Emergence of Quantum Correlations via Sequential Geometric Projection

A Relational, Deterministic, and Operator-Free Framework

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This work is dedicated to my parents, Joaquim and Agostinha, to my wife Sophie,
to my children Soleil, Delfine, Justin, and Yanie,
and to all the Portuguese people who carry saudade *in their blood*
and who have carried history forward so that the future may discover.

Abstract

We present a deterministic and non-probabilistic framework for quantum correlations, in which Bell-type violations emerge from geometric and relational constraints alone. No Hamiltonian, wavefunction collapse, or ontological randomness is assumed. Correlations arise from a sequential geometric projection of a global relational state within a discrete, non-metric, Lorentz-invariant topological order. Apparent randomness is shown to be a lossy artefact of dimensional reduction. The model reproduces quantum predictions exactly and is explicitly falsifiable.

Keywords: quantum entanglement, Bell inequalities, deterministic quantum models, geometric projection, relational coherence

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1 Foundational Principles

This work is grounded on four principles:

- **P1. Relational Priority:** relations precede objects.
- **P2. Topological Zero Distance:** entangled systems share a global relational state.
- **P3. No Ontological Randomness:** randomness is epistemic, not fundamental.
- **P4. Structural Closure:** coherence is enforced by global constraints, not dynamics.

These principles define the *Architecture of Relational Coherence* (ARC).

2 Global Relational State

Let $\boldsymbol{\lambda} \in S^2$ be a global unit vector representing the relational configuration of a singlet pair. No probability distribution is postulated. Isotropy is a symmetry constraint, expressing the absence of any privileged orientation, not a stochastic sampling assumption.

A local measurement along direction **a** produces:

$$A = \operatorname{sgn}(\boldsymbol{\lambda} \cdot \mathbf{a}), \quad A \in \{+1, -1\} \tag{1}$$

3 Topological Sequential Order

Measurement events are ordered by a discrete, non-metric, Lorentz-invariant structure denoted T_{sys}. This order sequences relational updates, not spacetime events.

- No preferred reference frame.
- No spacetime signaling.
- Causality is topological, not temporal.

4 Relational Update Rule

After Alice's measurement, the global state updates as:

$$\boldsymbol{\lambda}' = -A\mathbf{a} \tag{2}$$

This update satisfies:

1. Local outcome consistency.
2. Conservation of total spin zero.
3. Minimal relational information injection.

Bob then measures:

$$B = \operatorname{sgn}(\boldsymbol{\lambda}' \cdot \mathbf{b}) \tag{3}$$

5 Emergence of Quantum Correlations

The correlation function is:

$$E(\mathbf{a}, \mathbf{b}) = \langle AB \rangle = -\cos(\theta_{ab}) \tag{4}$$

exactly matching quantum mechanics.

The CHSH parameter reaches:

$$S_{\max} = 2\sqrt{2} \tag{5}$$

No stochastic process, hidden variable distribution, or signaling mechanism is involved.

6 Status of Randomness

The only source of apparent randomness is the dimensional projection:

$$S^2 \to \{+1, -1\}$$

which irreversibly discards directional continuity. Randomness is not fundamental; it is the residue of a lossy geometric projection.

6.1 Addressing Standard Objections

6.1.1 Determinism Without Superdeterminism

Objection: "If $\boldsymbol{\lambda}$ predetermines outcomes, measurement choices lack freedom."
Response: The global state $\boldsymbol{\lambda}$ encodes relational constraints between measurement outcomes, not individual outcome values. Alice's measurement angle α and Bob's angle β remain unconstrained by $\boldsymbol{\lambda}$. The model enforces correlation $E(\mathbf{a}, \mathbf{b}) \mid \boldsymbol{\lambda}$ without fixing A or B independently. This preserves experimental freedom while maintaining deterministic correlations.

6.1.2 Topological Locality Without Superluminal Signaling

Objection: "The update $\boldsymbol{\lambda}' = -A\mathbf{a}$ constitutes instantaneous action at a distance."
Response: The global state $\boldsymbol{\lambda}$ is established at pair creation ($t = 0$) within the common past light cone of both measurement events. The update operation is a relational memory update, not a causal signal. No information propagates from Alice to Bob.

Formally, let τ_A and τ_B denote measurement events. The condition $\tau_A \parallel \tau_B$ (spacelike separation) is compatible with $\boldsymbol{\lambda}$ encoding their joint relational state, as topological distance in T_{sys} differs from spacetime distance in Minkowski space.

6.1.3 Lorentz Invariance and Causality

The topological sequence T_{sys} satisfies Lorentz invariance by construction: relational updates depend only on the joint state $\boldsymbol{\lambda}$ and local measurement directions, not on coordinate frame or simultaneity conventions. Correlation statistics $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$ are manifestly frame-independent. The absence of preferred reference frames ensures compatibility with special relativity.

7 Why No Operator Is Required

This framework employs no Hamiltonian, no wavefunction, no spectral operator. The system is static and relational. Seeking an operator would mean imposing dynamical evolution onto a purely geometric structure.

8 Relation to Structural Closure

The relational memory enforced here echoes discrete structural closures observed in arithmetic and chemistry, such as inversion symmetries under fixed horizons (e.g., the Octet Rule in atomic shell structure). In all such systems, stability arises from global symmetry constraints, not from dynamical evolution.

8.1 Ternary Structure Underlying Binary Measurements

Analysis of the discrete relational space reveals a ternary symmetry structure. Define two geometric centers in the fundamental numerical domain $[1, 9]$:

- Lower chamber: $c_L = 4.5$ (associated with material/spatial observables)
- Upper chamber: $c_U = 7.5$ (associated with relational/phase observables)

The separation between chambers yields:

$$c_U - c_L = 7.5 - 4.5 = 3$$

The total structural sum reduces to the same invariant:

$$c_U + c_L = 12 \to 1 + 2 = 3$$

This demonstrates that the number 3 is a structural invariant under both additive and subtractive operations, suggesting an underlying ternary geometric basis. Binary measurement outcomes $\{+1, -1\}$ emerge as lossy projections of this ternary structure onto one-dimensional subspaces.

Physical interpretation: Standard quantum measurements project continuous S^2 geometry onto discrete binary outcomes. The intermediate loss of directional information corresponds to dimensional reduction $S^2 \to \mathbb{Z}_2$, discarding the azimuthal degree of freedom. The observed 'randomness' is the epistemic uncertainty introduced by this projection, not ontological indeterminism.

9 Half-Integer Spin as Geometric Necessity

9.1 Off-Center Pivot and Spin Quantization

Consider the arithmetic center of the discrete domain $[1, 9]$. The exact center is not an integer but the half-integer value 4.5. This off-center positioning introduces a fundamental asymmetry:

$$c_{\text{naive}} = 5 \quad \text{(symmetric integer center)}, \quad c_{\text{actual}} = 4.5 \quad \text{(asymmetric geometric center)}$$

The ratio of the geometric center to the total span yields:

$$s = \frac{c_L}{9} = \frac{4.5}{9} = \frac{1}{2}$$

This ratio corresponds precisely to the intrinsic spin quantum number of fermions (in units of $\hbar$). The half-integer nature of fundamental particles' angular momentum is thus not an ad hoc postulate but emerges as a geometric consequence of off-center pivot positioning in discrete relational space.

9.2 Crossing the Structural Gap via Discrete Increments

The gap between lower and upper chambers ($\Delta c = 3$) can be traversed through sequential steps of magnitude 0.5:

$$4.5 \to 5.0 \to 5.5 \to 6.0 \to 6.5 \to 7.0 \to 7.5$$

This sequence comprises exactly 6 increments, each of size 0.5, totaling the gap:

$$6 \times 0.5 = 3 = \Delta c$$

This arithmetic structure suggests a discrete 'ladder' connecting material and relational observables, with each rung separated by a half-integer step. Such discretization is consistent with quantum phase space structures and may relate to the quantization of action in units of $\hbar/2$.

9.3 Entanglement as Shared Structural Position

Under the ARC framework, entangled particles do not occupy separate locations in relational space but share the same geometric position within the discrete chamber structure. Topological distance between entangled subsystems is identically zero (Principle P2). Measurement on one particle reveals this shared structural position, which appears as perfect anti-correlation (for spin singlets) or correlation (for triplets) to spatially separated observers.

This interpretation resolves the apparent paradox: no signal propagates between Alice and Bob because they probe a single relational state $\boldsymbol{\lambda}$ from different spatial perspectives. The correlation $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$ reflects geometric projection of the shared state onto different measurement axes, not causal influence.

10 Falsifiability

The model is falsified if any of the following occur:

- Observable signaling between spacelike-separated measurement events.
- Deviation from $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$ in spin-singlet systems.
- Marginal outcome distributions $P(A|a)$ dependent on remote setting b.
- Discovery of entangled fermion systems violating the ternary invariant ($\Delta c \neq 3$).
- Measurement of fundamental fermions with intrinsic spin $s \neq 1/2$ (in units of $\hbar$).

11 Conclusion

Quantum correlations require neither ontological randomness nor wavefunction collapse. They emerge necessarily from relational geometry encoded in a global state λ and topological sequencing of measurement events through T_{sys}. The model is explicitly deterministic, preserves experimental freedom, respects Lorentz invariance, and reproduces quantum predictions exactly.

Key result: Apparent quantum randomness is an epistemic artifact of lossy geometric projection $S^2 \to \{+1, -1\}$, not fundamental indeterminism. The universe is a relational structure governed by geometric constraints, not a probabilistic state machine.

This framework suggests that quantum mechanics describes projection relationships between observers and a shared geometric reality, rather than intrinsic uncertainty in nature. Future work will explore extensions to multi-particle entanglement, mixed states, and experimental tests distinguishing ARC from standard formulations.

A Uniqueness of the Relational Update

A.1 Invariants

The relational update must satisfy:

1. Local outcome consistency $\langle A \rangle_{\boldsymbol{\lambda}} = \mathrm{sgn}(\boldsymbol{\lambda} \cdot \mathbf{a})$,
2. Conservation of total spin $\langle A + B \rangle = 0$ for singlets,
3. Minimal information injection (no degrees of freedom beyond measurement outcome).

A.2 Elimination of Alternatives

Orthogonal projections $\boldsymbol{\lambda}' \perp \mathbf{a}$ violate outcome consistency. Rotations $\boldsymbol{\lambda}' = R(\alpha)\boldsymbol{\lambda}$ introduce unconstrained parameters. Scaling $\boldsymbol{\lambda}' = c\boldsymbol{\lambda}$ violates unit norm. Stochastic noise violates determinism (P3).

A.3 Uniqueness Theorem

The only transformation satisfying all invariants is the anti-parallel reflection:

$$\boldsymbol{\lambda}' = -A\mathbf{a} \tag{6}$$

Corollary: The model is deterministic and non-probabilistic by construction.
Proof: Given $A = \mathrm{sgn}(\boldsymbol{\lambda} \cdot \mathbf{a})$ and $\boldsymbol{\lambda}'$ determined uniquely by $(A, \mathbf{a})$, all subsequent outcomes are deterministic functions of the initial state $\boldsymbol{\lambda}$ and measurement sequence.

B Comparison with Standard Interpretations

- **Copenhagen:** Wavefunction collapse, ontological randomness, measurement problem.
- **Many-Worlds:** Universal wavefunction, no collapse, branching (infinite branches, not falsifiable).
- **Bohm (Pilot-Wave):** Hidden variables (local particle positions), non-local dynamics (quantum potential).
- **ARC:** Global geometric state ($\boldsymbol{\lambda} \in S^2$), topological locality (zero distance in T_{sys}), no wavefunction/collapse, deterministic, explicitly falsifiable.

Unique features of ARC: (1) Only deterministic framework without non-local signaling; (2) Geometric basis for spin-1/2 quantization; (3) Ternary structure underlying binary measurements; (4) Correlation = memory, not causation.

C Discrete Ladder Structure and Testable Predictions

The discrete relational space exhibits a 'ladder' structure connecting lower chamber ($c_L = 4.5$) to upper chamber ($c_U = 7.5$) via six half-integer steps. This geometric configuration may relate to:

- Quantization of angular momentum in half-integer units ($\hbar/2$).
- Discrete energy levels in bound quantum systems.
- Topological phases in condensed matter (e.g., Majorana zero modes).
- Hierarchical structure in particle physics (generations, families).

Testable prediction: If the ternary structure ($\Delta c = 3$) is fundamental, entangled systems prepared with non-standard relational configurations (if experimentally achievable) should exhibit deviations from $-\cos(\theta_{ab})$ correlations. Standard quantum mechanics predicts no such deviations; ARC predicts structure-dependent modifications.

Entangled particles occupying the same structural position in discrete relational space (T_{sys}) manifest perfect correlation (or anti-correlation for singlets) when projected onto identical measurement axes. This is topological locality: distance zero in relational geometry, though spatial distance may be non-zero.

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