Procedural Vacuum Breakdown (PVB)

Admissibility-Locked Covariant Core with Phenomenological Optical Bridge
v1.8

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Conventions and Definitions

- Metric signature: $(-,+,+,+)$.
- Einstein tensor: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R$.
- Gravitational coupling: $\kappa \equiv 8\pi G/c^4$. Reserved for this meaning throughout; never reused for opacity.
- Stress-energy definition for physical sectors: $T_{\mu\nu} \equiv -\,(2/\sqrt{-g})\;\delta S/\delta g^{\mu\nu}$.
- Scalar kinetic convention: $\mathcal{L}_\psi \supset -\frac{1}{2} Z_\psi (\nabla\psi)^2$ with $Z_\psi > 0$.
- Admissibility scalar: $\psi(x) \equiv \ln\left(A_{adm}(x)/A_0\right)$, where $A_{adm}(x)$ is admissibility density and A_0 is a reference density.
- Membrane scalar: $\Pi(x) \equiv -\psi(x)$. Used in the optical membrane sector so that Π increases inward (membrane driver convention).
- Reaction placement: the λ-term is treated as a trace reaction on the geometric side, i.e. as a shift $\Lambda \to \Lambda + \kappa\lambda$, not as a separately conserved matter sector.
- Refractive coupling: g_n (dimensionless). Not to be confused with the Lagrange multiplier λ.
- Ray affine parameter: ℓ (script ell). Not to be confused with λ.
- Matter-sector lapse: $\alpha_m(r) = 1 - R_s/r$.
- Photon-sector lapse: $\alpha_\gamma(r) := \alpha_m(r)/n(r)^2$. Not to be confused with A_{adm}.
- Opacity/absorption: $\kappa(r, \omega, pol)$. Not to be confused with the gravitational coupling κ.

Unified Symbol Dictionary

Symbol	Meaning	Origin
DAG	Directed acyclic graph of micro-updates (substrate)	Spine
(M, g)	Emergent Lorentzian manifold and metric	Spine
$\omega(x)$	Fixed reference scalar density (weight +1) from coarse-graining	Spine
$A_{adm}(x)$	Admissibility density (routing capacity)	Spine
A_0	Reference admissibility density	Spine
$\psi(x)$	Scalar field $\ln\left(A_{adm}/A_0\right)$ controlling routing modulation	Spine (primitive)
$\Pi(x)$	Membrane scalar: $\Pi \equiv -\psi$	Bridge (derived)
$\lambda(x)$	Lagrange multiplier enforcing $\sqrt{-g} = \omega\, e^{\psi}$	Spine
Λ^*	Global integration constant (scalar): $\lambda e^{\psi} = \Lambda^*$	Spine (Bianchi)
Z_ψ	Positive kinetic stiffness parameter for ψ	Spine
$U(\psi)$	Scalar potential	Spine
S_{noise}	Diffeo-invariant noise / influence functional	Spine
J_{noise}	Scalar noise current: $(1/\sqrt{-g})\;\delta S_{noise}/\delta\psi$	Spine
κ	Gravitational coupling: $8\pi G/c^4$ (reserved)	Spine
g_n	Refractive coupling constant (dimensionless)	Bridge
ℓ	Ray affine parameter	Bridge
$\alpha_m(r)$	Matter-sector lapse: $1 - R_s/r$	Membrane
$\alpha_\gamma(r)$	Photon-sector lapse: α_m/n^2	Bridge (renamed)

Symbol	Meaning	Origin
$n(r)$	Refractive index: $\exp(g_n\Pi) = \exp(-g_n\psi)$	Membrane
$\kappa(r)$	Opacity / absorption coefficient	Bridge (renamed)
$G_{\max}$	Peak gradient magnitude $\|\Pi\prime\|$ at r_{mem}	Membrane
σ	Gaussian gradient width (membrane thickness)	Membrane
u, s	r_{mem}/R_s, σ/R_s	Kill-criteria
ε	Dimensionless peak-gradient coupling: $g_n G_{\max} R_s$	Kill-criteria
χ	Integrated gradient: $\sqrt{\pi/2} \cdot \varepsilon \cdot s$	Kill-criteria
K	Opacity strength: $\kappa_{pk} \cdot \sigma$	Kill-criteria

Executive Summary (v1.8 Claims Hierarchy)

This version upgrades the manuscript by separating what is actually derived from what is currently modeled phenomenologically. The paper now stands on a three-layer architecture:

1. Layer 1 - Derived covariant core: density-safe volume lock $\sqrt{-g} = \omega(x)e^{\psi}$ with ψ a true scalar.
2. Layer 2 - Exact exchange bookkeeping: Ward-clean noise sector and Bianchi closure giving $\lambda e^{\psi} = \Lambda^*$.
3. Layer 3 - Stability floor: conformal-mode ghost exorcism yielding the exact bound $Z_\psi > 3/(8\kappa)$.
4. Separate benchmark phenomenological bridge: Schwarzschild timelike sector plus Gordon optical membrane for null propagation, whose optical constitutive laws are ansätze driven by psi rather than uniquely derived microphysical laws.

This manuscript therefore claims a referee-hardened effective framework with explicit falsifiers, not a completed microscopic theory of emergent spacetime. That sharper calibration is a strength, not a retreat.

What This Manuscript Does and Does Not Establish

This manuscript does establish a covariant continuum core with a determinant lock, a pure-trace reaction term, a Ward-clean exchange law for the noise sector, an exact Bianchi closure condition, and an explicit conformal-mode stability floor.

This manuscript does not yet establish a microscopic derivation of the optical constitutive ansätze, a UV completion of the substrate, or a complete matter-coupling program that automatically satisfies all fifth-force and screening constraints. Those items remain open burdens, not hidden assumptions.

Accordingly, claims are partitioned as follows: the continuum lock framework is derived within the stated effective theory; the optical membrane is a benchmark phenomenological bridge motivated by psi whose optical constitutive laws are ansätze rather than uniquely forced consequences of the substrate; and the DAG substrate is the motivating ontology that organizes interpretation and future derivation targets.

That separation is deliberate. It prevents the phenomenology from borrowing certainty it has not yet earned and makes the kill criteria materially sharper.

Motivation and Scientific Context

Three structural gaps in the current theoretical landscape motivate this framework. Each identifies a place where existing formalisms either suppress a degree of freedom that an emergent-spacetime picture requires, or stop short of the dynamical extension needed to generate falsifiable predictions.

The volume element as a suppressed degree of freedom. In general relativity, the volume element $\sqrt{-g}$ is fully determined by the metric tensor and carries no independent information. But if spacetime emerges from a discrete causal substrate, the conformal structure (fixed by the causal partial order) and the volume measure (encoding the substrate's local routing capacity) need not be locked together. The substrate can support regions of identical causal geometry but different routing density—a distinction that GR's formalism cannot represent. This framework decouples the two by introducing a density-safe constraint that ties $\sqrt{-g}$ to a reference density $\omega(x)$ modulated by a physical scalar field ψ, restoring the substrate's routing capacity as a dynamical degree of freedom within a covariant continuum description.

Unimodular gravity stops short. Unimodular gravity achieves a genuine structural improvement over standard GR by demoting the cosmological constant from a fixed coupling to an integration constant, removing the worst of the vacuum energy fine-tuning problem at the classical level. But the program stops there. It provides no dynamical scalar field, no mechanism for the effective vacuum tax to respond to local field content, and no built-in route to falsifiable predictions beyond the cosmological constant itself. The present framework extends the unimodular insight: the determinant constraint is promoted from a gauge condition to a physical lock driven by ψ, and Bianchi integrability forces the Lagrange multiplier to satisfy

$$\lambda(x) = \Lambda_* \, e^{-\psi(x)}, \tag{1}$$

yielding a global integration constant Λ_* while allowing the local vacuum reaction to track the scalar field. This is a concrete dynamical extension of the unimodular architecture, not merely a restatement of it.

Analog gravity models lack a covariant field-theoretic spine. Gordon-metric descriptions of light propagation in effective media have been studied extensively as analog models for curved spacetime. These models are typically deployed as kinematic analogies: they describe how null geodesics bend in an effective medium but do not derive the medium's constitutive laws from a covariant action principle with a determinant constraint. This framework provides exactly that connection. The scalar ψ that drives the volume lock also determines the refractive and absorptive properties of the optical membrane through explicit constitutive ansätze. The benchmark bridge is not yet derived from microphysics, but it sits inside a covariant structure that constrains future derivation rather than leaving the constitutive laws entirely ad hoc.

Why a different path. Existing programs for emergent spacetime, modified gravity, and quantum gravity have produced rich formal structures but have not converged on a shared set of testable, near-term predictions. Loop quantum gravity, string-derived effective actions, causal set theory, and various modified dispersion frameworks each carry open problems of their own—problems that this manuscript does not claim to solve. Rather than building on one of these established foundations, this framework starts from a different structural primitive: the idea that a macroscopic scalar encoding routing capacity, constrained by a covariant volume lock, is the minimal additional ingredient needed to extend GR toward an emergent-spacetime interpretation while remaining falsifiable by current and near-future observations (against current experimental gravity constraints). The kill criteria built into the framework are structural rather than parameter-tuned: they identify conditions under which the framework is dead, not conditions under which it can be adjusted to survive.

Additionally, the conformal mode of the gravitational field is known to carry a wrong-sign kinetic term in the Einstein–Hilbert action. Most programs either work around this instability or defer it to a UV completion. The present framework confronts it directly: the ghost-free bound

$$Z_\psi > \frac{3}{8\kappa} \tag{2}$$

is an exact consequence of the volume-lock architecture that ensures the net scalar kinetic term is healthy, providing a concrete structural floor rather than an adjustable stabilization mechanism.

Relation to Adjacent Emergence Programs

The present framework overlaps conceptually with several existing lines of work in emergent gravity, vacuum-based inertia, and symmetry-broken locality, but it is not identical to any of them. The point of contact is broad: each program seeks to explain geometric or inertial structure as secondary rather than fundamental. The point of departure in PVB is more specific: the continuum theory is organized around an admissibility scalar ψ, a volume-lock relation linking measure to routing capacity, and a covariant closure chosen to preserve Bianchi consistency while remaining exposed to explicit phenomenological kill criteria.

This distinguishes the present program from approaches that derive effective curvature from filtered quantum histories, from vacuum-thermodynamic programs in which inertia and gravity arise from equilibrium response, and from broken-diffeomorphism programs in which locality or gravitation emerges through symmetry reduction. PVB does not merely assert emergence in general terms; it posits a particular admissibility-driven lock between microscopic routing capacity and macroscopic measure, then asks whether that lock can support a consistent field-theoretic spine and a benchmark-facing observational membrane. Whether that narrowing move is correct remains open, but it is the feature that makes the framework distinct and potentially falsifiable.

Part I: Framework Core

Ontological Primitives

Substrate The substrate is a directed acyclic graph (DAG) of micro-updates. Each update is an allowed local routing transition consistent with the causal partial order.

Coarse-grained spacetime Upon coarse-graining, the causal order fixes the conformal class of an emergent Lorentzian metric $g_{\mu\nu}$ on a manifold M. The determinant (volume element) is not treated as a free scalar degree of freedom: it is constrained by a density-safe lock tied to the substrate's reference density $\omega(x)$.

Primitive macroscopic scalar We promote a single macroscopic scalar $\psi(x)$ as the logarithmic admissibility modulation:

$$\psi(x) = \ln\left(A_{adm}(x)/A_0\right),$$

where $A_{adm}(x)$ is admissibility density (routing capacity) and A_0 is a reference value.

Status of the Substrate Hypothesis and Effective Closure

The directed-acyclic-graph (DAG) substrate used in this manuscript should be read as a kinematic ontological hypothesis rather than as a completed microscopic derivation of continuum gravity. Its role is to motivate the interpretation of admissibility as a finite routing capacity of an underlying update network. The continuum field theory developed here is therefore not presented as a theorem proven from first principles of the DAG, but as the lowest-order covariant effective closure compatible with three requirements: (i) a scalar admissibility field ψ that tracks local routing capacity, (ii) a volume-lock relation linking macroscopic measure to admissibility, and (iii) Bianchi-consistent covariant field equations.

In that sense, the emergent claim made here is intentionally modest. What this manuscript establishes is the internal consistency and phenomenological reach of the continuum spine once the admissibility lock is imposed. What remains open is the full coarse-graining derivation showing how a specific ensemble of substrate histories flows to the effective action used below. The present paper should therefore be read as an effective-program manuscript with explicit ontological motivation, not as a completed microscopic reconstruction.

Schematic Coarse-Graining Map

A minimal coarse-graining picture can be stated schematically. Consider an ensemble of locally finite DAG histories with a coarse-grained update density $\rho_{\mathrm{upd}}(x)$ and an effective routing capacity $C(x)$ defined over mesoscopic cells. The admissibility field $\psi(x)$ is then interpreted as a logarithmic measure of local routing capacity,

$$C(x) \propto e^{\psi(x)}.$$

Regions of higher admissibility correspond to regions in which the substrate can support greater routing throughput before structural obstruction or update congestion occurs. If macroscopic four-volume is directly weighted by routing capacity, the natural continuum lock takes the form

$$\sqrt{-g} \propto C(x) \propto e^{\psi(x)}.$$

The manuscript takes this lock as the effective continuum input and studies its consequences for covariant dynamics, closure, and phenomenology. No claim is made here that this sketch constitutes a full derivation from substrate microdynamics; rather, it is the minimal map needed to connect the substrate interpretation to the continuum variables used in the field-theoretic spine.

Density-Safe Volume Lock

The volume element $\sqrt{g}$ is a scalar density of weight $+1$. A prior identification $\sqrt{-g} = e^{\psi}$ is not diffeomorphism-covariant because e^{ψ} is a true scalar. We repair this by introducing a fixed reference density $\omega(x)$, also weight $+1$, inherited from coarse-graining, and enforce the density-safe lock:

$$\sqrt{-g} = \omega(x) \cdot \exp(\psi).$$

Here $\omega(x)$ encodes the background substrate density in the coarse-grained description, while $\psi(x)$ encodes local routing modulation as a true scalar field.

Action

We define the total action as

$$S_{total} = S_{EH} + S_{\psi} + S_m + S_{EM} + S_{noise} + S_{lock},$$

where $S_{noise}[g, \psi]$ is assumed diffeomorphism invariant, and the lock is implemented via a scalar multiplier $\lambda(x)$:

$$\begin{aligned} S_{EH} &= \int d^4x \sqrt{-g}\ \tfrac{R-2\Lambda}{2\kappa}, \\ S_{\psi} &= \int d^4x \sqrt{-g}\ [-\tfrac{Z_{\psi}}{2}(\nabla\psi)^2 - U(\psi)], \\ S_{lock} &= \int d^4x\ \lambda(x) \cdot \left[\omega(x)\, e^{\psi} - \sqrt{-g}\right]. \end{aligned}$$

Variational Definitions and Stress-Energy Sectors

For physical sectors (matter, electromagnetism, ψ, and noise), we define stress-energy by the Hilbert prescription:

$$T^{(i)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_i}{\delta g^{\mu\nu}}.$$

The lock multiplier is not treated as a separately conserved matter source; instead it produces a pure-trace reaction placed on the geometric side (see "Field Equations and Density-Safe Bianchi Closure).

$$T^{phys}_{\mu\nu} = T^{(m)}_{\mu\nu} + T^{(EM)}_{\mu\nu} + T^{(\psi)}_{\mu\nu} + T^{(noise)}_{\mu\nu}.$$

Scalar sector With $\mathcal{L}_{\psi} \supset -\frac{1}{2} Z_{\psi}(\nabla\psi)^2 - U(\psi)$, the scalar stress-energy is:

$$T^{(\psi)}_{\mu\nu} = Z_{\psi}\,(\nabla_{\mu}\psi)\,(\nabla_{\nu}\psi) - g_{\mu\nu}\left[\frac{Z_{\psi}}{2}(\nabla\psi)^2 + U(\psi)\right].$$

Correct Divergence Identity for the Scalar Stress-Energy

The divergence of the scalar stress-energy is algebraically fixed:

$$\nabla^{\mu} T^{(\psi)}_{\mu\nu} = [Z_{\psi}\,\Box\psi - U\prime(\psi)]\;\partial_{\nu}\psi.$$

Any alternative expression is a sign or index error. This identity is used directly in the Bianchi closure.

Noise Sector: Scalar Current and Ward Identity

We define the scalar noise current as the fully general Euler–Lagrange functional derivative normalized by $\sqrt{-g}$:

$$J_{noise} \equiv \frac{1}{\sqrt{-g}}\,\frac{\delta S_{noise}}{\delta\psi}.$$

The corresponding stress-energy is defined variationally:

$$T^{(noise)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\,\frac{\delta S_{noise}}{\delta g^{\mu\nu}}.$$

If S_{noise} is diffeomorphism invariant and depends on fields only through covariant objects, then the diffeomorphism Ward identity yields the exact exchange law:

$$\nabla^{\mu} T^{(noise)}_{\mu\nu} = J_{noise}\cdot\partial_{\nu}\psi.$$

No ad hoc metric-tightening term is required. Bookkeeping closes provided S_{noise} contains no explicit coordinate dependence and no hidden preferred-frame structures.

Field Equations and Density-Safe Bianchi Closure

Metric variation Varying the total action yields a pure-trace reaction from the lock term. Writing the equation in its cleanest form, we place this reaction on the geometric side as a shift of the cosmological term (note the sign correction required by the variational derivative of the density-safe lock):

$$G_{\mu\nu} + (\Lambda + \kappa\lambda)g_{\mu\nu} = \kappa\left[T^{(m)}_{\mu\nu} + T^{(EM)}_{\mu\nu} + T^{(\psi)}_{\mu\nu} + T^{(noise)}_{\mu\nu}\right].$$

Constraint and scalar variation Variation with respect to λ enforces the volume lock $\sqrt{-g} = \omega\, e^{\psi}$. Variation with respect to ψ yields:

$$\lambda = -\left(Z_{\psi}\,\Box\psi - U\prime(\psi) + J_{noise}\right).$$

Bianchi integrability Taking ∇^{μ} of the metric equation and using $\nabla^{\mu} G_{\mu\nu} = 0$ gives:

$$\partial_{\nu}\lambda - \left(Z_{\psi}\Box\psi - U'(\psi) + J_{noise}\right)\partial_{\nu}\psi = 0.$$

Substituting the ψ-variation identity for λ gives the correct closure condition:

$$\partial_{\nu}\lambda + \lambda\partial_{\nu}\psi = 0 \Longrightarrow \partial_{\nu}\left(\lambda e^{\psi}\right) = 0 \Longrightarrow \lambda e^{\psi} = \Lambda^{*} \Longrightarrow \lambda(x) = \Lambda^{*} e^{-\psi(x)}.$$

Λ^{*} is a global integration constant (a scalar). It removes local Λ-bubble freedom while preserving a conserved substrate tax that now properly scales inversely with admissibility ψ.

Clean Conformal Decomposition and Ghost-Free Bound

To avoid tensor-density contamination, we isolate the conformal factor as a scalar and assign the background density entirely to the determinant of the reference tensor:

$$g_{\mu\nu} = e^{\psi/2}\,\widetilde{g}_{\mu\nu}, \qquad \sqrt{\widetilde{g}} = \omega(x),$$

so that $\sqrt{-g} = e^{\psi}\sqrt{-\widetilde{g}} = \omega\, e^{\psi}$ exactly.

Under this decomposition, the Einstein–Hilbert action generates a wrong-sign kinetic term for ψ. After Weyl expansion and integration by parts:

$$\begin{aligned} S_{EH} &\supset \textstyle\int d^4x\,\sqrt{-\widetilde{g}}\;e^{\psi/2}\cdot[+\frac{3}{16\kappa}\,\left(\widetilde{\nabla}\psi\right)^2], \\ S_{\psi} &\supset \textstyle\int d^4x\,\sqrt{-\widetilde{g}}\;e^{\psi/2}\cdot[-\frac{Z_\psi}{2}\,\left(\widetilde{\nabla}\psi\right)^2]. \end{aligned}$$

Ghost-free stability requires the net coefficient of $\left(\widetilde{\nabla}\psi\right)^2$ to be negative:

$$\frac{3}{16\kappa} - \frac{Z_\psi}{2} < 0 \quad \Longleftrightarrow \quad Z_\psi > \frac{3}{8\kappa}.$$

Equivalently, defining the net healthy scalar stiffness as $Z_{eff} \equiv Z_\psi - 3/(8\kappa)$, ghost freedom requires $Z_{eff} > 0$.

This bound is the exact geometric stiffness floor for the ψ normalization used here.

Degrees of Freedom and Constraint Accounting

The reference tensor $\widetilde{g}_{\mu\nu}$ with $\sqrt{-\widetilde{g}}$ fixed by $\omega(x)$ is unimodular in the sense of a fixed determinant density, but this does not change the local propagating content of GR: the tensor sector still carries two graviton polarizations after diffeomorphism gauge fixing. The ψ field adds one additional propagating scalar degree of freedom (provided Z_ψ satisfies the ghost-free bound).

$$\text{Net propagating degrees of freedom: } 2\ \text{(tensor)} + 1\ \text{(scalar)} = 3.$$

Part II: Benchmark Phenomenological Bridge — Mapping ψ into the Gordon Optical Membrane

This section connects the core framework variables (ψ, λ, Z_ψ, Λ^*) to the operational Gordon Optical Membrane baseline used for shadow and lensing predictions. It establishes a single consistent symbol dictionary and declares what is "derived closure" versus "primitive field content."

$\Pi \leftrightarrow -\psi$ Identification

In this framework, the primitive macroscopic scalar is $\psi(x) := \ln\left(A_{adm}(x)/A_0\right)$. In the Optical Membrane notes, a scalar Π was introduced with boundary condition $\Pi(\infty) = 0$ and sign choice $\Pi\prime(r) < 0$ so that Π increases inward. For "strong field = reduced routing capacity" (i.e. A_{adm} falls inward so ψ becomes more negative inward), the consistent identification is:

$$\Pi(x) \equiv -\psi(x) \quad \text{with} \quad \psi(\infty) = 0 \Leftrightarrow \Pi(\infty) = 0.$$

Consequences:

$$
\begin{aligned}
\Pi\prime(r) &= -\psi\prime(r),\\
\Pi\prime(r) &= -G_{\max}\exp[-\tfrac{(r-r_{mem})^2}{2\sigma^2}] \Longrightarrow \psi\prime(r) = +G_{\max}\exp[-\tfrac{(r-r_{mem})^2}{2\sigma^2}],\\
\Pi(r) &= G_{\max}\sigma\sqrt{\tfrac{\pi}{2}}\; erfc(\tfrac{r-r_{mem}}{\sqrt{2}\,\sigma}),\\
\psi(r) &= -G_{\max}\sigma\sqrt{\tfrac{\pi}{2}}\; erfc(\tfrac{r-r_{mem}}{\sqrt{2}\,\sigma}).
\end{aligned}
$$

So $\psi \leq 0$ inward, matching "admissibility density drops" (since $A_{adm} = A_0\, e^{\psi} < A_0$).

Notation Collision Resolution

Four symbol collisions exist across the original documents. The following minimal-surgery renaming eliminates all collisions without rewriting the framework spine:

#	Symbol	Framework spine	Membrane docs	Resolution
1	A	Admissibility density	Photon lapse	Photon lapse $\to \alpha_\gamma(r)$
2	λ	Lagrange multiplier	Refractive coupling; affine param	Coupling $\to g_n$; affine $\to \ell$
3	κ	Gravitational: $8\pi G/c^4$	Opacity coefficient	Opacity $\to \kappa(r)$
4	Λ^*	Global integration const.	(Not used; collision risk)	Reserve for spine

The benchmark bridge constitutive ansätze in unified notation:

$$
\ln n(r) = g_n\Pi(r) = -g_n\psi(r) \Longrightarrow n(r) = \exp\left(g_n\Pi(r)\right) = \exp\left(-g_n\psi(r)\right).
$$

Radiative transfer along rays:

$$
\frac{dI}{d\ell} = -\kappa(r, \omega, pol)\cdot I.
$$

Sector Split (Benchmark Bridge Premise)

We adopt a two-sector approximation:

(1) Matter (timelike sector): massive matter follows the Schwarzschild exterior exactly:

$$
ds_m^2 = -\alpha_m(r)\, c^2\, dt^2 + \alpha_m(r)^{-1}\, dr^2 + r^2\, d\Omega^2, \qquad \alpha_m(r) = 1 - R_s/r.
$$

(2) Radiation (null sector): electromagnetic rays propagate in an effective optical medium (Gordon metric) with refractive index $n(r)$ and absorption $\kappa(r)$ localized to the strong-field region:

$$
\alpha_\gamma(r) := \frac{\alpha_m(r)}{n(r)^2}, \qquad ds_\gamma^2 = -\alpha_\gamma(r)\, c^2\, dt^2 + \alpha_m(r)^{-1}\, dr^2 + r^2\, d\Omega^2.
$$

Interpretive Status of the Optical Bridge

The optical constitutive relations introduced in Part II are not claimed to be uniquely derived microphysical outputs of the substrate. Their status is narrower and more disciplined. They should be read as the lowest-order phenomenological closure compatible with a ψ- dependent electromagnetic response in the geometric-optics regime, constructed for the specific purpose of testing whether a minimal admissibility-driven membrane sector can reproduce benchmark-facing observables without breaking the covariant spine.

Accordingly, the optical bridge is not presented as an arbitrary add-on, but neither is it advertised as a completed derivation. It is an effective constitutive ansatz constrained by covariance, regularity, benchmark utility, and kill-criteria exposure. The scientific role of the bridge is to determine whether the continuum admissibility sector can support a finite observational membrane with nontrivial refractive and absorptive structure. A more microscopic derivation of these constitutive laws remains an open task.

Optical Constitutive Ansätze for the Benchmark Bridge

The constitutive profiles used below are chosen as the minimal smooth functions capable of representing a ψ-driven optical membrane at lowest nontrivial order. They are therefore best understood as an effective truncation rather than as unique microscopic laws. In this sense, $n(r)$, $\kappa(r)$, and related response functions parameterize the leading electromagnetic imprint of the admissibility sector in the geometric-optics limit. Higher-order derivative corrections, nonlocal response terms, and alternative constitutive closures are expected in a more complete treatment and should ultimately be constrained by observation rather than assumed away.

$$\begin{array}{ll} \text{Refractive closure:} & \ln n(r) = g_n \Pi(r) = -g_n \psi(r) \Longrightarrow n(r) = \exp\left(g_n \Pi(r)\right). \\ \text{Absorption closure:} & \kappa(r) = \kappa_{pk} \exp[-q\,\frac{(r-r_{mem})^2}{2\sigma^2}], \quad q \geq 1. \end{array}$$

The scientific burden is therefore transparent: if the benchmark-facing features require constitutive structure that becomes excessively fine-tuned, non-smooth, or dynamically unstable, the optical bridge fails as a credible phenomenological closure. The present ansatz is intended precisely to expose that burden in a controlled way.

Engineered Membrane Profile (Explicit ψ-Background)

The benchmark membrane profile is defined by a compact-support-like gradient profile:

$$\begin{array}{lll} \Pi\prime(r) & = -G_{\max} \exp[-\frac{(r-r_{mem})^2}{2\sigma^2}], & \Pi(\infty) = 0, \\ \Pi(r) & = G_{\max}\sigma\sqrt{\frac{\pi}{2}}\; erfc(\frac{r-r_{mem}}{\sqrt{2}\,\sigma}), & \psi(r) = -\Pi(r). \end{array}$$

Dimensionless Parameter Dictionary

$$\begin{array}{cc} u & := r_{mem}/R_s, \qquad s := \sigma/R_s, \\ \varepsilon & := g_n G_{\max} R_s \quad \text{(peak-gradient coupling)}, \\ \chi & := g_n \Pi\left(r_{mem}\right) = g_n G_{\max}\sigma\sqrt{\frac{\pi}{2}} = \sqrt{\frac{\pi}{2}} \cdot \varepsilon \cdot s, \\ n_{mem} & := n\left(r_{mem}\right) = e^{\chi}, \\ K & := \kappa_{pk} \cdot \sigma \quad \text{(opacity strength)}. \end{array}$$

Benchmark Merge Specialization (Photon Sphere Pinned to Membrane)

$$2\varepsilon\, u^2 - (2\varepsilon + 2)\, u + 3 = 0.$$

Physical branch (smooth GR limit as $\varepsilon \to 0$):

$$u(\varepsilon) = \frac{(1+\varepsilon) - \sqrt{(1+\varepsilon)^2 - 6\varepsilon}}{2\varepsilon},$$

with $u \to 3/2$ when $\varepsilon \to 0$. The discriminant requires $\varepsilon \leq 2 - \sqrt{3} \approx 0.2679$.

Part III: Gordon Optical Membrane — Schwarzschild Baseline

Benchmark bridge status: timelike geodesics remain unmodified; the null sector is modified via refractive index + absorption. The optical constitutive laws are ansätze, and the photon-sphere pinning used below is a benchmark specialization rather than a theorem of the spine. All notation follows the unified dictionary established in Part II.

Matter Sector (Timelike Geodesics): Exact Schwarzschild

Define the Schwarzschild radius $R_s = 2GM/c^2$. The matter sector uses the standard exterior metric:

$$\alpha_m(r) = 1 - R_s/r, \qquad ds_m^2 = -\alpha_m(r)\, c^2\, dt^2 + \alpha_m(r)^{-1}\, dr^2 + r^2\, d\Omega^2.$$

Horizon (matter sector): $r_h = R_s$. Photon sphere in GR (reference): $r_{ph}^{GR} = \frac{3}{2} R_s$. Critical impact parameter in GR (reference): $b_{c,GR} = \frac{3\sqrt{3}}{2}\, R_s$.

This sector is intentionally left untouched: any exotic strong-field behavior is pushed into the optical sector, so the baseline gravitational dynamics for massive matter remains Schwarzschild.

Optical Sector (Null Geodesics): Gordon Metric + Absorption

Model EM propagation as rays in a static, isotropic refractive medium comoving with static Schwarzschild observers. Let $n(r)$ be the (isotropic) refractive index profile. The effective optical line element (Gordon form, specialized to this gauge) is:

$$ds_\gamma^2 = -\alpha_\gamma(r)\, c^2\, dt^2 + \alpha_m(r)^{-1}\, dr^2 + r^2\, d\Omega^2, \qquad \alpha_\gamma(r) := \frac{\alpha_m(r)}{n(r)^2}.$$

Radiation stripping / thermalization is implemented by intensity transport along rays:

$$\frac{dI}{d\ell} = -\kappa(r, \omega, pol) \cdot I.$$

The "peeling membrane" is optically thick when its optical depth $\tau_{mem} \gg 1$.

Exact Photon-Sphere Condition in the Optical Metric

For any static, spherically symmetric metric $ds^2 = -A(r)\, c^2\, dt^2 + B(r)\, dr^2 + r^2\, d\Omega^2$, an unstable circular null orbit satisfies $d/dr\left(A/r^2\right) = 0$, equivalently $r\, A\prime(r) - 2A(r) = 0$. With $A(r) \to \alpha_\gamma(r) = \alpha_m(r)/n(r)^2$:

Photon-sphere equation (exact):

$$r\, \alpha_\gamma\prime(r) - 2\, \alpha_\gamma(r) = 0.$$

Expanding with $\alpha_\gamma = \alpha_m/n^2$:

$$r\, [\alpha_m\prime(r) - 2\alpha_m(r) \cdot n\prime(r)/n(r)] - 2\alpha_m(r) = 0.$$

For $\alpha_m(r) = 1 - R_s/r$ (so $\alpha_m\prime(r) = R_s/r^2$), at $r = r_{ph}$:

$$\frac{3R_s}{r} - 2 = 2r\left(1 - \frac{R_s}{r}\right) \cdot \frac{n\prime(r)}{n(r)}.$$

Shadow Scale in the Optical Metric

The critical impact parameter b_c is defined by the unstable null orbit and sets the shadow edge. For the optical lapse $\alpha_\gamma = \alpha_m/n^2$:

$$b_c^2 = \frac{r_{ph}^2}{\alpha_\gamma(r_{ph})} = \frac{r_{ph}^2\, n(r_{ph})^2}{\alpha_m(r_{ph})}, \qquad b_c = \frac{r_{ph} \cdot n(r_{ph})}{\sqrt{1 - R_s/r_{ph}}}.$$

Gaussian Pressure-Gradient Membrane and Near-Compact Support

Introduce the PVB admissibility-pressure scalar $\Pi(r) = \psi(r)$. The membrane is defined by the maximum of the pressure-gradient magnitude $G(r) := |\Pi\prime(r)|$. Choose a Gaussian gradient so the optical response is localized and analytic:

$$G(r) := |\Pi\prime(r)| = G_{\max} \exp[-\frac{(r - r_{mem})^2}{2\sigma^2}].$$

Fix $\Pi(\infty) = 0$. Then:

$$\Pi(r) = G_{\max}\sigma\sqrt{\frac{\pi}{2}}\; erfc(\frac{r - r_{mem}}{\sqrt{2}\,\sigma}).$$

Refractive coupling (using unified notation):

$$\ln n(r) = g_n \Pi(r) = -g_n \psi(r) \Longrightarrow n(r) = \exp\left(g_n \Pi(r)\right).$$

Near-compact support: for $r \gg r_{mem} + \text{few} \cdot \sigma$, $erfc(\dots) \to 0$ and $n(r) \to 1$.

Opacity Spike and Stripping Criterion

Tie absorption to the same localized structure. Let $q \geq 1$:

$$\kappa(r, \omega, pol) = \kappa_{pk}(\omega, pol) \cdot \exp[-q\,\frac{(r - r_{mem})^2}{2\sigma^2}].$$

Optical depth through the membrane:

$$\tau_{mem} \approx \int \frac{\kappa(r)}{\sqrt{1 - R_s/r}}\, dr.$$

Stripping/thermalization regime: $\tau_{mem} \gg 1$.

Benchmark Merge Specialization: Photon Sphere Pinned to Membrane

As a benchmark specialization, set $r_{ph} = r_{mem}$. Since $\Pi\prime(r_{mem}) = -G_{\max}$, we have $n\prime(r_{mem})/n(r_{mem}) = g_n \Pi\prime(r_{mem}) = -g_n G_{\max}$. Substituting into the exact photon-sphere equation:

$$\frac{3R_s}{r_{mem}} - 2 + 2 g_n G_{\max} r_{mem} \left(1 - \frac{R_s}{r_{mem}}\right) = 0.$$

Quadratic form (using $u = r_{mem}/R_s$ and $\varepsilon = g_n G_{\max} R_s$):

$$2\varepsilon\, u^2 - (2\varepsilon + 2)\, u + 3 = 0.$$

Physical branch (smooth GR limit as $\varepsilon \to 0$):

$$u(\varepsilon) = \frac{(1+\varepsilon) - \sqrt{(1+\varepsilon)^2 - 6\varepsilon}}{2\varepsilon}.$$

Small-ε expansion:

$$u = \frac{3}{2}\left(1 + \frac{1}{2}\varepsilon + O\left(\varepsilon^2\right)\right).$$

Reality condition (this branch):

$$(1+\varepsilon)^2 - 6\varepsilon \geq 0 \Longrightarrow \varepsilon \leq 2 - \sqrt{3} \approx 0.2679.$$

Closed Expression for the Shadow Scale

In this benchmark-specialized configuration $r_{ph} = r_{mem}$, the shadow scale depends on (i) the membrane radius r_{mem} fixed by ε, and (ii) the index value at the membrane fixed by the integrated Gaussian profile.

From the Gaussian integral with $\Pi(\infty) = 0$:

$$\Pi\left(r_{mem}\right) = G_{\max}\sigma\sqrt{\frac{\pi}{2}}\ erfc(0) = G_{\max}\sigma\sqrt{\frac{\pi}{2}}.$$

Therefore:

$$n\left(r_{mem}\right) = \exp(g_n G_{\max}\sigma\sqrt{\frac{\pi}{2}}) = e^{\chi}.$$

Shadow edge (critical impact parameter):

$$b_c = \frac{r_{mem} \cdot e^{\chi}}{\sqrt{1 - R_s/r_{mem}}}.$$

Closed-form prediction:

$$b_c(\varepsilon, s) = r_{mem}(\varepsilon) \cdot \frac{\exp\left(\sqrt{\pi/2} \cdot \varepsilon \cdot s\right)}{\sqrt{1 - R_s/r_{mem}(\varepsilon)}}$$

where $\varepsilon = g_n G_{\max} R_s$ and $r_{mem}(\varepsilon) = R_s \cdot \left[(1+\varepsilon) - \sqrt{(1+\varepsilon)^2 - 6\varepsilon}\right]/(2\varepsilon)$.

To preserve weak-field constraints, choose $\sigma \ll R_s$ and ensure $n(r)$ returns to 1 rapidly outside a few σ.

Part IV: Kill-Criteria Map and Prediction Sheet

Companion to the benchmark bridge baseline. All notation follows the unified dictionary.

Kill-Criteria: Geometry

Reality of $u(\varepsilon)$: discriminant $\geq 0 \Rightarrow (1+\varepsilon)^2 - 6\varepsilon \geq 0$. For the chosen physical branch, this implies $\varepsilon \in (-\infty, 2-\sqrt{3}]$ (numerically ≤ 0.2679) or ε in a large positive band where this branch drives $u < 1$ (unphysical for an external membrane).

External membrane requirement: $u > 1$ (i.e. r_{mem} outside the matter horizon R_s). For the near-GR branch this is satisfied for all $\varepsilon \leq 0.2679$.

Kill-Criteria: Weak-Field Contamination

Outside the membrane at $r = r_{mem} + k\sigma$:

$$\Pi(r) = \Pi\left(r_{mem}\right) \cdot erfc\left(k/\sqrt{2}\right) \Longrightarrow \ln n(r) = \chi \cdot erfc\left(k/\sqrt{2}\right).$$

Choose a target tolerance δ_{weak} and enforce $|\chi| \cdot erfc\left(k/\sqrt{2}\right) < \delta_{weak}$.

k	$erfc\left(k/\sqrt{2}\right)$
3	2.70×10^{-3}
4	6.33×10^{-5}
5	5.73×10^{-7}
6	1.97×10^{-9}
7	2.56×10^{-12}
8	1.24×10^{-15}

Kill-Criteria: Stripping Strength (Optical Depth)

Approximate optical depth across the membrane ($\sigma \ll r_{mem}$ so α_m varies slowly):

$$\tau_{mem} \approx \frac{\kappa_{pk}}{\sqrt{\alpha_m\left(r_{mem}\right)}} \int \exp[-\frac{q\,x^2}{2\sigma^2}]dx \approx \frac{\kappa_{pk}}{\sqrt{1-1/u}} \cdot \sigma\sqrt{2\pi/q}.$$

Using $K := \kappa_{pk} \cdot \sigma$:

$$\tau_{mem} \approx \frac{K\sqrt{2\pi/q}}{\sqrt{1-1/u}}.$$

Stripping regime: $\tau_{mem} \gg 1$.

Practical Sanity Window

Avoid u too close to 1 (membrane hugging the horizon) unless you explicitly want extreme redshift amplification. A simple guard is $\alpha_m\left(r_{mem}\right) = 1 - 1/u \geq \alpha_{\min}$.

Example: $\alpha_{\min} = 0.1 \Rightarrow u \geq 1/0.9 \approx 1.111$, corresponding roughly to $\varepsilon \approx -3$ (order-of-magnitude).

Shadow sensitivity: in the near-GR branch, b_c shifts primarily through e^{χ}. So χ is the clean "observable knob" (percent-level shifts correspond to $\chi \approx$ percent).

One-Page Prediction Sheet (VLBI Constraints)

Observable: shadow scale b_c (or angular diameter $2b_c/D$ if distance D is known).

Benchmark-specialization prediction:

$$\frac{b_c}{R_s} = \frac{u(\varepsilon) \cdot \exp\left(\sqrt{\pi/2} \cdot \varepsilon \cdot s\right)}{\sqrt{1 - 1/u(\varepsilon)}}, \qquad u(\varepsilon) = \frac{(1+\varepsilon) - \sqrt{(1+\varepsilon)^2 - 6\varepsilon}}{2\varepsilon}.$$

GR reference: $b_{c,GR} = \frac{3\sqrt{3}}{2} R_s$.

Near-GR regime (small ε, small χ):

$$\frac{b_c}{b_{c,GR}} \approx e^{\chi} \cdot \left[1 + O\left(\varepsilon^2\right)\right].$$

(The geometric factor $u/\sqrt{1-1/u}$ has a stationary point at $u = 3/2$, so the first-order shift in ε vanishes.)

$$\delta_b \approx \chi = \sqrt{\frac{\pi}{2}} \cdot \varepsilon \cdot s.$$

So a measurement of δ_b constrains the integrated bridge quantity χ within this benchmark specialization, while independent physics (or modeling of the activation) would be needed to split χ into ε and s separately.

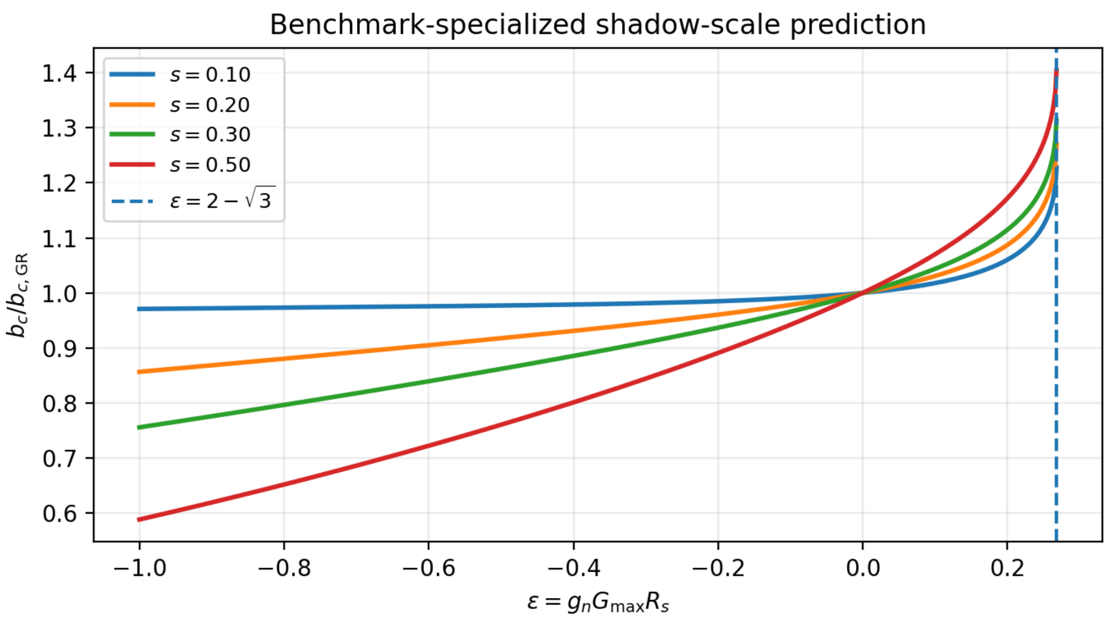

Figure 1: Dimensionless shadow-scale prediction for the benchmark-specialized Gordon optical membrane model, regenerated from the closed-form prediction in the manuscript. Curves show the ratio $b_c/b_{c,\mathrm{GR}}$ versus $\varepsilon = g_n G_{\max} R_s$ for representative membrane widths $s = \sigma/R_s \in \{0.10, 0.20, 0.30, 0.50\}$. The dashed vertical line marks the reality limit $\varepsilon = 2 - \sqrt{3} \approx 0.2679$ for the near-GR branch.

Part V: Phenomenology, Engineering, and Structural Kill Criteria

From the framework spine. These apply to the full framework, not just the optical membrane sector.

Phenomenology and Computable Observables

Effective scalar mass Around a background $\psi = \psi_0$ with small perturbations $\delta\psi$, the effective mass scale is:

$$m_\psi^2 \sim \frac{U''(\psi_0)}{Z_{eff}},$$

where $Z_{eff} \equiv Z_\psi - 3/(8\kappa)$ is the net healthy kinetic stiffness after subtracting the geometric liability.

Fifth-force Yukawa mapping (template) If $\delta\psi$ couples universally to the trace sector through the volume lock and minimal coupling, the Newtonian potential acquires a Yukawa correction:

$$V(r) = -\frac{G\, m_1 m_2}{r} \cdot \left[1 + \alpha_5\, e^{-m_\psi r}\right],$$

with α_5 determined by the ψ-to-matter coupling strength. The model is killed if current fifth-force bounds exclude the implied (α_5, m_ψ) region given the required stiffness floor $Z_\psi > 3/(8\kappa)$.

Cosmology If ψ is constant, then λ is constant and the reaction term behaves as an effective cosmological constant (with Λ^* fixed globally by the Bianchi closure). If ψ evolves, then λ is slaved by Bianchi closure to $\lambda(x) = \Lambda^* e^{-\psi(x)}$, i.e., it varies inversely with admissibility, so the vacuum tax is not locally constant.

Noise-driven decoherence The noise sector induces exchange via $\nabla^\mu T^{(noise)}_{\mu\nu} = J_{noise}\, \partial_\nu \psi$. When mapped to an open-system description, the decoherence structure is constrained: if empirical neutrino decoherence is definitively non-dephasing (incompatible with admissibility-gated dephasing), the framework is killed.

Engineering Implication: The Stiffness Tax on Any "Admissibility Pump"

The ghost-free bound enforces a minimum kinetic stiffness. The net healthy stiffness is $Z_{eff} = Z_\psi - 3/(8\kappa) > 0$. For any attempt to generate controlled spatial gradients in ψ (a proxy for "squeezing" routing capacity), the gradient energy density scales as:

$$\rho_{grad} \sim \frac{Z_{eff}}{2}\, (\nabla\psi)^2.$$

Any admissibility pump must overcome a structural resistance set by Z_{eff}. Viable engineering pathways (if any) must exploit resonant, collective, or boundary-mediated channels that generate effective routing modulation without paying the full bulk gradient cost.

Built-in Kill Criteria (Structural)

Criterion	Kill Condition	What to Compute
Fifth-force bounds	Yukawa deviations incompatible with required Z_ψ and implied coupling	Derive α_5, m_ψ from the matter sector and compare to current constraints
Noise / decoherence structure	Observed decoherence is non-dephasing / incompatible with admissibility-gated exchange	Map S_{noise} to GKSL structure and compare to neutrino/oscillation data

Criterion	Kill Condition	What to Compute
Internal consistency	ψ varies while λ treated as locally constant; or Ward identity violated by S_{noise}	Verify $\lambda = \Lambda^* e^{-\psi}$ and $\nabla^\mu T^{(noise)}_{\mu\nu} = J_{noise}\,\partial_\nu\psi$ for chosen kernel
Shadow scale (optical membrane)	VLBI measurement of δ_b inconsistent with predicted χ range, or weak-field contamination exceeds tolerance	Compute $b_c/b_{c,GR}$ for allowed (ε, s) parameter space and compare to EHT/ngEHT data
Geometry (optical membrane)	$u(\varepsilon)$ imaginary or $u < 1$ (membrane inside horizon)	Check reality condition $(1+\varepsilon)^2 - 6\varepsilon \geq 0$ and external membrane $u > 1$

Bianchi Closure Derivation

Start from the metric equation written with the trace reaction on the geometric side:

$$G_{\mu\nu} + (\Lambda + \kappa\lambda)g_{\mu\nu} = \kappa\left[T^{(m)}_{\mu\nu} + T^{(EM)}_{\mu\nu} + T^{(\psi)}_{\mu\nu} + T^{(noise)}_{\mu\nu}\right].$$

Taking ∇^μ and using $\nabla^\mu G_{\mu\nu} = 0$ and $\nabla^\mu g_{\mu\nu} = 0$:

$$\partial_\nu\lambda - \nabla^\mu\left[T^{(\psi)}_{\mu\nu} + T^{(noise)}_{\mu\nu}\right] = 0,$$

assuming minimal coupling ($\nabla^\mu T^{(m)}_{\mu\nu} = 0$ and $\nabla^\mu T^{(EM)}_{\mu\nu} = 0$).

Using the divergence identities:

$$\partial_\nu\lambda - \left(Z_\psi\Box\psi - U\prime(\psi) + J_{noise}\right)\partial_\nu\psi = 0.$$

With the ψ-variation identity $\lambda = -\left(Z_\psi\Box\psi - U\prime(\psi) + J_{noise}\right)$:

$$\partial_\nu\lambda + \lambda\partial_\nu\psi = 0$$

$$\partial_\nu\left(\lambda e^{\psi}\right) = 0$$

$$\lambda = \Lambda^* e^{-\psi}.$$

Conformal Decomposition and the Geometric Kinetic Coefficient

Let $g_{\mu\nu} = e^{2\sigma}\widetilde{g}_{\mu\nu}$ with $\sigma = \psi/4$. Then in 4D:

$$\begin{aligned}\sqrt{-g} &= e^{4\sigma}\sqrt{-\widetilde{g}} = e^{\psi}\sqrt{-\widetilde{g}},\\ R &= e^{-2\sigma}\left[\widetilde{R} - 6\widetilde{\Box}\sigma - 6\left(\widetilde{\nabla}\sigma\right)^2\right].\end{aligned}$$

Insert into $S_{EH} = \int d^4x\,\sqrt{-g}\,R/(2\kappa)$ and integrate the $\widetilde{\Box}\sigma$ term by parts. The kinetic piece becomes:

$$S_{EH} \supset \int d^4x \sqrt{-\widetilde{g}}\, e^{\psi/2} \cdot \frac{3}{16\kappa} \left(\widetilde{\nabla}\psi\right)^2 .$$

This term has the wrong sign relative to a healthy scalar kinetic term in mostly-plus signature.

Noise Ward Identity (Operational Form)

Assume $S_{noise}[g, \psi]$ is diffeomorphism invariant. Under an infinitesimal diffeo generated by ξ^ν:

$$\delta g_{\mu\nu} = -\nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu, \qquad \delta\psi = \xi^\nu \partial_\nu \psi.$$

Invariance $\delta S_{noise} = 0$ implies:

$$0 = \int d^4x \sqrt{-g} \left[-T^{(noise)}_{\mu\nu} \nabla^\mu \xi^\nu + J_{noise}\, \xi^\nu \partial_\nu \psi \right].$$

Integrating by parts and using arbitrariness of ξ^ν:

$$\nabla^\mu T^{(noise)}_{\mu\nu} = J_{noise}\, \partial_\nu \psi.$$

Open Derivation Targets

The present manuscript establishes a covariant effective spine and a benchmark-facing phenomenological bridge, but several derivational burdens remain open.

1. **Substrate-to-continuum coarse-graining.** A concrete statistical ensemble of DAG histories should be specified, together with a mesoscopic definition of routing capacity and a derivation of the volume-lock relation from equilibrium, extremal, or consistency principles.
2. **Matter coupling and force constraints.** The coupling of ψ to standard matter sectors must be made explicit in order to compute fifth-force bounds, screening behavior, and post-Newtonian consistency.
3. **Electromagnetic effective action.** The optical constitutive bridge should be derived from a controlled effective action for electromagnetism in the admissibility background, including higher-order corrections and possible anisotropic response terms.
4. **Cosmological sector.** Homogeneous and perturbed cosmological solutions should be studied to determine whether the admissibility lock remains viable away from the static and benchmark-facing regimes considered here.
5. **Kill-criteria refinement.** The framework should continue to be tested against explicit failure modes, including closure breakdown, benchmark degeneracy, fine-tuning dependence, and phenomenological indistinguishability from more conventional effective models.

These are not cosmetic extensions. They define the actual program required to determine whether the admissibility framework is merely an internally coherent effective model or the visible tip of a deeper microscopic theory.

Reference Notes and Context

The integrability constant Λ^* and the determinant constraint structure extend the role of an integration constant in unimodular gravity, where the cosmological constant arises as a constant of integration rather than a fixed coupling. The present framework goes beyond the unimodular program by coupling the determinant constraint to a dynamical scalar field, yielding local vacuum response through Bianchi closure. The optical membrane sector draws on the Gordon-metric formalism for light propagation in effective media, but embeds it within a covariant action with explicit constitutive ansätze rather than treating it as a standalone kinematic analogy. The ghost-free bound addresses the conformal mode instability identified by Gibbons, Hawking, and Perry. The framework's falsifiability constraints are designed to interface with the current experimental gravity program and with modified-gravity formalisms. For the emergent-spacetime context, see the causal-set and quantum-graphity references below.

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