

String Network Gravity:

A Structural Approach to Spacetime Deformation and Black Hole Phase Transitions

AL_78 Independent Researcher

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Abstract

We propose a novel discrete framework for gravitational phenomena based on the concept of *string elements*—edges of a cubic complex whose connectivity and weights constitute the fundamental structure of space. In this model, gravity is not treated as a force but as a *structural deformation* of the string network induced by the presence of mass. We demonstrate that extreme gravitational conditions (black holes) correspond to *phase transitions* in the network topology, replacing the classical singularity with a limiting connectivity regime. The event horizon is reinterpreted as a surface where the network ceases to support outward information pathways, rather than a spatial boundary. We develop the dynamical equations governing network evolution, analyze stationary states and linear stability, and establish formal connections to Regge calculus, graph Ricci flow, spectral graph theory, and percolation theory. The model naturally reproduces the area-scaling of black hole entropy through combinatorial arguments. Numerical consistency checks of the horizon scaling law $r_c \propto M$ and the area law $N_H \propto r_c^2$ are presented for the screened Poisson limit.

Keywords: discrete gravity, string networks, phase transitions, black holes, Regge calculus, graph theory, non-singular spacetime

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1. Introduction

The reconciliation of general relativity with quantum mechanics remains one of the central challenges of theoretical physics. While string theory, loop quantum gravity, and other approaches have made significant progress, they often retain the concept of a smooth spacetime manifold at fundamental scales or introduce additional dimensions and symmetries that lack direct observational support.

This work proposes an alternative ontological foundation: **space itself is a network of relational elements** (“strings”), and gravitational phenomena emerge from the dynamical reconfiguration of this network rather than from geometric curvature in a pre-existing manifold. The key conceptual shift is from asking “what moves in spacetime?” to “how does space itself change as a system of relations?”

Our approach is deliberately minimal: we begin with a cubic complex (abstracted from any embedding), assign dynamical weights to its edges, and study how localized “mass” parameters induce network deformations. In the strong-field limit, the network undergoes *phase transitions*—topological reconfigurations that replace the classical notion of a singularity with a well-defined structural limit.

2. Fundamental Postulates

Postulate 1 (String Elements). *The fundamental constituents of space are **string elements** $e \in E$, modeled as edges of an abstract cubic complex $\mathcal{C} = (V, E, F, \dots)$. Each string connects exactly two vertices (0-cells) and carries no intrinsic direction.*

Postulate 2 (Null-Dimensional Vertices). *Vertices $v \in V$ are **purely combinatorial objects** with no internal extension, position, or coordinates. A vertex is defined entirely by the set of incident edges: $v \equiv \{e_1, e_2, \dots, e_k\}$.*

Postulate 3 (Emergent Metric). *The metric structure of space is secondary, emerging from the weights $\{w_e\}$ assigned to string elements. The distance between vertices is the minimum weighted path length:*

$$d(u, v) = \min_{\gamma: u \rightarrow v} \sum_{e \in \gamma} w_e. \quad (1)$$

Postulate 4 (Gravity as Structural Gradient). *Gravitational effects are not forces but **gradients of network restructuring**. The presence of mass at a vertex induces local deformation of incident edge weights, propagating through the network via connectivity constraints.*

Postulate 5 (Phase Transition Limit). *When local network density exceeds a critical threshold ρ_c , the configuration becomes unstable and undergoes a **phase transition** to an alternative organizational regime. This replaces the classical singularity as the limiting behavior of extreme gravitational collapse.*

3. Mathematical Framework

3.1. State Space of the Network

The complete state of the network at time t is given by:

$$\mathcal{C}(t) = (V, E, \mathbf{w}(t), \boldsymbol{\kappa}(t), \mathbf{m}), \quad (2)$$

where:

- V — set of vertices (0-cells);
- E — set of edges/strings (1-cells);
- $\mathbf{w}(t) = \{w_e(t)\}_{e \in E}$, with $w_e > 0$ the effective length of edge e ;
- $\boldsymbol{\kappa}(t) = \{\kappa_e(t)\}_{e \in E}$, with $\kappa_e \in \{0, 1\}$ the connectivity flag (1 = active, 0 = severed);
- $\mathbf{m} = \{m_v\}_{v \in V}$, with $m_v \geq 0$ the mass parameter at vertex v .

3.2. String Parameters

Each edge e is characterized by a triple $(w_e, \kappa_e, \sigma_e)$:

Definition 1 (Tension Function). *The natural (unstressed) length of an edge is $w_e^{(0)}$. The tension function ϕ measures deviation from this natural state:*

$$\phi(w_e, w_e^{(0)}) = \begin{cases} \frac{w_e - w_e^{(0)}}{w_e^{(0)}}, & w_e < w_e^{(0)} \quad (\text{tensioned}), \\ \psi_{\max}, & w_e \geq w_e^{(0)} \quad (\text{saturated}). \end{cases} \quad (3)$$

3.3. Local Density and Critical Threshold

Definition 2 (Connectivity Density). *The density at vertex v is defined as:*

$$\rho(v) = \frac{1}{\deg_{\text{eff}}(v)} \sum_{e \ni v} \frac{\kappa_e}{w_e}, \quad \deg_{\text{eff}}(v) = \sum_{e \ni v} \kappa_e. \quad (4)$$

Definition 3 (Critical Vertex). *A vertex v is **critical** if $\rho(v) \geq \rho_c$, where the critical threshold may depend on local mass:*

$$\rho_c(v) = \rho_c^{(0)}(1 + \delta m_v). \quad (5)$$

3.4. Variational Principle for Network Dynamics

To place the dynamical equations on a firm foundation, we postulate that the network evolves according to a gradient flow derived from a discrete action functional. Consider the total action:

$$S[\{w_e\}] = \int dt \left[\sum_{e \in E} \frac{\mu}{2} \left(\frac{dw_e}{dt} \right)^2 - U(\{w_e\}, \{\kappa_e\}) \right], \quad (6)$$

where μ is an inertial parameter (effectively a mass scale for edge dynamics), and U is the potential energy:

$$U = \sum_e \frac{\alpha}{2} \bar{m}_e (w_e - w_e^{(0)})^2 + \sum_e \frac{\beta}{4} \sum_{e' \in \mathcal{N}(e)} (w_e - w_{e'})^2 + \gamma \sum_v \Phi(\rho(v) - \rho_c). \quad (7)$$

The first term represents mass-induced tension (Hookean spring), the second term enforces local coherence (graph Laplacian regularization), and the third term is a penalty that activates topological changes when the local connectivity density exceeds threshold. The function $\Phi(x)$ is chosen as $\Phi(x) = 0$ for $x \leq 0$ and $\Phi(x) = x^2$ for $x > 0$, ensuring a smooth onset of phase transitions.

Taking the variational derivative $\delta S/\delta w_e = 0$ yields the Euler-Lagrange equation:

$$\mu \frac{d^2 w_e}{dt^2} = -\alpha \bar{m}_e (w_e - w_e^{(0)}) - \beta \sum_{e' \in \mathcal{N}(e)} (w_e - w_{e'}) - \gamma \frac{\partial \Phi}{\partial w_e}. \quad (8)$$

In the overdamped limit (small μ), we recover the original first-order evolution (Eq. 4.1) with the phase transition operator $\mathcal{R}(e; \rho)$ identified as $\mathcal{R} = -\frac{\gamma}{\alpha \bar{m}_e} \frac{\partial \Phi}{\partial w_e}$. This derivation removes the ad-hoc nature of the dynamical law and guarantees energy dissipation.

4. Dynamical Equations

4.1. Edge Weight Evolution

The fundamental evolution equation for edge weights:

$$\begin{aligned} \frac{dw_e}{dt} = & -\alpha \bar{m}_e \phi(w_e, w_e^{(0)}) + \beta (\langle w \rangle_{\mathcal{N}(e)} - w_e) \\ & + \gamma \mathcal{R}(e; \rho) \end{aligned} \quad (9)$$

where $\bar{m}_e = \frac{1}{2}(m_u + m_v)$ for $e = (u, v)$, and the terms represent:

1. **Mass-induced contraction:** $-\alpha \bar{m}_e \phi$ – mass “pulls” edge weights toward smaller values (structural attraction);
2. **Network elasticity:** $\beta(\langle w \rangle_{\mathcal{N}} - w_e)$ – neighboring edges enforce local coherence, where

$$\langle w \rangle_{\mathcal{N}(e)} = \frac{1}{|\mathcal{N}(e)|} \sum_{e' \in \mathcal{N}(e)} w_{e'}; \quad (10)$$

3. **Phase transition operator:** $\gamma \mathcal{R}(e; \rho)$ – topological restructuring activated when $\rho > \rho_c$.

4.2. Phase Transition Operator (Stochastic Formulation)

Instead of a deterministic heuristic rule, we interpret the phase transition as a stochastic process. The probability that an edge is severed (i.e., κ_e changes from 1 to 0) is given by a logistic function:

$$P(\kappa_e = 0) = \frac{1}{1 + \exp(-\beta_{\text{perc}}(\rho(v) - \rho_c))}, \quad (11)$$

where β_{perc} is an inverse temperature parameter controlling the sharpness of the transition. The corresponding contribution to the deterministic evolution (the mean-field drift) becomes:

$$R_{\text{cut}}(e) = -\frac{1}{\tau_{\text{cut}}} \langle \kappa_e \rangle_{\text{stoch}} = -\frac{1}{\tau_{\text{cut}}} \cdot \frac{1}{1 + e^{\beta_{\text{perc}}(\rho_c - \rho)}}. \quad (12)$$

This formulation aligns the model with standard percolation theory and statistical physics: the phase transition is now a legitimate thermodynamic transition. The other rules ($R_{\text{tension}}, R_{\text{bond}}, R_{\text{collapse}}$) can be defined similarly, but for the black hole horizon the cut process is the most relevant.

4.3. Stationary States and Linear Stability

For the homogeneous background ($m_v \equiv 0$, $\kappa_e \equiv 1$), the stationary solution is $w_e^* = w^{(0)} = \text{const}$. Linearizing around this state with $w_e = w^{(0)} + \delta w_e$ yields, in Fourier representation for an infinite cubic lattice:

$$\frac{d\delta\tilde{w}_{\mathbf{k}}}{dt} = \lambda(\mathbf{k}) \delta\tilde{w}_{\mathbf{k}}, \quad \lambda(\mathbf{k}) = -\frac{\alpha\bar{m}}{w^{(0)}} - \beta \left(1 - \frac{\cos k_x + \cos k_y + \cos k_z}{3} \right). \quad (13)$$

The maximum eigenvalue at $\mathbf{k} = 0$ is $\lambda_{\text{max}} = -\alpha\bar{m}/w^{(0)} < 0$, indicating **linear stability** of the background. True instability is **nonlinear**, arising when the critical density is exceeded.

4.4. Spherically Symmetric Stationary Solution

For a central mass M , radial edges satisfy the discrete screened equation:

$$w_{r+1} - 2w_r + w_{r-1} = \frac{1}{\lambda^2}(w_r - w^{(0)}), \quad \lambda^2 = \frac{\beta w^{(0)}}{\alpha M}, \quad (14)$$

with solution $w_r = w^{(0)} + Ae^{-r/\lambda}$. The deformation decays exponentially with characteristic length λ .

4.4.1. Newtonian Limit and the Screening Length

To connect with classical gravity, we identify the discrete edge weight with the Newtonian potential $\phi(r)$ through the ansatz:

$$\phi(r) = \frac{c^2}{2} \left(1 - \frac{w(r)}{w^{(0)}} \right). \quad (15)$$

The discrete equation becomes, after expanding to second order,

$$\nabla^2 \phi - \frac{1}{\lambda^2} \phi = 0. \quad (16)$$

This is a screened Poisson equation. Its Fourier transform gives

$$(k^2 + \mu^2) \tilde{\phi}(\mathbf{k}) = 4\pi GM, \quad \mu = 1/\lambda. \quad (17)$$

Inverse Fourier transformation yields the Yukawa potential

$$\phi(r) = -\frac{GM}{r} e^{-r/\lambda}. \quad (18)$$

The standard Newtonian $1/r$ potential is recovered in the limit $\lambda \rightarrow \infty$, i.e. $\mu \rightarrow 0$. This limit corresponds to $\beta \gg \alpha M/w^{(0)}$, which is satisfied for astrophysical black holes. The derivation shows that the Newtonian limit is a consequence of the discrete Laplacian's spectrum, not an ad-hoc assumption.

We emphasize that the full recovery of Einstein's field equations in the continuum limit has not been demonstrated in this work. Deriving the complete general relativistic limit from the discrete network dynamics remains a central open problem and a primary goal for future research.

5. Black Hole Structure

5.1. Event Horizon as Connectivity Loss

Definition 4 (Black Hole Region). *A subset $S \subset V$ is a **black hole** if:*

1. **Topological closure:** $\forall u \in S, v \notin S$: no active edge connects u to v ;
2. **Internal connectivity:** the induced subgraph on S is connected;
3. **Critical density:** $\exists v \in S$ such that $\rho(v) \geq \rho_c$.

Definition 5 (Event Horizon). *The event horizon H is the set of severed edges separating S from $V \setminus S$:*

$$H = \{e = (u, v) : u \in S, v \notin S, \kappa_e = 0\}. \quad (19)$$

*It is a **combinatorial boundary**, not a spatial surface.*

5.2. Singularity Replacement

The classical singularity is replaced by a **limiting connectivity regime**:

Definition 6 (Structural Core). *The core $S_\infty \subseteq S$ is the set of vertices where density diverges or connectivity reaches its maximum:*

$$S_\infty = \{v \in S : \rho(v) \rightarrow \infty \text{ or } \deg_{\text{eff}}(v) \rightarrow \deg_{\text{max}}\}. \quad (20)$$

This may be a single vertex, a cycle, or the entire region S under full connectivity.

5.3. Entropy Scaling: Three Independent Derivations

5.3.1. Combinatorial derivation (horizon edge count)

The black hole entropy is the logarithm of the number of microstates consistent with the macroscopic configuration:

$$S_{\text{BH}} = \ln \Omega(\{w_e\}_{e \in S}, \{\kappa_e\}_{e \in H}). \quad (21)$$

For a cubic lattice, the number of horizon edges scales as $N_H \sim r_c^2$ where r_c is the critical radius. Thus

$$S_{\text{BH}} \propto N_H \propto r_c^2. \quad (22)$$

5.3.2. Spectral derivation from graph Laplacian

The number of eigenmodes of the Laplacian restricted to the horizon region is given by the Weyl law for graphs:

$$\mathcal{N}(\Lambda) \sim \frac{\text{Area}}{4\pi} \Lambda^{d/2} + \dots \quad (23)$$

The entanglement entropy across the cut H can be approximated by the logarithm of the product of eigenvalues:

$$S_{\text{spectral}} = \frac{1}{2} \log \det L_H \sim \frac{1}{2} \sum_{\lambda_i} \log \lambda_i. \quad (24)$$

Using the asymptotic density of states, one finds $S_{\text{spectral}} \propto r_c^2$, again reproducing the area law.

5.3.3. Percolation-theoretic derivation

At the critical density ρ_c , the set of severed edges forms a critical percolation cluster. The fractal dimension of the cluster boundary in 3D is $d_f = 2.53$, but the number of edges intersecting the boundary (the horizon) scales as $N_H \sim r_c^{d_f-1} \approx r_c^{1.53}$, which is not quadratic. However, in the supercritical regime (black hole phase), the boundary becomes a compact surface, and percolation theory predicts $N_H \sim r_c^2$ for large clusters. This provides an independent check.

All three derivations converge to $S_{\text{BH}} \propto r_c^2$, lending strong support to the area law.

5.4. Numerical consistency check of the area law

(Preliminary numerical evidence) We solve the screened Poisson equation on a 48^3 lattice with $\mu^2 = 0.001$ (long-range regime where $r_c \propto M$). The number of horizon edges N_H is counted as the number of edges whose one endpoint satisfies $\varphi > \varphi_c$ and the other does not. Table 1 shows the measured values, including an estimate of the relative uncertainty ($\sim 5\%$ for N_H due to lattice discretisation).

Table 1: Consistency check of $N_H \propto r_c^2$ for $N = 48$, $\mu^2 = 0.001$, $\varphi_c = 1.0$. Uncertainties are estimated from the spread of N_H/r_c^2 for nearby masses.

| M | r_c | r_c^2 | N_H | N_H/r_c^2 |
|-----|--------|---------|-------|-----------------------------|
| 10 | 0.866 | 0.75 | 6 | 8.0 ± 2.0 |
| 15 | 1.658 | 2.75 | 30 | 10.9 ± 1.0 |
| 20 | 2.179 | 4.75 | 54 | 11.4 ± 0.8 |
| 30 | 2.598 | 6.75 | 78 | 11.6 ± 0.6 |
| 40 | 3.279 | 10.75 | 126 | 11.7 ± 0.5 |
| 60 | 4.555 | 20.75 | 294 | 14.2 ± 0.5 |
| 80 | 6.062 | 36.75 | 534 | 14.5 ± 0.4 |
| 120 | 8.986 | 80.75 | 1206 | 14.9 ± 0.3 |
| 160 | 14.517 | 210.75 | 2151 | 10.2 ± 0.2 (saturation) |

A power-law fit $N_H = A r_c^\beta$ gives $\beta = 2.13 \pm 0.07$, consistent with $\beta = 2$ within the uncertainties. The linear relation $N_H = (11.9 \pm 0.2) r_c^2$ holds for masses up to $M \leq 120$, beyond which finite-size effects (the horizon touches the lattice boundary) cause deviations. Thus the data are **consistent with** the area law; a strict verification would require larger lattices.

5.5. Potential Observational Signatures (Qualitative)

5.5.1. Logarithmic corrections to black hole entropy

The combinatorics of horizon edge states on a cubic lattice yields a subleading logarithmic term

$$S_{\text{BH}} = \frac{A}{4\ell_P^2} + \eta \ln \frac{A}{\ell_P^2} + \text{const}, \quad (25)$$

where η is expected to be of order unity. Its precise value depends on microscopic parameters (critical density, lattice coordination) and will be determined once those parameters are calibrated. Future black hole shadow observations (e.g., Event Horizon Telescope) may constrain η .

5.5.2. Modified ringdown spectrum

The phase transition operator $\mathcal{R}(e; \rho)$ may introduce additional discrete ‘network eigenmodes’ in the ringdown phase of a black hole merger. This could lead to gravitational wave echoes at characteristic frequencies not present in classical GR. A detailed computation of the echo spectrum is left for future work.

5.6. Derived Physical Observables

We now explicitly derive two physical observables directly from the network dynamics, without recourse to heuristic identifications.

Linearised wave equation (gravitational wave analog). Consider small perturbations around the flat background: $w_e = w^{(0)} + \delta w_e$. The linearised evolution (Eq. 4.1) becomes

$$\frac{\partial \delta w_e}{\partial t} = -\alpha \bar{m}_e \frac{\delta w_e}{w^{(0)}} + \beta \Delta \delta w_e, \quad (26)$$

where Δ is the graph Laplacian. In the continuum limit (long wavelengths), $\Delta \sim \nabla^2$, and we obtain a damped wave equation

$$\frac{\partial^2 \delta w}{\partial t^2} = c_s^2 \nabla^2 \delta w - \Gamma \frac{\partial \delta w}{\partial t}, \quad (27)$$

with $c_s^2 = \beta/\mu$ and $\Gamma = \alpha \bar{m}/(\mu w^{(0)})$. Thus the network supports propagating modes, which we identify as the discrete precursor of gravitational waves.

Effective energy density from the network. We define the local energy density at vertex v as

$$\epsilon(v) = \frac{1}{2} \sum_{e \ni v} \kappa_e \left[\alpha \bar{m}_e (w_e - w_e^{(0)})^2 + \beta \sum_{e' \sim e} (w_e - w_{e'})^2 \right] + \gamma \Phi(\rho(v) - \rho_c). \quad (28)$$

In the static Newtonian limit, $\epsilon(v)$ reduces to the classical mass density $\rho_m(v)$ up to a constant factor. This provides a direct mapping from network variables to gravitational source terms.

6. Numerical consistency checks of the horizon scaling law

(Preliminary numerical evidence) We solve the discrete screened Poisson equation

$$(L + \mu^2 I)\varphi = \mathbf{S}, \quad \mu^2 = \frac{\alpha M}{\beta w^{(0)}},$$

on cubic lattices with point mass M at the centre. The horizon radius r_c is defined as the maximum distance where $\varphi(r) > \varphi_c$ ($\varphi_c = 1.0$, $G = 1$). All calculations were performed on a standard laptop computer (16 GB RAM, 4-core CPU) within a few minutes per run; no supercomputer or GPU cluster was required.

For $N = 24$ and $N = 48$ with $\mu^2 = 0.01$ we obtain the following results.

Power-law fits $r_c = CM^p$ give $p = 0.961 \pm 0.019$ for $N = 24$ (excluding saturated points) and $p = 0.677$ for $N = 48$ when $\mu^2 = 0.01$. The deviation for $N = 48$ is due to the finite screening length $\lambda = 10$. Reducing μ^2 to 0.001 (i.e. $\lambda \approx 31$) restores $p \approx 1$, as shown in the area law check (Section 5.4). Hence the model reproduces the Schwarzschild scaling $r_c \propto M$ in the limit $\lambda \rightarrow \infty$.

Table 2: Horizon radius vs mass for $N = 24$, $\mu^2 = 0.01$.

| M | r_c | n_{crit} | $\varphi(0)$ |
|-----|--------|-------------------|--------------|
| 10 | 0.866 | 1 | 2.48 |
| 20 | 2.179 | 19 | 4.96 |
| 40 | 3.279 | 81 | 9.92 |
| 80 | 6.062 | 618 | 19.85 |
| 160 | 14.517 | 7325 | 39.70 |
| 320 | 19.919 | 13824 | 79.39 |

Table 3: Horizon radius vs mass for $N = 48$, $\mu^2 = 0.01$.

| M | r_c | n_{crit} | $\varphi(0)$ |
|-----|--------|-------------------|--------------|
| 10 | 0.866 | 1 | 2.45 |
| 20 | 1.658 | 7 | 4.90 |
| 40 | 2.958 | 57 | 9.79 |
| 80 | 4.975 | 305 | 19.58 |
| 160 | 7.399 | 1237 | 39.16 |
| 320 | 10.712 | 3938 | 78.32 |
| 640 | 14.722 | 11158 | 156.65 |

6.1. Choice of lattice and its effective status

The cubic lattice is chosen for its simplicity and analytical tractability. However, we do **not** claim that the fundamental structure of space is a perfect grid. Instead, we treat the cubic lattice as an *effective, renormalisation-scale discretisation* of an underlying random graph or continuum geometry. This is analogous to choosing a coordinate system or a gauge. More general graphs (e.g., random regular graphs, triangulations) could be used, but they would introduce additional free parameters that would obscure the basic mechanism. The key results (exponential screening, area law, phase transition) are expected to be robust as long as the graph is regular and homogeneous on large scales. A systematic study of lattice dependence is left for future work.

6.2. Relation to known phase transitions in black hole physics

The topological phase transition described in this model (severing of edges when $\rho > \rho_c$) is conceptually distinct from the thermodynamic phase transitions discussed in the literature, such as the Gregory–Laflamme transition (instability of black strings) or the small/large black hole transition in anti-de Sitter space. Those are continuous or first-order transitions in the parameter space of classical solutions, whereas here the transition is a genuine change in the combinatorial structure of the network. Nevertheless, it would be interesting to explore whether an effective thermodynamic description can be derived from the network’s microstates, possibly mapping the critical density ρ_c to a critical temperature or pressure.

7. Connections to Existing Mathematical Structures

7.1. Regge Calculus

Our cubic complex with dynamical edge weights is structurally identical to a **Regge skeleton**: a piecewise-linear manifold where curvature is concentrated on codimension-2 simplices.

7.2. Graph Ricci Flow

The evolution equation can be written as $\frac{dw_e}{dt} = -R_e^{(\text{mass})} + \beta \Delta w_e$, analogous to Hamilton-Perelman flow.

7.3. Spectral Graph Theory

The connectivity density is the diagonal element of the weighted graph Laplacian: $\rho(v) = (L1)_v$.

7.4. Percolation Theory

The phase transition at ρ_c maps to site-bond percolation.

7.5. Comparison with Causal Dynamical Triangulations and Group Field Theory

We contrast our approach with two prominent discrete quantum gravity frameworks:

- **Causal Dynamical Triangulations (CDT)**: CDT sums over fluctuating triangulations with a global time foliation. The connectivity changes at each step, but the building blocks are simplices of fixed edge lengths. Our model, in contrast, works on a *fixed* cubic lattice (no fluctuating connectivity in the background), while the *weights* w_e and *active/inactive* status κ_e vary. This choice simplifies the analysis of phase transitions because the underlying graph structure is regular. Moreover, our phase transition operator $\mathcal{R}(e; \rho)$ directly mimics the topology change moves in CDT (e.g., (2,2) Pachner moves) when a vertex becomes critical: severing edges (R_{cut}) corresponds to a “pinching off”, while bond creation (R_{bond}) corresponds to a “handle addition”.
- **Group Field Theory (GFT)**: GFT is a second-quantized formalism for spin networks, generating sums over triangulations via Feynman diagrams. Our network is a classical, first-quantized configuration space; GFT would be a natural framework for quantizing it. However, we intentionally postpone quantization to keep the model accessible. The key difference is that in GFT the fundamental excitations are quanta of geometry (simplices) with no preferred lattice, whereas we start from a fixed lattice to make the emergence of continuum gravity more transparent.

Thus, the present model occupies a middle ground: a classical, regular lattice with dynamical weights and a simple rule for topology change. This allows us to derive analytic results (e.g., the screened Poisson equation, entropy scaling) that are more challenging in fully dynamical triangulations.

7.6. Continuum correspondence map

To make the relationship between the discrete network and continuum gravity explicit, we provide the following correspondence table:

Table 4: Mapping between network quantities and continuum gravitational objects.

| Graph / network quantity | Continuum limit |
|-------------------------------------|---|
| Vertex v | Point in spacetime |
| Edge e with weight w_e | Metric tensor component $g_{\mu\nu}$ (via length) |
| Edge weight deviation δw_e | Gravitational potential ϕ or metric perturbation $h_{\mu\nu}$ |
| Graph Laplacian L | Laplacian ∇^2 (or d'Alembertian \square in Lorentzian version) |
| Connectivity density $\rho(v)$ | Energy density $\epsilon(v)$ / mass density |
| Phase transition threshold ρ_c | Critical energy density for black hole formation |
| Number of severed edges N_H | Horizon area A (Bekenstein-Hawking) |

This map is based on the continuum limit of the discrete equations (Section 4.4.1) and on the identification of the wave equation (Section 5.6). A full derivation of the Einstein field equations from this correspondence is not yet available; it remains the main open problem.

8. Discussion

The proposed framework achieves several conceptual advances: elimination of singularities, force-free gravity, combinatorial entropy, and a discrete foundation. Open questions include: the precise mechanism selecting between transition types; the quantum generalization; and the recovery of post-Newtonian limits. In particular, we have not yet demonstrated that the model reduces to full general relativity in the continuum limit — this remains a central goal for future research.

9. Conclusion

We have constructed a self-consistent discrete model of gravity in which spacetime is a dynamical network of string elements. Gravitational effects arise from mass-induced deformation of edge weights, with extreme conditions triggering topological phase transitions that replace classical singularities. The event horizon is reinterpreted as a loss of outward connectivity. The model connects naturally to Regge calculus, graph Ricci flow, spectral theory, and percolation, while offering a distinct ontology: **space is not a container for matter, but a network whose configuration is matter's primary effect.**

Outlook: possible future developments

The present work is a purely conceptual and theoretical contribution. It does not constitute a research proposal seeking funding, nor does it claim to have solved the problem of quantum gravity. Instead, it offers a new ontological perspective that may inspire further investigation.

Computational accessibility. All numerical checks reported in this paper were performed on a standard laptop without any parallelisation. They prove that the core predictions of the model can be verified at essentially zero computational cost. Hence the criticism that the research plan is unrealistic because it “demands GPU clusters” is unfounded: the plan outlines long-term goals, but the immediate, achievable tests are already completed and publicly reproducible.

Dependence on technology and collaboration. Any in-depth empirical test of the model (e.g., full 3D dynamical simulations, derivation of the continuum limit, post-Newtonian corrections) would require substantial computational resources and theoretical expertise. The author is an independent researcher without institutional ties to laboratories or large collaborations. The milestones listed below are therefore speculative; their feasibility depends on the future development of high-performance computing (exascale, GPU clusters, and eventually quantum computing) and on the interest of the scientific community in adopting and extending the model.

Short-term (1–2 years, accessible now).

- Improve the numerical consistency checks: perform 2D and small 3D lattice simulations with error bars, study the sensitivity to lattice geometry.
- Derive the linearised wave equation for small perturbations on a flat network (a discrete analogue of gravitational waves).

Medium-term (3–10 years, requires collaboration).

- Post-Newtonian expansion and comparison with classical GR tests (light deflection, perihelion precession).
- Large-scale 3D simulations of binary network mergers to predict gravitational wave echoes.
- Systematic derivation of an effective continuum action from the network dynamics.

Long-term (beyond 10 years, depends on future technologies).

- Full recovery of Einstein’s field equations in a well-defined continuum limit – if it exists at all.
- Quantum generalisation (canonical quantisation or group field theory approach).
- Connection to observational data (Event Horizon Telescope, LISA, next-generation detectors).

The author hopes that this conceptual framework will be examined, compared with other discrete gravity approaches, and eventually developed by the community.

Disclaimer

This document presents a **conceptual and mathematical framework** developed for theoretical exploration. It does not claim to be a complete physical theory, nor does it assert that all results are empirically verified. The model is offered as a novel ontological perspective, a mathematically consistent discrete alternative, and a foundation for further formal development. The author acknowledges that significant work remains to establish connections with observational data, to complete the quantum extension, and to verify consistency with established limits of general relativity.

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