R-layer Mode Theory II:
Excitations, Fermionic Twist, and Emergent Gauge Fields

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March 2026

Abstract

This work develops the excitation structure of the R-layer introduced in Volume I. We show that linear fluctuations of the mode field form localized bound states, that twist degrees of freedom naturally generate fermionic behavior and spin-1/2 structure, and that collective phase flows give rise to emergent gauge fields. Non-Abelian internal rotations of multi-component modes lead to SU(N)-type gauge structures. These results extend the R-layer framework toward a unified description of particles and interactions.

1 Introduction

Volume I of the R-layer Mode Theory established the microscopic foundation of the framework: a binary mode variable $\sigma(x)$, its expectation value $\phi(x)$, tunneling dynamics, curvature generation, and domain formation. These ingredients provided a unified origin for quantum fluctuations, gravitational curvature, and early-universe asymmetry.

Having established the geometric and dynamical structure of the R-layer, the next natural step is to investigate the excitations supported by this substrate. Fluctuations of the mode field, twist degrees of freedom, and collective phase flows give rise to particle-like behavior, fermionic structure, and gauge interactions. This constitutes the focus of the present work (Volume II).

2 Linear Excitations and Bound States

We consider small fluctuations around the background mode field:

$$\phi(x) \to \phi(x) + \delta\phi(x).$$

Expanding the unified Lagrangian to quadratic order yields:

$$\Box\,\delta\phi + V''(\phi)\,\delta\phi = 0, \qquad V''(\phi) = 2\alpha + 12\beta\phi^2.$$

If ϕ is approximately constant, excitations behave as massive modes with

$$m_{\text{eff}}^2 = V''(\phi_0).$$

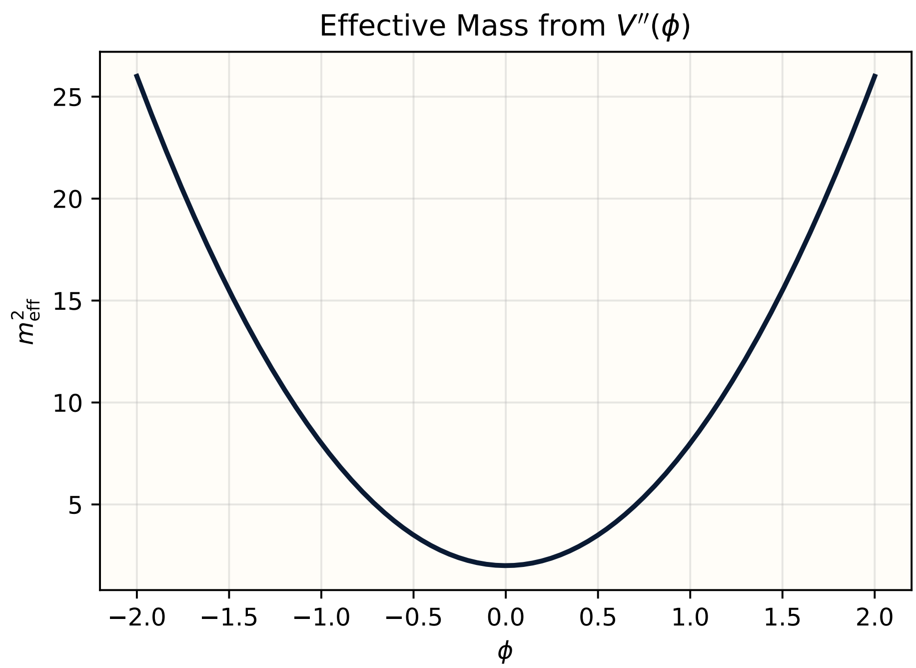

Figure 1: Effective mass $m_{\text{eff}}^2 = V''(\phi)$ as a function of ϕ.

For a domain-wall profile $\phi(x) = \tanh x$, the fluctuation equation reduces to:

$$-\frac{d^2}{dx^2}\psi + U(x)\psi = E\psi, \qquad U(x) = V''(\phi(x)).$$

This potential supports localized bound states:

3 Fermionic Behavior from Mode Twisting

We introduce a complex mode field:

$$\Phi(x) = \phi(x)e^{i\theta(x)}.$$

A key property of the twist is:

$$\theta \to \theta + 2\pi \quad \Rightarrow \quad \Phi \to -\Phi,$$

which reproduces the double-valuedness characteristic of spin-1/2 systems.

A two-component spinor can be defined as:

$$\Psi(x) = \begin{pmatrix} \phi(x) \\ \phi(x)e^{i\theta(x)} \end{pmatrix}.$$

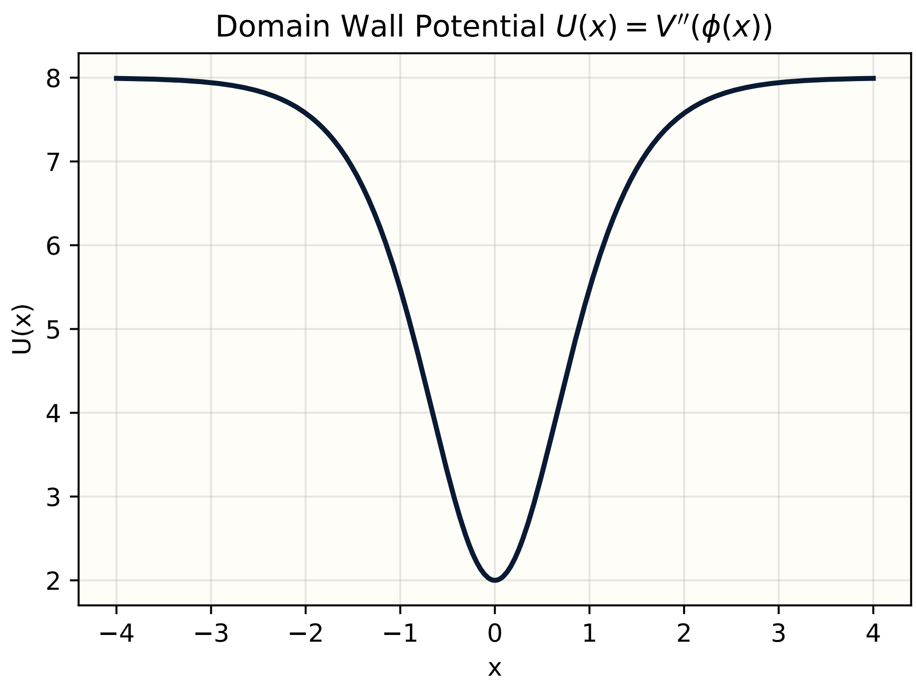

Figure 2: Domain-wall potential $U(x) = V''(\phi(x))$ for $\phi(x) = \tanh x$.

4 Gauge Fields as Collective Phase Flows

Under a local phase transformation

$$\theta(x) \to \theta(x) + \alpha(x),$$

the naive kinetic term is not invariant. Introducing a covariant derivative

$$D_\mu \Phi = (\partial_\mu + iA_\mu)\Phi$$

restores invariance.

For multi-component modes:

$$\mathbf{\Phi}(x) \to U(x)\mathbf{\Phi}(x), \qquad U(x) \in SU(N),$$

5 Composite Structures and Stability

Localized excitations can combine to form composite structures. The effective potential for two modes is:

$$V_{\text{eff}}(d) = V_0 + Ae^{-d/\xi} - Be^{-2d/\xi}.$$

Three-mode configurations:

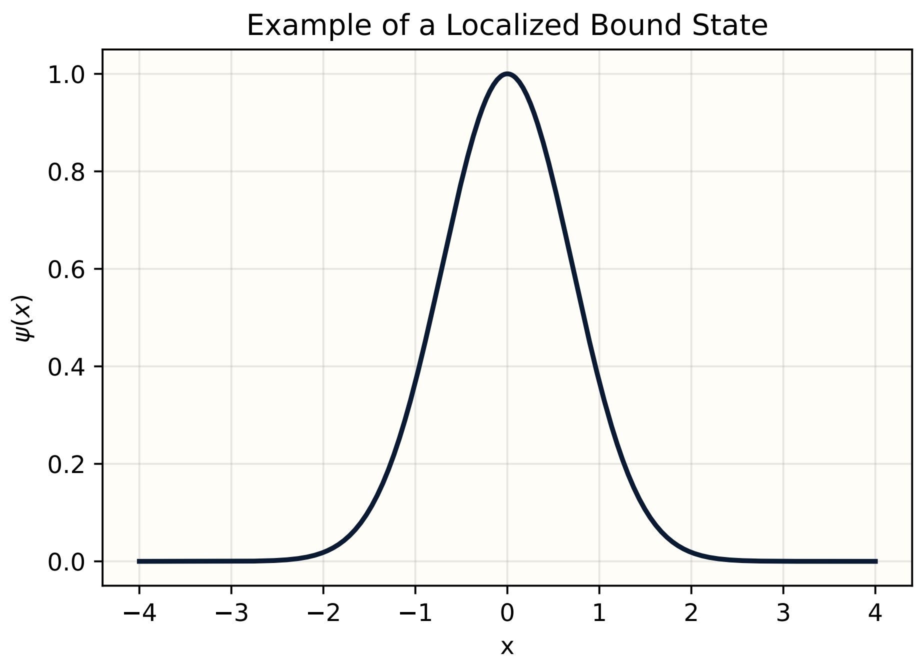

Figure 3: Example of a localized bound state in the domain-wall potential.

6 Thermodynamic and Informational Aspects

The R-layer supports a rich set of configurations involving domains, twist defects, and curvature peaks. These structures naturally carry entropy and information.

Strong curvature regions act as information-storage sites, reminiscent of black-hole thermodynamics.

Non-equilibrium evolution of the R-layer provides a microscopic picture for early-universe asymmetry and structure formation.

7 Discussion and Outlook

Volume II extends the R-layer Mode Theory from geometric and quantum behavior to particle-like excitations, fermionic twist, and emergent gauge fields. Future work (Volume III) will explore cosmological evolution and observational signatures of the R-layer.

References

[1] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, W. H. Freeman (1973).

[2] S. Weinberg, *The Quantum Theory of Fields*, Cambridge University Press (1995).

[3] G. 't Hooft, "Dimensional reduction in quantum gravity," arXiv:gr-qc/9310026.

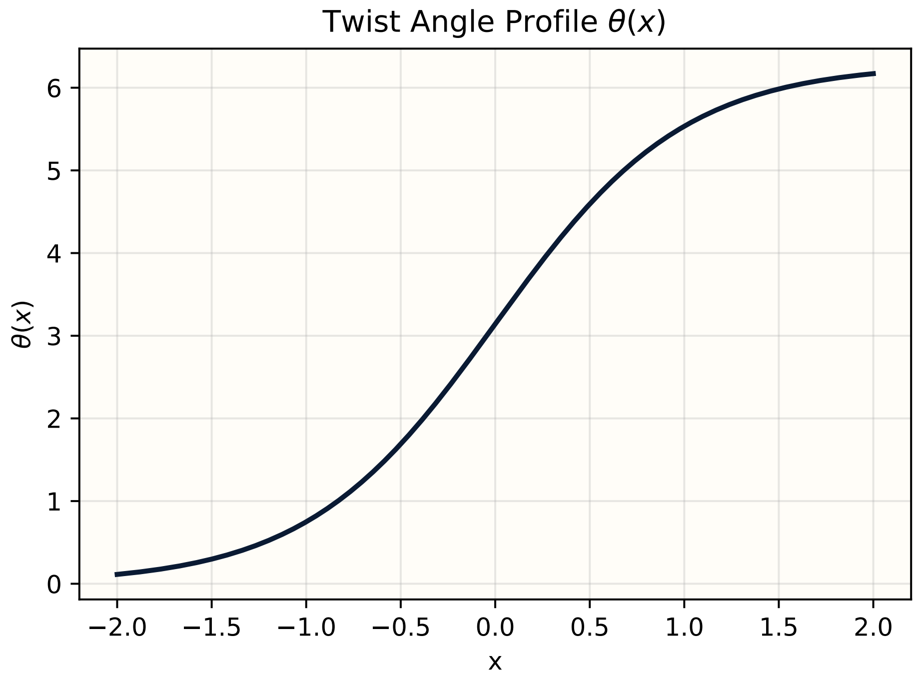

Figure 4: Smooth twist angle profile $\theta(x)$ interpolating between 0 and 2π.

[4] W. H. Zurek, "Decoherence and the quantum origins of the classical," Rev. Mod. Phys. 75, 715 (2003).

[5] J. D. Bekenstein, "Black holes and entropy," Phys. Rev. D 7, 2333 (1973).

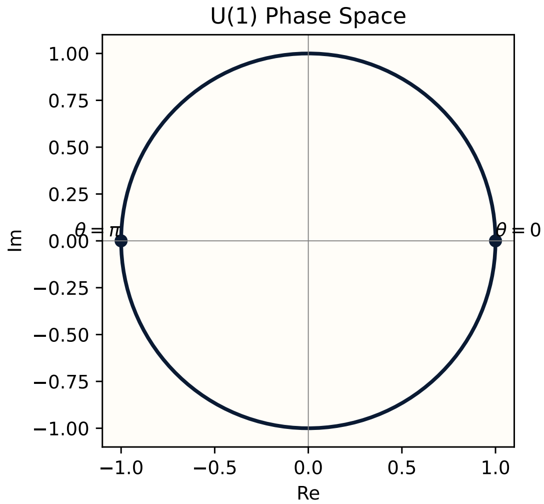

Figure 5: U(1) phase traced by $\Phi = \phi e^{i\theta}$.

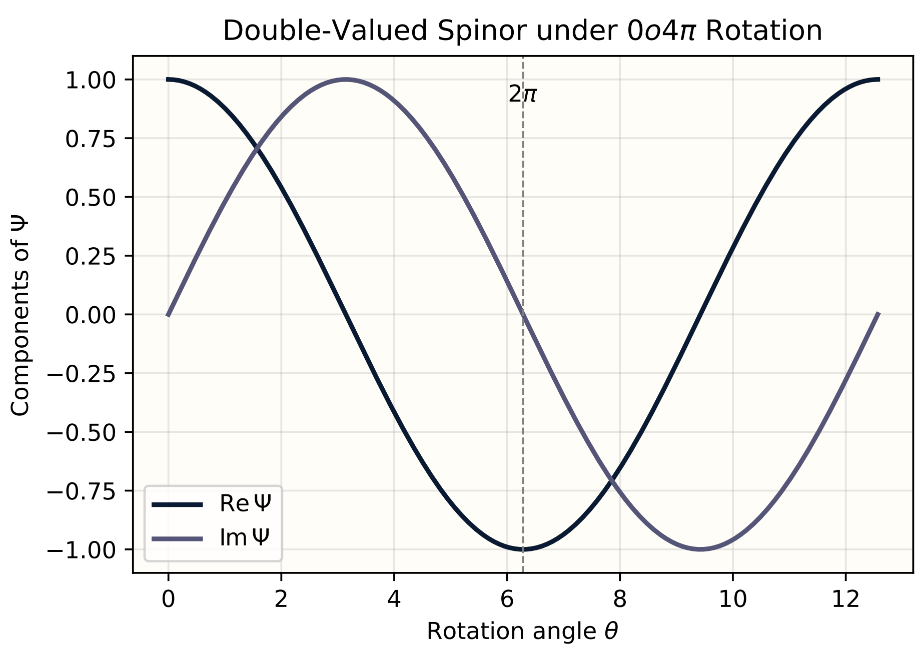

Figure 6: Double-valued spinor behavior under $0 \to 4\pi$ rotation.

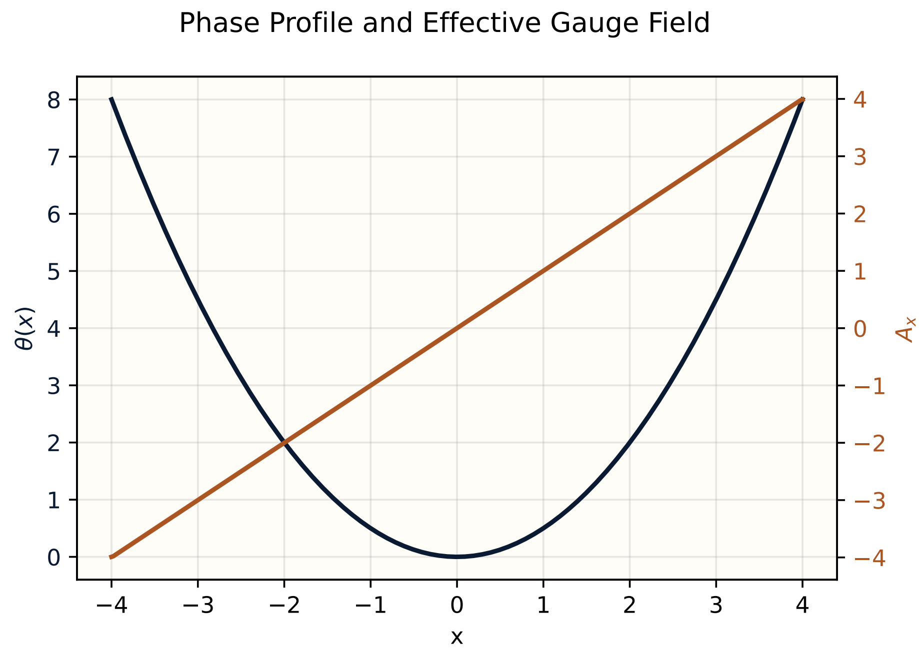

Figure 7: Phase profile $\theta(x)$ and effective gauge field $A_x \sim \partial_x \theta$.

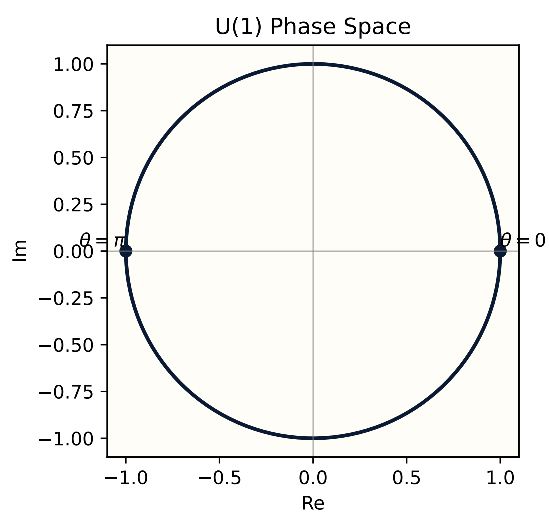

Figure 8: U(1) phase space traced by $\Phi = \phi e^{i\theta}$.

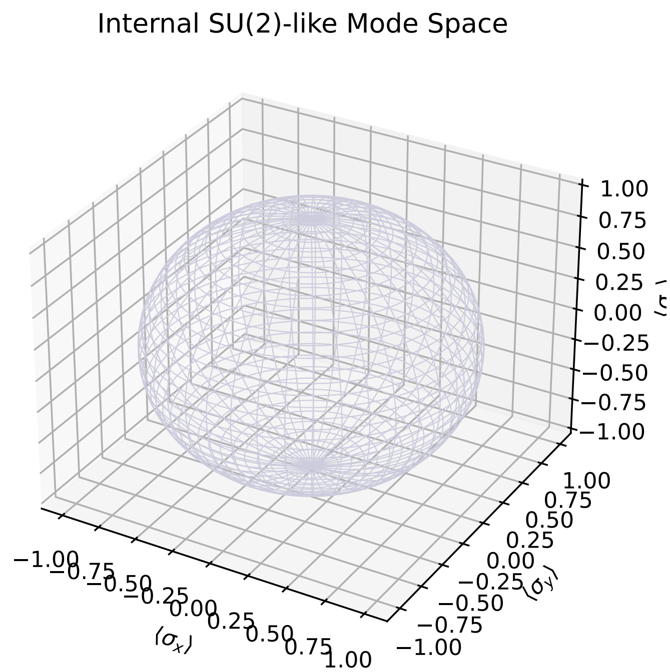

Figure 9: Internal SU(2)-like mode space.

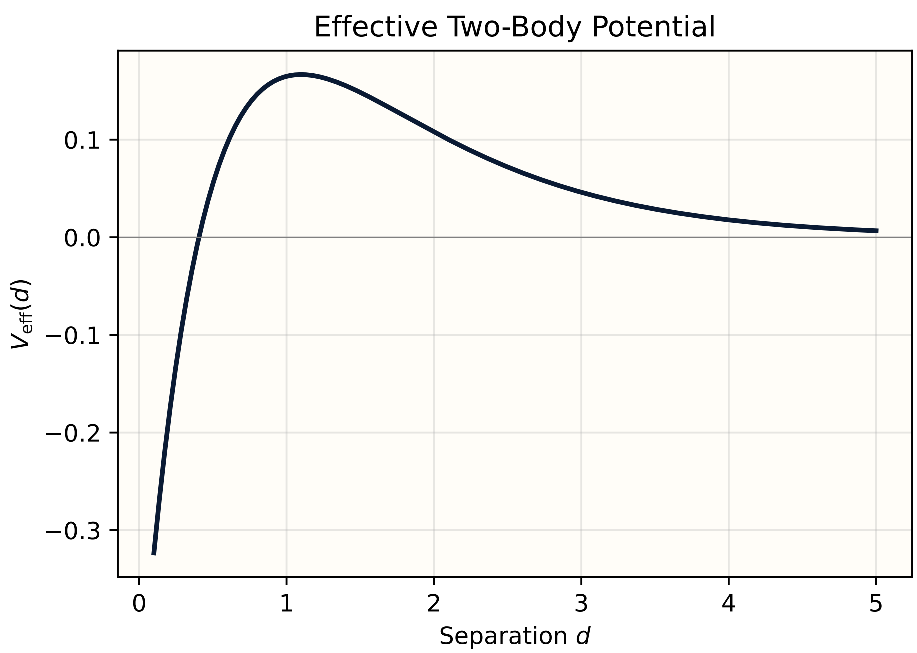

Figure 10: Effective two-body potential.

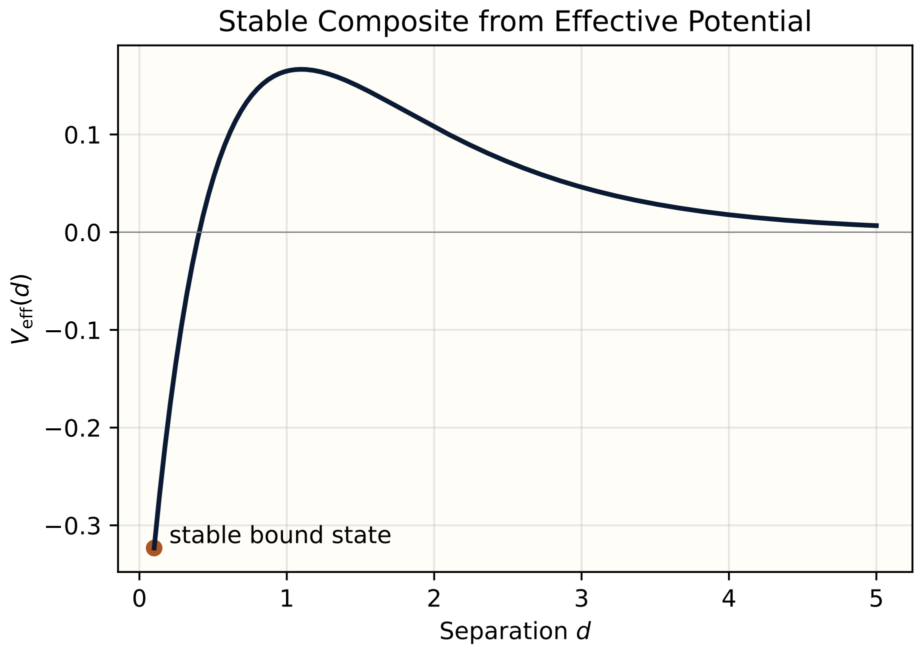

Figure 11: Stable bound state at the potential minimum.

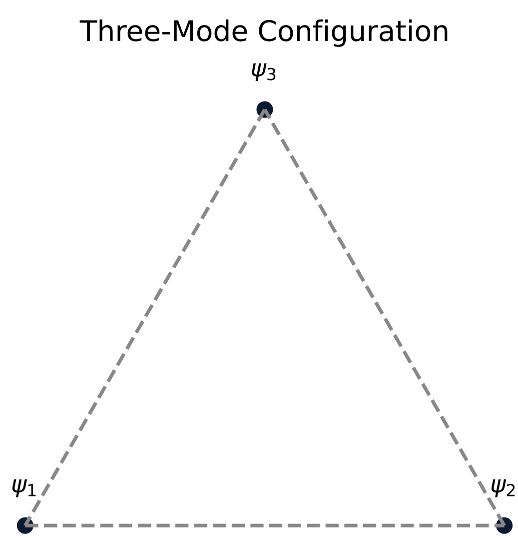

Figure 12: Three-mode geometric configuration.