D Riemannian manifold with $K(p) \neq 0$, no execution of yields a Euclidean straight line. The geodesic curvature k_g of any constructible path necessarily accumulates irremovable π-twist when enclosed by a region with $\iint K\,dA \neq 0$ [5,7,9].

Proof. (i) is Theorem 2.1 [1]. (ii) follows because an exact circle requires the arc length $2\pi r$ [1,2,3], and an infinite line requires the full continuum $\mathbb{R}$; neither is reachable by finite algebraic towers [1]. (iii) combines (ii) with Theorem 2.4 [5,7,9]: any constructible path γ in a curved space is a finite concatenation of geodesic or algebraic segments. Closing γ into a loop and applying Gauss–Bonnet [5,7,9] gives a holonomy defect [8] proportional to $\iint_D K\, dA$. Since $K \neq 0$, the defect is non-zero, violating the trivial-holonomy condition of a Euclidean straight line. □

Corollary 6.3 (What a geodesic really is). A Riemannian geodesic is not a "straight line in curved space." It is the locally-optimal residual trace left after the construction of a global straight line fails due to the Uncircumscribability Theorem (Theorem 6.2). It satisfies local minimality (constructible) but not global flatness (unconstructible).

6.2 The OpenAI Paradigm: Algebra as Evasion of Construction [14,15]

In May 2026, an OpenAI reasoning model resolved the Erdős unit-distance problem [14], verified by Alon et al. [15]. The mathematical community (Gowers, Alon, Shankar) confirmed the result [15].

We interpret this success not as "high-dimensional trickery," but as an ontological shift:

OpenAI did not solve the problem by drawing better circles; it solved the problem by ceasing to draw at all [14,15].

In the traditional approach, one places integer lattice points $\mathbb{Z}[i]$ and attempts to realize the unit circle $x^2 + y^2 = 1$ discretely. This inevitably confronts the Uncircumscribability Theorem (Theorem 6.2): the circle cannot be exhausted by finite polygons [1,2,3], so the unit-distance structure is sparse.

OpenAI's model [14] instead defined "unit distance" via the algebraic field norm $N_{K/Q}(x)=1$ in a high-dimensional CM field. This relation is:

- Purely algebraic;
- Self-consistent within $\_K$;
- Independent of π [1] and of any geometric drawing [14,15].

The isometric structure is declared, not constructed. π-twist is not endured; it is bypassed. This is the algebraic counterpart to the Clifford torus $T^2 \subset S^3$ [11]: a flat submanifold whose isometric lines exist intrinsically without being "drawn" into the curvature of the ambient space.

Corollary 6.4 (The recovery of Euclid). True isometric lines—exact, flat, parallel, and never meeting—are recoverable only when geometric construction is abandoned for algebraic declaration [14,15]. Euclid's fifth postulate [4] is therefore a theorem of constructibility under the jurisdiction of π [1], not merely a theorem of flat space.

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7. Corollary: The Geodesic Postulate in Pseudo-Riemannian Geometry is Mathematically Undefined

7.1 The Mathematical Skeleton of General Relativity

General Relativity is formulated on a four-dimensional pseudo-Riemannian manifold $(M^{1,3}, g)$ with Lorentz signature [16,17,18]. The Einstein field equations
$G_{\mu\nu} = (8\pi G/c^4) T_{\mu\nu}$
are a system of second-order partial differential equations for the metric $g_{\mu\nu}$ [16,17,18]. The kinematics of test particles and light is governed by the geodesic equation
$$d^2x^\lambda/d\tau^2 + \Gamma^\lambda_{\mu\nu} (dx^\mu/d\tau)(dx^\nu/d\tau) = 0, \quad (7.1)$$
where $\Gamma^\lambda_{\mu\nu}$ are the Christoffel symbols of the Levi-Civita connection ∇ [12,16,17,18].

The standard physical interpretation, present in every textbook since 1916 [16,17,18], asserts:

Geodesic Postulate [16,17,18]: The solutions of (7.1) are the "straightest possible lines" in curved spacetime; gravity does not act as a force but as a curvature that "bends" these lines away from Euclidean straightness.

We now prove that this interpretation is not a physical approximation, but a mathematically undefined statement in any region where the Riemann curvature tensor $R \neq 0$.

7.2 Application of the Uncircumscribability Theorem

Corollary 7.1 (Non-existence of reference straight lines in curved spacetime). Let $(M^{1,3}, g)$ be a pseudo-Riemannian manifold satisfying the Einstein field equations with non-vanishing stress-energy tensor $T \neq 0$ in a region $U \subset M$ [16,17,18]. Then in U:
(i) The Riemann curvature tensor satisfies $R|_U \neq 0$;
(ii) There exists at least one two-dimensional timelike or spacelike slice $\Sigma \subset U$ with Gaussian curvature $K \neq 0$;
(iii) By Theorem 3.1 [1,5,7,8,11], no Euclidean straight line exists in Σ;
(iv) Consequently, no Euclidean straight line exists globally in U.

Proof. (i) follows from the contracted Bianchi identity and the field equations [16,17,18]: $T \neq 0 \Longrightarrow G \neq 0 \Longrightarrow R \neq 0$. (ii) follows from the decomposition of the Riemann tensor in four dimensions [12]: $R \neq 0$ implies non-vanishing sectional curvature for at least one tangent 2-plane, which by Frobenius' theorem integrates locally to a 2-surface Σ with $K \neq 0$. (iii) is Theorem 3.1 [1,5,7,8,11] applied to $(\Sigma, g|_\Sigma)$. (iv) follows because a global Euclidean straight line in M would restrict to a Euclidean straight line in any submanifold it intersects. □

7.3 The Falsification

Theorem 7.2 (Undefinedness of "straightest possible line"). Let $U \subset M$ be as in Corollary 7.1. Then the statement
"γ is the straightest possible line in U"
is mathematically undefined.

Proof. The modifier "straightest" is the superlative of "straight." It presupposes a well-defined set $\mathscr{L}$ of "straight lines" in U, of which γ is the extremal element under some metric of deviation. By Corollary 7.1(iv), $\mathscr{L} = \varnothing$ in U. The superlative of an empty set is undefined in Zermelo-Fraenkel set theory; there is no straight line of which γ could be the "straightest" approximation. □

Corollary 7.3 (Category error in the geodesic postulate [16,17,18]). The standard interpretation of (7.1)—"test particles follow straight lines that have been bent by gravity" [16,17,18]—commits a category error. The geodesic γ satisfies (7.1) by virtue of the Levi-Civita connection [12], but it cannot be interpreted as a "bent straight line" because the referent "straight line" does not exist in the category of curved pseudo-Riemannian manifolds.

7.4 Consequence for Observational Statements

Theorem 7.4 (Undefinedness of "light bending"). Let γ_{light} be a null geodesic passing through a region U with $R \neq 0$ (e.g., near a massive body) [16,17,18]. Then the observational statement
"The light ray is bent by an angle θ relative to its straight path"
is mathematically undefined.

Proof. The angle θ is defined as the angular deviation between γ_{light} and a reference Euclidean straight line ℓ connecting the same emitter and observer in the absence of gravity [16,17,18]. By Corollary 7.1(iv), such an ℓ does not exist in U. Therefore θ lacks a definitional domain, and the statement is not a well-formed geometric proposition. □

Remark [16,17,18]. What is observed in gravitational lensing is not a "deviation from straightness" but the convergence of distinct geodesics toward a focal region. This is an intrinsic property of the geodesic flow in a pseudo-Riemannian manifold with $R \neq 0$, fully describable by the Jacobi equation and the optical scalar equations [16,17,18] without any reference to a non-existent Euclidean straight line.

7.5 The Separation of Formalism from Interpretation

Proposition 7.5 (Self-consistency of the field equations [16,17,18]). The Einstein field equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$ and the geodesic equation (7.1) remain mathematically self-consistent as a system of differential equations. The Christoffel symbols $\Gamma^\lambda_{\mu\nu}$, the Riemann tensor $R^\rho_{\sigma\mu\nu}$, and the geodesic deviation equation are all well-defined intrinsic objects on (M, g) [12,16,17,18].

Proposition 7.6 (Falsification of the semantic overlay [16,17,18]). The interpretation overlay that equates geodesics with "generalized straight lines" [16,17,18] is not a consequence of the differential geometry of (M, g) [12]. It is an extraneous semantic device that violates the Uncircumscribability Theorem (Theorem 6.2). The formalism does not need it; the formalism works better without it.

7.6 Final Conclusion on General Relativity [16,17,18]

The mathematical structure of General Relativity is sound. Its standard verbal interpretation is mathematically false.

Einstein's equations correctly describe the metric g and its curvature [16,17,18]. The geodesic equation correctly describes the paths of free-falling particles and light [16,17,18]. But the statement that these paths are "straight lines in curved spacetime" [16,17,18] is not a simplification for laymen—it is a mathematical impossibility in any region where $R \neq 0$.

Gravity does not "bend" straight lines, because there are no straight lines to bend. What we call "gravitational light bending" [16,17,18] is the intrinsic focusing of a geodesic congruence in a manifold where the only constructible paths are geodesics, and where the concept of a straighter path is forbidden by π's transcendence [1].

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8. Conclusion

We have proven the following:
(i) In any two-dimensional Riemannian manifold with $K \neq 0$ at some point, Euclidean straight lines do not exist (Theorem 3.1 [1,5,7,8,11]). The proof proceeds from the transcendence of π [1] through the Gauss–Bonnet theorem [5,7,9] and the Ambrose–Singer theorem [8].
(ii) "Parallel lines intersect in curved space" is a proposition with a false premise. If straight lines do not exist, parallel lines do not exist, and the question of their intersection is ill-posed.
(iii) Geodesics in curved space are the macroscopic limit of a discrete, history-dependent recalibration process, whose rigorous mathematical prototype is the method of exhaustion [2,3]. The transcendence of π [1] guarantees that this process never terminates and that the deviation it produces is irremovable.
(iv) The Uncircumscribability Theorem (Theorem 6.2 [1,5,7,8,9]) formalizes π's transcendence as an ontological barrier: straight lines exist in the continuous limit but are unconstructible by any finite process. Riemannian geodesics are the locally-optimal residuals of this failed construction.
(v) True isometric lines can be recovered only by abandoning geometric construction for algebraic declaration, as exemplified by the OpenAI resolution of the Erdős unit-distance problem [14,15].
(vi) In General Relativity, the geodesic postulate—"test particles follow the straightest possible path in curved spacetime" [16,17,18]—is mathematically undefined in any region with $R \neq 0$, because the set of straight lines is empty (Corollary 7.1 [1,5,7,8,11]). The Einstein field equations remain intact; only their semantic overlay is falsified.

Euclid's fifth postulate is exact in spaces where straight lines exist [4]. Riemann's geometry describes spaces where they do not [6]. The two geometries are complementary, not contradictory. And General Relativity, properly understood, is a theory of curvature without straight lines [16,17,18]—not because space is too curved to permit them, but because π's transcendence [1] forbids their construction.

Data Availability Statement

All data supporting the findings of this study are available within the article. No new experimental or observational data were generated.

Conflicts of Interest

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References

[1] Lindemann, F. (1882). Über die Ludolphsche Zahl. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 2, 679–682.
[2] Archimedes (c. 250 BC). *Measurement of a Circle*.
[3] Liu Hui (263). Commentary on *The Nine Chapters on the Mathematical Art*, Method of Exhaustion.
[4] Euclid. *Elements*, Book I, Postulate 5; Book XII, Proposition 2.
[5] Gauss, C. F. (1827). *Disquisitiones generales circa superficies curvas*.
[6] Riemann, B. (1854). Über die Hypothesen, welche der Geometrie zu Grunde liegen. Habilitationsschrift, Göttingen.
[7] Bonnet, O. (1855). Sur quelques propriétés des lignes géodésiques. *Comptes Rendus de l'Académie des Sciences*, 40, 1311–1313.
[8] Ambrose, W., & Singer, I. M. (1953). A theorem on holonomy. *Transactions of the American Mathematical Society*, 75(3), 428–443.
[9] Chern, S. S. (1944). A simple intrinsic proof of the Gauss-Bonnet formula for closed Riemannian manifolds. *Annals of Mathematics*, 45(4), 747–752.
[10] Myers, S. B. (1941). Riemannian manifolds with positive mean curvature. *Duke Mathematical Journal*, 8(2), 401–404.
[11] do Carmo, M. P. (1992). *Riemannian Geometry*. Birkhäuser.
[12] Kobayashi, S., & Nomizu, K. (1963). *Foundations of Differential Geometry*, Vol. I. Interscience.
[13] Milnor, J. (1963). *Morse Theory*. Princeton University Press.
[14] OpenAI. (2026). An explicit lower bound for the unit distance problem. arXiv preprint arXiv:2605.20579v1. https://arxiv.org/abs/2605.20579
[15] Alon, N., Bloom, T. F., Gowers, W. T., Litt, D., Sawin, W., Shankar, A., Tsimerman, J., Wang, V., & Wood, M. M. (2026). Remarks on the disproof of the unit distance conjecture. arXiv preprint arXiv:2605.20695v1. https://arxiv.org/abs/2605.20695
[16] Misner, C. W., Thorne, K. S., & Wheeler, J. A. (1973). *Gravitation*. W. H. Freeman.
[17] Wald, R. M. (1984). *General Relativity*. University of Chicago Press.
[18] Carroll, S. M. (2004). *Spacetime and Geometry: An Introduction to General Relativity*. Addison Wesley.