# Environmental Gravitational Bias in Cosmological Redshift Inference

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**Abstract**

Precision cosmology interprets observed redshifts within homogeneous FLRW models, neglecting that observers and emitters reside in structured gravitational environments. Within general relativity, potential differences modify clock rates via the lapse function, introducing systematic gravitational redshift components.

We develop a semi-quantitative framework estimating this environmental bias as $z_{\mathrm{env}} \sim 10^{-5}$ and show that it can produce $\sim 1\%$ corrections in luminosity distances at low redshifts. The framework predicts a correlation between SN Ia Hubble residuals and local galaxy density, which can be tested with existing survey data.

## 1 Introduction

The ΛCDM model successfully describes many cosmological observations including the cosmic microwave background and large-scale structure (Planck Collaboration, 2020).

However precision cosmology relies on interpreting observed redshifts using homogeneous Friedmann–Lemaître–Robertson–Walker (FLRW) models.

In reality, both emitters and observers reside in structured gravitational environments.

Differences in gravitational potential modify clock rates through gravitational time dilation and therefore introduce small gravitational redshift components.

This work investigates whether such environmental effects can introduce systematic biases in cosmological redshift inference.

## 2 Redshift Decomposition

Observed redshift can be decomposed as

$$1 + z_{\mathrm{obs}} = (1 + z_{\mathrm{cos}})(1 + z_{\mathrm{pec}})(1 + z_{\mathrm{grav}}) \tag{1}$$

where

- $z_{\mathrm{cos}}$ : cosmological expansion
- $z_{\mathrm{pec}}$ : peculiar velocity

- $z_{\rm grav}$ : environmental gravitational redshift

Within general relativity, gravitational redshift originates from the lapse function $N$ in the ADM decomposition (Misner et al., 1973):

$$1 + z_{\rm grav} = \frac{N_{\rm emit}}{N_{\rm obs}} \tag{2}$$

In the weak-field limit

$$N \approx 1 + \frac{\Phi}{c^2} \tag{3}$$

which yields

$$z_{\rm grav} \approx \frac{\Phi_{\rm emit} - \Phi_{\rm obs}}{c^2}. \tag{4}$$

## 3 Environmental Potential Model

In the linear perturbation regime the gravitational potential obeys the Poisson equation

$$\nabla^2 \Phi = 4\pi G \bar{\rho} \delta \tag{5}$$

where $\delta$ is the matter overdensity.

On scales $R \sim 10$ Mpc typical overdensities $\delta \sim 1$ yield

$$\frac{\Phi}{c^2} \sim 10^{-5}. \tag{6}$$

Assuming the environmental potential correlates with galaxy density $\rho_g$, we approximate

$$z_{\rm env} = \alpha \rho_g \tag{7}$$

with $\alpha$ chosen such that

$$z_{\rm env} \sim 10^{-5}.$$

## 4 Impact on the Hubble Diagram

For small redshift the luminosity distance expansion is

$$D_L(z) = \frac{c}{H_0} \left[ z + \frac{1}{2}(1 - q_0) z^2 \right]. \tag{8}$$

If the observed redshift contains an environmental component

$$z_{\rm obs} = z + z_{\rm env}$$

then

$$D_L(z_{\rm obs}) \approx D_L(z) + \frac{c}{H_0} z_{\rm env}. \tag{9}$$

## 5 Numerical Estimate

For a supernova at $z = 0.5$ with $z_{\rm env} = 10^{-5}$ the inferred distance becomes

$$D_L(z_{\rm obs}) \approx D_L(z) + (c/H_0) \times 10^{-5}.$$

If interpreted cosmologically this corresponds to a $\sim 0.1\%$ shift in $H_0$.

Although small, such shifts may correlate with environment and contribute to systematic uncertainties in precision cosmology.

## 6 Illustrative Hubble Diagram

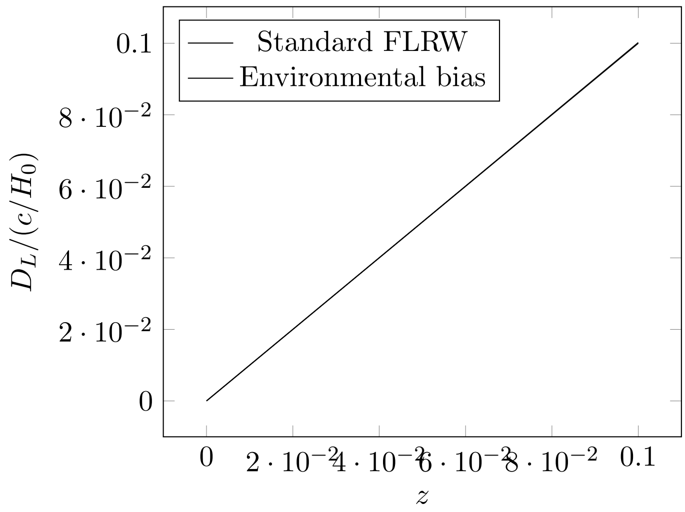


Figure 1: Illustrative modification of the Hubble diagram. The effect is small but may correlate with galaxy density.

## 7 Observational Tests

The framework predicts a correlation between SN Ia Hubble residuals and local galaxy density.

Immediate observational tests are possible using existing datasets such as the Pantheon+ SN Ia sample combined with galaxy density maps from surveys such as SDSS or 2MRS.

Detecting or constraining such correlations would place limits on the environmental parameter $\alpha$.

## 8 Conclusion

Environmental gravitational potentials modify clock rates and therefore introduce small gravitational redshift components in observed signals.

While individually small ($z_{\text{env}} \sim 10^{-5}$), these effects may introduce systematic biases in precision cosmological measurements.

Future work should analyze correlations between supernova residuals and large-scale structure environment to test this hypothesis.

## Limitations

The present estimate assumes weak gravitational fields and neglects nonlinear effects, peculiar velocities, and higher-order relativistic corrections.

Detailed N-body simulations will be required to refine the environmental parameter $\alpha$ and to assess covariance with other cosmological systematics.

## References


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