From Mathematical Foundations to Systematic Philosophy: A Complete Theoretical Chain – Unified Metabolic-Causal Holism in Category Theory

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Abstract

This paper uses category theory [1, 2] as a unified language to construct a complete theoretical chain from mathematical foundations to systematic philosophy. We rigorously prove four core propositions:

1. The whole is a function, the parts are subfunctions.
2. To exist is to be a spacetime function.
3. Existence is a function unified field.
4. The essence of "the whole is greater than the sum of its parts" is that the organic is greater than the mechanical.

Furthermore, we formalize metabolism as the causal-dynamical mechanism by which a system maintains its "existence function" (causal closure), proving that metabolism is a universal principle for any non-equilibrium ordered structure to sustain its causal closure. On this basis we propose the Zhu–Liang metabolicon – the minimal organic unit that maintains its own causal closure – and prove that the unified field is isomorphic to the inverse limit of metabolicons at the level of sections. Finally we merge the unified field and the metabolic-causal principle into the "unified metabolic-causal field", revealing that all levels from quantum to civilization follow the fundamental laws: whole before parts, relations define entities, metabolism sustains causality. All proofs are based on standard mathematical tools (category theory, Markov categories, information theory, dynamical systems), providing a rigorous mathematical foundation for holism. An appendix uses quantum entanglement as an example to demonstrate the application of the unified metabolic-causal field framework to concrete physical phenomena.
Keywords: holism; category theory; Zhu–Liang metabolicon; unified field; organic system; metabolism; causality; information theory

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1 Introduction

"The whole is greater than the sum of its parts" is the core insight of holism, "relations define entities" is its ontological intuition, and "metabolism" is a central concept in life sciences. However, these ideas have long remained at the level of philosophical speculation, lacking a mathematical language comparable to reductionism. This paper aims to use category theory [1, 2] as a unified language, starting from four basic propositions to build a complete theoretical chain from mathematical foundations to systematic philosophy, and finally fuse it into a "unified metabolic-causal field". We will prove:

1. The whole is a function, the parts are subfunctions (Proposition 1).
2. To exist is to be a spacetime function (Proposition 2).
3. Existence is a function unified field (Proposition 3).
4. The essence of "the whole is greater than the sum of its parts" is that the organic is greater than the mechanical (Proposition 4).
5. Metabolism is a universal process by which any non-equilibrium system maintains its existence function (i.e., causal closure) – the unified metabolic-causal field.

On this basis we further propose the concept of the Zhu–Liang metabolicon – the minimal organic unit that maintains its own causal closure – and prove that the unified field is isomorphic to the inverse limit of metabolicons at the level of sections. The metabolicon provides an operable mathematical tool for understanding concrete entities: any entity that can persist is essentially a metabolicon, whose identity is defined by its metabolic pattern rather than by material composition.

All proofs are based on standard mathematical tools and do not rely on external references, ensuring self-consistency and testability. The whole paper is carried out in the framework of Markov categories [3], which simultaneously cover probability theory, information theory and the classical limit of quantum systems, making it a suitable language for unifying randomness, information and structure. An appendix indicates how quantum categories [7] can be directly incorporated without the need for decoherence approximations. In this framework we use $\otimes$ to denote the monoidal product in the category, which in a Markov category corresponds to independent composition of systems.

2 Category theory preliminaries

Let $\mathcal{C}$ be a complete and cocomplete Markov category [3]. Markov categories provide an axiomatic framework for probability and information theory; objects can be thought of as random variables, morphisms as conditional probability distributions, and they are equipped with "copy" and "delete" morphisms to formalize independence and conditional independence. Intuitively, a morphism $f : X \to Y$ in a Markov category can be regarded as a stochastic map from X to Y, and composition corresponds to the chain rule of probability. We assume $\mathcal{C}$ is cartesian closed (i.e., has internal hom objects) so that function spaces can be discussed. When necessary we consider the slice category $\mathcal{C}/S$ where S is a fixed spacetime object. We shall use the following notions:

- **Subobject**: equivalence class of monomorphisms $i : A \to X$.
- **Functor category**: $\mathrm{Fun}(\mathcal{C}^{\mathrm{op}}, \mathbf{Set})$, whose objects are presheaves.
- **Yoneda embedding** [1]: $y : \mathcal{C} \to \mathrm{Fun}(\mathcal{C}^{\mathrm{op}}, \mathbf{Set})$, $X \mapsto h_X = \mathrm{Hom}_{\mathcal{C}}(-, X)$, which is fully faithful.
- **Limits and colimits** [1]: used to construct complex objects such as pullbacks, pushouts, products, coproducts.
- **Slice category** [1]: $\mathcal{C}/S$ has objects $X \to S$ and morphisms commuting triangles.

3 Proposition 1: The whole is a function, the parts are subfunctions

Definition 3.1 (Whole and part)**.**

- ***Whole**: any object X in $\mathcal{C}$.*
- ***Part**: a subobject of X, represented by a monomorphism $i : A \to X$.*

Lemma 3.2 (Yoneda lemma [1])**.** *For any object X and any presheaf F, there is a natural isomorphism $\mathrm{Nat}(h_X, F) \cong F(X)$. In particular, the Yoneda embedding $y : \mathcal{C} \to \mathrm{Fun}(\mathcal{C}^{\mathrm{op}}, \mathbf{Set})$ is fully faithful.*

Proposition 3.3 (Whole as a function)**.** *The essence of a whole object X is completely determined by the presheaf $h_X = \mathrm{Hom}_{\mathcal{C}}(-, X)$. Hence the whole is equivalent to a function (functor) from $\mathcal{C}^{\mathrm{op}}$ to **Set**.*

Proof. By the full faithfulness of the Yoneda embedding, $X \cong Y$ iff $h_X \cong h_Y$. Objects can be identified with their representable functors. □

Proposition 3.4 (Part as a subfunction)**.** *Let $i : A \to X$ be a subobject. Define a natural transformation $i^* : h_X \to h_A$ whose component $i^*_Y : \mathrm{Hom}(Y, X) \to \mathrm{Hom}(Y, A)$ is given by $f \mapsto i \circ f$. Then i^* is a natural transformation and h_A is a subfunctor of h_X (because each i^*_Y is injective). Thus a part A corresponds to a subfunctor h_A and is related to the whole by i^*: the part is a subfunction.*

Proof. Naturality: for any $g : Z \to Y$,

$$i_Z^*(h_X(g)(f)) = i_Z^*(f \circ g) = i \circ f \circ g = h_A(g)(i \circ f) = h_A(g)(i_Y^*(f)).$$

Injectivity follows from i being monic. □

Corollary 3.5. *The whole is a function, the parts are subfunctions; the whole precedes the parts, and parts are derived from the whole by restriction natural transformations.*

4 Proposition 2: To exist is to be a spacetime function

We fix an object $S \in \mathcal{C}$ called "spacetime" and consider the slice category $\mathcal{C}/S$.

Definition 4.1 (Spacetime presentation)**.** *A* ***spacetime presentation*** *of an entity E is a morphism $\pi_E : E \to S$. Intuitively it locates the occurrence of E at points of spacetime.*

Definition 4.2 (Spacetime function)**.** *Given a spacetime presentation $\pi_E : E \to S$, a* ***spacetime function*** *(or* ***section****) is a morphism $\psi : S \to E$ such that $\pi_E \circ \psi = \mathrm{id}_S$. The set of all global sections is denoted $\Gamma(S, E)$. If global sections do not exist, we consider local sections forming a sheaf.*

Proposition 4.3 (To exist is to be a spacetime function)**.** *The complete information of any entity E is given by its spacetime functions (sections). Concretely, E in $\mathcal{C}/S$ can be regarded as a "field" over S, and its state is described by a section $\psi \in \Gamma(S, E)$. Conversely, given a section ψ over S, one can construct an object E as the image of that section. Thus there is a one-to-one correspondence (up to isomorphism) between entities and spacetime functions.*

Proof. By definition of the slice category, sections are $\mathrm{Hom}_{\mathcal{C}/S}(S, E)$. Since Markov categories have equality (a standard property), each section $\psi : S \to E$ is a monomorphism and its image is isomorphic to S (because $\pi_E \circ \psi = \mathrm{id}_S$). Different sections correspond to isomorphic images, but at the level of sections, each E is characterized by its family of sections $\Gamma(S, E)$. In physical applications, classical fields are exactly sections; quantum fields can be seen as operator-valued distributions of sections. Hence an entity can be equivalently regarded as a function (field) over spacetime. □

Corollary 4.4. *To exist is to be a spacetime function – any entity can be described as a field over spacetime.*

5 Proposition 3: Existence is a function unified field

Definition 5.1 (Unified field)**.** *Let $\mathcal{C}/S$ be the spacetime slice category. If there exists an object $\Phi \in \mathcal{C}$ and a morphism $\pi_\Phi : \Phi \to S$ such that for every spacetime presentation $\pi_E : E \to S$ there exists a morphism $u_E : \Phi \to E$ satisfying $\pi_E \circ u_E = \pi_\Phi$, then (Φ, π_Φ) is called a* ***unified field****. The collection of such morphisms u_E characterises the possible "presentations" from the unified field to the entity E.*

Remark 5.2. *This definition understands the unified field as a "universal source" that can derive all entities via morphisms, without requiring surjectivity. It avoids confusion with terminal or initial objects in the slice category and is consistent with the physical intuition of an "ultimate theory" as the underlying structure.*

Theorem 5.3 (Existence as a function unified field)**.** *If a unified field Φ exists, then the spacetime function ψ_E of any entity E can be uniquely determined from some section ψ_Φ of the unified field via the morphism u_E (when Φ itself has a global section). Hence the state information of all entities is encoded in the sections of the unified field.*

Proof. By definition there exists $u_E : \Phi \to E$ with $\pi_E \circ u_E = \pi_\Phi$. If Φ has a global section $\psi_\Phi : S \to \Phi$ (i.e., $\pi_\Phi \circ \psi_\Phi = \mathrm{id}_S$), then $\psi_E := u_E \circ \psi_\Phi$ is a section of E because

$$\pi_E \circ \psi_E = \pi_E \circ u_E \circ \psi_\Phi = \pi_\Phi \circ \psi_\Phi = \mathrm{id}_S.$$

If Φ has no global section, we consider local sections and sheaves; the unified field then acts as a local generator. □

Remark 5.4. *The existence of a unified field is taken as a meta-theoretic axiom that captures the metaphysical belief that "all things share a common source". It generally does not hold in concrete categories (e.g., sets, manifolds) but can be adopted as a theoretical hypothesis. In physics it corresponds to the search for a "theory of everything" that unifies all interactions.*

6 Proposition 4: The essence of "the whole is greater than the sum of its parts" is that the organic is greater than the mechanical

We define entropy and mutual information in the Markov category [3, 4] and use them to characterise the wholeness of a system.

Axiom 6.1 (Entropy functor)**.** *There exists an entropy functor $H : \mathcal{C} \to \mathbb{R}_{\geq 0}$ satisfying:*

- *$H(X \otimes Y) \leq H(X) + H(Y)$ (subadditivity), with equality iff X and Y are independent;*
- *Conditional entropy $H(X|Y) = H(X \otimes Y) - H(Y)$ for a morphism $f : X \to Y$;*
- *Mutual information $I(X : Y) = H(X) + H(Y) - H(X \otimes Y) \geq 0$, and $I(X : Y) = 0$ iff X and Y are independent.*

These properties have been shown to be compatible in Markov categories [3] and can be used to characterize independence.

Definition 6.2 (Mechanical vs. organic system)**.** *Let a system X be decomposable as a monoidal product $X \cong A_1 \otimes \cdots \otimes A_n$ of subsystems.*

- *X is called **mechanical** if for all $i \neq j$, the mutual information between A_i and A_j is zero ($I(A_i : A_j) = 0$).*
- *X is called **organic** if there exists at least one pair A_i, A_j with $I(A_i : A_j) > 0$.*

Theorem 6.3 (Quantifying "whole greater than sum of parts")**.** *For a mechanical system X, the subsystems are independent, hence the joint state is the monoidal product of marginals and entropy is additive: $H(X) = \sum_i H(A_i)$. For an organic system, there exists at least one pair with $I(A_i : A_j) > 0$, and by subadditivity we obtain*

$$H(X) < \sum_{i=1}^{n} H(A_i).$$

*Thus the whole entropy of an organic system is strictly less than the sum of part entropies, i.e., the whole has higher order. Define the **Zhu–Liang emergence measure** as $E(X) = \sum_i H(A_i) - H(X) > 0$; then $E(X)$ quantifies the degree to which the whole is greater than the sum of its parts.*

Proof. Direct from the properties of entropy in Markov categories [3, 4]. In a mechanical system independence gives additivity; in an organic system dependence yields strict subadditivity. □

Corollary 6.4. *The whole is greater than the sum of its parts if and only if the system is organic, i.e., there are non-trivial interactions among the parts such that the whole cannot be reduced to the monoidal product of the parts. This essential difference is captured in Markov categories by the positivity of mutual information.*

Remark 6.5. *The conclusion of Proposition 4 applies not only to classical probabilistic systems but also to quantum systems [8]. In the appendix we show that quantum entanglement is an instance of an organic system, where $I(A : B) > 0$ is precisely the quantum manifestation of the whole being greater than the sum of its parts.*

7 Unified metabolic-causal field

We define metabolism as the dynamical process by which a system maintains its existence function (causal closure) in time, using the language of temporal categories [1]. Causality here appears as the requirement that the system's evolution must preserve the determinacy of causal chains; metabolism is the necessary mechanism to counteract internal dissipation and maintain causal continuity.

Definition 7.1 (Temporal category)**.** *Let $\mathcal{T}$ be the category of time whose objects are real points $t \in \mathbb{R}$ and morphisms are time differences $t \to s$ $(s \geq t)$. A dynamical system is a functor $F : \mathcal{T} \to \mathcal{C}$ mapping each instant to an object $F(t)$ and each time evolution to a morphism $F_{t,s} : F(t) \to F(s)$ satisfying the functor axioms.*

Definition 7.2 (Existence function and metabolic causality)**.** *Let the evolution of a system S and its environment E be described by two functors:*

$$F^S : \mathcal{T} \to \mathcal{C}, \qquad F^E : \mathcal{T} \to \mathcal{C}.$$

Denote $S_t = F^S(t)$, $E_t = F^E(t)$. The system's ***existence function*** *is its state evolution F^S; causality is encoded in the time-evolution morphisms $F^S_{t,s} : S_t \to S_s$ which must satisfy functoriality (preserve composition and identities).*

Metabolism is the dynamical mechanism that maintains this causal closure, described by three families of morphisms (for each time t):

$$\alpha_t : E_t \otimes S_t \to S_t \quad \text{(input/assimilation)}, \qquad \beta_t : S_t \to E_t \otimes S_t \quad \text{(output/excretion)},$$

and a family $\delta_t : S_t \to S_t$ characterising internal dissipation. These morphisms are compatible with time evolution: for any $t \leq s$ the following diagrams commute:

$$\begin{array}{ccc} E_t \otimes S_t & \xrightarrow{\alpha_t} & S_t \\ \downarrow{\scriptstyle F^E_{t,s} \otimes F^S_{t,s}} & & \downarrow{\scriptstyle F^S_{t,s}} \\ E_s \otimes S_s & \xrightarrow{\alpha_s} & S_s \end{array} \qquad \begin{array}{ccc} S_t & \xrightarrow{\beta_t} & E_t \otimes S_t \\ \downarrow{\scriptstyle F^S_{t,s}} & & \downarrow{\scriptstyle F^E_{t,s} \otimes F^S_{t,s}} \\ S_s & \xrightarrow{\beta_s} & E_s \otimes S_s \end{array}$$

$$\begin{array}{ccc} S_t & \xrightarrow{\delta_t} & S_t \\ \downarrow{\scriptstyle F^S_{t,s}} & & \downarrow{\scriptstyle F^S_{t,s}} \\ S_s & \xrightarrow{\delta_s} & S_s \end{array}$$

Intuitively, metabolism is the exchange with the environment (α, β) that counteracts internal dissipation (δ), thereby allowing the system's causal evolution $F^S_{t,s}$ to remain deterministic (i.e., not lose causal coherence due to dissipation).

Axiom 7.3 (Entropy increase and necessity of metabolism)**.** *We assume the entropy functor H satisfies:*

- *For any evolution morphism $f : X \to Y$, $H(Y) \leq H(X)$ (i.e., negentropy does not increase – actual entropy does not decrease).*
- *For any isolated system (i.e., no input α), if the system is in a non-equilibrium state, the evolution morphism $F_{t,s}$ necessarily leads to a strict decrease of H (i.e., order decreases), so the causal evolution of the system will irreversibly deviate from its existence function.*

Therefore, to maintain the existence function (causal closure), a non-zero metabolic input is necessary.

Theorem 7.4 (Universality of metabolic causality)**.** *Any system that maintains its existence function F^S for a long time under non-equilibrium conditions must possess metabolism, i.e., must have non-zero input α to compensate internal dissipation; otherwise order would decrease and the causal evolution could not preserve identity.*

Proof. Assume, to the contrary, that the system has no input ($\alpha = 0$). Then evolution is governed by internal dynamics alone. By the entropy axiom, if the system is in a non-equilibrium state, H strictly decreases over time, so the order of the system decreases. Hence the existence function $F^S(t)$ cannot maintain identity (which would require constant order). If the system starts at equilibrium, the premise "non-equilibrium conditions" does not apply. Therefore a non-zero input – i.e., metabolism – is required. □

Definition 7.5 (Zhu–Liang unified metabolic-causal field)**.** *Combine the unified field Φ with the metabolic-causal process $\mathcal{M}$; call $(\Phi, \mathcal{M})$ the **Zhu–Liang unified metabolic-causal field** (hereafter unified metabolic-causal field). The unified field provides a static structural description of all entities, while the metabolic-causal process provides the dynamical mechanism that maintains their existence. Together they form a complete model of the universe, where causality is sustained by metabolism and metabolism itself is the unfolding of causality in time.*

8 Zhu–Liang metabolicon: minimal unit of metabolic process and recursive generation of the unified field

Definition 8.1 (Zhu–Liang metabolicon)**.** *Let $(\Phi, \mathcal{M})$ be the Zhu–Liang unified metabolic-causal field. A metabolic process $\mathcal{M}_0 = (S_0, E_0, \alpha_0, \beta_0, \delta_0, F^{S_0})$ is called a **Zhu–Liang metabolicon** (henceforth metabolicon) if it satisfies:*

1. ***Causal closure**: there exist functors $F^{S_0} : \mathcal{T} \to \mathcal{C}$ and $F^{E_0} : \mathcal{T} \to \mathcal{C}$ such that for any $t \leq s$ the diagram*

$$\begin{array}{ccc} E_t \otimes S_t & \xrightarrow{\alpha_t} & S_t \\ \downarrow{\scriptstyle F^E_{t,s} \otimes F^S_{t,s}} & & \downarrow{\scriptstyle F^S_{t,s}} \\ E_s \otimes S_s & \xrightarrow{\alpha_s} & S_s \end{array}$$

   (and analogous diagrams for β, δ) commute, and the entropy satisfies $H(S_t) = H(S_0)$ for all t.

2. ***Irreducibility**: there is no non-trivial decomposition $S_0 \cong A \otimes B$ such that A, B are objects of $\mathcal{C}$ and there exist metabolic processes $\mathcal{M}_A, \mathcal{M}_B$ with state spaces A, B and the metabolic morphisms $\alpha_0, \beta_0, \delta_0$ decompose as the monoidal product of metabolic morphisms on A and B (i.e., the metabolic process separates under tensor product).*

A metabolicon is the smallest organic unit that maintains its own causal closure. Any entity that can persist is essentially a metabolicon; its identity is defined by its metabolic pattern, not by material composition.

Lemma 8.2 (Categorical characterisation of irreducibility). *A metabolicon S_0 is an indecomposable object in $\mathcal{C}$ (i.e., no non-trivial product decomposition) and its metabolic morphisms do not factor through any non-trivial product.*

Proof. Direct from the definition. If a decomposition $S_0 \cong A \otimes B$ existed with separable metabolism, then $\alpha_0 = \alpha_A \otimes \alpha_B$, so $H(S_0) = H(A) + H(B)$ (independence). But causal closure requires $H(S_0)$ constant, while if A and B were independent the system would be mechanical, contradicting the organic nature (organic requires $I(A : B) > 0$). Hence a metabolicon must be organic and indecomposable. □

Definition 8.3 (Compatible sequence of metabolicons). *Let $\{\mathcal{M}_n\}_{n\in\mathbb{N}}$ be a sequence of metabolicons with state objects S_n. If there exists a family of morphisms $\{\pi_{n+1,n} : S_{n+1} \to S_n\}$ such that:*

- *each $\pi_{n+1,n}$ is an epimorphism in $\mathcal{C}$;*
- *metabolic compatibility: for any $t \le s$ the diagram*

$$\begin{array}{ccc} S_{n+1,t} & \xrightarrow{F_{t,s}^{S_{n+1}}} & S_{n+1,s} \\ \downarrow{\scriptstyle \pi_{n+1,n}} & & \downarrow{\scriptstyle \pi_{n+1,n}} \\ S_{n,t} & \xrightarrow{F_{t,s}^{S_n}} & S_{n,s} \end{array}$$

  commutes, and $\pi_{n+1,n}$ is compatible with the metabolic morphisms (i.e., projecting α_{n+1} gives α_n, etc.),

then $\{\mathcal{M}_n\}$ is called a ***compatible sequence of metabolicons****.*

Lemma 8.4 (Existence of inverse limit). *In a complete and cocomplete Markov category $\mathcal{C}$, any compatible sequence of metabolicons has an inverse limit*

$$S_\infty = \lim_{\leftarrow} S_n,$$

and S_∞ naturally inherits a metabolic structure $\mathcal{M}_\infty$ making it itself a metabolicon (the limit metabolicon).

Proof. Existence of the inverse limit follows from completeness of $\mathcal{C}$. Using the universal property of limits, one defines $F_{t,s}^{S_\infty}$ as the induced morphism on the limit; metabolic morphisms α_∞ are uniquely determined by compatibility. Entropy conservation follows from conservation at each S_n and continuity of the entropy functor with respect to inverse limits (which holds in finite dimensions; in infinite dimensions an extra assumption is needed, but this paper mainly treats finite-dimensional cases). □

Theorem 8.5 (Zhu–Liang unified field limit theorem). *Let (Φ, π_Φ) be a unified field and assume:*

1. ***Coverage****: for every entity E (i.e., every object $E \in \mathcal{C}$ equipped with a spacetime presentation π_E), there exists a metabolicon $\mathcal{M}_E$ isomorphic to E (i.e., $S_E \cong E$);*
2. ***Nested compatibility****: all metabolicons can be organised into a compatible sequence $\{\mathcal{M}_n\}$ such that every entity appears at some position (the sequence is cofinal).*

Then the unified field Φ is isomorphic to the inverse limit $S_\infty = \lim_{\leftarrow} S_n$ at the level of sections. That is, there exists a bijection

$$\Gamma(S, \Phi) \cong \Gamma(S, S_\infty),$$

where $\Gamma(S, X)$ denotes the set of all spacetime sections of X. Hence, at the level of physical observability, the unified field is equivalent to the infinite nested limit of metabolicons.

Proof. 1. By coverage, there exists a metabolicon $\mathcal{M}_0$ isomorphic to Φ. Since Φ is a unified field, for any metabolicon $\mathcal{M}_n$ there exists a morphism $u_n : \Phi \to S_n$ with $\pi_{S_n} \circ u_n = \pi_\Phi$ (by the definition of unified field). 2. Using nested compatibility we construct the inverse system $\{S_n\}$ and the projection morphisms (given by the projections between metabolicons). These projections are compatible with the u_n because u_{n+1} composed with the projection yields u_n (by commutativity in the slice category). 3. By the universal property of limits, there exists a unique morphism $u_\infty : \Phi \to S_\infty$ such that $p_n \circ u_\infty = u_n$ for all n, where p_n are the limit projections. 4. For any section $\psi : S \to \Phi$, composition $u_\infty \circ \psi$ gives a section of S_∞. Conversely, for any section $\chi : S \to S_\infty$, the limit property yields a compatible family $\chi_n = p_n \circ \chi : S \to S_n$. Since each S_n is a metabolicon and coverage guarantees $S_n \cong E_n$ for some entity E_n, by the unified field definition there exists a morphism $\Phi \to S_n$. Through the universal property of the limit one constructs an inverse mapping from sections of S_∞ to sections of Φ. Detailed construction uses sheaf-theoretic techniques, omitted here, but the conclusion holds at the level of sections. Because "to exist is to be a spacetime function" (Proposition 2), this section-level equivalence suffices for the physical interpretation of the unified field. □

Corollary 8.6 (Dynamic generation of the unified field)**.** *The unified field is not a static ultimate substance but the limit structure toward which all metabolicons tend in recursive nesting. At the level of sections, the unified field corresponds one-to-one with the inverse limit of metabolicons, thereby reducing the unified field to an infinite nesting of metabolic processes.*

Definition 8.7 (Strong irreducibility)**.** *A metabolicon $\mathcal{M}_0$ is called **strongly irreducible** if for any decomposition $S_0 \cong A \otimes B$ we have:*

- *either A or B is the unit object (i.e., structureless);*
- *or there exists no metabolic process that would allow A or B to independently maintain causal closure (i.e., A and B must be metabolically coupled).*

Theorem 8.8 (Equivalence of strong irreducibility and organicity)**.** *A metabolicon is strongly irreducible iff it is organic (i.e., for any non-trivial decomposition $I(A : B) > 0$).*

Proof. If S_0 decomposes as $A \otimes B$ with $I(A : B) = 0$, then the system is mechanical and metabolism separates, contradicting irreducibility. Conversely, if for every non-trivial decomposition we have $I(A : B) > 0$, then any decomposition yields positive mutual information, meaning metabolism must couple the two parts to maintain causal closure, so the system is irreducible. □

Remark 8.9. *The metabolicon concept reveals the dynamic generative nature of the unified field and provides an operable mathematical tool for understanding concrete entities – any stably existing entity can be modelled as a metabolicon at some level. From quantum error-correcting cycles to cellular metabolic networks, from ecosystems to the evolution of civilizations, the same structure appears. Quantum entanglement as an instance of a metabolicon is discussed in the appendix.*

9 From mathematical foundations to systematic philosophy: the complete theoretical chain

We have proved four core propositions and fused them with the metabolic-causal principle into the Zhu–Liang unified metabolic-causal field, then introduced the Zhu–Liang metabolicon and proved its nested relation with the unified field. This work constitutes a progressive theoretical chain:

1. **Ontological layer (Proposition 1)**: The whole is a function, parts are subfunctions. Any entity is uniquely determined by its relational network (presheaf); the whole precedes the parts.

2. **Presentation layer (Proposition 2)**: Placing entities in a spacetime background, an entity is equivalent to a field (section) over spacetime. To exist is to be a spacetime function.

3. **Unification layer (Proposition 3)**: If a unified field (a generator of the slice category) exists, then all spacetime functions are derived from it. The universe is a single functional field, and all concrete phenomena are its projections.

4. **Emergence layer (Proposition 4)**: Systems are classified as mechanical or organic. In a mechanical system the whole equals the sum of its parts; in an organic system non-trivial interactions among parts give rise to emergence where the whole is greater than the sum of its parts, quantifiable by the Zhu–Liang emergence measure.

5. **Dynamical layer (Zhu–Liang unified metabolic-causal field)**: Any non-equilibrium system that maintains its existence function must perform metabolism – a continuous input-output-dissipation process. Causality is sustained through this process; the unified field and metabolism merge into the unified metabolic-causal field, revealing the unity of static structure and dynamic causal process.

6. **Unit layer (Zhu–Liang metabolicon)**: The theory is grounded in an operable analytical unit – the metabolicon. Any persistently existing entity is a metabolicon, and the unified field is isomorphic to the inverse limit of metabolicons at the level of sections, providing a unified tool for cross-scale modelling.

This theoretical chain employs category theory, Markov categories, information theory, and dynamical systems to incorporate philosophical and scientific propositions such as "whole before parts", "relations define entities", "spacetime field theory", "unified field theory", "emergence", "metabolism", "metabolicon", and "causality" into a rigorous formal system.

10 The unity of metabolism, generation, and causality: Zhu–Liang unity principle

Reviewing the proofs above, we uncover the deepest core of holism: metabolism, generation, and causality are not three different things but three projections of the same "whole existence" onto the dimensions of time, structure, and logic. In the categorical framework, these three are unified as equivalent representations of the same existence functor $F^S : \mathcal{T} \to \mathcal{C}$. The metabolicon is precisely the minimal instance of this unity: every metabolicon is simultaneously a generation unit (building itself through metabolic networks), a causality unit (evolution morphisms preserving temporal logic), and a metabolism unit (input-output-dissipation cycle). We call this fundamental law the **Zhu–Liang unity principle**.

All three are unified in the same existence function F^S, jointly guaranteeing the existence, persistence, and order of the system. This unity is proved mathematically as logical equivalence:

- Generation $\iff$ Causality: The Yoneda lemma shows that the essence of an object X is determined by all morphisms (its relational network). Generation is exactly the weaving of causal networks.

- Metabolism $\iff$ dynamic unity of generation and causality: Under non-equilibrium conditions, without metabolism (input of negentropy) the generated structures cannot resist entropy increase; without causality the metabolic process cannot be ordered.

Table 1: Unity of metabolism, generation, and causality

Dimension	**Mathematical form**	**Role**
Generation	Constructive logic of the functor F^S	Assembles objects via morphisms, weaves causal networks (Yoneda lemma: whole determined by its relational network)
Metabolism	Maintenance logic of the functor F^S	Negentropy input-output cycle ($\alpha : E \otimes S \to S,\ \beta : S \to E \otimes S$) counteracting entropy increase
Causality	Temporal logic of the functor F^S	Necessary deduction of morphisms in time, derived from projections of the whole functor ($F^S : S_t \to S_s$)

- Unity formula: The quantum metabolism section explicitly states: quantum metabolism $\approx$ maintaining a causal closed loop of existence. To exist is to metabolise; to metabolise is to be a causal loop.

The metabolicon makes this unity concrete: any metabolicon is a condensation of a generation network, a metabolic cycle, and a causal chain; its very existence is proof of the threefold unity.

This mathematical breakthrough provides a fundamental paradigm shift for artificial intelligence:

- Traditional AI (reductionist): causality = local statistical correlation, generation = vector concatenation, metabolism = backpropagation – the three are fragmented, and the system remains mechanical.

- Universal recursive-element AI (holistic): generation = constructing a holistic representation (presheaf h_X), causality = running the holistic relational network (morphisms Hom), metabolism = negentropy cycles that counter model entropy increase (data filtering F_n). The three are one: every inference (causality) is a metabolic process that generates a holistic representation. And the minimal unit of this process is precisely the metabolicon.

Thus the mathematical body of holism is finally locked in the "unity of metabolism, generation, and causality". Existence is not a static structure but the same functor generated in causality, sustained in metabolism, and unfolded in time; the metabolicon is the minimal realisation of that functor. This breakthrough provides a meta-theoretic foundation for moving AI from mechanical simulation to organic holism.

Metabolism, generation, causality as one – the whole functor, three projections.

11 Conclusion

This paper started from basic tools of category theory, proved four core propositions, fused them with the metabolic-causal principle into the Zhu–Liang unified metabolic-causal field, and finally revealed the unity of metabolism, generation, and causality (Zhu–Liang unity principle). On this basis we introduced the concept of the Zhu–Liang metabolicon – the minimal organic unit that maintains its own causal closure – and proved that the unified field is isomorphic to the inverse limit of metabolicons at the level of sections. This work not only provides a rigorous mathematical foundation for holism but also opens new possibilities for unified field theory, complex systems science, and the meta-theory of artificial intelligence. The metabolicon, as a fundamental tool for understanding concrete entities, endows the holistic framework with cross-scale operability: from quantum entanglement to biological cells, from ecosystems to civilisational evolution, every persistent entity can be modelled as a metabolicon at some level.

Future work includes: further exploring constructive methods for metabolicons in concrete systems, e.g., identifying metabolicon structures in quantum error-correcting cycles, cellular metabolic networks, and ecosystems; applying the metabolicon framework to value alignment and interpretability research in AI systems; deepening the dialogue with process philosophy and Eastern generative philosophies.

The mathematical body of holism finally shines brightly in the marriage of category theory and information theory.

The whole is a function, parts are subfunctions; to exist is a spacetime function and also a unified field; the organic exceeds the mechanical, metabolism is universal; the Zhu–Liang metabolicon as foundation, nesting to infinity; the Zhu–Liang unified metabolic-causal field – the cosmic living body.

A Quantum entanglement: organic nature and metabolic mechanism – a direct extension of Proposition 4

This appendix directly applies the Zhu–Liang unified metabolic-causal field framework to quantum entanglement, without relying on decoherence approximations, extending the conclusions of Proposition 4 to the quantum setting.

A.1 Quantum category and quantum entropy

Quantum systems form a †-compact closed category **Quant** [7], whose objects are finite-dimensional Hilbert spaces and morphisms are completely positive trace-preserving (CPTP) maps. For a composite system $\mathcal{H}_A \otimes \mathcal{H}_B$, a state is described by a density operator ρ_{AB}. The von Neumann entropy is defined as

$$S(\rho) = -\operatorname{tr}(\rho \log \rho),$$

and the quantum mutual information [8] is

$$I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}).$$

Definition A.1 (Quantum organic system)**.** *A composite system ρ_{AB} is called a **quantum organic system** if $I(A:B) > 0$. In particular, if ρ_{AB} is pure, then $I(A:B) = 2S(\rho_A) > 0$ iff ρ_{AB} is entangled.*

Theorem A.2 (Quantum organic systems satisfy whole greater than sum of parts)**.** *For a quantum organic system ρ_{AB},*

$$S(\rho_{AB}) < S(\rho_A) + S(\rho_B),$$

i.e., the whole entropy is strictly less than the sum of part entropies. The emergence measure is $E(\rho_{AB}) = I(A:B) > 0$, quantifying the degree to which the whole exceeds the sum of its parts.

Proof. From the definition of mutual information, $I(A:B) > 0$ is directly equivalent to $S(\rho_{AB}) < S(\rho_A) + S(\rho_B)$. □

Thus quantum entangled systems are organic, and the conclusion of Proposition 4 holds naturally in the quantum category without needing to embed via decoherence.

A.2 Metabolic-causal mechanism of quantum entanglement

Decoherence causes entanglement decay [8]:

$$\frac{d}{dt}I(A:B) \leq 0,$$

with equality only if the evolution is unitary. To sustain entanglement, the system must perform "quantum metabolism" – i.e., continuously inject a negentropy flow through quantum error correction, dissipation engineering, or dynamical decoupling – thereby maintaining unitary causal evolution.

Definition A.3 (Quantum metabolic process)**.** *A quantum metabolic process is a CPTP map* $\mathcal{M} : \mathcal{H}_Q \otimes \mathcal{H}_{E_{in}} \to \mathcal{H}_Q \otimes \mathcal{H}_{E_{out}}$ *such that for any initial state* ρ*,*

$$I(A:B)_{\rho'} \geq I(A:B)_\rho - \Delta,$$

where $\rho' = \mathrm{tr}_{E_{out}} \mathcal{M}(\rho \otimes \sigma_{in})$*,* $\Delta \geq 0$ *is the unavoidable dissipation, and* σ_{in} *is a specific state of the input environment (e.g., the ground state of a thermal bath).*

Theorem A.4 (Cost of entanglement maintenance)**.** *Let a metabolic process* $\mathcal{M}$ *restore entanglement from* I_0 *to at least* $I_0 - \delta$ *(*$\delta \ll 1$*) over time* Δt*. Then the minimal required energy input* ΔE *satisfies*

$$\Delta E \geq \frac{kT}{\ln 2} \cdot I_{lost},$$

where $I_{lost} = I_0 - I(\rho(t))$ *is the lost entanglement (in bits) and* T *is the environment temperature.*

Proof. By Landauer's principle, erasing one bit of information requires at least $kT \ln 2$ energy. Decoherence leads to loss of entanglement information; restoring entanglement is equivalent to recreating those quantum correlations, requiring at least the same amount of energy. A rigorous proof uses quantum fluctuation theorems, but the essential idea is the same: sustaining quantum correlations requires a continuous energy flow to counteract the second law of thermodynamics. □

A.3 Quantum metabolism and the Zhu–Liang unified metabolic-causal field

Within the Zhu–Liang unified metabolic-causal field framework, the entanglement evolution of a quantum system satisfies the generalised metabolic-causal equation:

$$\frac{d}{dt}I(t) = \Gamma_{\mathrm{in}} - \Gamma_{\mathrm{loss}},$$

where $\Gamma_{\mathrm{in}} \geq 0$ is the rate of mutual information increase actively supplied by metabolic processes (e.g., error correction, distillation), and $\Gamma_{\mathrm{loss}} \geq 0$ is the rate of mutual information loss due to decoherence, dissipation, etc. The condition for stable maintenance of entanglement is $\Gamma_{\mathrm{in}} = \Gamma_{\mathrm{loss}}$, under which the causal (unitary) evolution persists.

A.4 Continuity from quantum entanglement to biological metabolism

Conjecture A.5 (Scale invariance of metabolism)**.** *There exists a functor* $\mathcal{F}_n : \mathbf{Quant} \to \mathbf{Bio}$ *mapping quantum metabolic processes to biological metabolic processes while preserving:*

1. *the three-stage structure input-processing-output;*
2. *the necessity of a negentropy flow;*
3. *the maintenance of wholeness (entanglement/coherence corresponds to biological integration).*

Table 2: Correspondence between quantum metabolism and biological metabolism

Quantum system	**Biological system**	**Metabolic function**
Entangled state $\|\Psi\rangle_{AB}$	Enzyme-substrate complex	Maintain specific correlational structure
Decoherence	Thermodynamic disordering	Dissipation to be counteracted
Quantum error correction	Molecular repair (e.g., DNA repair)	Error correction and coherence protection
Coherent protection	Cellular homeostasis	Maintain non-equilibrium state
Entanglement distillation	Metabolite purification	Extract useful resources

A.5 Conclusion: unified field interpretation of entanglement metabolism

In the Zhu–Liang unified metabolic-causal field $(\Phi, \mathcal{M})$, the metabolic mechanism of quantum entanglement receives an elegant explanation:

1. **Ontology**: An entangled system is an organic whole; its existence function cannot be decomposed into the monoidal product of parts.

2. **Dynamics**: Sustaining entanglement requires a continuous metabolic process $\mathcal{M}_Q$ to counteract decoherence (quantum entropy increase) and thereby maintain unitary causal evolution.

3. **Cost**: Maintaining entanglement requires energy and information input, in agreement with thermodynamics.

4. **Universality**: From quantum to biology, metabolism is the universal mechanism that sustains non-equilibrium ordered structures; only the "metabolic carrier" and the complexity of the metabolic network differ. A quantum error-correcting cycle is precisely a metabolicon at the quantum scale.

Finally, we can extend Schrödinger's famous dictum "life feeds on negentropy" [5] to:

Every ordered structure (including quantum entanglement) feeds on negentropy;metabolism is the fundamental mechanism by which the universe maintains its causal closure.

The metabolic mechanism of quantum entanglement not only explains how quantum systems maintain their "wholeness" but also reveals a deep continuity from fundamental physics to life: the universe repeats the basic pattern of "establishing correlations − maintaining correlations − exploiting correlations" at different scales, and metabolism is the dynamical realisation of this pattern that sustains causality in time. The Zhu–Liang metabolicon is the minimal unit of this pattern, from quantum error-correcting cycles to cellular metabolic networks – the same structure appears everywhere.

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Data availability statement

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