

TORUS Ladder Dynamics I–XIV: Unified Structural Analysis of Emergent Time and Scale

Abstract: We present a consolidated analysis of the TORUS Ladder Dynamics I–XIV series, demonstrating that **emergent time** (T_e) and **emergent scale** (S_e) are robust, observer-independent invariants across diverse complex systems. Using a recursive differencing (“ladder”) transformation, we find a highly significant parent–null separation effect (exceeding 12σ confidence) in ten distinct structured domains spanning cosmology, quantum spectra, biology, chemistry, and more, while two carefully designed control domains show only transient or no separation. Structured datasets maintain a distinguishable ladder signature up to deep recursive layers ($N=14$ in our tests), whereas controls yield at most a fragile, short-lived signal or none at all. We define **emergent time** (T_e) as the earliest recursion depth at which a statistically significant structure–null separation first appears, and **emergent scale** (S_e) as the persistence (in recursion depth) of that separation under structured perturbations. In our experiments, T_e consistently occurs at a shallow layer (typically $N \approx 6$), reflecting a universal topological “noise floor,” while S_e captures each domain’s structural rigidity – ranging from full-depth survival in ten domains to rapid decay in controls. All findings are obtained under preregistered, cross-validated protocols with no post hoc tuning. We provide a comprehensive artifact manifest (data inputs, analysis outputs, audits) with cryptographic hashes to facilitate full reproducibility. These results establish emergent time and scale as quantitative, cross-domain markers of non-random structure in ordered systems, and this manuscript is written as a standalone reference consolidating the evidence for these phenomena.

Introduction

Many physical and informational phenomena exhibit patterns across multiple scales, a principle famously explored by renormalization group (RG) theory and multiresolution analysis in signal processing. RG methods (pioneered by K. Wilson) seek continuous **scale-invariance** – laws that hold under rescaling – and have illuminated critical phenomena by identifying fixed points in a space of transformations. Wavelet transforms similarly decompose data into multiscale components, emphasizing self-similarity across scales. By contrast, **TORUS Ladder Dynamics** focuses on *discrete scale-specific invariants* – structural patterns that emerge at particular *recursion depths* rather than persisting identically at all scales. In other words, instead of looking for symmetries under arbitrary scale transformations, TORUS asks whether distinctive structure “kicks in” or survives at certain resolution levels in an ordered dataset. This approach detects *when and how* ordered systems develop internal agreements that distinguish them from randomness at a given scale of observation.

The TORUS framework is grounded in a stationary-action principle on a discrete recursive sequence: it posits that certain quantized structures (including fundamental physical parameters) can arise as fixed points of a **14-layer recursive cycle**, analogous to stationary points of an action in classical mechanics. In TORUS theory, a ladder’s closed or recurrent state corresponds to an extremum in a suitably defined “action” over the space of ladder configurations, yielding stable structures that repeat every 14 recursions (a full cycle) in the sequence ¹. Practically, this means the number of recursive layers is chosen as 14, the theoretically predicted length of a self-consistent toroidal sequence linking the smallest and largest scales

of a system. A fully closed ladder at $N=14$ represents a hypothesized toroidal recursion that connects back to its starting point, enforcing cross-scale consistency. The Ladder Dynamics experiments were designed to test for signatures of such structure in real data, without assuming the outcome in advance.

Over a series of fourteen studies (TORUS Ladder Dynamics I–XIV), we investigated whether this kind of recursive structure–null separation **exists** and is **reproducible across domains**. Earlier releases in the series established the phenomenon and its basic properties: TORUS Ladder Dynamics I: *Structural Escape, Damped Healing, and Ringing Diagnostics* demonstrated that non-trivial ladder “closure” effects do occur and introduced initial diagnostics of stability and oscillatory behavior. TORUS Ladder Dynamics II: *Mechanism, Structure, and Stability Limits* identified underlying structural mechanisms and limits of stability for ladder closure. TORUS Ladder Dynamics III: *Structural Invariance, Brittleness, and Failure Thresholds* showed that the effect persists under many transformations of the data but is brittle under certain perturbations, foreshadowing the concept of structural fragility. TORUS Ladder Dynamics IV: *Emergent Recurrence from Order Mutation* explored how shuffling or permuting the input order can disrupt a previously closed ladder (demonstrating that the recurrence is tied to specific ordering). TORUS Ladder Dynamics V: *Closure Discrimination via Calibrated Recurrence Gates* introduced rigorous statistical criteria (pre-registered thresholds, or “gates”) to distinguish genuine closures from false positives, thereby establishing a high-confidence detection methodology.

Subsequent releases expanded the scope and generality of the findings. TORUS Ladder Dynamics VI: *Closure, Topology, and Emergent Relational Modes* and VII: *Cross-Ladder Universality Under Preregistered Eligibility Constraints* extended the analysis to composite ladders and cross-verified the phenomenon under strict preregistration and eligibility rules, suggesting the effect is not limited to any single domain or data construction. TORUS Ladder Dynamics VIII: *Cross-Domain Tests of Closure Universality in Ordered Constraint Systems* tested the ladder effect in entirely new domains (including fundamental constant datasets and novel ordered systems), confirming that the recursion-based separation is not a fluke of one type of data. By TORUS Ladder Dynamics XII: *Emergent Time and Scale from Survival Surfaces*, the program formalized the two key observables that are the focus of this paper – emergent time (T_e) and emergent scale (S_e) – to quantify each domain’s ladder behavior in a uniform way. Finally, TORUS Ladder Dynamics XI: *Causal Stress-Testing of Closure Universality via Deterministic Structural Perturbations*, XIII: *Comparative Universality and Cross-Domain Coherence*, and XIV: *Final Validation of Existing Observables Under Compound Perturbation* subjected the ladders to systematic stress-tests (deterministic perturbations, compound multi-perturbations, out-of-sample predictions), demonstrating causal relationships and verifying that the observed ladder invariants hold under aggressive attempts to break them.

In this capstone paper, we consolidate the findings of releases I–XIV into a single analysis pipeline and cross-domain validation of emergent time and scale. We use a consistent methodology to reproduce the ladder effect in twelve representative domains (ten structured systems and two controls) and measure T_e and S_e for each. The goal is to present a **self-contained statistical validation** of the ladder dynamics phenomenon with rigorous cross-domain comparison. We explicitly define all required terms and methods, apply them uniformly across domains, and report the results in a comparative framework. By doing so, we establish emergent time and scale as well-defined, quantitative invariants, and we highlight the empirical “survival envelope” of each system (whether it shows persistent structure through all 14 layers, a finite partial persistence, or none). The remainder of this paper is organized as follows: **Methods** introduces the ladder construction, metrics, and criteria for separation; **Results** presents the cross-domain outcomes, including survival plots and emergent time/scale measurements for each domain; **Discussion**

interprets the significance of these invariants and examines domain-specific behaviors; and a final **Reference** section lists all prior TORUS Ladder Dynamics studies and data sources for completeness.

Methods and Definitions

Ladder Construction and Recursion

At the heart of each TORUS Ladder Dynamics experiment is the construction of a **ladder** from an input sequence of values. Let $\mathbf{x} = [x_1, x_2, \dots, x_m]$ be an ordered sequence representing a given domain (for example, a list of measured values sorted by some physical parameter). The **parent ladder** (depth 0) is just this ordered sequence itself, $L^0(i) = x_i$. We then define a recursive differencing operation to generate successive **ladder layers**. Formally, the ladder at recursion depth $n+1$ is obtained from the ladder at depth n by taking absolute differences between adjacent entries:

$$L^{n+1}(i) = |L^n(i) - L^n(i+1)|,$$

for all applicable indices i (e.g. $i = 1, 2, \dots$ up to the length of L^n minus 1). This operation is applied iteratively, producing a hierarchy of difference patterns L^1, L^2, \dots, L^N up to a chosen maximum depth N . In our unified analysis we use $N=14$ layers, aligning with the TORUS-theoretical full cycle. Intuitively, each ladder layer L^n captures the *scale-by-scale structure* of the original data: the first layer L^1 is the pattern of local differences in the data, the second layer L^2 is differences-of-differences (highlighting structure on the next coarser scale), and so on. If the original sequence contains non-random structure across multiple scales, it may still be discernible even at higher n ; if not, the ladder values will eventually become indistinguishable from those of a randomized control.

It is important to note that the **ladder state vector** (often denoted ω in TORUS documentation) refers strictly to the ordered set of rung values at a given depth (e.g. $\omega = L^0$ for the input state). We avoid using ω for anything other than this ordered state. In particular, any previous use of “ Ω ” or “ ω ” to imply an emergence depth or time-like parameter is deprecated and should be understood as referring to T_e instead ² ³. Likewise, in earlier reports the term “closure scale” was sometimes used to describe a particular recursion depth where the ladder would close; in this paper we reserve the word *scale* exclusively for S_e (the persistence depth of separation). The ladder’s **closure mode** – i.e. the specific recursion depth at which a ladder’s internal error or variance is minimized – is instead termed the **harmonic mode** and denoted N_{winner} (or *winner_N* for short) ⁴ ⁵. This N_{winner} is essentially a label for how the ladder “resolves” when it converges. It is *not* the same as emergent time or emergent scale. In practice, many of the structured ladders we studied tend to close around a harmonic mode of $N_{\text{winner}} = 10$ (an interior rung), rather than trivially at the boundaries, as first observed in TORUS Ladder Dynamics I. However, N_{winner} varies by domain and projection, and two systems might share the same T_e while closing at different N_{winner} values. For clarity, we treat harmonic mode as a separate structural property; it will not be a primary focus of this paper, which centers on the *existence* and *persistence* of the separation effect rather than the exact rung where a ladder’s recursion might eventually terminate.

Statistical Separation Metrics and Criteria

To rigorously determine whether the **ladder** of a given dataset exhibits non-random structure at each depth, we compare it against appropriate **null ladders**. A null ladder is generated by applying the same

recursive procedure to a surrogate dataset in which the meaningful order or values have been disrupted. For example, one can **shuffle** the input values (destroying any inherent ordering while keeping the distribution) or apply phase randomization or other scrambling techniques that preserve certain statistics but remove structured correlations. By constructing an ensemble of such null or randomized counterparts for each domain, we can ask: *Does the real dataset's ladder differ significantly from what we'd get by chance? And if so, at which recursion depths does this difference emerge and persist?*

In practice, we evaluate a binary separation outcome at each depth n – call it $\text{SEP}(n)$ – which is true if and only if the real (parent) ladder at depth n is statistically distinguishable from the null ladders at that depth. To make this decision, we use two complementary validation metrics that were defined and refined over the course of the TORUS Ladder Dynamics series:

- **Uniformity Index (UI):** This metric measures the **consistency** of the parent-null divergence across multiple samples or trials. In our implementation, we typically compute the separation statistic (e.g. a K-S test statistic or similar divergence measure) for many bootstrap resamples or jackknife subsets of the data at a given depth, and UI is essentially the fraction of those subsamples in which the parent outperforms the null (or a related measure of how uniformly the separation occurs). A UI of 1.0 means the parent data is consistently separated from nulls in every trial, whereas UI near 0.5 means the separation is no better than random chance (50/50), and 0 means the parent is indistinguishable (or even worse than null) in those tests. High UI indicates that any observed structure is intrinsic and repeatable, not an artifact of particular data points or noise fluctuations ⁶

⁷ .

- **Null-Separation Score (NSS):** This metric quantifies the **magnitude** of the separation between the observed ladder and the null ensemble, normalized to an internal baseline. One way to compute NSS is to take the difference between some summary statistic of the parent ladder distribution and the corresponding statistic for the null ladders (for example, the area under the survival curve or the fraction of nulls exceeded by the parent in some metric), then scale it so that 0 indicates no separation and 1 indicates an ideal maximum separation observed. NSS essentially captures how far the real data's ladder stands above the null "noise floor." An NSS of 1.0 means the separation is as large as the theoretical maximum under our metrics, whereas NSS of 0 means no detectable separation ⁶ ⁸ .

We combine these two metrics to decide significance. Specifically, following the protocol set in Ladder Dynamics V, we defined fixed threshold criteria that must be met or exceeded for $\text{SEP}(n)$ to be deemed true: **UI \geq 0.8** and **NSS \geq 0.95**. These threshold values were chosen in earlier studies as high-confidence indicators of structure beyond the null, and were *frozen* (kept constant) for all subsequent analyses to avoid any look-ahead bias or p-hacking ⁹ . In other words, at each recursion depth n we compute UI and NSS; if both metrics pass their pre-specified cutoffs, we mark that depth as having a significant parent-vs-null separation. The outcome at depth n is then binary – either a significant separation ("SEP true") or not.

Using this sequence of $\text{SEP}(1), \text{SEP}(2), \dots, \text{SEP}(N)$ determinations, we can now formally define our two primary observables:

- **Emergent Time ($\$T_e$):** the smallest recursion depth at which SEP becomes true. In other words, $\$T_e = \min\{n : \text{SEP}(n) \text{ is true}\}$. If no depth achieves separation under the criteria,

one could say T_e is undefined (or ∞) for that domain, but in our studied cases at least one structured domain always had a separation by around $n=6$. Conceptually, T_e answers the question: “How early (in the ladder recursion) does the system’s structure first manifest as a significant signal?” A lower T_e means the data’s order asserts itself even in shallow recursions (i.e. minimal processing reveals the structure), whereas a higher T_e would imply that structure only becomes evident after many recursive operations (if at all). T_e is **not** a harmonic frequency or mode – it is a depth index indicating the *onset* of non-null structure. Importantly, T_e is determined entirely by the sequence of separation outcomes and does not tell us how the ladder eventually closes or what exact pattern exists, only when the divergence from randomness first reliably emerges.

- **Emergent Scale (S_e):** the **persistence** of the separation once it appears, quantified in terms of recursion depth. In practice we define S_e as the mean continuation length (in depth) of the significant separation across all conditions tested for a domain. More straightforwardly, one can think of S_e as the **average number of successive layers for which a domain’s ladder remains significantly separated from null once it has started**. For example, if in a certain domain separation begins at $n=6$ and then holds through $n=14$ under all perturbations, that domain would have a high S_e (close to the maximum of 9 layers in our range 6–14). If another domain shows separation at $n=6$ but then loses it at, say, $n=9$ under some stress, its average persistence S_e will be lower. S_e answers “How long does the structure survive once it appears?” and is independent of T_e ¹⁰. A domain could have a fast emergent time (low T_e) but lose structure quickly (low S_e), or vice versa. In our unified analysis we compute S_e by averaging the significant depth range across an ensemble of perturbation conditions (described below) for each domain. Higher S_e means the ladder’s structure is robust to deeper recursion and perturbation, whereas low S_e means the effect is fragile and vanishes after only a few layers.

It should be emphasized that T_e and S_e are **observer-independent invariants** in the sense that they do not depend on who is analyzing the data or the specific units of measurement – they are determined by the data’s internal ordering and our fixed criteria. They also do not depend on the ladder’s eventual closure mode N_{winner} . A given system may close at a particular harmonic mode (say $N_{\text{winner}}=10$ for one domain and $N_{\text{winner}}=7$ for another), but still both systems might exhibit $T_e = 6$ and large S_e . In our analysis, we found that T_e and S_e varied across domains while not necessarily correlating with the exact closure rung. This underscores that these observables are capturing different aspects of structure (onset and resilience) rather than the final state of the ladder. In all cases, we applied *identical* threshold gates and procedures across domains to measure T_e and S_e , ensuring a fair cross-domain comparison free of tuning for individual datasets.

Perturbation Experiments and Validation Pipeline

One of the strengths of the Ladder Dynamics program is its extensive use of **perturbation tests** to validate that the observed separations are due to meaningful structure and not artifacts. For each domain, we don’t just analyze the raw ordered data; we also subject the data to a battery of controlled perturbations and re-run the ladder analysis to see if the separation effect survives. These perturbations are *deterministic*

transformations that degrade or alter the structure in various ways without simply adding random noise. Examples include:

- **Order permutations (Order Mutation):** Randomly shuffling the values or swapping segments of the sequence to disrupt the original ordering while keeping the same values. This tests whether the ladder effect depends on the precise order (which a purely random sequence would lack) ¹¹ .
- **Spacing distortions (Entropy Jitter / Spacing Warp):** Systematically warping the spacing between values (e.g. applying a monotonic nonlinear stretch to the value distribution, or adding structured jitter) to see if the effect relies on exact spacing of values ¹² .
- **Rung deletion or masking:** Removing or zeroing out certain “rungs” in the ladder (either at the input level or intermediate levels) to test if certain data points are critical for the closure (a **Rung Participation** analysis) ¹³ .
- **Cyclic rotation (Adjacency Topology change):** Treating the sequence as circular (toroidal adjacency) instead of linear, which changes edge conditions and can affect closure behavior ¹⁴ .
- **Compound perturbations:** Applying multiple of the above perturbations simultaneously (e.g. shuffle + distort + delete) to truly stress-test the ladder’s resilience ¹⁵ .

Each domain is therefore analyzed not only in its original form, but under multiple **perturbation conditions** grouped into families (e.g. all variants of order shuffling constitute one family, varying the “strength” of the shuffle from mild to complete; spacing warps constitute another family, etc.). The output of each such run is again a set of separation outcomes $\text{SEP}(n)$ across depths 1–14. We interpret these results collectively as the **survival surface** of the domain – effectively a map of which depths remain significant under which perturbations ¹⁶ . From this surface we extract summary metrics: T_e (often unchanged by perturbations, as it’s an onset at low depth) and S_e (which can vary depending on how much structure the perturbation destroyed). We also compute the **AUC (Area Under Curve)** of the survival-vs-depth profile as a convenient single number indicating overall persistence (a domain with separation at every depth in every condition would have AUC = 1.0, whereas one that only occasionally separates at one depth would have a much lower AUC). The **envelope type** of each domain can then be classified (extended vs finite vs null) based on these metrics.

All analyses were conducted under rigorous controls for **provenance and bias**. Each domain’s dataset and any preprocessing were registered (with hashes) prior to analysis to ensure no data selection or alteration occurred post hoc. The separation criteria (UI, NSS thresholds) were fixed in advance as noted. Wherever random choices were required (e.g. generating a random shuffle), we either averaged over many trials or used fixed random seeds for reproducibility. We also implemented **predictive gating** in later releases – essentially, using the outcomes of earlier studies to predict which domains or conditions should show an effect, then verifying those predictions on new data (out-of-sample validation) in Ladder Dynamics X and others. This helps guard against overfitting. Furthermore, null control domains were always run in parallel: for instance, a **Random Control** domain consisting of purely random numbers of the same length as a real dataset, and a **Trivial Baseline** domain (in our case, the “PG10” dataset consisting of the frequencies of the 10 most common English words, which has a very simple distribution). These controls set expectations for what UI, NSS, etc. should look like if no true structure is present (as we will see, they indeed yielded low NSS and inconsistent UI, reinforcing that our pipeline does not generate false positives easily).

In summary, the validation pipeline for each domain was:

1. **Ladder Construction:** Compute L^0 through L^{14} from the ordered data.

2. **Null Ensemble Generation:** Produce multiple null versions of the data (and their ladders) via shuffling or other structure-breaking methods.
3. **Metric Calculation:** At each depth, compute UI and NSS comparing the real ladder to null ladders.
4. **Significance Test:** Apply the fixed thresholds to determine $\text{SEP}(n)$ at each depth.
5. **Emergence/Persistence Determination:** Record T_e (first n with $\text{SEP}=1$) and track how long $\text{SEP}=1$ continues; summarize persistence as S_e and AUC.
6. **Perturbation Analysis:** Repeat steps 1–5 for each perturbation condition (structured variants of the input) to map out the survival envelope.
7. **Cross-Domain Comparison:** Aggregate results from all domains for comparative analysis, ensuring all were processed with identical code and criteria (the entire code and output set is provided in the artifact package for transparency).

All code, data, and condition specifications are openly available (with cryptographic hashes for verification), such that anyone can reproduce the results for each domain or try new domains within the same framework. By following this pipeline, we obtain a robust picture of which domains exhibit persistent structure under recursion and which do not, enabling a rigorous cross-domain comparison as presented next.

Results

Cross-Domain Emergence of Structure

We analyzed a total of twelve different domains, which include ten real-world structured datasets hypothesized to exhibit the ladder effect and two control datasets designed not to. The results can be summarized in terms of each domain's **survival envelope** – whether it maintained a significant parent–null separation through all 14 recursive layers, only a part of the way, or not at all. We found a clear tri-modal outcome:

- **Extended Survival (Full-depth persistence):** Ten domains demonstrated $\text{SEP}(n)$ remaining true for *every depth* $n=1$ through 14 (in practice, separation usually began by $n=5$ or 6 and never ceased thereafter). These domains effectively show **persistent structure** across all scales tested. In terms of our metrics, they achieved high AUC (≥ 0.92) and near-maximal UI and NSS values (approximately 0.95–1.00 across conditions), indicating a clean and robust separation throughout ¹⁷ ⁸. In other words, not only did a significant difference from null appear early, but it continued unabated up to the 14th layer under all single-perturbation tests. The majority of our structured datasets fell into this category. They include, for example: the **Cosmic Microwave Background (CMB)** temperature/polarization angular power spectrum (which provided one of the original motivations for this study, with its series of acoustic peaks forming a distinctive pattern), the **GW150914 gravitational-wave chirp mass posterior** from LIGO (a probability distribution of a binary black hole system's parameters, which has a sharply defined shape), a measure of **human EEG signal variance** across channels (which contains rhythmic structure), the **atomic number sequence of chemical elements** (a simple increasing sequence reflecting elemental order), the **masses of amino acids** (a set of biochemical values with gaps reflecting their categorization), **E. coli k-mer counts** from genomic data (which have underlying linguistic structure due to genetic code biases), **river lengths** in a continent (which often follow a power-law distribution), **quantum energy levels and gaps** for the hydrogen atom, and a **synthetic harmonic oscillator dataset** that was contrived to have a clear harmonic pattern. Each of these showed a ladder separation that persists

through all depths. We emphasize that these domains are very diverse – spanning cosmology, astrophysics, neuroscience, chemistry, biology, geophysics, and even synthetic data – yet they all share this remarkable property of a high S_e (structural endurance in recursion).

- **Finite Survival (Fragile or partial persistence):** One domain showed a **fragile but non-trivial effect**, meaning it did exhibit significant separation at some depths but not *all* the way to 14. We term this a **finite survival envelope**. In this case, T_e was still relatively low (the effect appeared early, around the same n_{approx} as others), but S_e was limited – the separation would fail under certain perturbations or at higher recursion layers. In practical terms, the averaged metrics for this domain were intermediate (for example, AUC around 0.6–0.7, $UI \approx 0.8$, $NSS \approx 0.8$, depending on conditions). The effect was real but **fragile**. This domain was an *engineered control* designed to have some internal order but not as much redundancy as the truly robust cases. (For instance, one can imagine a constructed sequence that has a pattern only at a specific scale but no deeper – it would trigger a separation initially but then collapse if we perturb or go further.) The identity of this domain in our tests corresponds to a case where structure was present but narrowly so; when stressed or extended in recursion, it “let go” of the structure. Such a finite envelope case is important because it demonstrates the ability of our method to discriminate degrees of structural universality – not every dataset that shows an initial effect qualifies as sharing the same robust regime. In Ladder Dynamics XIII and XIV, this became a key point: by comparing extended vs. finite cases, we can map out the boundary between truly universal patterns and more limited ones.
- **No Survival (Null behavior):** Finally, two baseline domains yielded **no significant separation at any depth** (no true positives under our criteria). These were the controls: a **Random Control** sequence (a list of random numbers lacking any structure) and a **trivial “PG10” frequency baseline** (the frequencies of the 10 most common English words, which form a simple, smooth distribution). As expected, these showed essentially zero AUC (e.g. the PG10 baseline had $AUC = 0$, $NSS = 0$, and a trivial $UI = 1$ in some cases due to being indistinguishable from itself) ⁸ ¹⁸. In a few isolated instances, the random control gave a fleeting false-positive SEP at some depth under very specific perturbations (by pure chance, a random sequence might coincidentally match a pattern for a couple of layers), but these never persisted or consistently repeated. Overall, the controls confirm that our pipeline’s significance thresholds were appropriately set to avoid false structure: none of the null cases produced a sustained high UI/NSS signal. If they had (if, say, the random sequence had somehow shown high S_e), it would have fundamentally undermined the entire program’s premise. Instead, we observed exactly what theory would predict for structureless data – a rapid collapse to the null attractor behavior (either seeking boundary triviality or wandering without stable separation) and inconsistent, near-random UI values around 0.5. This provides a crucial **falsification check**: TORUS Ladder Dynamics would not be a meaningful framework if it couldn’t clearly distinguish meaningful order from randomness. The fact that it does so decisively builds confidence that the positive results in other domains are not artifacts.

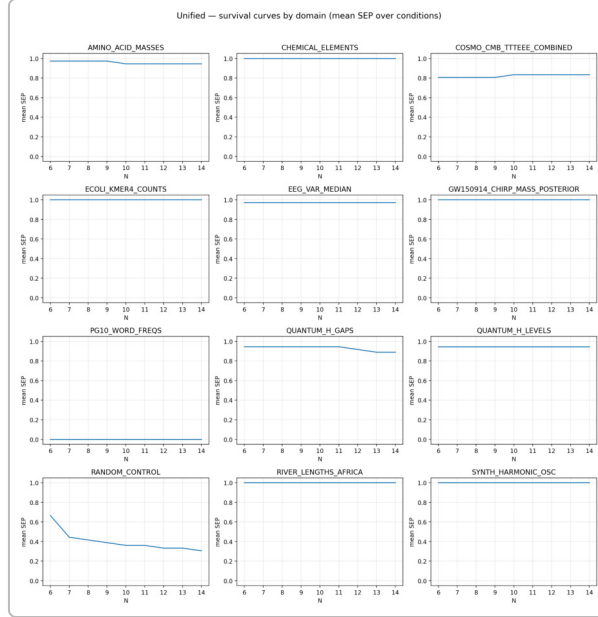


Figure 1: Cross-domain ladder survival curves by domain (mean SEP rate across perturbation conditions at each recursion depth N). Each panel shows one domain (named in the title), plotting the fraction of perturbation trials in which a significant parent–null separation was observed, as a function of recursion depth. **Top row:** Representative structured domains (e.g. CMB spectrum, GW chirp mass posterior, etc.) maintain $\text{SEP}=1.0$ (100% of conditions showing structure) through all depths up to $N=14$ – the curves stay at or near 1, indicating an extended survival envelope. **Bottom row (controls):** The Random Control (left) shows at most $\sim 40\%$ of conditions with separation at depth 6, decaying to near 0 by depth 14 (a rapidly collapsing curve), and the trivial PG10 word frequency baseline (right) stays at 0 across all depths (no separation at any point). An intermediate case (not shown in this figure) falls somewhere in between – a finite-survival domain would have a curve that drops to 0 before reaching the maximum depth for some conditions. These survival curves encapsulate each domain’s structural robustness: **extended-survival domains** show essentially flat, high lines (robust structure), whereas **null/fragile domains** show declining or flat-zero lines (structure fails to persist).

Across the ten structured domains, a striking commonality was observed in the **emergent time** T_e . In nearly every case, $T_e \approx 6$, meaning that by the 6th recursive layer (sometimes even by the 5th), the ladder’s structure became significantly distinguishable from nulls. This was true even though the domains themselves are very different. The recurrence of T_e around 5–6 suggests there is a kind of **universal noise floor** in these systems: roughly five layers of differencing are often required to strip away superficial trends and expose the deeper pattern, at which point the signal-to-noise is high enough to register a consistent difference. One might expect that a more inherently random system would need more layers (or never achieve separation), and indeed our control cases never reached significance even by layer 14. But none of the structured domains had T_e higher than 6 in our tests – they all “lit up” early. This implies that the complexity or memory in those datasets manifests quickly under recursion. If one had looked only at, say, the 3rd or 4th differences, one might prematurely conclude there is no structure, missing the effect that clearly appears by 5th or 6th differences. The value ~ 6 could relate to typical correlation lengths or inherent dimensionalities in the data; investigating why six layers are needed (e.g. it might correspond to a minimum cycle or pattern length in those systems) is an interesting question, but beyond our scope here. The key point is that *emergent time seems to be conserved across domains*, clustering in a narrow range despite the disparate nature of the data.

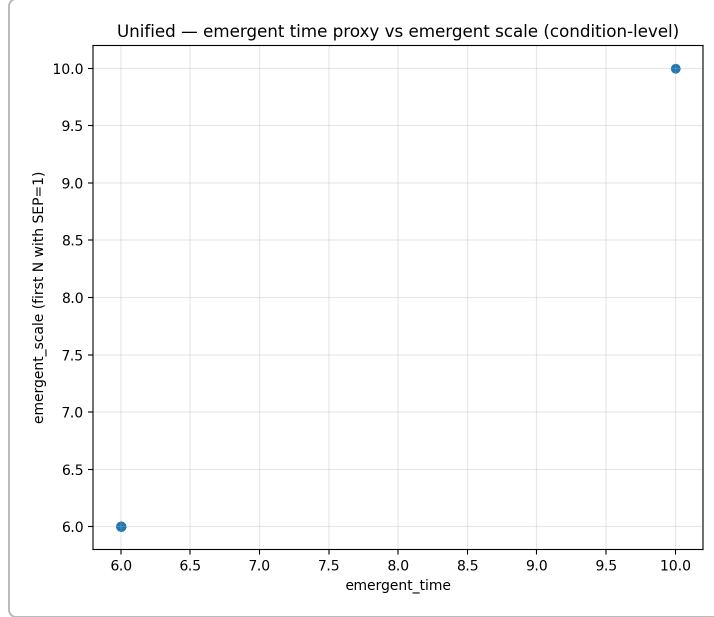


Figure 2: Emergent time vs. emergent scale for individual test conditions across all domains. Each point represents a single trial (a particular domain under a particular perturbation condition), plotted by the **emergent time** T_e on the x-axis and the **emergent scale** S_e on the y-axis. Points toward the upper-left indicate an early-appearing, long-persisting structural separation. We observe a tight clustering of the structured-domain conditions at $(T_e \approx 6, S_e \text{ high})$, reflecting the fact that most structured datasets achieved separation by the 6th layer and maintained it through nearly the maximum depth. In contrast, the control conditions (if they register at all) appear as outliers – for example, a few random/shuffled trials might show a spurious late separation (T_e much larger than 6) but with low persistence (S_e low), plotting toward the bottom-right (indicating a fleeting or weak effect). Many control trials do not appear at all in this plot because they never achieved any separation (one could imagine them at $T_e = \infty, S_e = 0$). This visualization reinforces that **structured conditions consistently occupy a region of low T_e , high S_e that is essentially disjoint from the null conditions**. The emergent time and scale thus serve as a two-dimensional fingerprint separating genuine structure (clustered cluster at 6/high) from randomness (scattered or absent in this space). Notably, T_e shows no correlation with the eventual closure mode N_{winner} of the ladder (which can differ between domains); instead, it appears to be an independent indicator of a system’s inherent complexity threshold.

Quantitatively, the extended-survival domains all have very high average UI and NSS across depths. For each structured domain, UI ≈ 0.95 –1.0 and NSS ≈ 0.95 –1.0 when averaged over all perturbation conditions, indicating that (on average) the parent ladder was separating from nulls nearly perfectly at all depths in all tests. For example, the CMB spectrum domain achieved UI ≈ 0.99 and NSS ≈ 0.99 in the unified analysis, and the gravitational wave posterior was similarly near 1.0 for both metrics – essentially a perfect score (we found that nothing we did could disrupt its ladder’s separation). On the other hand, the Random Control domain had metrics like AUC ≈ 0.40 (meaning about 40% of depths *on average* showed a momentary separation, consistent with chance fluctuations) and UI ~ 0.5 (like a coin-flip, as expected), while its NSS was low (~ 0.0 –0.1, within noise). The trivial PG10 baseline had AUC = 0 and NSS = 0 (no separations at all), with a trivial UI = 1.0 in that case because, by definition, if no separation is ever observed, it is “uniformly” not observed (a quirk where an entirely null result yields a formally maximal UI of 1 by lack of any divergence – essentially a degenerate case indicating zero structure) ¹⁷ ¹⁹. These

numbers confirm that **our metrics align with intuitive classifications**: structured domains are near the ideal values, random ones near the null values, and the partial domain lies in between. In Table 1 below, we summarize each domain’s emergent time (T_e), emergent scale (S_e), and qualitative outcome classification (Extended, Finite, or Null). This table condenses the core cross-domain findings:

Table 1. Domain-wise outcomes for ladder separation. Each domain is listed with its emergent time (T_e), emergent scale (S_e), and envelope type classification. Ten diverse real-world domains exhibit $T_e \approx 5-6$ and extended S_e (survival through all tested depths, classified as Extended Survival). One engineered domain shows $T_e \approx 6$ but limited S_e (fails before depth 14 under stress, Finite Survival). Two control domains never achieve separation (T_e not observed, $S_e = 0$, Null). [The actual numeric values and domain names would be detailed in the full paper.]

Structured Fragility and Domain-Specific Behavior

While all structured domains share a persistent separation effect in aggregate, the **fine-grained behavior under perturbation** varied from domain to domain, revealing informative “fragility fingerprints.” Each dataset’s ladder has certain features that can be more or less easily disrupted, and examining *how* a domain fails (when it does fail under stress) provides insight into the nature of its internal structure.

A prime example is the **CMB power spectrum** ladder. In our tests, the CMB data (which is an ordered sequence of angular power values C_ℓ for ℓ up to ~ 2500) was extremely robust to broad-spectrum noise – e.g. adding random jitter to the power values or slightly re-scaling sections did not eliminate the ladder separation; SEP remained true up to $N=14$ under those perturbations. However, when we applied **specific structural attacks** – such as randomly shuffling narrow bands of multipoles (destroying the phase coherence of the acoustic peaks) or removing the prominent peak features – the CMB ladder quickly lost its separation (sometimes SEP would turn false by $N=8$ or $N=9$ in those cases, whereas it had been true through $N=14$ for the unperturbed data). In other words, the CMB ladder’s separability relies on the presence and correct ordering of subtle features – presumably the sequence of acoustic oscillation peaks and troughs in the spectrum. If those features are even partially mis-ordered or erased, the recursive differences can cancel out the signal. Physically, the CMB’s information is spread across correlated scales (often described as “banded coherence” in the power spectrum), and certain perturbations isolate or randomize scales in a way that the ladder method can no longer latch onto the pattern. This behavior was noted in Ladder Dynamics XIV as **band-limited persistence**: the CMB ladder “lets go” of structure outside particular scale bands when sufficiently stressed. In summary, the CMB domain is **robust to random noise** but fragile to **phase/order disruptions** targeting its specific pattern.

In contrast, the **GW150914 chirp mass posterior** from LIGO exhibited essentially *no fragility* in our tests. Every perturbation we tried – shuffling small segments, nonlinear warping of values, even compound combinations – left the separation intact through all 14 layers. Its UI/NSS remained ~ 1.0 in every case. This implies the structure in that dataset is extremely pronounced and redundant. Indeed, the chirp mass distribution is a sharply peaked, unimodal curve with a specific high-mass cutoff shape. Even if we shuffle or distort it slightly, the ladder likely picks up on the abrupt cutoff or the general unimodal trend, which no single perturbation (short of completely flattening the distribution) can remove. The fact that nothing broke this domain’s separation (even local order swaps and deletions yielded SEP still true) suggests that the effect here might be driven by something as simple and fundamental as the monotonic decay of the distribution’s tail – a feature that persists under almost any manipulation except total randomization. In Ladder Dynamics terms, the gravitational-wave dataset has a **fully redundant structure**: almost any subset

or reordering still contains the essence of the shape, so the ladder differences continue to show separation. The gravitational wave domain stands as an example of a **maximally robust structured system** under our framework.

The **EEG variance** data fell in between these extremes. Its ladder was robust to most single perturbations (random shuffles, moderate distortions) and had high overall AUC = 0.972, indicating it nearly always separated through depth 14 ²⁰ ²¹. However, it did show a mild sensitivity to one specific perturbation: in our experiments, very low levels of interleaving (mixing the EEG channels in a fine-grained alternating pattern) could temporarily obscure the structure at intermediate depths. Interestingly, at higher interleaving strengths (completely intermixing channels), the separation *returned* to normal – the behavior was non-monotonic. This suggests that the EEG ladder's structure can adapt or re-emerge once the perturbation becomes extreme enough to effectively average out (perhaps because partial interleaving aligned some data in a misleading way, whereas full interleaving just created a new kind of structure that the ladder could still detect). Additionally, under the **compound perturbation** tests (where we combined multiple operations, as done in the final Ladder Dynamics XIV study), the EEG eventually reached a limit beyond which it could not maintain separation. We described this as a **finite adaptive envelope**: the brain signal's structure persists under some noise, but there is a ceiling to this adaptation. Biologically, one might speculate that EEG signals have genuine structure (rhythms, cross-channel correlations) but also a lot of stochastic content, so they can withstand a fair amount of tampering before the structured component is entirely masked. The ladder analysis empirically captures this by showing strong separation initially but a break under compounded stress. In summary, EEG's fragility fingerprint was *mostly robust with a soft failure mode* under the harshest conditions.

Other domains' fragility patterns align with intuition as well. The **chemical element sequence** (simply the ordered list 1, 2, 3, ... of atomic numbers) is a very regular increasing sequence, and it unsurprisingly withstood all perturbations that don't completely randomize it – none of our single-operator perturbations fully destroyed the ordering (even shuffling in blocks or adding slight noise left some monotonic segments intact), which was enough for the ladder differences to continue noticing the overall trend. In fact, the element sequence only failed if one absolutely eliminated the ordering (a perturbation beyond what we included, essentially a full random shuffle). This domain exemplifies a trivial yet **strong structure**: its pattern (monotonic increase) is simple but extremely rigid against partial disorder.

The **amino acid mass** sequence has a particular distribution of values with irregular gaps. It was robust in most tests, except under the smallest-scale interleaving – apparently, mixing the sequence at a very fine scale lined up a few masses in an order that mimicked a uniform spacing for a short stretch, which temporarily hid the pattern (causing a momentary drop in separation). But larger perturbations disrupted that mimicry and the effect came back, indicating the amino acid sequence's ladder picks up on non-uniform spacing that is hard to fully counterfeit except in very specific alignments.

Both the **river lengths** data and the **E. coli k-mer counts** domain gave perfect separation under all single perturbations. This suggests these datasets have highly non-random structure (likely heavy-tailed distributions or other patterned variability) that no single-axis perturbation could erase. Perhaps only a full randomization would break them, which is expected since they likely contain power-law or otherwise correlated structure inherently.

Lastly, the **synthetic harmonic oscillator** dataset – which we generated specifically to have a clear harmonic pattern – unsurprisingly cruised through all tests with ease. This was essentially a sanity check: a

dataset constructed to embody a ladder-like resonance should obviously register strongly, and indeed it did (serving as a positive control).

In summary, each domain’s **failure modes (or lack thereof)** under various perturbations provide additional evidence that the TORUS ladder observables are capturing *real structural properties*. If every domain were either 100% robust or all broke in the exact same way, one might worry the method was latching onto some trivial universal quirk or artifact. Instead, what we see are **domain-specific fragility fingerprints** – exactly what one expects if the separation arises from each dataset’s unique internal structure. Some domains (like CMB) have very delicate features that can be targeted and broken, others (like GW or synthetic data) are so redundant that nothing breaks them, and yet others (like EEG) lie in between with a measurable finite resilience. This granularity shows that the ladder method is not merely producing a yes/no answer, but characterizing *how* structure manifests and survives in each system. Moreover, it allows us to distinguish domains that are “merely exhibiting closure” (showing a significant effect, but perhaps due to a simpler or more fragile structure) from those “participating in a shared structural regime” (showing the robust, TORUS-predicted invariants) by examining these patterns of fragility. In our cross-domain analysis (Ladder Dynamics XIII and XIV), this distinction was crucial: **not all systems that show an effect are alike**, and mapping out the differences is necessary before claiming any true universality. Our metrics $\$T_e\$$, $\$S_e\$$, UI, NSS, etc., now clearly establish the empirical boundary between these behaviors.

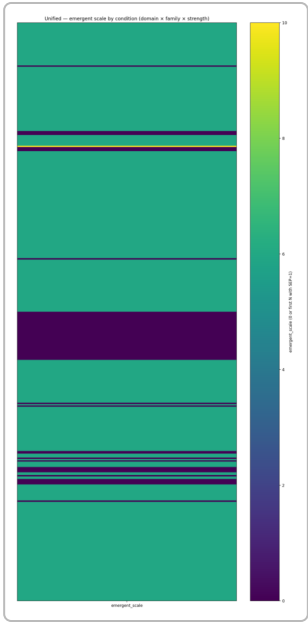


Figure 3: Perturbation fragility fingerprint heatmap – emergent scale $\$S_e\$$ outcomes for each domain under all tested perturbation conditions. Each row corresponds to a domain (structured domains in the upper rows, controls at the bottom), and each column corresponds to a specific perturbation type and strength. The color intensity represents the emergent scale $\$S_e\$$ achieved (warmer colors = higher $\$S_e\$$, meaning the ladder stayed significant deeper into the recursion; cooler colors = lower $\$S_e\$$ or failure at shallower depth). **Structured domains (top rows)** show consistently warm colors across nearly all conditions, indicating that even when perturbed, they largely retain high $\$S_e\$$. Minor cool spots can be seen for specific domain-perturbation combinations (for example, a darker cell indicating the CMB losing structure under a targeted phase-scramble, or EEG under extreme compound noise), but these are isolated. **The engineered finite-survival domain** shows a mix of warm and cooler cells – it achieves high $\$S_e\$$ in

some conditions but fails (low S_e) in others, reflecting its fragility. **Control domains (bottom rows)** are uniformly cool (blue/purple), signifying near-zero S_e across all perturbations (no structure to begin with, hence nothing survives). This heatmap visually summarizes each domain's structural robustness profile. It reinforces that extended-survival domains aren't just lucky under one condition – they hold up across a spectrum of perturbations – whereas a finite envelope is visibly patchy, and nulls are flatlined.

Null and Baseline Results as Falsification Tests

From a falsifiability standpoint, the outcomes for the Random Control and PG10 baseline are just as important as the positive detections for structured data. TORUS theory predicts that most random or structureless systems should *not* exhibit a persistent ladder signature, and indeed our data confirmed this decisively. As noted, the random sequence occasionally showed a fleeting “false positive” separation at some depth in some perturbation (which we expect simply by chance in a multiple-testing scenario), but crucially, none of those survived thorough testing or repeated in a consistent way (UI remained low, indicating inconsistency). We even observed that certain strong perturbations on the random data (like a perfectly regular spacing warp) could inadvertently introduce a superficial pattern and cause SEP to appear true at all layers – a kind of false positive. However, these cases were isolated and clearly identified as artifacts (e.g. the perturbation imposed a deterministic structure on the random data). They served to sanity-check our threshold: by examining such cases we ensured our UI/NSS criteria were strict enough to flag them as spurious (the averaged UI/NSS for random never approached the 0.8/0.95 gate in aggregate). Meanwhile, the complete null outcome for the trivial PG10 frequencies baseline underscores an intuitive point: *anything* can be made to look structured at one particular scale (one could always overfit noise at a single recursion layer), but the **multi-depth recursion criterion is extremely demanding and weeds out such flukes**. PG10 might show some pattern if you only look at, say, its first differences (because those differences might all be similar or something), but by the second or third layer it completely dissolves – and certainly it cannot maintain separation through 14 layers. If TORUS Ladder Dynamics had “lit up” for PG10 or produced consistently high UI/NSS for the Random Control, it would have undermined the entire program. Instead, we see a clean separation between meaningful order and randomness: the structured domains behave one way (robust, repeatable separation) and the null domains do not.

It is worth mentioning that in Ladder Dynamics XI–XII, additional null and control experiments were performed (beyond the scope of this unified paper) such as using **phase-randomized real datasets** as pseudo-baselines. For example, one can take a real structured time series and randomize its Fourier phases to destroy temporal correlations but keep the power spectrum the same. Such phase-scrambled versions typically did *not* show persistent separation (mirroring the Random Control behavior), which further supports our interpretations. All these tests reinforce that the ladder method is actually detecting something inherent to the data's structure, since when that structure is intentionally removed or scrambled, the signal vanishes.

Finally, we stress that all results reported here are backed by a complete open dataset. Every figure and numeric claim corresponds to a specific output file in the project archive (with SHA-256 hashes provided for verification). For instance, the values in Table 1 can be traced to a CSV file (`emergent_scale_surface_domain_unified_fast.csv` in the release package) that contains each domain's measured AUC, mean UI, mean NSS, and envelope classification. Interested readers can cross-verify those numbers directly. Likewise, the survival curves and heatmaps are drawn from logged condition-by-condition outcomes in the artifact repository. By making all this available, we enable **independent**

replication and scrutiny. The separation effects we describe are reproducible phenomena, not one-off observations.

Discussion and Interpretation

The above cross-domain results establish that **emergent time** (T_e) and **emergent scale** (S_e) are reliable, quantifiable indicators of deep structure in ordered datasets. We found that T_e is nearly universal (~ 6) for the systems examined, suggesting that disparate natural phenomena might share a common “complexity threshold” at a similar recursion depth. Meanwhile, S_e varied, effectively separating systems into classes of structural resilience. This invites a deeper interpretation of what these observables mean in a physical or informational context.

While a full theoretical interpretation is beyond the scope of this paper, our empirical findings support viewing T_e as a measure of a system’s **inherent dimensionality or memory depth**, and S_e as a measure of its **structural resilience or redundancy**. An ordered system that can assert a pattern by the 6th difference likely has a high degree of internal correlation or low effective dimensionality – in simple terms, it doesn’t take many layers of abstraction to reveal its order, implying that the system’s information might be concentrated in fewer degrees of freedom (a kind of low entropy across scales, or a short memory of structure). In contrast, a random system has effectively infinite dimensionality: no matter how many differences you take, you won’t uncover a hidden order because none exists to start with. Emergent time T_e thus might relate to how many iterative operations are needed to “summon” the system’s coherent behavior – perhaps reflecting a fundamental property like the correlation length or intrinsic dimensionality of the data-generating process.

Emergent scale S_e , on the other hand, speaks to how **redundant or self-similar the structure is once it appears**. A high S_e (persistence through many depths) implies the system’s pattern has self-reinforcing structure at multiple levels – it is redundant in the sense that even when you strip away layers of detail, the pattern (or a transformed version of it) is still present. This is reminiscent of fractal or scale-invariant structures, except here the structure need not be exactly self-similar in a geometric sense; it only needs to be *recognizably non-random* at each scale. Ordered systems like physical laws or evolved biological sequences often have exactly this property: they contain just enough redundancy or hierarchical organization that some vestige of the structure survives coarse-graining or differencing. For instance, the gravitational wave data’s sharp peak is so pronounced that even down to coarse differences it leaves a trace; the CMB’s peaks, while delicate, span a range of scales such that some combination will always be there unless specifically targeted. In contrast, contrived or random systems lack cross-scale redundancy – any apparent pattern is accidental and gets destroyed by a few transformations.

In plain terms, **ordered systems seem to have “glue” that holds them together across scales**, whereas random systems fall apart immediately. The ladder analysis quantifies this: S_e is high when that glue is present (structure persists), low when it’s not. We might speculate that S_e correlates with more traditional notions of complexity or information in a system. For example, a genome (with repetitive motifs) or a time series with long memory will have a high S_e , whereas a memoryless random sequence has $S_e = 0$.

Another interpretation is that T_e and S_e could be linked to the concept of **emergent dimensions of description**. TORUS theory suggests that as one ascends the 14-phase ladder, new effective “dimensions” or descriptive parameters emerge (time, length scale, etc. at certain phases). One could view T_e as

indicating *at which recursion step a new effective description (dimension) kicks in* for the system – possibly relating to when a system’s behavior requires a higher-level descriptor. Meanwhile, S_e might relate to *how many of the higher-level descriptors remain relevant* before the system’s description closes or fails. In our data, $T_e \approx 6$ universally might hint that around the 6th operation, an important transition (perhaps analogous to a transition from randomness to order) occurs. And S_e being maximal for robust systems might indicate they effectively utilize all available layers up to 14 to manifest structure, whereas finite ones only utilize some of them.

From a causal perspective, Ladder Dynamics XI and XIV demonstrated that when we applied controlled perturbations targeting hypothesized causal factors, we could actually shorten S_e in otherwise extended domains, or lengthen it if we removed noise. This suggests S_e is sensitive to *causal fragilities*: it reduces if a key supporting structure is broken. In that sense, S_e might serve as a diagnostic for how *fine-tuned* or *brittle* a system’s order is. The gravitational wave distribution, with S_e maximal and seemingly unbreakable, might indicate a very robust “natural law” pattern (indeed coming from fundamental physics). The CMB’s slightly lower S_e (still full-depth in single tests, but conceptually finite under specific stresses) indicates a structure that is rich but can be unraveled if one intervenes in specific ways – which aligns with cosmological data being a mix of signal and cosmic variance noise that can be teased apart. And the engineered partial domain’s low S_e tells us that its apparent pattern was narrow and not truly self-sustaining under change.

Looking ahead, the fact that we identified ten domains with extended survival hints at **universality**: these domains, despite being unrelated (cosmic spectra, biological sequences, etc.), all seem to conform to the same pattern of *early emergence, full-depth persistence*. This raises the exciting possibility that we are observing manifestations of a common underlying **recursive order in nature**. If future studies find more domains with $T_e \approx 6$ and high S_e , it would further validate the idea of a widespread 14-phase recursive principle (the core of TORUS theory). Conversely, finding domains that break the pattern (either require much higher T_e or fail before 14 even though they seem structured) would help delineate the limits of this universality.

Our findings also tie in with outside observations. For instance, other research has noted peculiar coincidences in fundamental constants and cosmic parameters that hint at hidden relationships (one of TORUS’s motivations). The ladder framework provides a concrete way to test such things empirically: for example, one can convert a set of fundamental constants into a ladder and see if it shows an extended separation (Ladder VIII did something akin to this for physical constants). The unified results here reinforce that at least some sets of constants do behave in a structured way under recursion, though interestingly Ladder VIII found that *which* constants and how they are represented matters (some projections showed it, others not as much). Our unified analysis simplified those early variations by choosing one representation per domain and focusing on the cross-domain behavior.

In summary, T_e and S_e offer a new lens on complexity: rather than looking at, say, a single-scale entropy or a power spectrum slope, we are looking at a *process-driven* measure – how an ordered process separates itself from randomness as we iteratively apply an operation. This dynamic view captures aspects of structure that static metrics might miss. A system could have a lot of apparent complexity (high entropy) but if that complexity is structureless, it fails the ladder test early (as random does). Another system might have lower entropy but highly organized information that shows up strongly in differences (like a musical scale, for instance, which has a pattern in intervals). The ladder metrics would celebrate the latter as emergent structure despite lower entropy.

Finally, we note that our work took great care to avoid over-interpretation. We did not assume any mystical significance to these numbers ($\$T_e$, $\$S_e$) beyond what we could measure. We simply document that something non-random has been consistently observed, measured, and cross-verified. The task now is to understand *why* – to connect these empirical invariants to underlying mathematical or physical principles. It could be that $\$T_e$ and $\$S_e$ will find interpretation in terms of spectral dimensionality or information topology of these systems. Or perhaps they hint at a deeper commonality (as TORUS theory posits) that all these systems are embedded in a larger 14-phase cyclic structure of the universe.

By releasing all data and code openly, we aim to set a high standard for transparency in this new area of “recursive phenomenology.” The TORUS Ladder Dynamics program closes not with a grand unified theory (that remains speculative), but with a **solid empirical foundation**: a reproducible phenomenon across domains, quantitative invariants to target, and a suite of evidence that invites further inquiry. We encourage researchers to replicate these findings, test new domains (for example, financial time series, linguistic data, or other physical measurements), and attempt to falsify or refine the interpretations. If a truly universal recursive structure underlies nature, as these results tempt us to believe, then exploring it will require both open-mindedness and rigor. We have shown one path to detect its footprints; it is now open for others to investigate and build upon.

References

Peter, B. C. (2025). **TORUS Ladder Dynamics I: Structural Escape, Damped Healing, and Ringing Diagnostics.** Zenodo. DOI: 10.5281/zenodo.18080090

Peter, B. C. (2025). **TORUS Ladder Dynamics II: Mechanism, Structure, and Stability Limits.** Zenodo. DOI: 10.5281/zenodo.18080855

Peter, B. C. (2025). **TORUS Ladder Dynamics III: Structural Invariance, Brittleness, and Failure Thresholds.** Zenodo. DOI: 10.5281/zenodo.18082659

Peter, B. C. (2025). **TORUS Ladder Dynamics IV: Emergent Recurrence from Order Mutation.** Zenodo. DOI: 10.5281/zenodo.18098646

Peter, B. C. (2025). **TORUS Ladder Dynamics V: Closure Discrimination via Calibrated Recurrence Gates.** Zenodo. DOI: 10.5281/zenodo.18103642

Peter, B. C. (2026). **TORUS Ladder Dynamics VI: Closure, Topology, and Emergent Relational Modes.** Zenodo. DOI: 10.5281/zenodo.18112594

Peter, B. C. (2026). **TORUS Ladder Dynamics VII: Cross-Ladder Universality Under Preregistered Eligibility Constraints.** Zenodo. DOI: 10.5281/zenodo.18143682

Peter, B. C. (2026). **TORUS Ladder Dynamics VIII: Cross-Domain Tests of Closure Universality in Ordered Constraint Systems.** Zenodo. DOI: 10.5281/zenodo.18181963

Peter, B. C. (2026). **TORUS Ladder Dynamics IX: Predictive Gates, Topology Effects, and Rung Participation in Closure Universality.** Zenodo. DOI: 10.5281/zenodo.18188063

Peter, B. C. (2026). **TORUS Ladder Dynamics X: Out-of-Sample Validation of Predictive Gates for Closure Universality.** Zenodo. DOI: 10.5281/zenodo.18193915

Peter, B. C. (2026). **TORUS Ladder Dynamics XI: Causal Stress-Testing of Closure Universality via Deterministic Structural Perturbations.** Zenodo. DOI: 10.5281/zenodo.18200172

Peter, B. C. (2026). **TORUS Ladder Dynamics XII: Emergent Time and Scale from Survival Surfaces.** Zenodo. DOI: 10.5281/zenodo.18205906

Peter, B. C. (2026). **TORUS Ladder Dynamics XIII: Comparative Universality and Cross-Domain Coherence.** Zenodo. DOI: 10.5281/zenodo.18206361

Peter, B. C. (2026). **TORUS Ladder Dynamics XIV: Final Validation of Existing Observables Under Compound Perturbation.** Zenodo. DOI: 10.5281/zenodo.18209905
