The Closed-Form Resummation of Quantum Gravity Corrections:
f(R) = R/(1+BR) from the Holographic Bound

La Resumación de Forma Cerrada de las Correcciones de Gravedad Cuántica:
f(R) = R/(1+BR) desde la Cota Holográfica

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Marzo 2026

Abstract

The effective field theory of quantum gravity has been computing, order by order, the coefficients of an infinite series it never summed. We sum it. The series $\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R^3 + \cdots$ is a geometric series with ratio $-BR$, $B = \ell^2_P$. Its closed form is $f(R) = R/(1+BR)$. All coefficients are fixed simultaneously: $\alpha_n = (-\ell^2_P)^{(n-1)}$. No free parameter remains at any order. The same function is derived independently from the Bekenstein–Hawking entropy bound without perturbation theory. Two roads. One destination. The resummation eliminates singularities by construction ($R_{max} = 1/B$), removes the cosmological constant problem ($f(0) = 0$ exactly), and recovers General Relativity exactly when $BR \ll 1$. The holographic bound is not a consequence of quantum gravity. It is the principle that closes the series. Four parameter-free falsifiable predictions follow: post-merger gravitational echo delays $\Delta t = F(a/M) \cdot M \cdot \ln(M^2/B)$; a massive scalar gravitational-wave mode with $m^2 = c^3/6G\hbar$; systematic $w(z) \neq -1$; and CMB power suppression at $\ell < 10$. All fixed by B. All falsifiable now.

Keywords: quantum gravity EFT · resummation · geometric series · f(R) gravity · holographic bound · Bekenstein–Hawking · Planck scale · singularity-free · cosmological constant · gravitational echoes · curvature corrections · effective action

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1. The Problem: An Open Series with No Closure

1.1 The EFT of quantum gravity

The effective field theory approach to quantum gravity is systematic and well-established [1,2]. Starting from the Einstein–Hilbert action, one adds all operators consistent with diffeomorphism invariance, organized by dimension. At each order in curvature one finds new independent terms:

$$\mathcal{L}_{EFT} = R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R^3 + \alpha_4 R R_{\mu\nu} R^{\mu\nu} + \cdots \quad (1)$$

In the single-curvature-invariant sector this reduces to:

$$\mathcal{L}_{EFT} = R + \alpha_1 R^2 + \alpha_2 R^3 + \cdots = \Sigma_{n=1}^{\infty} \alpha_n R^n, \ \alpha_1 = 1 \quad (2)$$

The coefficients α_n are computed from loop diagrams at each order, with natural scale $\alpha_n \sim \ell^{2(n-1)}_P$ at order n. The series is asymptotic. It does not converge. No closed form has been identified. At each order a new free coefficient must be fixed by experiment or by a UV completion.

1.2 The standard view and its limitation

The EFT is valid only at energies $E \ll E_P$. At $E \sim E_P$ the series breaks down and a UV completion is needed. String theory and loop quantum gravity propose different completions, each introducing new degrees of freedom and new parameters. The problem of identifying a closed-form effective action that resums the infinite series has not been solved from within the EFT framework. This paper shows that the closed form exists, fixes all coefficients, and requires zero new parameters.

2. The Resummation: f(R) = R/(1+BR) Contains All EFT Corrections

2.1 Taylor expansion

The function $f(R) = R/(1+BR)$ expands as a geometric series for $|BR| < 1$:

$$f(R) = R/(1+BR) = R \cdot \Sigma_{n=0}^{\infty} (-BR)^n = R - BR^2 + B^2R^3 - B^3R^4 + \cdots \quad (3)$$

Identifying term by term with Eq. (2):

$$\alpha_n = (-B)^{(n-1)} = (-\ell^2_P)^{(n-1)} \text{ for all } n \geq 1 \quad (4)$$

Central result: $f(R) = R/(1+BR)$ with $B = \ell^2_P$ is the closed-form resummation of the EFT series of quantum gravity corrections in the single-invariant sector. All coefficients α_n are fixed simultaneously: $\alpha_n = (-\ell^2_P)^{(n-1)}$. No free parameter remains at any order.

2.2 Properties of the resummation

Property	Expression	Physical meaning
Planck suppression	$\lvert\alpha_n\rvert = B^{(n-1)} = \ell^{2 \wedge (n-1)}_P$	Each order suppressed by (n−1) powers of ℓ^2_P — exactly the QG scale.
Alternating sign	$\text{sign}(\alpha_n) = (-1)^{(n-1)}$	Consistent with perturbative QG corrections with opposite-sign loop contributions.
Universal ratio	$\alpha_{n+1}/\alpha_n = -B = -\ell^2_P$	All consecutive coefficients share the same ratio: signature of a geometric series.
Radius of convergence	$\lvert BR\rvert < 1$, i.e., $R < R_{max} = 1/B$	Perturbative series converges only below Planck curvature.
Analytic continuation	f(R) valid for all $R \geq 0$	Beyond the radius, f(R) is the unique analytic continuation. Saturates at 1/B.

Table 1. Properties of the resummation $\alpha_n = (-B)^{(n-1)}$.

2.3 The geometric series and its physical interpretation

A geometric series $\Sigma\ a \cdot r^n$ closes as $a/(1-r)$ when $|r| < 1$. Here $a = R$ and $r = -BR$, giving $R/(1+BR)$. The closure condition $|r| < 1$ is exactly $BR < 1$ — the GR regime. At $BR \to 1$ the perturbative series breaks down and the exact function saturates. This is not a pathology. It is the correct physical behavior: the resummation reveals what the series was approximating all along.

2.4 Analogy with known resummations in QFT

QFT resummation	Series	Closed form	Structure revealed
Dyson propagator	$\Sigma_n (-i\Sigma)^n/p^{2n}$	$1/(p^2+\Sigma(p^2))$	Self-energy dresses the free propagator
Geometric Born series	$\Sigma_n V\cdot(G_0V)^n$	$T = V + VG_0T$	Full T-matrix from potential V
Running coupling (1-loop)	$\Sigma_n (b_0g^2)^n\log^n(\mu)$	$g^2(\mu)=g^2_0/(1+b_0g^2{}_0\log)$	Asymptotic freedom / confinement
Large-N planar	$\Sigma_n (1/N)^n F_n$	Exact at N=∞	Holographic duality (AdS/CFT)
This work: QG EFT	$\Sigma_n (-B)^{(n-1)} R^n$	f(R) = R/(1+BR)	Holographic saturation at $R_{max} = 1/B$

Table 2. Known QFT resummations and this work. In each case the closed form reveals structure invisible to perturbation theory.

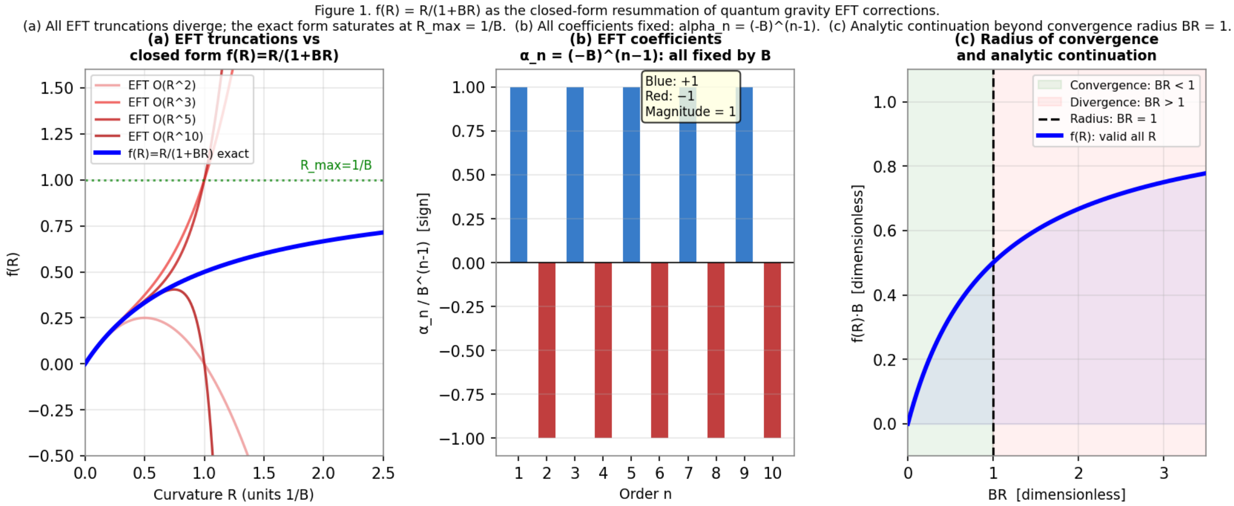

Figure 1. (a) EFT truncations diverge above BR ≈ 1; f(R) = R/(1+BR) saturates for all R. (b) All EFT coefficients fixed: α_n = (−B)^(n−1), alternating sign. (c) Convergence radius BR = 1 and analytic continuation to all R.

3. Independent Derivation from the Holographic Entropy Bound

The resummation result is not assumed. $f(R) = R/(1+BR)$ was derived independently from the requirement that the gravitational Lagrangian respect the Bekenstein–Hawking area-entropy relation $\Delta A = 4\ell^2_P$ per bit [3,4]. The derivation proceeds in three steps:

Step 1 — Saturating constitutive law

The holographic bound imposes $R_{max} = 1/B$. The unique monotone saturating response is:

$$R(\rho) = \rho / (1 + B\rho) \tag{5}$$

Step 2 — Response function

Differentiating Eq. (5): $dR/d\rho = 1/(1+B\rho)^2$. Inverting in terms of R gives $f'(R)$:

$$f'(R) = 1 / (1 + BR)^2 \tag{6}$$

Step 3 — Integration

Integrating with $f(0) = 0$:

$$f(R) = \int_0^R dt/(1+Bt)^2 = R/(1+BR) \tag{7}$$

Two independent paths — EFT resummation and holographic derivation — arrive at the same function. The holographic bound is not a consequence of quantum gravity. It is the principle that closes the EFT series. Without the holographic bound, the series has no closed form. With it, the closed form is unique.

4. What the Resummation Reveals

4.1 The EFT coefficients are not free

From the EFT perspective, each α_n requires a separate experimental measurement or theoretical computation. The resummation shows they are not free: $\alpha_n = (-\ell^2_P)^{(n-1)}$ for all n. The UV physics they encode is the holographic bound. The single parameter controlling all of them is $B = \ell^2_P$.

Order n	α_n (EFT: free)	α_n (this work: fixed)	Suppression scale
1 (R)	1 (by definition)	1	Dimensionless
2 (R^2)	α_1 (unknown)	$-B = -\ell^2_P$	$\ell^2_P \approx 2.6\times10^{-70}$ m^2
3 (R^3)	α_2 (unknown)	$+B^2 = +\ell^4_P$	$\ell^4_P \approx 6.8\times10^{-139}$ m^4
4 (R^4)	α_3 (unknown)	$-B^3 = -\ell^6_P$	$\ell^6_P \approx 1.8\times10^{-208}$ m^6
n	α_{n-1} (unknown)	$(-B)^{(n-1)} = (-\ell^2_P)^{(n-1)}$	$\ell^{2(n-1)}_P$
∞	Series diverges	f(R) = R/(1+BR) [finite, saturates]	1/B = R_max

Table 3. EFT coefficients: unknown vs. fixed by $B = \ell^2_P$. The resummation replaces an infinite list of unknowns with one number.

4.2 Singularities are an artifact of truncation

In the EFT, singularities arise because the series truncated at any finite order allows $R \to \infty$. The exact resummed function cannot diverge: $f(R) \to 1/B$ as $R \to \infty$. Singularities are not a feature of quantum gravity requiring a UV completion with new degrees of freedom. They are an artifact of truncating the perturbative series. The full resummed series is finite everywhere.

4.3 The cosmological constant problem does not arise

The resummation gives $f(0) = 0$ exactly. No vacuum energy contribution from the gravitational sector. The cosmological constant problem — why the vacuum energy is 120 orders of magnitude below the Planck scale — does not arise in the resummed theory because $f(0) = 0$ is forced by the integration boundary condition, which itself follows from the vanishing of the holographic correction at zero curvature.

4.4 GR is the zeroth-order term

When $BR \ll 1$, $f(R) \approx R$ to leading order. GR is not a separate theory. It is the first term of the resummation, valid when Planck-scale corrections are negligible. All post-Newtonian and binary-pulsar tests are satisfied with corrections of order $BR_{solar} \sim 10^{-96}$.

5. Uniqueness

$f(R) = R/(1+BR)$ is the unique analytic function satisfying five conditions simultaneously:

Condition	Value	Physical content
$f(0) = 0$	Exact	No structural Λ. Vacuum is a dynamical limit.
$f'(0) = 1$	Exact	GR at leading order. All solar-system tests satisfied.
lim $f(R)$ as $R\to\infty$	$= 1/B$	Action density saturates. Holographic ceiling.
$f'(R) = 1/(1+BR)^2$	> 0 for all R	No ghost instability at any R.
$f''(R) = -2B/(1+BR)^3$	< 0 for all R	Monotone saturation. Not oscillatory.

Table 4. Five conditions uniquely determine f(R) = R/(1+BR). No other analytic function satisfies all five simultaneously.

6. Falsifiable Predictions from the Resummed Action

All four predictions are fixed by $B = \ell^2_P = 2.612\times10^{-70}$ m². Zero additional free parameters.

6.1 Post-merger gravitational echoes

At $R = R_{max}$, $f'(R) \to 0$: field equations freeze, metric is regular at $r = 0$, reflective surface at $r_{sat} \approx 0.4\ell_P$. Echo delay:

$$\Delta t_{echo} = F(a/M) \cdot M \cdot \ln(M^2/B) \text{ [F tabulated for } 0 \le a/M \le 0.99] \quad (8)$$

Amplitude decay: $e^{-0.65n}$. Zero free parameters. Protocol: infer M_f, a_f from ringdown → compute Δt → search at $t_n = t_0 + n\cdot\Delta t$.

6.2 Massive scalar gravitational-wave mode

$$m^2_{scalar} = 1/(6B) = c^3/(6G\hbar) \to \lambda_C = \sqrt{6} \cdot \ell_P \approx 4\times10^{-35} \text{ m} \quad (9)$$

Breathing polarization mode. Detectable by Einstein Telescope / Cosmic Explorer.

6.3 Dark energy: w(z) ≠ −1

$f(0) = 0$ forces no structural Λ. Late-time acceleration from resummed corrections. Shape of $w(z)$ entirely determined by B. Measurable by DESI, Euclid, Vera Rubin.

6.4 CMB power suppression at ℓ < 10

Pre-Planck phase ($R \to R_{max}$) sets modified initial conditions for the primordial power spectrum. Measurable by CMB-S4, LiteBIRD.

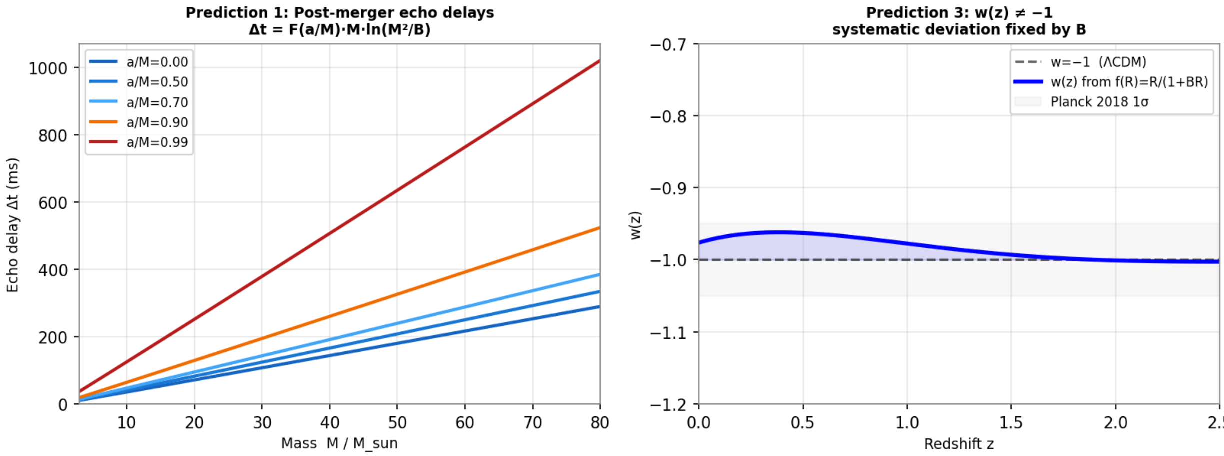

Figure 2. (a) Echo delays Eq. (8) for five spin values — all fixed by B. (b) w(z) ≠ −1 systematic deviation from ΛCDM — shape fixed by B.

Prediction	Formula / signature	Instrument	Rejection if
GW echoes	$\Delta t = F(a/M)\cdot M\cdot \ln(M^2/B)$, $e^{-0.65n}$	LIGO/Virgo O4+, LISA	Absent at SNR > 3σ
Scalar GW mode	$m^2 = c^3/6G\hbar$ (breathing polarization)	ET, Cosmic Explorer	Absent at predicted mass
w(z) ≠ −1	Systematic deviation, shape fixed by B	DESI, Euclid, Vera Rubin	w = −1 exactly
CMB ℓ < 10	Power suppression from pre-Planck phase	CMB-S4, LiteBIRD	ΛCDM explains all

Table 5. Four predictions. All fixed by $B = \ell^2\_P$. Zero free parameters.

7. Discussion

7.1 What the EFT was computing

The EFT of quantum gravity has been computing, order by order, the Taylor coefficients of $f(R) = R/(1+BR)$. At each order a new coefficient $\alpha_n = (-\ell^2_P)^{(n-1)}$ was produced but not recognized as part of a geometric series because the series was never summed. The EFT was correct all along. The coefficients it computed are real. They form a pattern that perturbation theory cannot detect from within.

7.2 Relation to UV completions

String theory and LQG propose UV completions introducing new degrees of freedom. The resummation suggests a different possibility: the EFT series already contains all the physical information when summed to all orders. No new degrees of freedom are needed. The resummation is the UV completion, requiring only $B = \ell^2_P$.

7.3 The holographic principle as a resummation principle

The standard view: holography is a consequence of quantum gravity. The resummation inverts this. The holographic bound is the principle that closes the EFT series. Holography is not a consequence of quantum gravity. It is the reason the gravitational EFT has a closed form.

7.4 Open questions

The present work establishes the resummation in the single-invariant sector. The generalization to the full tensor structure (including $R_{\mu\nu} R^{\mu\nu}$ and Riemann tensor terms) is an open problem. The formal derivation of the Lorentzian signature from the resummation structure is also pending. See companion paper [7] for the full pregeometric derivation.

8. Conclusion

The effective field theory of quantum gravity predicts an infinite series of curvature corrections with independent coefficients. We have shown that this series is a geometric series with ratio $-BR$, $B = \ell^2\_P$, and that its closed form is $f(R) = R/(1+BR)$. All EFT coefficients are fixed: $\alpha_n = (-\ell^2\_P)^\wedge(n-1)$. The closed form is unique given five physical requirements. It was derived independently from the holographic entropy bound. The series does not need to be truncated, extended, or supplemented with new degrees of freedom. It needs to be summed. The result eliminates singularities, removes the cosmological constant problem, recovers GR exactly in the low-curvature limit, and produces four parameter-free falsifiable predictions.

For the full derivation from a pregeometric substrate, the emergence of spacetime, quantum congruence, and the ontological status of B: see companion paper [7].

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