What Does a Physical Clock Measure in Curved Spacetime?

An Operational Clarification Within Standard General Relativity

IRCBHC – Independent Research Collaboration on Black Hole and Cosmology Concepts

Abstract

In general relativity (GR), common teaching phrases such as "time runs slower in a gravitational field" can blur a crucial operational distinction: a physical clock records a measurable quantity along its worldline, whereas coordinate time is a labeling convention. This paper provides a compact, GR-literate clarification suitable for advanced undergraduates and instructors. The reading of an ideal clock is modeled by the invariant proper time along its worldline, and "time dilation" is interpreted strictly as a comparison of accumulated clock readings between worldlines under a specified protocol. We connect this operational content to the 3+1 decomposition via the lapse function N, present a Schwarzschild example yielding $d\tau = N(r)\,dt$ for stationary clocks, and add a weak-field numerical estimate and a flat-spacetime Rindler analogy. Brief connections to precision practice (GPS, Pound–Rebka) and to physics-education research on time concepts in relativity are included. No claims beyond standard GR are made.

Important Reminder

Scope and limitations (explicit). This educational research note is restricted to *standard GR*. No alternative theories, no cosmology, and no data fitting are introduced. We avoid metaphysical claims about the "existence" or "flow" of time. The goal is operational clarity: what clocks measure and how GR encodes clock comparisons.

1 Educational motivation: where students and readers stumble

Despite the geometric precision of GR, classroom shorthand often blends two distinct notions:

1. the *measured* quantity recorded by a physical clock carried along a worldline, and
2. a *coordinate* time label used to parametrize a foliation or chart.

When this distinction is not made explicit, readers may infer that "time itself" is a universal entity that "flows" at different rates. Physics education research (PER) documents persistent difficulties with time concepts in relativity, including conflation of coordinate descriptions with measurable quantities and confusion about observers, reference frames, and simultaneity [9, 10].

In GR instruction, these difficulties are amplified by strong-field narratives, where coordinate choices can produce misleading stories unless the operational definition is kept in view [11, 5]. Our aim is to provide a compact operational framework and classroom-ready examples that prevent this drift.

Pedagogical contribution

The contribution of this note is an instructional synthesis:

- a strict operational statement of what a physical clock measures;

- a direct bridge to the 3+1 lapse function N as a clock-comparison encoder;
- minimal examples (Schwarzschild, weak-field, Rindler) unified by a single operational message.

Common Misconception

Misconception. “GR says time itself slows down.”
Operational correction. GR predicts that **clocks** on different worldlines can accumulate different proper times when compared by a specified protocol.

Important Reminder

Comparison protocol (once and for all).
Time dilation is not a local diagnosis. It is defined only relative to a comparison protocol, such as reunion of clocks or a specified signal-exchange convention.

2 Proper time: the clock-measured quantity

In GR, the reading of an ideal clock is modeled by the invariant proper time accumulated along its worldline. For a timelike curve,

$$d\tau^2 = -\frac{1}{c^2}\, g_{\mu\nu}\, dx^\mu dx^\nu. \tag{1}$$

This invariant provides a coordinate-independent model for clock output [1, 2, 3, 5].

2.1 Operational interpretation

A physical clock is a local periodic process producing countable ticks (events). Operationally,

$$\tau \propto \#(\text{ticks}), \tag{2}$$

up to calibration. “Time dilation” therefore refers to a comparison of accumulated tick counts between worldlines, not to a local modification of a universal time entity.

Important Reminder

Guardrail.
No single observer’s wristwatch reads the coordinate label t in general; observers read their own proper time τ.

3 The 3+1 decomposition and the lapse as a clock-comparison encoder

Consider a foliation by spacelike hypersurfaces Σ_t. In this subsection only, we temporarily set $c = 1$ for notational simplicity; elsewhere in the paper we keep c explicit. The standard 3+1 split reads

$$ds^2 = -N^2 dt^2 + h_{ij}\big(dx^i + \beta^i dt\big)\big(dx^j + \beta^j dt\big), \tag{3}$$

where N is the lapse, β^i the shift, and h_{ij} the induced spatial metric [6, 7].

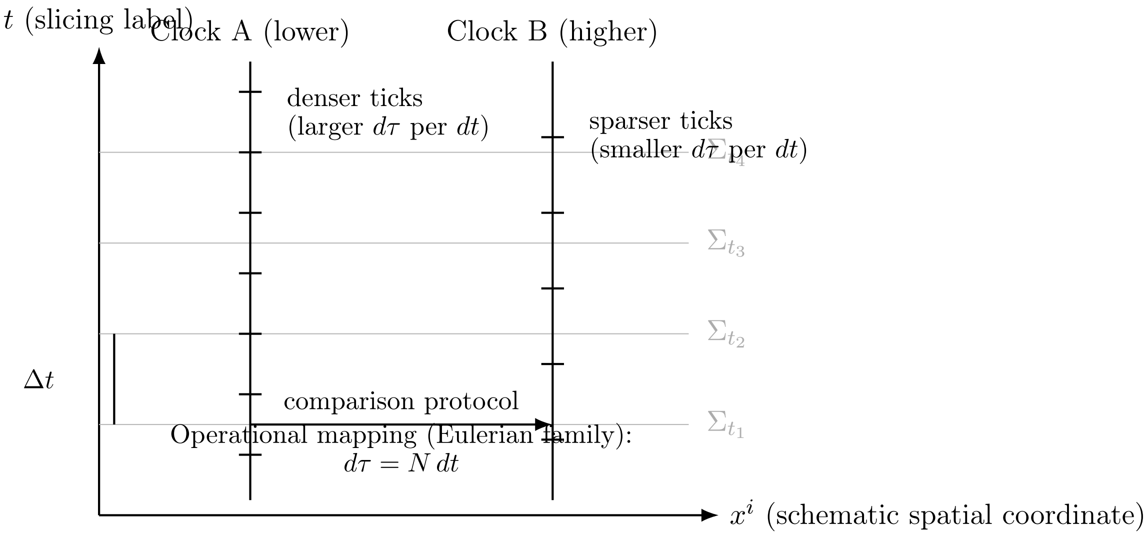

Figure 1: Schematic operational picture: two clocks on different worldlines accumulate different proper times. "Time dilation" is defined only via a comparison protocol (e.g., reunion or a specified signal-exchange convention). In a chosen 3+1 slicing, the lapse N encodes the conversion between the slicing label t and the proper time recorded by a specified observer family (Sec. 3).

3.1 Key equation: $d\tau = N\,dt$ for Eulerian observers

For Eulerian observers whose worldlines are normal to Σ_t (equivalently, $dx^i = -\beta^i dt$), Eq. (3) gives

$$d\tau = N\,dt. \tag{4}$$

The lapse is not a directly measured scalar. It parametrizes the mapping between the coordinate label t and the proper time τ for a chosen slicing and its associated observer family.

4 Example 1: stationary clocks in Schwarzschild spacetime

The Schwarzschild metric outside a spherical mass M is [1, 3, 4]

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1}dr^2 + r^2d\Omega^2. \tag{5}$$

For a stationary clock ($dr = d\theta = d\phi = 0$),

$$d\tau = \sqrt{1 - \frac{2GM}{rc^2}}\,dt. \tag{6}$$

4.1 Comparing two stationary clock rates

Two stationary clocks at radii r_1 and r_2 satisfy

$$\frac{d\tau_1}{d\tau_2} = \sqrt{\frac{1 - \frac{2GM}{r_1c^2}}{1 - \frac{2GM}{r_2c^2}}}. \tag{7}$$

Common Misconception

Clarification. An infinite ratio can arise for stationary (accelerated) observer families as $r \to r_s$ in Schwarzschild slicing; it does not imply a breakdown of free-fall clock readings.

5 Example 2: weak-field two-height estimate

In the weak-field limit,

$$\frac{d\tau}{dt} \approx 1 + \frac{\Phi}{c^2}, \tag{8}$$

where Φ is the Newtonian potential [8]. For a small height separation h near Earth, $\Delta\Phi \approx gh$, so over one day $T = 86400$ s,

$$\Delta\tau \approx T\frac{gh}{c^2}. \tag{9}$$

Important Reminder

Class activity. (i) Derive Eq. (9).
(ii) Evaluate for $h = 1000\,\mathrm{m}$.
(iii) Discuss measured versus conventional quantities.

6 Example 3: Rindler observers (flat spacetime)

In Rindler coordinates,

$$ds^2 = -(a\xi)^2 d\eta^2 + d\xi^2 + dy^2 + dz^2, \tag{10}$$

with $\xi > 0$ [2, 1]. For observers at fixed ξ,

$$d\tau = a\xi\, d\eta. \tag{11}$$

Important Reminder

Clock-rate differences can arise for a chosen observer family even in flat spacetime. Curvature is a distinct question, diagnosed by invariants.

7 Conclusion

Within standard GR, a physical clock measures the proper time accumulated along its worldline. Time dilation is a relational statement comparing clock readings between worldlines under a specified protocol. The 3+1 lapse function provides a compact bridge between geometry and measurement language by encoding the mapping between coordinate labels and proper time for a chosen observer family. The examples presented form a minimal, coherent instructional set that avoids coordinate-dependent narratives.

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