Action Functional Incorporating π for Quantum-Modified $f(Q)$ Gravity

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Abstract

We propose a novel action functional that integrates the mathematical constant π to connect modified $f(Q)$ gravity with quantum-mechanical phase dynamics. This construction utilizes complex-analytic structures, including poles and essential singularities, alongside angle-amplitude relationships where π arises naturally. The action is formulated with mathematical rigor, featuring key equations in dedicated environments. We demonstrate how this functional captures quantum-gravitational effects, such as by imposing periodic phase identifications modulo 2π, akin to quantum interference conditions, and how non-metricity in $f(Q)$ gravity couples to these quantum phases. The significance of complex singularities is clarified by associating pole-type singularities with discrete spectra or stable states, and essential singularities with non-perturbative or chaotic regimes. Drawing inspiration from a recent discrete-action graph model featuring layered graph structures and emergent $SU(3)/U(1)$ symmetries, we highlight how discrete time steps of $\hbar$ can be modeled effectively through a π-periodic phase functional in the continuum limit. Finally, we propose observational strategies, including precision interferometry and cosmological surveys, to provide falsifiable tests for this quantum-modified $f(Q)$ gravity framework.

1 Introduction

The unification of quantum mechanics and gravity stands as one of the most profound challenges in theoretical physics. Modified gravity theories, such as $f(Q)$ gravity—where the action depends on a function of the non-metricity scalar Q—provide a framework for exploring deviations from General Relativity (GR) that may accommodate quantum effects. In $f(Q)$ gravity, a subset of symmetric teleparallel theories, the fundamental action involves an integral of a function of Q, the non-metricity invariant that quantifies deviations from Levi-Civita parallel transport [2, 7]. This formulation yields second-order field equations, avoiding the higher-derivative instabilities common in $f(R)$ gravity, positioning $f(Q)$ as a viable platform for quantum modifications [8]. Meanwhile, quantum mechanics introduces elements absent in classical gravity, including complex phases, interference, and discrete action quanta ($\hbar$). A pivotal insight from quantum theory is that physical amplitudes depend on the action modulo $2\pi\hbar$: phases differing by 2π yield identical interference patterns [3]. For example, paths interfere constructively if their action difference ΔS is an integer multiple of $2\pi\hbar$, but destructively if $\Delta S = (2n+1)\pi\hbar$ [9]. This 2π-periodicity underpins phenomena like the Aharonov-Bohm effect, where a magnetic flux shift of one quantum $\Phi_0 = h/q$

induces a 2π phase shift, leaving interference unaltered [4]. These insights inspire embedding π (and thus 2π periodicity) directly into the gravity action to mirror quantum phase behavior.

Recent advancements indicate that incorporating discrete action quanta can lead to significant outcomes. In the graph-theoretic discrete action framework by Abishev and Berkimbayev [1], time evolves in discrete increments, each accruing action $\Delta S = \hbar$. This generates a layered configuration space hierarchy: at layer C_1, a single oriented loop with $U(1)$ phase $e^{iEt/\hbar}$ [1]; at C_2, a second edge bifurcates the action, yielding phase-space pairs with commutator $[x, p] = i\hbar$ and local $U(1)$ gauge symmetry [1]; by C_3, three edges form a triangle with phase closure $\varphi_{12} + \varphi_{23} + \varphi_{31} = 0$, fostering emergent $SU(3)$ gauge structure and an Einstein-Yang-Mills action [1]. This emergence of gauge symmetries from discrete quantum steps underscores the role of phase periodicity and discrete action in fundamental interactions.

In this work, we introduce a continuum action functional embedding analogous principles via a 2π-periodic structure. Our total action S_{tot} merges the $f(Q)$ term with a novel π-dependent functional enforcing periodic action (or phase) identification in $2\pi\hbar$ units. Using complex analysis, we incorporate singularity structures reflecting physical spectra: poles linked to quantized states (e.g., bound or stable levels), and essential singularities to quantum-gravitational regimes where perturbation fails. We develop this with mathematical precision and explore encapsulated quantum-gravitational effects. We also delineate experimental probes, such as interference tests sensitive to Planck-scale phases or cosmological observations of $f(Q)$ modifications. Building on discrete action and graph evolution ideas [1], we translate them into a continuous, π-bridged functional between classical geometry and quantum phase.

The paper is structured as follows. Section 2 defines the action and its structure. Section 3 delves into the complex analysis framework, elucidating poles, essential singularities, and angle-amplitude theorems. Section 4 examines physical implications and observational tests. Section 5 concludes with a summary and future directions. We adopt metric signature $(+, -, -, -)$ and units with $c = 1$.

2 Functional Formulation with π-Periodic Structure

Our proposal centers on an action augmenting standard $f(Q)$ gravity with a term enforcing 2π-periodicity in a quantum phase functional. The total action is:

$$S_{\text{tot}}[g_{\mu\nu}, \Psi] = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, f(Q) + \alpha \int d^4x \sqrt{-g}\, \mathcal{L}_{\text{phase}}(\theta)\,, \tag{1}$$

where $f(Q)$ is differentiable in the non-metricity scalar Q, and $\mathcal{L}_{\text{phase}}(\theta)$ involves a dimensionless phase $\theta(x)$. Ψ denotes matter or quantum fields coupling to θ; initially, it encompasses degrees of freedom for phase dynamics. G is Newton's constant, normalizing to GR for $f(Q) = Q$ [7]. The dimensionless α modulates the new term's strength. $\theta(x)$ imposes symmetry under $\theta \to \theta + 2\pi$.

The non-metricity scalar Q in symmetric teleparallel geometry is:

$$Q \equiv Q_{\alpha\mu\nu} P^{\alpha\mu\nu}\,, \tag{2}$$

with $Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}$ the non-metricity tensor and $P^{\alpha\mu\nu}$ the superpotential ensuring scalar invariance [2, 10]. For $\alpha = 0$ and $f(Q) = Q$, Eq. (1) recovers the symmetric teleparallel GR equivalent (STEGR), as Q differs from Ricci scalar R by a boundary term [7]. We focus on $\alpha \neq 0$ and nonlinear $f(Q)$, yielding dynamics beyond GR [8].

For $\mathcal{L}_{\text{phase}}(\theta)$, we choose:

$$\mathcal{L}_{\text{phase}}(\theta) = \Lambda_0 \left[1 - \cos\theta(x)\right]\,, \tag{3}$$

where Λ_0 has energy density dimensions. The $1 - \cos\theta$ is 2π-periodic, minimized at $\theta = 2\pi n$. This form recalls Josephson junctions or periodic potentials, enforcing phase alignment without unbounded growth; for small θ, $\mathcal{L}_{\text{phase}} \approx \frac{1}{2}\Lambda_0\theta^2$ [11]. Here, minima align θ to $2\pi n$, identifying configurations differing by 2π as equivalent.

We define $\theta(x)$ as proportional to local action density per action quantum:

$$\theta(x) = \frac{2\pi}{\hbar}\ell^4 \mathcal{L}_G(x)\,, \tag{4}$$

with $\mathcal{L}_G(x) = \frac{1}{16\pi G} f(Q)$, and ℓ a length scale (e.g., Planck length) rendering θ dimensionless. $\theta(x)$ gauges phase advance from gravitational action; large $\mathcal{L}_G$ implies rapid θ change. The $2\pi/\hbar$ factor ensures θ advances by 2π when action density over ℓ^4 equals $\hbar$. Generalization to include matter is possible, but we emphasize gravitational coupling.

Substituting yields:

$$\mathcal{L}_{\text{phase}} = \Lambda_0 \left[1 - \cos\left(\frac{2\pi\ell^4}{\hbar} f(Q)\right)\right]\,, \tag{5}$$

absorbing constants into Λ_0. The full action is:

$$S_{\text{tot}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, f(Q) + \alpha\Lambda_0 \int d^4x\sqrt{-g}\left[1 - \cos\left(\frac{2\pi\ell^4}{\hbar} f(Q)\right)\right]. \tag{6}$$

Key features include:

(i) **Mathematical rigor:** Terms are diffeomorphism scalars. The cosine is analytic in $f(Q)$. Expansion:

$$1 - \cos\left(\frac{2\pi\ell^4}{\hbar} f(Q)\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)!}\left(\frac{2\pi\ell^4}{\hbar} f(Q)\right)^{2n}\,, \tag{7}$$

derived from Taylor series $\cos x = \sum(-1)^n x^{2n}/(2n)!$. At low curvature (small Q), leading term adds $\frac{\alpha\Lambda_0(2\pi\ell^4/\hbar)^2}{2} f(Q)^2$, a quadratic $f(Q)$ modification. Higher orders introduce infinite powers, signaling non-polynomial nature. The action remains finite and smooth, as $\cos x$ is entire.

(ii) **Equations of motion:** Varying S_{tot} w.r.t. metric and connection gives:

$$\frac{1}{16\pi G} E^{(Q)}_{\mu\nu} + \alpha\Lambda_0 E^{(\cos\theta)}_{\mu\nu} = T^{(\text{matter})}_{\mu\nu}\,, \tag{8}$$

where $E^{(Q)}_{\mu\nu}$ are $f(Q)$ equations [2]. For $E^{(\cos\theta)}_{\mu\nu}$, consider variation:

$$\delta S_{\text{phase}} = -\alpha\Lambda_0 \int \delta\left(\sqrt{-g}\cos\theta\right) d^4x = -\alpha\Lambda_0 \int \left[\frac{1}{2}\sqrt{-g} g^{\rho\sigma}\delta g_{\rho\sigma}\cos\theta + \sqrt{-g}\sin\theta\,\delta\theta\right] d^4x\,,$$

with $\delta\theta = \frac{2\pi\ell^4}{\hbar} f'(Q)\delta Q$, and δQ from metric/connection variations [10]. Thus, $E^{(\cos\theta)}_{\mu\nu}$ includes $\sin\theta$ terms coupled to Q derivatives. Contribution vanishes at cosine extrema, i.e., $\frac{2\pi\ell^4}{\hbar} f(Q) = 2\pi n$, recovering $f(Q)$ equations. Deviations induce restoring forces toward quantized action.

(iii) **Physical meaning of α:** α governs phase quantization influence. Small α approximates classical $f(Q)$; order-unity α yields macroscopic effects like quantum stiffening. Likely $\alpha \ll 1$ to match observations.

Eq. (6) embodies our π-periodic gravitational action. Next, we explore its complex-analytic interpretation.

3 Complex Singularity Framework and Angle–Amplitude Mapping

Complex analysis illuminates theories by associating singularities with spectra and stability. The $\cos(2\pi\ell^4 f(Q)/\hbar)$ introduces analytic structure in complexified $f(Q)$. We discuss singularities, their physical mappings, and π-involving angle-amplitude relations.

3.1 Pole Singularities and Quantized States

Poles, where functions diverge as $(z - z_0)^{-m}$, often denote discrete spectra or resonances. In scattering, S-matrix poles signify bound states or resonances [5]. Analogously, complexified action poles may indicate quantized $f(Q)$ solutions in path integral $Z = \int \mathcal{D}[g, \Psi] \exp(iS_{\text{tot}}/\hbar)$.

Consider perturbations $Q(x) = Q_0 + \delta Q(x)$ around Q_0 with $\frac{2\pi\ell^4}{\hbar} f(Q_0) = 2\pi n$. Expanding cosine to second order:

$$1 - \cos\theta \approx \frac{1}{2}\theta^2\big|_{\theta\to 0} = \frac{1}{2}\left(\frac{2\pi\ell^4}{\hbar} f'(Q_0)\delta Q\right)^2,$$

yielding effective mass $M^2 \propto \alpha\Lambda_0(2\pi\ell^4 f'(Q_0)/\hbar)^2$ for δQ. Propagator poles at $\omega^2 = M^2 + k^2$ imply discrete modes around quantized backgrounds, akin to harmonic oscillators [12].

In path integrals, $f(Q) = n\hbar/\ell^4$ stationarizes phase, dominating semiclassically. Multiple saddles (n) suggest multi-sheeted phase space, with n as winding number, potentially yielding interference between branches [13].

3.2 Essential Singularities and Quantum Chaos

Essential singularities exhibit wild behavior, assuming most complex values nearby (Picard's theorem) [6]. $\cos w$ has essential singularity at $w = \infty$, as $\cos w = (e^{iw} + e^{-iw})/2$ oscillates direction-dependently. Here, $w = \frac{2\pi\ell^4}{\hbar} f(Q)$, so essential singularity at $f(Q) \to \infty$ (high curvature).

Physically, this signals perturbation breakdown and quantum chaos. Similar to instantons e^{-1/g^2} [14], infinite Q powers indicate non-perturbative effects. Bounded $1 - \cos\theta \in [0, 2]$, the term oscillates without divergence, potentially taming singularities via averaging [16].

Argument winds infinitely at essential singularities, suggesting erratic phase θ and observables in quantum-gravity regimes, akin to spacetime foam [15].

3.3 Angle–Amplitude Theorems Involving π

Interference maxima/minima link angles (phases) to amplitudes (intensities): constructive at $2\pi n$, destructive at $(2n+1)\pi$ [3]. Gravitationally, $\Delta S = 2\pi\hbar$ unobservable, $\Delta S = \pi\hbar$ maximal disparity.

In $f(Q)$, null-holonomy yields deficit angles. Model may quantize deficits: $\oint Q_{\alpha\mu\nu} dx^\alpha = 2\pi k_{\mu\nu}$, paralleling flux quantization [4].

Emergent $SU(3)$: phase closure $\sum \varphi = 2\pi m$ enables gauge structure [1], linking angles to field strengths.

Thus, singularities map to quanta/chaos, angles to quantized geometries/interference.

4 Empirical Tests and Observational Signatures

A viable quantum-gravity–inspired modification must be operationally testable. Our π-functional action (6) implies (i) a quantized phase/winding sector tied to 2π, (ii) residual non-metricity dynamics in $f(Q)$ that modifies propagation and quasi-static potentials, and (iii) analyticity constraints on response functions that translate singularity structure (zeros/poles/branch cuts) into measurable spectral patterns. To keep the discussion falsifiable, we introduce a minimal phenomenological dictionary of parameters:

$$\Theta \equiv \{\Lambda_\pi,\ \epsilon_\pi,\ n_\pi,\ \alpha_Q,\ \lambda_Q,\ \varphi_\pi,\ \gamma_\pi\}, \tag{9}$$

where Λ_π is the characteristic "π-scale" (energy or inverse length), ϵ_π the amplitude of phase-sector deformation, n_π an effective scaling exponent, (α_Q, λ_Q) encode the $f(Q)$-induced fifth-force amplitude and range in the weak-field limit, and $(\varphi_\pi, \gamma_\pi)$ capture potential oscillatory phase offsets and damping in response functions. The following subsections outline concrete probes and the corresponding null tests.

4.1 Interferometric Phase Experiments

Interferometers are natural "phase microscopes." In our framework the accumulated phase admits a topological-plus-dynamical decomposition,

$$\Delta\Phi = 2\pi N + \Delta\Phi_{\rm dyn} + \delta\Phi_\pi, \qquad N \in \mathbb{Z}, \tag{10}$$

where $\delta\Phi_\pi$ encodes the π-functional correction. A minimal effective form consistent with a winding bias is

$$\delta\Phi_\pi \simeq \epsilon_\pi \left(\frac{\mathcal{E}}{\Lambda_\pi}\right)^{n_\pi} \sin(\Delta\Phi_{\rm dyn} + \varphi_\pi)\,, \tag{11}$$

with $\mathcal{E}$ the interferometer's characteristic energy scale (e.g., photon energy for optical interferometry or kinetic energy for matter waves). In practice, this predicts a small but structured distortion of the fringe probability distribution away from a purely sinusoidal dependence.

4.1.1 Optical interferometry: fringe-shape distortion and phase-wrap statistics

Let $I(\phi)$ be the normalized intensity as a function of controlled phase ϕ. Standard optics yields $I_0(\phi) = \frac{1}{2}[1 + \mathcal{V}\cos(\phi)]$. Our correction generates higher harmonics and a phase-wrap bias,

$$I(\phi) \approx \frac{1}{2}[1 + \mathcal{V}\cos(\phi) + c_2\cos(2\phi) + s_2\sin(2\phi)]\,, \tag{12}$$

where $(c_2, s_2) = \mathcal{O}(\epsilon_\pi(\mathcal{E}/\Lambda_\pi)^{n_\pi})$. A direct falsification route is a harmonic analysis of high-visibility fringes in stabilized cavities or long-baseline setups, searching for statistically significant nonzero (c_2, s_2) after controlling known systematics (polarization leakage, etalon effects, servo artifacts) [17].

4.1.2 Atom interferometry: mass-dependence and geometry dependence

For atom interferometers, the phase is sensitive to inertial and gravitational potentials, enabling cross-checks: if $\delta\Phi_\pi$ couples through $\mathcal{E} \sim mc^2$ or through kinematic invariants, different atomic species (Rb/Cs/Sr) and different pulse sequences yield distinct scalings. A useful discriminant is a mass-ratio test:

$$\mathcal{R}_{12} \equiv \frac{\delta\Phi_\pi^{(1)}}{\delta\Phi_\pi^{(2)}} \approx \left(\frac{\mathcal{E}_1}{\mathcal{E}_2}\right)^{n_\pi}, \tag{13}$$

measured at fixed geometry to suppress gravitational-gradient contamination. A null result across species bounds n_π-dependent deformations without assuming a specific UV completion.

4.1.3 Complex-analysis signature: poles/zeros of transfer functions

A distinctive feature of our construction is that analyticity/singularity data can be operationalized. For a linear response $\chi(\omega)$ (optomechanical susceptibility, cavity transfer function, or atomic polarizability), define the argument change on a contour $\mathcal{C}$:

$$\Delta_{\mathcal{C}} \arg \chi \;=\; \oint_{\mathcal{C}} d\omega \, \frac{d}{d\omega} \arg \chi(\omega) \;=\; 2\pi \, (N_Z - N_P) \, , \tag{14}$$

where N_Z and N_P count zeros and poles inside $\mathcal{C}$. Our π-functional sector predicts that small deformations can reorganize $(N_Z - N_P)$ across controlled parameter sweeps (e.g., cavity detuning, modulation depth), producing quantized jumps in phase-lag winding that are absent in standard linear optics. Measuring $\arg \chi(\omega)$ with network-analysis techniques provides a clean falsifiable diagnostic: no quantized jump pattern implies strong bounds on ϵ_π at the relevant Λ_π.

4.2 Time Quantization and Atomic Clocks

If fundamental "ticks" exist, they appear as either (a) energy-dependent phase noise, (b) small periodic modulations in clock comparisons, or (c) non-Gaussian tails in Allan deviation at short integration times. A minimal clock-comparison observable is the fractional frequency ratio,

$$y_{AB}(t) \equiv \frac{\nu_A(t)}{\nu_B(t)} - \left\langle \frac{\nu_A}{\nu_B} \right\rangle , \tag{15}$$

for two distinct transitions A, B (e.g., optical vs microwave). We model a "granularity-induced" correction as a superposition of a colored-noise floor and a weak coherent component:

$$y_{AB}(t) = y_{AB}^{\rm sys}(t) + \eta_\pi \left(\frac{\nu_{\rm ref}}{\Lambda_\pi} \right)^{n_\pi} \cos(\Omega_\pi t + \varphi_\pi) + \xi_\pi(t), \tag{16}$$

where $\xi_\pi(t)$ is an a priori predicted non-white stochastic process (e.g., with a power-law spectrum). This is falsifiable by multi-clock networks: one searches for a common Ω_π line appearing coherently across geographically separated comparisons, while ξ_π is constrained by deviations from standard noise models [18].

4.2.1 High-frequency spectroscopy: line-shape non-analyticities

Granularity can also manifest as tiny non-analytic features in line shapes (cusps, micro-splittings, or oscillatory residuals) after subtracting known QED and systematic contributions. A generic parametrization for a transition line profile $L(\omega)$ is

$$L(\omega) = L_{\rm SM}(\omega) \left[1 + \epsilon_\pi \left(\frac{\omega}{\Lambda_\pi} \right)^{n_\pi} \cos\!\left(\gamma_\pi \ln \frac{\omega}{\omega_0} + \varphi_\pi \right) \right] , \tag{17}$$

which predicts log-periodic residuals (a natural footprint of complex singularity structure). Absence of such residuals in ultra-high-resolution spectroscopy directly excludes portions of $(\epsilon_\pi, \Lambda_\pi, n_\pi, \gamma_\pi)$.

4.3 Cosmological Implications

At the background level, the $f(Q)$ sector modifies the effective Friedmann equation, while the π-functional sector can induce oscillatory components in the effective dark-energy equation of state. A deliberately conservative parametrization is

$$H^2(a) = H_0^2 \left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{\rm DE} \, \exp\!\left(3 \int_a^1 \frac{1 + w(a')}{a'} \, da' \right) \right] + \delta H_Q^2(a), \tag{18}$$

with an oscillatory $w(a)$,

$$w(a) = w_0 + w_1 \sin(\omega \ln a + \varphi_\pi)\,, \tag{19}$$

and $\delta H_Q^2(a)$ capturing the leading $f(Q)$ correction in a model-independent way. This produces correlated signatures across $H(z)$, distance ladders, and growth:

$$f\sigma_8(z) \quad \text{and} \quad P(k,z) \;\; \text{acquire oscillatory residuals and phase shifts.} \tag{20}$$

The falsification strategy is not "fit anything" but to demand phase coherence: the same (ω, φ_π) must improve (or be consistent with) supernovae, BAO, and growth data simultaneously, while respecting CMB constraints [19].

4.3.1 Perturbations: scale-dependent slip and ISW phase

Modified gravity generically yields a gravitational slip, $\Phi \neq \Psi$. Define

$$\eta_{\rm slip}(k,a) \equiv \frac{\Phi(k,a)}{\Psi(k,a)} - 1, \tag{21}$$

and an effective Newton constant $\mu(k,a)$ through the Poisson equation. In our framework, $\eta_{\rm slip}$ and μ can inherit mild oscillatory dependence in $\ln a$ consistent with (19). This can be tested via cross-correlations sensitive to the integrated Sachs–Wolfe (ISW) effect and weak lensing; the key falsifiable feature is a correlated phase relation between ISW residuals and growth residuals.

4.4 Laboratory Tests of Modified Gravity

In the quasi-static weak-field regime, $f(Q)$ deviations can be cast as corrections to the Newtonian potential. To capture a broad class including oscillatory tails motivated by the π-sector, we write

$$V(r) = -\frac{G m_1 m_2}{r}\left[1 + \alpha_Q e^{-r/\lambda_Q} \cos\!\left(\frac{r}{\lambda_\pi} + \varphi_\pi\right)\right], \tag{22}$$

where λ_π is an effective oscillation length (not necessarily equal to λ_Q). This form is sharply falsifiable in short-range tests: torsion-balance experiments, micro-cantilever force measurements, and Casimir-force residual analyses can set upper limits on α_Q as a function of (λ_Q, λ_π) [20].

4.4.1 Null channels and systematics control

A robust strategy is to engineer null channels where standard forces cancel. For a differential setup with two compositions A, B, define the residual acceleration difference

$$\Delta a(r) \equiv a_A(r) - a_B(r). \tag{23}$$

Standard gravity yields $\Delta a \simeq 0$ up to known corrections; a composition-dependent coupling in the effective theory would generate $\Delta a \neq 0$ with the same (λ_Q, λ_π) scaling. If experiments find no signal in Δa while observing a putative signal in the common mode, the latter is more likely systematic—this constitutes an internal falsification check.

4.5 Astrophysical Signals

Astrophysical environments probe strong fields and long baselines simultaneously.

4.5.1 Gravitational-wave propagation and dispersion

Modified gravity can affect the friction term (amplitude damping) and phase evolution in the GW waveform. A generic parametrization is

$$\tilde{h}(f) = \tilde{h}_{\rm GR}(f)\, \exp[-\mathcal{D}(f)]\, \exp[i\, \delta\Psi(f)]\,, \tag{24}$$

where $\mathcal{D}(f)$ and $\delta\Psi(f)$ are model-predicted functions. In our setting, $\delta\Psi(f)$ may include log-periodic or oscillatory components akin to (17), rooted in the same singularity/winding structure. Joint inference across events tests coherence: the same Θ must explain multiple mergers at different masses and redshifts.

4.5.2 Black-hole echoes: discreteness as a time-domain comb

If near-horizon microstructure exists, late-time echoes can appear as secondary bursts separated by an approximately constant delay Δt. To connect to the π-sector, we parametrize

$$\Delta t \approx \Delta t_0 \left[1 + \epsilon_\pi \left(\frac{M}{M_\pi}\right)^{-n_\pi}\right], \tag{25}$$

where M is the remnant mass and M_π sets the scale. Searching for an event-population correlation of Δt with M is a decisive falsification lever: random echo-like features with no coherent scaling do not support the model [21].

4.5.3 Pulsars, black-hole shadows, and multimessenger consistency

Binary pulsars constrain radiative and conservative sectors simultaneously (periastron precession, Shapiro delay, orbital decay). Black-hole imaging probes photon rings and spacetime structure. Although each channel alone may be degenerate with astrophysical uncertainties, the combined requirement of a single parameter set Θ across timing, imaging, and GW observations sharply reduces freedom. Any claimed deviation must survive multi-messenger cross-checks.

4.6 Data Analysis Strategy and Falsifiability Criteria

We emphasize a disciplined inference philosophy. Let $\mathcal{D}$ denote a dataset, and $\mathcal{M}_\pi$ our model. Parameter inference uses the posterior

$$p(\Theta|\mathcal{D}, \mathcal{M}_\pi) \propto p(\mathcal{D}|\Theta, \mathcal{M}_\pi)\, p(\Theta|\mathcal{M}_\pi), \tag{26}$$

and model comparison employs a Bayes factor K against a baseline $\mathcal{M}_0$ (GR+ΛCDM+SM clocks/optics). The model is falsified (at a given confidence prescription) if:

$$\epsilon_\pi \to 0 \text{ and } \alpha_Q \to 0 \text{ are consistently preferred across independent channels,} \tag{27}$$

or if any detected anomaly fails coherence tests (wrong scaling with $\mathcal{E}$, wrong phase relation across datasets, or inconsistency between weak-field and strong-field constraints). In other words, the absence of the correlated, phase-coherent patterns predicted by (11), (19), and (22) rules out the model's distinctive content.

5 Conclusion

We proposed a π-functional action (6) that intertwines (i) modified teleparallel/non-metricity gravity of the $f(Q)$ type and (ii) quantum-phase structure controlled by winding/argument principles. Conceptually, π plays a dual role: it is the geometric constant organizing 2π-periodic phases and, via complex analysis, the book-keeping factor that converts contour winding into integer-valued invariants. In this sense, singularity data (zeros, poles, and branch-cut structure) become physical: they encode discrete quanta, instability thresholds, and possible "chaotic" sectors through analyticity constraints, offering a bridge between gravitational kinematics and quantum interference.

Crucially, the framework is not merely aesthetic: we outlined a hierarchy of falsifiable empirical routes. Interferometric experiments test fringe-shape distortions, phase-wrap statistics, and quantized winding changes of transfer-function phases (14), extending standard quantum-gravity phenomenology ideas [18, 17]. Precision timekeeping probes granularity via coherent lines and non-white residuals (16). Cosmology confronts oscillatory dark-energy and $f(Q)$-driven growth/slip patterns (19)–(21) [19]. Laboratory short-range gravity searches constrain oscillatory potentials (22) with internal null checks [20]. In strong fields, gravitational-wave propagation and potential echo combs provide complementary tests (24)–(25) [21]. The theory is therefore falsifiable by the absence of correlated, cross-channel phase-coherent signals, summarized by (27).

Several immediate tasks remain for a complete program: (1) derive channel-by-channel predictions for $(\mathcal{D}(f), \delta\Psi(f))$ from specific $f(Q)$ choices and the explicit π-functional kernel; (2) establish stability/causality conditions and a controlled perturbation expansion; (3) compute weak-field PPN-like limits in the same parameterization used by experimental analyses; and (4) extend the construction to include gauge fields and matter-sector couplings while preserving analyticity and contour-invariant consistency. These steps turn the present synthesis—inspired by discrete and analytic viewpoints [1]—into a quantitatively testable research pipeline.

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