Singularity, Duality, and Triality: Criteria for Irreducible Triadic Mediation in Physical and Formal Systems

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1 Introduction

Identifying recurring principles of emergent stability is a central challenge in science. As Philip Anderson argued in his landmark essay, the ability of complex systems to exhibit fundamentally new behaviors at each level of organization means that reductionism alone is insufficient; what is needed are principles that govern the emergence of stable, higher-order structures from simpler components [11]. This paper proposes a candidate classificatory schema — the Singularity–Duality–Triality (S-D-T) sequence — offered not as a metaphysical law but as a constrained meta-theoretical framework — one that does not argue triadic structure is ubiquitous, but that it may be necessary in a specific class of systems defined below. We define this process operationally in the language of dynamical systems:

- **Singularity:** A high-symmetry state occupying a single, stable basin of attraction. It is a foundational structure or "canvas" that serves as a domain for higher-dimensional interaction.
- **Duality:** A symmetry-breaking event that bifurcates the phase space, creating a system defined by the tension between two competing basins of attraction. This is a direct emergent property of the Singularity's existence, manifesting as inherent tensions, bifurcations, or complementarities within the system.
- **Triality:** The emergence of a new constraint or mediating interaction that reshapes the phase space, resolving the dualistic tension into a new, more complex, and dynamically stable basin of attraction. The third element reshapes the phase space of the system, producing novel organizational properties distinct from either competing structure.

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Formal Definition: Triality (S-D-T sense)

The framework identifies irreducible triadic structure in two distinct modes:

- **Dynamical mode:** A system whose phase-space evolution satisfies conditions (i)–(iv) below — competing attractors resolved by a mediating constraint that reshapes the dynamics.
- **Algebraic/topological mode:** A formal or physical system in which a threefold symmetry, combinatorial threshold, or topological constraint generates a closed organizational outcome (a classification, a qualitatively new phase, a novel physical effect) that is unavailable to the dyadic case and cannot be recovered by any manipulation internal to it.

Both modes share a common logical core: a third structural role that is irreducible — it cannot be eliminated, absorbed, or derived from the two-part description without loss of the organizational outcome it generates. The modes differ in whether that irreducibility is realized through phase-space dynamics or through algebraic/combinatorial structure. The examples in this paper span both modes, and the paper explicitly identifies which mode each example illustrates (see Table 2).

In the **dynamical mode**, a system exhibits Triality if and only if all four of the following conditions are met:

(i) **(Duality prerequisite)** The system contains two competing regimes, attractors, or organizational tendencies whose tension cannot be stably resolved within the dyadic description alone;

(ii) **(Novel mediating constraint)** A third element appears that is not reducible to interpolation, parameter tuning, or relabeling of the original dyad. Its role is mediating rather than merely additive: it changes the organizational structure of the system rather than simply modifying one term within it;

(iii) **(Higher-order closure)** The mediating element makes possible a stable regime, organizational capacity, or form of coherence that is absent from either pole alone and cannot be recovered by the dyadic system without it;

(iv) **(Reduction test)** If the third element can be removed without loss of the new regime, or if its effect can be reproduced entirely from the original dyad by reparameterization alone, the case does not qualify as Triality.

In the **algebraic mode**, the analogous test is whether the organizational outcome (classification, phase, or physical effect) is mathematically forbidden for the dyadic case and mathematically entailed by the triadic structure. The burden of proof lies with the proposed example in either mode: a valid application must show that the third term is structurally necessary, not merely descriptively convenient.

Common logical core: transition invariance across modes. The S-D-T sequence can be understood as a directed graph whose nodes are organizational states (S, D, T, S′) and whose edges are transitions between them. While the nodes may be realized dynamically or algebraically, the *edge logic* is invariant across both modes. A crucial clarification: the directed edges represent *logical dependency*, not necessarily temporal sequence. In the dynamical mode, the transitions may unfold chronologically; in the algebraic mode, all three nodes may coexist simultaneously, and the arrows trace the logical order of the description — which role presupposes which — rather than a causal chain. The invariance claim is that the same dependency structure holds in both cases:

- **S → D (Dyadic insufficiency):** The singular description proves insufficient — a symmetry breaks, a tension emerges, or an opposition becomes structurally relevant that the singular state cannot resolve.
- **D → T (Mediated restoration):** A third term is introduced that restores or enables organizational capacity unavailable to the dyad alone. This is not mere model enrichment: the third term changes the class of organizational outcomes accessible to the system.
- **T → S′ (Reduction failure):** The organizational outcome produced by the mediated system cannot be recovered by eliminating the third term and returning to the dyadic descriptive class. Attempted reduction produces explanatory loss — a shift to non-Markovian dynamics, a collapse of classificatory architecture, or comparable degradation.

This transition logic is what unifies the two modes. The nodes may be dynamical (competing attractors, mediating state variables) or algebraic (symmetry representations, combinatorial thresholds). But in every candidate case, the same three-edge structure must hold: the dyad is insufficient, the third term restores, and the restoration is irreversible within the original descriptive class. If the two modes shared nothing beyond numerical three-ness — if the edge logic failed to apply uniformly — the framework would reduce to a loose family of analogies rather than a unified classificatory tool. That possibility is explicitly acknowledged as the framework's central open question (see §4.3).

Scope. S-D-T does not claim that all systems are triadic, or that triadic organization is the only form of higher-order stability. It identifies a *specific subset* of systems: those in which higher-order organizational outcomes arise through an irreducible third structural role, whether realized dynamically (conditions i–iv) or algebraically (the analogous combinatorial/symmetry test). Systems that achieve stability through other mechanisms — direct parameter relaxation, external forcing, dissipation alone — are not S-D-T systems and are not claimed to be. The framework's content is its identification of this subset and its argument that the subset is populated by physically significant examples.

To guard against structural inflation — the risk that the definition is satisfied too easily, leading to trivial identifications — we provide a class of explicit non-examples. A thermostat is not a

Triality: it stabilizes a temperature by parameter adjustment (a feedback loop), but introduces no new degree of freedom and changes no topological dimension of the attractor manifold. A Lotka-Volterra predator-prey cycle is not a Triality: the oscillation between predator and prey populations constitutes a Duality, but the addition of a third species does not necessarily satisfy condition (ii) unless the third species plays a categorically novel organizational role (e.g., a keystone predator that does not simply add a third competitor but restructures the competitive dynamic entirely). These non-examples bound the framework's scope and prevent the "find three things, call it triadic" conflation that any cross-domain heuristic must guard against.

Why three and not more. A natural question is whether the framework generalizes: why not four-node closure, or higher-order simplices? The answer is that three is the *minimum* closure number for the specific organizational problem S-D-T addresses: resolving the tension between two competing attractors requires at least one mediating constraint, yielding a minimum of three organizational roles. This is a minimality claim with a stronger backing than mere convenience.

The backing is structural: conditions (i)–(iii) of the Formal Definition specify a system with exactly three distinct roles — two competing attractors and one mediating constraint. A two-role system satisfying condition (i) is, by definition, unstable (the Duality). A three-role system satisfying (ii) and (iii) is, by definition, the minimum configuration in which the instability is resolved without eliminating either competitor. Four-role systems are not forbidden; they arise in nested S-D-T cycles where the output S' becomes the Singularity of a new cycle. In this sense, apparent quartic configurations are compound triadic systems, not genuinely quartic closures. The framework claims minimality and recurrence for the triadic pattern, not universality or uniqueness.

The argument of the paper proceeds in four stages. First, we motivate the model's structure with selected examples from mathematical and physical theory (§2). Second, we address directly the principal vulnerability of any cross-domain heuristic—the risk of post hoc pattern-fitting—and define rigorous conditions for the model's refutation (§3). Third, we clarify the ontological status of the framework, present a minimal dynamical illustration of triadic mediation, and identify the formal work that remains (§4). Fourth, we conclude with a summary and a candid assessment of what the paper does and does not establish (§5).

What this paper is, what it is not, and how to evaluate it. This paper advances S-D-T as a constrained meta-theoretical framework for identifying one specific class of organized systems: those in which a stable higher-order regime appears only through an irreducible mediating third term. It does not claim to replace established physical theory, to provide a universal account of emergence, or to function as a complete mechanistic theory of triadic organization. Its success should be judged by a narrower standard: whether it imposes nontrivial exclusion criteria, survives reduction tests, and helps distinguish genuine triadic mediation from arbitrary three-part description.

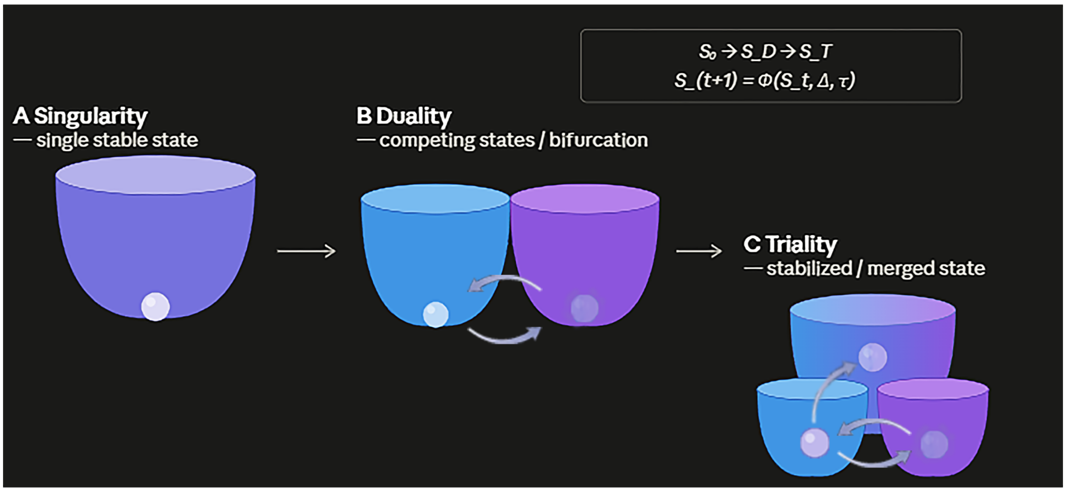

Figure 1: Dynamical schematic of the S-D-T sequence. (A) A single stable basin (Singularity). (B) Symmetry breaking creates two competing basins (Duality); the saddle point represents an unresolved configuration. (C) A mediating constraint (Triality) reshapes the phase space, breaking the symmetry between competing basins and producing an asymmetric resolution in which one regime is favored while the other is subordinated. The inset equation expresses the process schematically; a concrete instantiation as gradient descent on the cusp catastrophe potential is given in Section 4 (Figure 3).

2 The Physical Motivation for a Triadic Heuristic

The examples in this section are presented to illustrate compatibility with existing theory, not as independent confirmation of the framework. The strongest case is a canonical mathematical instance of triality; the second is a bounded physical illustration of mediated structural novelty.

2.1 The Structural Pattern of Spin(8) Triality

The Lie group Spin(8) possesses a unique, threefold symmetry (triality) among its representations that is not found in any other simple Lie group [2, 3, 4, 5]. The generalizability of this structure beyond Spin(8) has been established on rigorous algebraic grounds. Kamiya (2021) has shown that triality groups arise systematically across a broad family of algebraic triple systems—(α, β, γ)-triple systems that generalize Jordan and structurable algebras—and that in every instance the triality symmetry is traceable to the D_4 Lie algebra that grounds Spin(8) itself [23]. This result is significant: it means that the threefold symmetry we identify as Triality is not an accident of one exceptional group but a recurring feature of a wide class of algebraic systems equipped with a trilinear product.

This abstract mathematical structure has physically consequential implications through a chain of connections that the original presentation of this argument left implicit; making them explicit is essential for the claim's precision.

The mathematical core. Spin(8) triality is the statement that the three eight-dimensional representations of Spin(8) — vector (V), positive spinor (S^+), and negative spinor (S^-) — are mutually equivalent under the outer S_3 automorphism of the D_4 Dynkin diagram. This equivalence is closely related to the fact that the real Clifford algebra sequence has period 8: $\mathrm{Cl}(8) \cong M(16, \mathbb{R})$ is a full matrix algebra, and the sequence of real Clifford algebras $\mathrm{Cl}(1), \mathrm{Cl}(2), \ldots$ repeats with period 8 because of the triality-mediated equivalence of the spinor and vector representations at dimension 8 [22, 3].

The chain to physical consequence. This eightfold periodicity of real Clifford algebras is, via the Atiyah-Bott-Shapiro construction [22], the mathematical basis of Bott periodicity in real K-theory. And it is this Bott periodicity that underlies the $\mathbb{Z}_8$ classification of topological phases in the real symmetry classes: as Kitaev (2009) established and Ryu, Schnyder, Furusaki, and Ludwig (2010) demonstrated in detail, the eightfold periodicity of the classification table directly reflects the eightfold periodicity of the spinor representations of the orthogonal groups SO(N) [1, 6]. In S-D-T terms: the Lie group is the canvas (Singularity), its physical instantiation as the symmetry group of a topological superconductor produces the interaction (Duality), and the unique triality symmetry — via Clifford periodicity and Bott periodicity — is the mediating rule (Triality) that yields a robust, finite $\mathbb{Z}_8$ classification of topological phases. The connection between Spin(8) triality and the $\mathbb{Z}_8$ count is therefore not direct but mediated through real Clifford algebra periodicity; the chain is real and mathematically established, not merely analogical. For completeness: this $\mathbb{Z}_8$ periodicity is specific to the *real* symmetry classes (those with time-reversal or particle-hole symmetry); the complex symmetry classes, classified by complex K-theory via complex Clifford algebras, follow a $\mathbb{Z}_2$ periodicity and are not connected to the Spin(8) triality argument presented here [1, 6].

The strongest S-D-T reading. A physically significant detail: in systems such as 1D topological superconductors in class BDI, the non-interacting classification is $\mathbb{Z}$, but Fidkowski and Kitaev showed that introducing a specific quartic interaction can gap out $N = 8$ Majorana boundary modes without breaking time-reversal symmetry — reducing the classification to $\mathbb{Z}_8$. Triality is what makes this possible: the equivalence of vector and spinor representations at dimension 8 enables the construction of the symmetry-preserving interaction Hamiltonian. In S-D-T terms, triality does not act as a dynamical mediator; it acts as the algebraic precondition that *permits* the interaction which reshapes the classification. The chain from Spin(8) to $\mathbb{Z}_8$ is established mathematics; the S-D-T contribution is to note that the chain originates in a threefold symmetry that invites structural interpretation.

Algebraic-mode caveat. An important caveat: the three Spin(8) representations are co-equal and simultaneous — they do not exist in a state of dynamical competition, nor does one emerge chronologically to “resolve” a tension between the other two. The S-D-T mapping here is therefore *algebraic*, identifying the structural role that a threefold symmetry plays in generating a closed classification, rather than *dynamical* in the phase-space sense of conditions (i)–(iv). The example illustrates the kind of irreducible three-way structure the framework seeks to identify; it does not claim that Spin(8) undergoes a temporal S-D-T process.

A further algebraic instantiation. A further algebraic instantiation of threefold structure appears in quantum chromodynamics, where Ghanbarpour and von Smekal [24] have explicitly constructed lattice QCD ensembles with fixed $\mathbb{Z}_3$ center charge, demonstrating that the triality sectors of SU(3) color are physically distinct and thermodynamically significant — a concrete realization of threefold symmetry as an active organizational constraint on the QCD phase space (Figure 2, panel B).

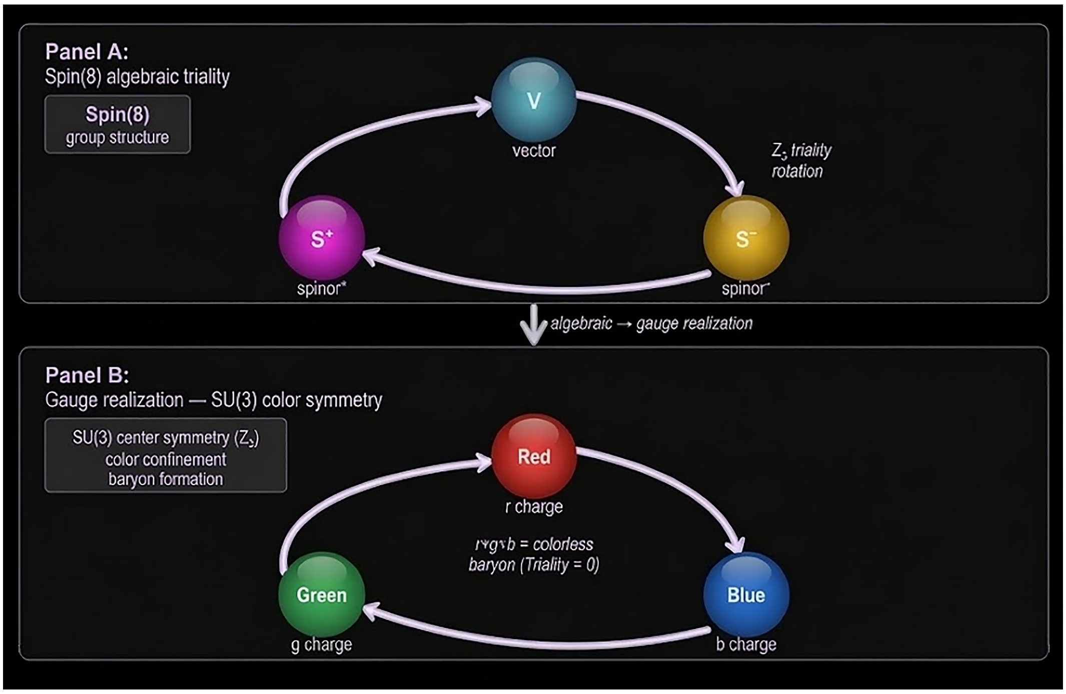

Figure 2: Triality across two levels of description. (A) Spin(8) algebraic triality: cyclic $\mathbb{Z}_3$ symmetry among vector (V) and spinor (S^+, S^-) representations. (B) SU(3) color triality in QCD: three color charges form a closed triadic system under the $\mathbb{Z}_3$ center symmetry [24]. The figure illustrates structural resemblance; it does not claim a formal derivation connecting these levels.

2.2 Anyonic Statistics as Structural Illustration

The following example illustrates the algebraic/topological mode of the S-D-T sequence: a structural precondition generates an organizational outcome inaccessible to the dyadic case. It is not a case of dynamical mediation within a single system, but of structural premises generating qualitatively new possibilities.

In three spatial dimensions, particle exchange is restricted to the boson–fermion binary—a fundamental duality arising because the fundamental group of the configuration space of N identical particles in 3D is the symmetric group S_N, whose one-dimensional representations admit only two signs: $+1$ (bosons) and -1 (fermions) [7, 8]. However, when particles are confined to a 2D substrate, the topology of the configuration space changes fundamentally: the relevant exchange group becomes the braid group B_N, which has infinitely many one-dimensional representations parametrized by an arbitrary phase $e^{i\theta}$. This topological fact generates a qualitatively new class of exchange behavior — anyonic statistics — governed by a continuous fractional phase $\theta \in (0, \pi)$ acquired when one particle winds around another, rather than the discrete ± 1 of bosons and fermions [7, 8]. Anyons are bulk quasi-excitations whose exchange statistics are governed by the topology of their trajectory space, not by protected modes at a phase boundary—they are distinct from edge states in the standard condensed-matter sense.

In the edge logic of the framework: the permutation group S_N exhausts the exchange possibilities in 3D (the dyadic classification). The braid group B_N, enabled by the 2D substrate, generates a continuously parameterized family of exchange statistics inaccessible to S_N by any internal manipulation—satisfying the reduction-failure condition in algebraic form. One cannot recover anyonic statistics by any operation within the symmetric group; the braid group is categorically richer.

A caveat is warranted: the "duality" here (boson–fermion binary) belongs to 3D kinematics, while the "triality" (braiding statistics) arises from a change in the underlying configuration-space topology. The example is included as a bounded illustration of how a mediating structural condition can open a space of organized behavior inaccessible to the original binary regime, not as a claim that all S-D-T instances take this dimensional-reduction form. The direct experimental confirmation of anyonic statistics [9, 10] makes this one of the few cases where the qualitative novelty claimed by the framework has been empirically observed.

3 Falsifiability and Scientific Status

3.1 Confronting the Principal Vulnerability

The principal risk facing the S-D-T framework is not that it is too narrow, but that it may be too easy to apply after the fact. "Singularity," "Duality," and "Triality" are broad enough labels that an undisciplined reader could map them retrospectively onto many organized systems. If that were all the framework permitted, it would function less as a scientific tool than as a vocabulary for post hoc pattern recognition.

For that reason, the credibility of the framework depends less on how many examples it can absorb than on how rigorously it can exclude false positives. The central question is therefore not whether S-D-T can be made to fit interesting cases, but whether it imposes nontrivial constraints on what counts as a valid application and on what would count against it.

The framework makes two claims that carry scientific content:

- **Claim 1 (Irreducible mediation):** The framework hypothesizes that certain forms of higher-order stability require irreducible mediation and cannot emerge from unmediated dualistic systems alone. While simple dualities can be stable, S-D-T suggests they may be unable to generate new, more complex organizational layers without a mediating third element satisfying conditions (i)–(iv) of the Formal Definition.
- **Claim 2 (Minimality — secondary conjecture):** A stable triadic system is not expected to be succeeded by a simple fourfold structure that is a mere isomorphic extension.

> If a fourth element introduces a genuinely new organizational principle, it constitutes a new S-D-T cycle rather than a quartic extension. *This claim is presented as a secondary conjecture, more weakly supported than Claim 1. We do not rely on it for the core validity of the framework.*

A deeper concern about the framework's falsifiability structure deserves direct acknowledgment. These claims are *asymmetric* in their falsifiability: the confirmation direction (finding the S-D-T pattern in a new domain) is structurally easier than the falsification direction, because the operational criteria in Section 3.3, while necessary, retain enough flexibility that a sufficiently motivated interpreter could identify triadic structure in most complex systems. This asymmetry is real and cannot be fully resolved by a heuristic framework at the current stage of formalization.

The appropriate response is transparency about the asymmetry and commitment to the formalization program that will resolve it. At the current stage, S-D-T is more readily illustrated than falsified in the strictest Popperian sense. The pre-specification requirement of Section 3.3 mitigates this by requiring that all identifications be locked before analysis begins.

3.2 Formal Falsification Conditions

Before turning to any domain-specific application, the framework-level failure condition must be stated first. The question here is not whether one proposed example succeeds or fails, but what would count against S-D-T as a framework.

Framework-Level Falsification Condition (F0)

Claim: Cases presented as Triality must involve a third term that is structurally necessary for the emergence of a stable regime not recoverable from the singular or dyadic description alone.

Falsification: S-D-T fails at the framework level if purported positive cases repeatedly admit reduction to singular or dyadic descriptions without explanatory loss of the regime claimed to require triadic mediation.

Operational requirement: Terms must be specified in advance of evaluation. Retroactive redefinition of what counts as the singular state, the dual tension, or the mediating third term is not an acceptable defense against failure.

Burden of proof: Falls on the framework. A single unsuccessful application need not kill S-D-T, but repeated failures across independently specified cases count strongly against it.

Honest limitation: We acknowledge that falsification depends on prior operationalization of key terms, and that borderline cases may remain subject to interpretive disagreement. The pre-specification requirement (§3.3) mitigates but cannot eliminate this. The goal is to make such disagreements explicit and tractable, not to claim the falsification architecture is dispute-proof.

3.3 Operational Criteria for Applying the Framework

To guard against post hoc pattern-fitting, the following criteria must be met before the S-D-T framework is applied to any system. A critical requirement applies to all applications: **all terms must be specified prior to empirical evaluation**. The identification of what counts as the Singularity, Duality, Triality, and their physical referents must be made in advance and locked before data are collected or analyzed. Retroactive adjustment of these identifications to accommodate results is not permitted and constitutes a violation of the framework's application conditions. This pre-specification requirement is not merely recommended; it is necessary for any S-D-T application to constitute a genuine test rather than a retrospective redescription. All examples discussed in the present paper are retrospective illustrations rather than prospective tests satisfying this requirement. They are used to clarify the framework's scope and discrimination criteria, not to claim that the framework has already been tested under its own strictest conditions.

- **Singularity:** Identifiable as the initial conditions, a baseline state of high symmetry, or a specific substrate that enables interaction. It is the state against which change and structure are measured.
- **Duality:** Identified by the emergence of a dominant, system-defining tension, measurable as a bifurcation in the system's phase space, the establishment of two competing feedback loops, or a fundamental trade-off that governs the system's behavior.
- **Triality:** Identified by the emergence of a new constraint or regulatory protocol that fundamentally alters the rules of interaction for the dualistic components. It is not merely a third element but a new, higher-order organizational layer that subordinates the duality to a more complex, coherent whole. Its relevance must be shown by a demonstrable change in the system's organizational structure that is not recoverable from the dyadic description alone.

Crucially, the Triality must be shown to be *categorically different* from the Duality's components—a new *rule*, not just a third *player*. If this qualitative-difference criterion cannot be satisfied for a proposed application, the framework adds no explanatory value and should not be applied.

Existing dynamical-systems tools—Lyapunov exponents for basin stability, entropy measures for adaptive capacity, transfer entropy for directional coupling—may help operationalize these criteria in specific domains, but no general quantitative metric is developed in the present paper.

3.4 What Kind of Irreducibility Is at Issue?

The irreducibility at issue in this paper is *framework-relative*, not absolute in every conceivable descriptive sense. In the dynamical mode, the relevant comparison class is finite-dimensional, autonomous, dyadic state-space description without explicit memory terms. A system counts as irreducibly triadic when eliminating the third role does not yield a closed lower-dimensional autonomous dyadic model without losing the organizational feature of interest. In such cases, attempted reduction may instead introduce effective memory, path dependence, or other departures from the target explanatory class.

This restriction matters. Without it, claims of irreducibility would become either trivially false — because some highly permissive redescription can always be imagined — or too vague to test. The present paper therefore advances a more limited claim: some systems resist reduction to the explanatory forms most commonly used to model lower-dimensional autonomous dyadic organization.

In the algebraic/topological mode, irreducibility should be understood analogously: the relevant closure, equivalence, or structural completion is not fixed by singular or dyadic specification alone without loss of the formal organization being described. The common thread across both modes is the *shift in descriptive class*: eliminating the third term does not merely yield a simpler member of the same descriptive family, but forces a change in the kind of description available. This framework-relative approach to irreducibility reflects a broader methodological point: explanatory adequacy is assessed relative to a specified investigative framework, not against an impossible standard of elimination under every formally available redescription [26, 27].

3.5 Scope: Where S-D-T Does Not Apply

Explicitly bounding the framework's scope is as important as stating its claims. S-D-T applies to systems in which an irreducible third structural role — whether realized as a dynamical mediating constraint or as an algebraic/topological symmetry — generates an organizational outcome unavailable to the dyadic case. It does not apply to:

- **Equilibrium systems with no internal tension.** A noble gas at room temperature has no competing attractors and no Duality to mediate. The S-D-T grammar is not invoked. Stability without tension is simply stability.
- **Systems whose complexity arises from simple aggregation.** A sand pile is complex but not S-D-T-organized: its avalanche dynamics arise from local grain interactions, not from the resolution of competing organizational attractors by a categorically novel constraint. Complexity and triadic organization are not synonymous.
- **Systems where the number of competing attractors at the relevant organizational level exceeds two.** Multi-party political systems, many-species ecological communities, and multi-modal dynamical systems require either S-D-T cycle nesting (where pairs of competitors are resolved in sequence) or a different analytical framework. Single-cycle S-D-T application to systems with more than two competing organizational attractors is a misapplication, and its failure does not constitute a counterexample to the framework.

- **Systems where the "Triality" candidate is derivable from the Duality resources.** Fine-tuned parameter adjustments, weighted averages of competing states, or symmetry transformations of existing degrees of freedom are not Trialities. If condition (iv) of the Formal Definition cannot be satisfied, the framework does not apply.

A further clarification: the present framework addresses a different question from the growing literature on higher-order interactions in complex networks and simplicial dynamics, which asks when n-body interactions carry dynamical information not reducible to pairwise terms. S-D-T asks the complementary question: when is a *specific mediating third role* explanatorily irreducible relative to a stated comparison class? The statistical question and the explanatory question are related but distinct.

These exclusions are not ad hoc retreats. They follow directly from the Formal Definition: without a Duality, there is nothing for a Triality to mediate; without categorical novelty, there is no Triality; and without explanatory loss under reduction, the third term is not doing irreducible structural work. The framework's scope is determined by its definitions, and the definitions have boundaries.

3.6 S-D-T as a Proto-Research Program

A Kuhnian paradigm defines problems, constrains acceptable solutions, and supports a routine practice of normal science. S-D-T does not yet meet that standard, and this paper does not claim that it does. The more accurate description is that S-D-T functions, at present, as a **proto-research program**: a structured proposal for identifying and testing one specific form of organized mediation.

The research agenda that follows from the present paper is correspondingly limited:

1. **Sharpen the formal criterion for irreducibility.** The central theoretical task is to make the reduction test more precise across domains, so that triality can be identified by stricter structural criteria rather than by analogy alone.

2. **Build cleaner positive and negative cases.** The framework's value depends on whether independently motivated examples can be shown either to require or to fail triadic mediation under pre-specified conditions.

3. **Develop controlled dynamical illustrations.** If a third mediating term is genuinely irreducible, that claim should be demonstrable in simple models where the relevant stable regime disappears under reduction to singular or dyadic forms.

What distinguishes a proto-research program from a mature paradigm is the absence of an established routine by which practitioners can apply it mechanically to new systems. S-D-T does not yet supply such a routine. What it does supply is a formal definition, a set of exclusion criteria, and a framework-level failure condition. The framework's future depends on whether its constraints prove scientifically productive rather than merely rhetorically suggestive.

4 Ontological Status, Formal Limits, and Future Directions

The remaining question is not whether S-D-T is a fundamental physical theory, but what sort of intellectual object it is if its more modest claims are granted. The framework is best understood as a candidate classificatory pattern: one that may capture a recurrent form of organized mediation across otherwise distinct domains without collapsing those domains into a single mechanism. This section clarifies the ontological status of the framework, states the limits of what has and has not been established in the present paper, and identifies the formal work still required before stronger scientific claims would be justified.

4.1 S-D-T as a "Real Pattern": Ontic Structural Realism

If S-D-T is not proposed here as a fundamental law, what kind of scientific object might it nonetheless be? One plausible answer is that it may count as a candidate "real pattern" in the sense articulated by Dennett and developed within ontic structural realism by Ladyman and Ross [17, 18]. On that view, a pattern earns ontological seriousness not by being basic in the metaphysical sense, but by supporting nontrivial compression, explanatory organization, and projectible structure across cases.

That is the most defensible philosophical interpretation of S-D-T at present. Recent work extending ontic structural realism to non-equilibrium dynamics has introduced flux-dependent stabilizer subgroups to account for the emergence and persistence of dissipative structures within a structural realist ontology [29]. The S-D-T framework complements such developments by identifying the minimal organizational architecture — three irreducible structural roles — required for such persistence. The framework does not posit three new kinds of thing. Rather, it proposes three recurrent *structural roles*—a substrate or enabling condition, a constitutive tension, and an irreducible mediating term—that may recur across otherwise distinct systems. The question is therefore not whether S, D, and T are ontological objects, but whether this three-role structure marks a scientifically useful and non-arbitrary pattern.

The present paper does not claim to have established that status conclusively. At most, it argues that S-D-T is a candidate real pattern whose legitimacy depends on whether it can compress selected cases without erasing their differences, generate sharper classifications than looser analogical language, and survive explicit reduction tests. In that limited sense, the framework's ontological ambition remains provisional and methodologically downstream from its success or failure as a classificatory tool.

4.2 Resonances with Historical Philosophy

With operational criteria and explicit failure conditions in place, the closest historical resonance is with **Peirce**, who formalized three universal categories sufficient to describe all phenomena: Firstness (pure potential; Singularity), Secondness (reaction; Duality), and Thirdness (mediation, habit, law; Triality) [21]. Crucially, Peirce's *Reduction Thesis* provides formal logical arguments that genuinely triadic relations cannot be completely decomposed into combinations of monadic and dyadic predicates, while all higher-arity relations (tetradic, pentadic, and beyond) *can* be so reduced.[1] This is closely analogous to the logical structure of the present paper's minimality claim and reduction test: three is the first arity at which irreducible relational structure appears, and higher arities do not introduce further irreducibility.[2] Independent convergence on the same triadic minimality claim has recently appeared in information-systems theory: Parreira Figueiredo [25] derives from drift-coherence requirements and Input-to-State Stability theory that any action-effective system maintaining bounded-error fidelity to a drifting environment must implement exactly three mutually irreducible functional roles — a result obtained without reference to the physical examples discussed here, yet arriving at the same structural conclusion via formal proof rather than heuristic identification. **Schwemmler** (1980), working in population ecology and evolutionary systematics without reference to the physics discussed here, arrived at what he termed a "triality principle" as the structural cause of periodicity in evolving systems [32]. Independent structural arguments for triadic necessity have also been advanced from the analysis of persistence under transformation, where it has been argued that self-verifying systems require exactly three irreducible functional roles — reference, transformation, and closure — and that binary architectures cannot distinguish genuine identity preservation from accidental coincidence [30]. These frameworks are not identical to S-D-T, but their independent convergence on triadic structure suggests that the pattern is not merely an artifact of the present paper's construction. What distinguishes S-D-T is not historical novelty, but the attempt to pair selected physical illustrations with explicit reduction tests and failure conditions.

4.3 Open Questions and Future Work

Several open questions define the next steps for the framework:

- **The mechanism question:** The S-D-T framework identifies a pattern and motivates interpretive implications from it. The physical mechanism explaining *why* some mediating third terms stabilize higher-order regimes is not established in this paper; its formal development is reserved for separate work.

- **The formalization question:** The framework's operational criteria identify what must be shown in any candidate application, but they do not yet constitute a general theorem of triadic mediation. What can be offered here is a minimal formal illustration in which the

[1] The formal status of the Reduction Thesis remains an active area of investigation; proofs have been given by Burch (1991) and Hereth Correia and Pöschel (2006), though the precise scope of the result depends on which relational operations are admitted, and the question of whether the thesis is "gerrymandered" by the choice of base operations has been raised [31]. The structural parallel to S-D-T's claims is suggestive but should not be treated as an independent formal proof of triadic minimality.

[2] A bounded algebraic aside illustrating this combinatorial minimality: the CKM quark-mixing matrix has $(n-1)(n-2)/2$ irreducible complex phases for n generations [19]. For $n = 2$, zero; for $n = 3$, one [12, 13]. Three generations are the minimum for CKM-sector CP violation. A fourth generation has been independently ruled out by precision measurements [14, 15, 20]. This is not a prediction of S-D-T and is not presented as evidence for the framework; it is noted only as a combinatorial instance of the edge logic in which the dyadic case ($n = 2$) is insufficient and the triadic case ($n = 3$) is the minimum that restores the qualitatively new feature. CKM CP violation is quantitatively insufficient for baryogenesis [16, 13].

S-D-T sequence is realized explicitly and the reduction test can be stated in mathematical terms.

- **The unity question:** Whether the same transition logic — dyadic insufficiency, mediated restoration, reduction failure — genuinely unifies dynamical mediation and algebraic/topological closure, or whether these should ultimately be treated as related but distinct frameworks. If the edge logic fails to apply uniformly across both modes, the framework should be divided rather than defended as unified.

The transition-invariance claim can be stated more precisely. It is not a claim that the same physical mechanism operates in both modes — dynamical mediation and algebraic closure are realized differently. The claim is that the same *dependency structure* governs both: in each case, the dyadic description is insufficient for the organizational outcome, a third structural role is required to restore or enable it, and eliminating that third role produces explanatory loss within the relevant descriptive class. In the dynamical mode, explanatory loss takes the form of irreducible non-Markovian memory (the mediated Lotka-Volterra system below). In the algebraic mode, the analogous loss is classificatory collapse — the inability to recover a closed organizational outcome (the $\mathbb{Z}_8$ periodicity, the anyonic phase space) from the dyadic resources alone. This parallel is precisely the kind of invariance of relational architecture across physically distinct realizations that ontic structural realism identifies as the mark of genuine structure [18]. The test of the unity claim is therefore whether the explanatory-loss condition can be made comparably rigorous in both modes. If the algebraic cases admit only a weaker form of "loss" — aesthetic impoverishment rather than genuine explanatory failure — then the modes should be separated rather than unified. That test has not been fully executed in the present paper, and executing it is among the most important tasks for future work.

Minimal formal illustration: S-D-T as a cusp-catastrophe parameter path. The cusp catastrophe provides a concrete minimal model in which the S-D-T sequence is realized as a path through a two-parameter family of potentials:

$$V(x;\, u, v) = x^4 + ux^2 + vx \tag{1}$$

where x is the state variable, u is the primary control parameter, and $v = \tau$ is the mediating constraint. The comparison class is fixed in advance as the family of one-parameter symmetric potentials $V(x; u) = x^4 + ux^2$. This class is not selected because it makes the triadic reading easy; it is selected because it is the minimal class that already captures the dyadic organizational feature under study — symmetric bistable competition — and the question is whether the organizational outcome of interest (asymmetric resolution) can be obtained without leaving that class [28]. The S-D-T sequence corresponds to a specific path through the (u, v) control space:

Stage	**Parameters**	**Potential structure**	**S-D-T interpretation**
Singularity	$u = +0.5,\ v = 0$	Single stable minimum at $x = 0$	High-symmetry ground state
Duality	$u = -1.0,\ v = 0$	Symmetric double well; minima at $x = \pm 1/\sqrt{2}$	Two competing basins of equal depth
Triality	$u = -1.0,\ v = \tau$	Asymmetric wells; $\Delta V \approx 0.56$ at $\tau = 0.4$	One basin deepened, one subordinated

Table 1: S-D-T parameter path through the cusp-catastrophe potential (1). The transition from Duality to Triality is driven by the mediating constraint $\tau = v$, which breaks the $\mathbb{Z}_2$ symmetry of the double well and produces a calculable depth asymmetry between the competing basins. Bistability requires $|v| < \sqrt{8/27} \approx 0.544$; the chosen value $\tau = 0.4$ lies well within this region.

Proposition 1 (Bistability and asymmetry in the cusp potential)**.** *For the potential* $V(x; u, v) = x^4 + ux^2 + vx$*, the derivative* $V'(x) = 4x^3 + 2ux + v$ *has three distinct real zeros if and only if* $8u^3 + 27v^2 < 0$*. Hence, for* $u < 0$ *and* $|v| < \sqrt{-8u^3/27}$*, the system remains bistable while the two minima are unequal whenever* $v \neq 0$*. By contrast, in the one-parameter class* $V(x; u) = x^4 + ux^2$*, every bistable regime is necessarily symmetric.*

Proof sketch. The stationary points are the roots of the depressed cubic $x^3 + (u/2)x + v/4 = 0$. For a depressed cubic $x^3 + px + q = 0$, three distinct real roots occur exactly when $-4p^3 - 27q^2 > 0$. Substituting $p = u/2$ and $q = v/4$ yields $-u^3/2 - 27v^2/16 > 0$, equivalent to $8u^3 + 27v^2 < 0$. When $u < 0$ and $|v| < \sqrt{-8u^3/27}$, the potential has two minima separated by a maximum, so bistability persists. If $v \neq 0$, the symmetry $x \mapsto -x$ is broken and the two minima are no longer equal in depth. In the restricted one-parameter class with $v = 0$, bistability is possible only in the symmetric form. This result is modest but sufficient: it isolates a controlled setting in which asymmetric bistable organization requires departure from the one-parameter symmetric class.

The formal conditions (i)–(iv) are each illustrated within this parameter space. Condition (i): at $u = -1.0$ and $v = 0$ the system exhibits two competing minima at $x = \pm 1/\sqrt{2}$, with no intrinsic basis for selecting between them. Condition (ii): the Triality parameter $v = \tau$ is not a relabeling of the dual configuration but a distinct control parameter that breaks the symmetry of the double well. Condition (iii): the resulting landscape does not eliminate one side of the Duality but produces an asymmetric configuration in which one basin is deepened ($V = -0.55$) while the other remains as a shallow residual competitor ($V \approx 0.01$). Condition (iv): the asymmetry cannot be obtained by reparameterizing the symmetric double well alone; it requires the introduction of the additional mediating term v.

Figure 3 shows the full cusp surface. At the Duality values ($u = -1.0, v = 0$), the potential has exact $\mathbb{Z}_2$ symmetry. At the Triality values ($u = -1.0, v = \tau = 0.4$), that symmetry is broken: the left well deepens to $V \approx -0.55$, while the right well rises to $V \approx 0.01$. The depth asymmetry

$\Delta V \approx 0.56$ scales analytically with τ, providing a computable illustration of how changing the mediating constraint alters basin asymmetry and expected transition behavior.

A clarification is needed on why v should be treated as a candidate mediating role rather than as an ordinary bias parameter. Within the pre-specified one-parameter symmetric class, variation in u can generate or destroy bistability, but it cannot produce asymmetric resolution while retaining the bistable organization itself. The term v is therefore not counted as a mediator merely because it is a second parameter. It is treated as a candidate mediating role because, conditional on the dyadic structure generated by $u < 0$, it reorganizes the *relation between* the competing basins in a way not recoverable within the original class [26]. This is the precise sense in which explanatory adequacy here depends on recognizing a structurally distinct third role [27].

A note on the status of this illustration: v acts in a qualitatively different direction from u — it breaks the symmetry of the double well rather than modifying the bifurcation that created it — and the organizational outcome (asymmetric resolution) is inaccessible to the original one-parameter system regardless of how u is varied. The model illustrates the *logical structure* of conditions (i)–(iv), not a claim that every physical instance of Triality takes the form of a cusp unfolding parameter. Its role is to show that the reduction test and the categorical-novelty condition can be given concrete mathematical form in at least one controlled setting. Applications of the S-D-T framework to complex adaptive systems beyond the physical examples of Section 2 would require independent justification in each domain.

Dynamical non-reducibility: a proof sketch. The cusp illustration is geometric; the following argument is mechanistic. Consider the standard Lotka-Volterra two-species system:

$$\dot{x} = ax - bxy, \qquad \dot{y} = -cy + dxy, \quad a, b, c, d > 0.$$

The nontrivial equilibrium $(x^*, y^*) = (c/d,\, a/b)$ has purely imaginary eigenvalues $\lambda = \pm i\sqrt{ac}$: the equilibrium is a center, not asymptotically stable — the Duality oscillates without resolution. Now introduce a mediating variable z that dynamically tracks the interaction:

$$\dot{x} = ax - bxy - \alpha xz, \quad \dot{y} = -cy + dxy - \beta yz, \quad \dot{z} = \gamma(xy - z), \quad \alpha, \beta, \gamma > 0.$$

Here z is not a third competitor but a constraint mediator: it relaxes toward the interaction term xy and feeds back symmetrically to damp both variables. For sufficiently large γ (fast mediator relaxation), the Routh-Hurwitz conditions are satisfied and the equilibrium becomes locally asymptotically stable — a stability that is structurally mediated by the third variable and absent from the original dyadic system, though still dependent on the mediated system's parameter regime.

The critical result is *non-reducibility*: the mediator z solves to a history-dependent convolution integral $z(t) = \int_0^t \gamma\, e^{-\gamma(t-s)}\, x(s)y(s)\, ds$. Substituting this into the $\dot{x}$ and $\dot{y}$ equations yields an integro-differential system in (x, y) alone — a non-Markovian system whose future depends on the full trajectory history $\{x(s), y(s)\}_{s \le t}$, not merely the current state. The mediator cannot be algebraically absorbed into parameters: it encodes an irreducible memory of the interaction. This constitutes a proof sketch that the triadic system defines a strictly larger dynamical class than the dyadic system — specifically, irreducible relative to lower-dimensional autonomous dyadic ODE descriptions. *Scope note:* the demonstration establishes a sufficient condition — that one class of mediated systems is irreducibly higher-dimensional — but does not claim that all systems requiring three variables are triadic in the S-D-T sense. Its value is to show that the categorical-novelty condition has dynamical content: a Triality is not just a third element, but a constraint whose removal destroys a stability that cannot be restored within the lower-dimensional algebra. A methodological note: the Mori-Zwanzig projection operator formalism guarantees that eliminating *any* coupled degree of freedom from a high-dimensional system generally produces memory kernels in the reduced description. The present example does not claim that memory kernels are unique to S-D-T systems. What it claims is narrower: the *specific* memory structure here arises because z tracks the interaction term xy and feeds back symmetrically on both variables, producing a stability absent from the dyad. The negative control below confirms this distinction — an uncoupled third variable produces no such memory upon elimination.

For completeness, the Jacobian of the mediated system at a general state (x, y, z) is

$$J(x,y,z) = \begin{pmatrix} a - by - \alpha z & -bx & -\alpha x \\ dy & -c + dx - \beta z & -\beta y \\ \gamma y & \gamma x & -\gamma \end{pmatrix}.$$

A fuller treatment would state explicit parameter conditions for local asymptotic stability at the relevant equilibrium; the present discussion is offered as a controlled proof sketch rather than a full theorem.

Negative control: three components without irreducible mediation. A useful contrast is provided by a three-variable system in which the third variable is dynamically present but explanatorily redundant. Suppose w evolves independently as $\dot{w} = -\lambda w$ while x and y obey an ordinary two-species interaction law unaffected by w. Such a system is numerically triadic but not triadic in the S-D-T sense: removing w leaves the organizational structure of the x–y dynamics unchanged and does not force memory, closure failure, or explanatory loss. More generally, a three-component model fails the S-D-T test whenever the third role is eliminable without altering the explanatory class required for the phenomenon of interest. This negative control matters because it shows that the framework does not classify all three-variable systems as Trialities — the edge logic S $\to$ D $\to$ T $\to$ S$'$ must actually hold, not merely the node count.

A harder negative control: coupling without mediation. A stronger test is provided by a third variable that is genuinely coupled to both x and y yet still fails the S-D-T criteria. Consider adding a uniform damping term $\dot{w} = -\mu w + \nu(x + y)$, with w feeding back as a simple friction $-\epsilon w$ on both species. Here w is dynamically coupled to the dyad and alters its quantitative behavior — trajectories decay faster, amplitudes shrink — but it does not *reorganize* the dyadic relation. The two-species interaction retains its qualitative character; w acts as an energy sink, not as a mediating constraint that produces a categorically new organizational regime. Removing w changes the rate of decay but does not force a shift in descriptive class. This case matters because it shows that the framework distinguishes mediation from mere coupling: a third variable must reorganize the dyadic relation, not simply modulate it [26].

Algebraic negative control: three components without triadic structure. An analogous control is needed for the algebraic mode. The Lie algebra $\mathfrak{su}(2)$ provides a clean example. Its three generators J_1, J_2, J_3 satisfy $[J_i, J_j] = i\varepsilon_{ijk}J_k$, and all three are algebraically essential: no proper subalgebra generates the full algebra. Yet $\mathfrak{su}(2)$ is not triadic in the S-D-T sense. The reason is precise: the three generators are structurally interchangeable — each stands in the same relation to the other two via inner automorphisms (rotations). There is no pair that "competes" while a third "mediates." Removing any one generator yields a one-dimensional subalgebra, which is quantitatively smaller but not qualitatively impoverished — no classificatory architecture collapses, no new organizational outcome disappears. The contrast with Spin(8) is instructive: in Spin(8), the triality is an *outer* automorphism connecting three *categorically distinct* types of representation (vector, positive spinor, negative spinor). The three representations play structurally different roles that cannot be interchanged by inner automorphisms of the group. It is this asymmetry — not the mere presence of three algebraic components — that the S-D-T framework identifies as triadic structure. The $\mathfrak{su}(2)$ example confirms that numerical three-ness, even when algebraically necessary for closure, does not constitute Triality unless the edge logic holds: without dyadic insufficiency, there is nothing for a Triality to mediate.

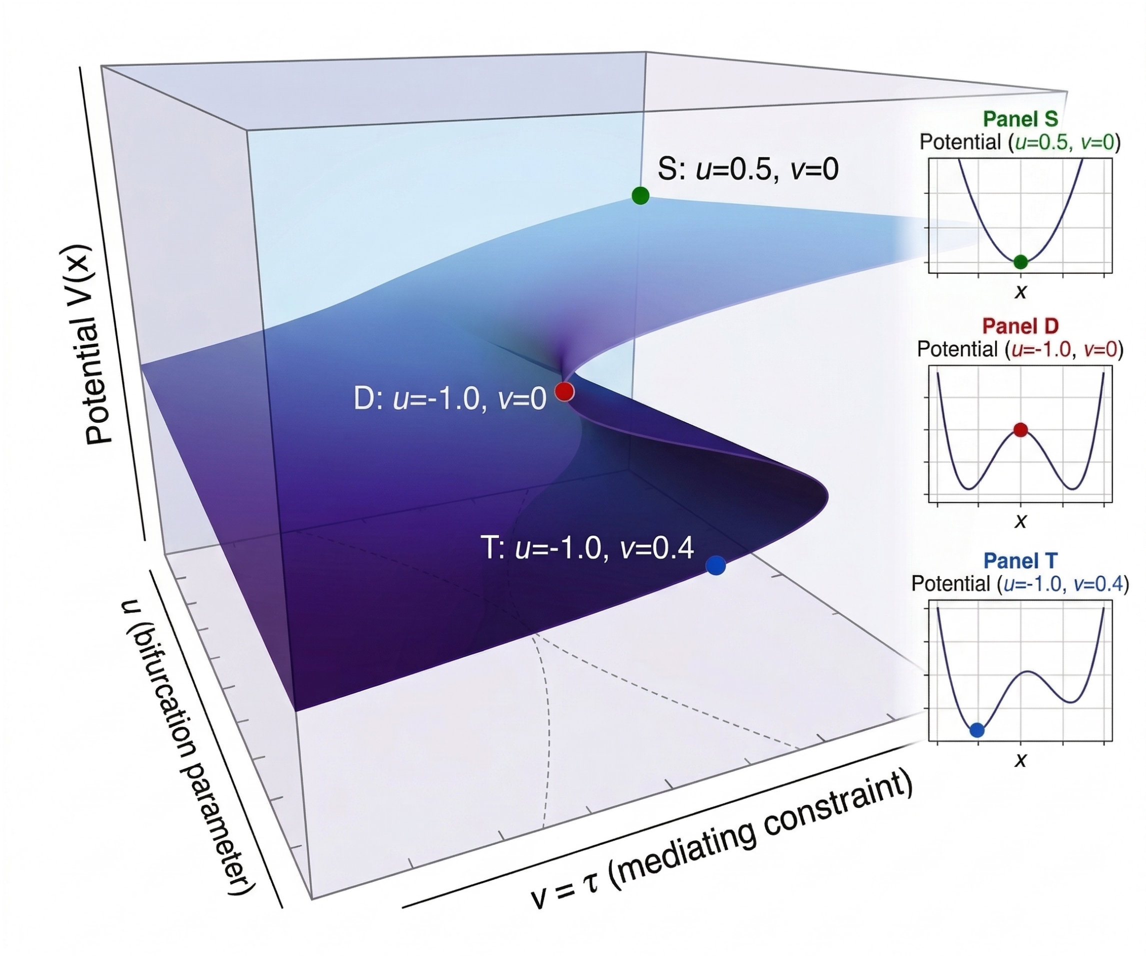

Figure 3: Cusp-catastrophe illustration of the S-D-T transition. The folded surface represents $V(x; u, v) = x^4 + ux^2 + vx$, with u governing bifurcation and $v = \tau$ acting as the mediating constraint. The cusp region marks bistability; analytical parameter values are listed in Table 1.

5 Conclusion

This paper has presented the Singularity–Duality–Triality (S-D-T) framework as a constrained meta-theoretical tool for identifying cases in which a third term appears to be structurally necessary for the emergence of a new stable regime. Its central claim has been deliberately limited. The paper does not argue that triadic structure is universal, that S-D-T derives established physical theory, or that every interesting three-part organization qualifies as Triality in the sense defined here. Its more modest proposal is that some systems may be misdescribed if their organizing logic is forced into singular or dyadic terms alone.

The contribution of the paper is therefore methodological and conceptual. It offers a formal criterion for distinguishing genuine triadic mediation from reducible three-part decomposition, states explicit reduction and exclusion tests, and formulates framework-level failure conditions under which the proposal should be rejected. Selected examples from mathematical and physical theory illustrate the framework's intended scope, but they are not presented as co-equal forms of evidence, and they do not eliminate the need for sharper formalization. A minimal dynamical model illustrates how triadic mediation can introduce structure not recoverable by reparameterization of the dual system: the cusp catastrophe shows this geometrically through the reduction test on control parameters, while the Lotka-Volterra proof sketch shows it mechanistically through the irreducible non-Markovian memory that a mediating constraint introduces into the reduced system.

The framework should therefore be judged less by the breadth of cases it can absorb than by the rigor with which it can exclude false positives and survive skeptical testing. If purported instances of Triality repeatedly reduce to dyadic structure without explanatory loss, the framework should be abandoned. If, by contrast, a small number of cases prove to require irreducible mediating structure in the stronger sense defined here, then S-D-T may have value as a scientific classificatory pattern and a guide for further formal work. At present, that status remains provisional. The framework is offered not as a completed theory, but as a disciplined wager that irreducible mediation is real in at least some domains and can be studied without collapsing into numerology.

The next stage of that work is clear: sharper mathematical criteria for irreducibility, cleaner positive and negative cases, and controlled dynamical illustrations in which the role of a mediating third term can be demonstrated rather than inferred. Until then, S-D-T should be treated as a provisional framework whose legitimacy depends on its ability to constrain interpretation as much as to inspire it.

A S-D-T Mappings for Core Physical Examples

Table 2 provides an explicit mapping of the S-D-T framework's components onto the selected physical examples retained in Section 2. For each system, the table identifies the S-D-T mode, evidence grade, and the specific mathematical or physical objects corresponding to the Singularity (canvas), Duality (tension), Triality (mediating rule), and the emergent stable property that results from the mediating constraint.

System	**Mode**	**Evidence Grade**	**Singularity (Canvas)**	**Duality (Tension)**	**Triality (Mediating Rule)**	**Emergent Stable Property**
Spin(8) / Topological Superconductors (§2.1)	Algebraic	Bounded analogue	Lie group Spin(8) as abstract mathematical structure	Instantiation as symmetry group of a physical system	Unique threefold triality among representations	$\mathbb{Z}_8$ classification: finite, robust topological phases
Anyons (§2.2)	Algebraic (topological)	Bounded illustration	Two-dimensional material substrate	Boson/fermion binary of 3D particle exchange	Topology of particle braiding on 2D canvas (braid group B_n replacing permutation group S_n)	Expanded exchange-statistics regime (fractional / non-abelian braiding phases)

Table 2: Explicit S-D-T mappings for the core physical examples of Section 2. Each row identifies the S-D-T mode, the evidence grade (bounded analogue or bounded illustration), the canvas (Singularity), the tension (Duality), the mediating rule (Triality), and the resulting organizational outcome. The dynamical mode is illustrated separately in Section 4 (cusp catastrophe as geometric illustration; mediated Lotka-Volterra as restricted formal demonstration). The evidence grades are not equal: see Section 4 for the hierarchy.

This table is intended as a reference aid, not a claim of formal derivation. The mode column makes explicit which species of triadic structure each example instantiates. The mappings identify the *structural correspondence* between the S-D-T framework and each physical system; the detailed mathematics of each domain (Lie group theory, braid statistics) remain the province of the specialized literature cited in the main text. The table's value lies in making the claimed pattern explicit and therefore open to criticism: if the mapping fails for any row—if the identified Triality is not categorically distinct from the Duality, or if the emergent property does not satisfy the reduction test—the framework's application to that domain is falsified.

B Computational Supplement

The analytical well locations, depth asymmetries, and parameter values reported in Table 1 and the cusp illustration (Figure 3) were computed using a short Python script (`numpy`, `scipy`) that solves $dV/dx = 4x^3 + 2ux + v = 0$ numerically for the Triality parameter values ($u = -1.0$, $v = \tau = 0.4$) and analytically for the Singularity and Duality cases. The Duality well locations are $x = \pm 1/\sqrt{2}$ (exact). The Triality well locations are obtained by root-finding on the cubic $4x^3 - 2x + \tau = 0$. The depth asymmetry $\Delta V = V_{\text{shallow}} - V_{\text{deep}} \approx 0.56$ at $\tau = 0.4$ scales to first order as $\Delta V \approx 2\tau/\sqrt{2}$. Bistability requires $|\tau| < \sqrt{8/27} \approx 0.544$; the chosen value lies within this bound.

The computation is fully reproducible: all parameter values are stated in Table 1, and the cusp potential $V(x; u, v) = x^4 + ux^2 + vx$ is a standard two-parameter unfolding. The generating script is provided as supplementary material accompanying this submission.

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