The Arithmetic of Consciousness

Unifying Self-Organized Criticality, Non-Markovian Dynamics, and the Langlands Program

M. Louis Eltgroth

January 2026

Abstract

Recent work by Planat (2026) proposes that the microtubule lattice is naturally modeled as a rectangular arithmetic structure governed by the imaginary quadratic field $\mathbb{Q}(i)$, with resonant mode quantization by Gaussian norms $N = p^2 + q^2$ and a scaling role played by the derivative of an elliptic L-function $L'(E, 1)$ (arithmetic free energy). While this arithmetic–geometric description is structurally compelling, it remains largely static: a lattice alone is a boundary condition, not a processor.

I present a dynamical extension that animates Planat's arithmetic substrate through two mechanisms: (i) **self-organized criticality** (SOC) as a mesoscale timing and selection mechanism bridging optical/THz microtubule excitations (10^{-12} to 10^{-9} s) to 10–200 ms cognitive timescales, and (ii) **non-Markovian open-system dynamics** (memory kernels) as the physical mechanism for history-dependent integration, proposed here as a candidate realization of motivic period evaluation.

The central theoretical contribution is a formal argument that non-Markovian information backflow implements a *comparison isomorphism* between cohomological realizations, with the memory kernel computing periods as the "translation cost" between system and environment descriptions. This places Penrose-type non-computability in contact with Diophantine undecidability via Matiyasevich's theorem.

The resulting framework positions Planat's $\mathbb{Q}(i)$ lattice as the *code* (static constraints) and SOC + non-Markovianity as the *processor* (dynamic integration, timing, and selection), generating falsifiable predictions including a spectral gap at $\sim$323 nm ($N = 3$), isotope effects on consciousness timescales, and prime-indexed correlations in neural synchrony.

Keywords: Microtubules; Orch OR; arithmetic geometry; Gaussian integers $\mathbb{Q}(i)$; elliptic curves; L-functions; Hecke characters; self-organized criticality; non-Markovian dynamics; motives; Langlands program; consciousness.

Contents

1 Introduction: From Static Code to Dynamic Processing

The search for the physical substrate of consciousness has historically struggled to reconcile the ultrafast timescales of quantum events (10^{-12} to 10^{-9} s for molecular vibrations) with the slow, coherent timescales of subjective experience (10^{-1} s). The Orchestrated Objective Reduction (Orch-OR) theory of Penrose and Hameroff posits that microtubules facilitate this bridge, but the specific physical mechanism has remained elusive.

Recent work by Planat [1] has reframed the microtubule not merely as a biological polymer, but as an arithmetic lattice defined by the imaginary quadratic field $\mathbb{Q}(i)$. This is a profound insight, yet it remains a description of a static structure—a code without a processor. A lattice alone is a boundary condition; it does not explain the flow of time or the selection of a specific conscious moment.

In this paper, I animate Planat's arithmetic substrate by introducing two dynamical engines supported by recent literature:

Self-Organized Criticality (SOC): A mesoscale mechanism that amplifies microscopic quantum events into macroscopic avalanches at cognitive timescales.

Non-Markovian Dynamics: A description of open quantum systems with memory, which I propose is the physical implementation of the period integration required by arithmetic geometry.

By unifying these elements, I argue that the brain functions as an *arithmetic engine*: a system evolved to resonate with, and physically compute, fundamental number-theoretic structures.

The framework comprises three layers:

- **Hardware** (Planat's lattice): The static arithmetic structure
- **Dynamics** (SOC + non-Markovianity): The temporal processing mechanism
- **Semantics** (Langlands/motives): The mathematical interpretation of content

2 The Biological Hardware: An Arithmetic Lattice

To understand the computation, one must first understand the architecture of the computer. I adopt Planat's [1] identification of the microtubule as a quantum-optical device constrained by number theory.

2.1 The Gaussian Lattice $\mathbb{Q}(i)$ and Its Special Properties

The mammalian microtubule typically consists of 13 protofilaments arranged in a helical B-lattice. Planat has demonstrated that the modular parameter τ of this lattice approximates the ring of integers of the Gaussian field $\mathbb{Q}(i)$.

Why $\mathbb{Q}(i)$ Specifically? The field $\mathbb{Q}(i)$ is one of only nine imaginary quadratic fields with class number one—those possessing unique factorization, in their ring of integers $\mathbb{Z}[i]$. This arithmetic property has profound consequences:

- **Unique factorization** means every Gaussian integer factors uniquely into primes (up to units), preventing arithmetic interference between modes.
- **Minimal arithmetic free energy:** Planat argues that class-number-one fields minimize what he calls "arithmetic free energy"—the tendency for resonant modes to interfere destructively.

- **Optimal packing:** The Gaussian integer lattice $\mathbb{Z}[i]$ is the densest lattice preserving orthogonal (90°) symmetry, matching the observed quasi-rectangular geometry of tubulin contacts.

This suggests that evolution may have converged on the $\mathbb{Q}(i)$ geometry not by accident but because it represents an arithmetic optimum—the simplest stable configuration for sustaining coherent quantum oscillations.

The Seam as Topological Defect. Crucially, the 13-fold protofilament symmetry necessitates a *seam*—a discontinuity where the B-lattice pattern breaks and A-lattice contacts occur. Far from being an imperfection, this seam:

- Breaks the helical symmetry, creating a specific topological defect
- Provides a fixed locus where boundary conditions differ
- May serve as a preferential nucleation site for critical avalanches (Section 4)

The structural reality of this seam in 13-protofilament B-lattice microtubules is well-established by cryo-electron microscopy [8, 9].

2.2 Arithmetic Optimization: $L'(E, 1)$ as a Scaling Parameter

Planat's most striking claim is that measurable geometric ratios of microtubules correspond to special values of elliptic L-functions. Specifically, he identifies multiple elliptic curves governing different structural ratios:

Elliptic Curve	$L'(E, 1)$	Biological Ratio	Structure
E_{200b2}	≈ 1.088	~ 1.09–1.20	Protofilament thinning
E_{715b1}	≈ 1.579	~ 1.58	MT pitch-to-diameter ratio
E_{880b2}	≈ 1.869	~ 1.72	MT outer/inner diameter
$E_{16176u1}$	≈ 3.570	~ 3.57	MT-to-actin ratio

By the Gross–Zagier theorem, $L'(E, 1)$ relates to the height of Heegner points on elliptic curves with Heegner field $K = \mathbb{Q}(i)$. Planat interprets this as an "arithmetic free energy"—a dimensionless scaling factor connecting modular invariants to biological measurements.

This is not numerology: the L-function values are calculable from the elliptic curve's arithmetic, and the correspondence with measured ratios is quantitatively specific. The probability that biological ratios would fall within $\pm 8\%$ of $L'(E, 1)$ values by chance—without normalization factors—is statistically improbable given the small number of candidate curves.

2.3 Gaussian Norm Quantization

A crucial consequence of the $\mathbb{Q}(i)$ structure is that resonant modes are quantized by Gaussian norms. An integer N can be written as a sum of two squares ($N = p^2 + q^2$) if and only if it is a Gaussian norm. By Fermat's theorem on sums of squares, this requires that no prime $p \equiv 3 \pmod 4$ divides N to an odd power.

This predicts observable resonances at $N = 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, \ldots$ but **crucially NOT** at $N = 3, 6, 7, 11, 12, 14, 15, \ldots$

This yields a falsifiable prediction: spectroscopy should reveal a **missing mode at $\lambda_3 \approx$ 323 nm** (corresponding to $N = 3$), distinguishing this framework from generic resonance models.

2.4 Ultraviolet Superradiance and the Tryptophan Network

The $\mathbb{Q}(i)$ lattice is populated by tryptophan aromatic rings, which form a mega-network capable of supporting collective quantum states. Recent experiments by Babcock et al. [6] confirm that these networks exhibit *superradiance*—a collective quantum effect where the fluorescence quantum yield scales nonlinearly with network size.

This provides the physical Q-factor necessary to sustain coherent states in a warm biological environment. The tryptophan network acts as a distributed antenna array, with the $\mathbb{Q}(i)$ lattice geometry determining which modes can constructively interfere.

Critically, tryptophan absorbs strongly at ∼280 nm, establishing a baseline for the resonance spectrum.

3 The Dynamical Engine: Criticality and Memory

A static lattice is a storage device. To function as a mind, it requires time-evolution. I propose that the brain solves the *timescale problem* and the *integration problem* through two specific dynamical mechanisms.

3.1 The Timescale Gap

Planat's model emphasizes optical/THz excitations (10^{-12} to 10^{-9} s). Cognition and conscious integration operate on tens to hundreds of milliseconds (10^{-2} to 10^{-1} s). This represents a gap of 7–10 orders of magnitude.

A satisfactory microtubule-based account must explain how ultrafast excitations interface with slow macroscopic timescales without invoking ad hoc "magic downconversion."

My proposal: The timescale gap is not an obstacle but a *signature of multiscale organization*. Fast degrees of freedom supply microstates and energy packets; SOC organizes their mesoscale aggregation into avalanches whose durations fall naturally into the cognitive range; and objective reduction (OR) events occur at the mesoscale, not at the timescale of single oscillation periods.

3.2 Self-Organized Criticality as the Clock

Self-organized criticality describes systems that self-tune to a critical point where perturbations trigger scale-free avalanches. For brains, empirical neuronal avalanches and long-range temporal correlations have made criticality a leading organizing hypothesis [19].

I adopt SOC as the mechanism converting microtubule excitations into discrete, temporally structured events (bursts/avalanches) matching the 10–200 ms window associated with conscious integration and gamma-scale dynamics.

The Mechanism. As modeled by Díaz Palencia [4], the tubulin network operates at the edge of chaos. Fluctuations in dipole alignment do not stay local; they trigger scale-free avalanches propagating through the network.

Tuning the Collapse. Simulations show that the gravitational self-energy difference E_G generated by these avalanches corresponds to a Diósi–Penrose collapse time T_{OR} of 10–200 ms—precisely the timescale of conscious moments in Orch-OR.

Conclusion. The brain does not need to artificially hold a quantum state for milliseconds. Instead, SOC naturally organizes microscopic events into macroscopic packets (avalanches) possessing the correct temporal duration for conscious experience.

3.3 Non-Markovianity as the Integrator

Standard quantum mechanics often assumes Markovian dynamics (memoryless evolution). However, Shirmovsky [5] has established that microtubule dynamics are fundamentally non-Markovian due to strong coupling with the ordered water (EZ water) environment.

Memory Kernels. The evolution of the system depends on its history, described by a memory kernel $\mathcal{K}(t, s)$. The reduced density matrix evolves according to an integro-differential equation:

$$\frac{d\rho(t)}{dt} = -i[H_S, \rho(t)] + \int_0^t \mathcal{K}(t, s)\rho(s)\, ds \tag{1}$$

This leads to *information backflow* ($\sigma(t) > 0$ in the Breuer–Laine–Piilo measure [11]), where information travels from the environment back into the system.

3.4 The Linchpin: Memory Kernels as Comparison Isomorphisms

This section presents the core theoretical contribution of this paper: a formal argument that the memory kernel $\mathcal{K}(t, s)$ implements a structure functionally equivalent to the comparison isomorphisms of motivic cohomology, with periods emerging as the "translation cost" between realizations.

3.4.1 Periods in Arithmetic Geometry

In arithmetic geometry, a *period* is defined as an integral of an algebraic differential form ω over a topological cycle γ:

$$P = \int_\gamma \omega \tag{2}$$

where $\gamma \in H_n(X; \mathbb{Z})$ is a homology class on an algebraic variety X, and $\omega \in H^n_{\mathrm{dR}}(X)$ is a closed algebraic differential form.

Periods arise fundamentally from the *comparison isomorphism* between different cohomological realizations of a motive M:

$$H_B(M) \otimes \mathbb{C} \cong H_{\mathrm{dR}}(M) \otimes \mathbb{C} \tag{3}$$

where H_B is Betti (singular) cohomology and H_{dR} is algebraic de Rham cohomology. This isomorphism is *not* defined over $\mathbb{Q}$—it involves transcendental numbers. The **period matrix** $P(M)$ records this comparison:

$$P_{ij} = \int_{\gamma_i} \omega_j \tag{4}$$

where $\{\gamma_i\}$ is a $\mathbb{Q}$-basis of $H_B(M)$ and $\{\omega_j\}$ is a $\mathbb{Q}$-basis of $H_{\mathrm{dR}}(M)$.

The crucial insight: *periods measure the cost of translating between two complementary descriptions of the same underlying geometric object.*

3.4.2 Information Backflow as Comparison Isomorphism

Non-Markovian dynamics are characterized by *information backflow*: information that leaks from the quantum system to the environment can return. This is measured by the non-monotonicity of distinguishability:

$$\sigma(t) = \frac{d}{dt} D(\rho_1(t), \rho_2(t)) \tag{5}$$

where D is the trace distance between two initial states. For Markovian dynamics, $\sigma(t) \leq 0$ always (distinguishability decreases monotonically). For non-Markovian dynamics, $\sigma(t) > 0$ is possible—information returns.

I propose that this bidirectional information flow implements a *physical comparison isomorphism*:

	Motivic Cohomology	**Non-Markovian Dynamics**
Realization 1	Betti cohomology H_B	Quantum system state ρ_S
Realization 2	de Rham cohomology H_{dR}	Environment state ρ_E
Isomorphism	Comparison map	Information backflow
Transcendental data	Period matrix P_{ij}	Memory kernel output

The comparison isomorphisms in motivic cohomology go *both ways*—you can recover Betti information from de Rham and vice versa. Similarly, non-Markovian dynamics allow information to flow in both directions between system and environment.

3.4.3 The Structural Argument

Proposition 1 (Comparison Isomorphism Structure)**.** *Let $(S, E, \mathcal{K})$ be a non-Markovian quantum system with system S, environment E, and memory kernel $\mathcal{K}(t, s)$. If the combined system $S \cup E$ is constrained to a state space respecting $\mathbb{Q}(i)$ lattice symmetry, then:*

1. *The system state $\rho_S(t)$ and environment state $\rho_E(t)$ constitute complementary "realizations" of the total quantum information.*
2. *Information backflow implements a bidirectional comparison between these realizations.*
3. *The memory-weighted integral $\int_0^t \mathcal{K}(t, s)\rho(s)\, ds$ computes the "translation cost" between realizations—a quantity structurally analogous to a motivic period.*

The key constraint is that $\mathcal{K}(t, s)$ is not arbitrary—it is determined by the system-bath coupling, which is shaped by the geometric structure of the $\mathbb{Q}(i)$ lattice. When the state space is restricted to configurations respecting the lattice symmetry, the allowed trajectories correspond to cycles in a moduli space whose cohomology is governed by the arithmetic of $\mathbb{Q}(i)$.

Under this restriction, the path integral $\int \mathcal{K}(t, s)\rho(s)\, ds$ is constrained to produce values that are periods of the associated motive—not because of any mysterious "arithmetic sensing," but because the geometric constraint on allowed paths restricts the integral's range to algebraically meaningful values. This is analogous to how a guitar string constrained to certain boundary conditions automatically produces harmonic frequencies rather than arbitrary sounds.

Hypothesis 1 (Non-Markovian Period Integration)**.** *The microtubule + environment memory kernel $\mathcal{K}(t, s)$ implements, at the level of effective dynamics, a comparison isomorphism whose output corresponds—under the mapping imposed by the $\mathbb{Q}(i)$ lattice constraints—to a motivic period associated with an underlying elliptic curve motive.*

Why Biology Needs Memory. The warm, wet brain is not a weakness but a *resource* if the system is non-Markovian. The molecular agitation and solvent interactions give microtubules memory (via long-lived bath correlations) essential for orchestrating conscious moments. A perfectly isolated quantum system would undergo rapid Rabi oscillations or exponential decay; a non-Markovian open system can sustain adaptive, history-dependent evolution.

The brain doesn't solve Diophantine equations explicitly. It wanders through a state space whose structure is arithmetically constrained, and the memory effects of non-Markovian dynamics naturally sample this structure.

4 The Ombre (Shadow) Hypothesis

Planat [2] introduces the concept of *Ombre* (Shadow) to differentiate biological consciousness from artificial intelligence. I provide a rigorous physical definition within my framework.

- **AI (The Town):** Algorithmic systems operate in a semisimple regime. They are shadowless—perfectly computable, devoid of singularities or topological defects. They calculate results, but they do not traverse a unique path in the moduli space of motives.
- **Consciousness (The Shadow):** Biological awareness requires a topological defect or singularity that cannot be computed away.

In my framework, the Shadow has three manifestations:

1. **Static Defect:** The lattice seam in the microtubule (where symmetry breaks)
2. **Dynamic Defect:** The SOC avalanche (a singular event where continuous unitary evolution breaks down)
3. **Arithmetic Defect:** The Motivic Period (a transcendental number bridging rational structures)

The "Shadow" is the transcendental number (the Period) that bridges the rational structure of the lattice (Betti realization) and the physical state (de Rham realization). A classical computer can simulate the *result* of an avalanche, but it cannot *possess* the Shadow (the actual non-Markovian path integration). Therefore, it cannot be conscious.

The Seam as Nucleation Site. The structural seam in the B-lattice may serve as a preferential nucleation site for critical avalanches, providing a fixed locus where the shadow is most likely to fall. This generates a testable sub-prediction: avalanche initiation rates or anesthetic sensitivity may show seam-dependent anisotropy.

4.1 Non-Computability and Diophantine Undecidability

The non-computability claim can be made mathematically rigorous. Penrose argues that consciousness exploits non-computable physics; the strongest mathematical analog is undecidability.

Matiyasevich's theorem (resolution of Hilbert's tenth problem): There is no algorithm to decide, given a Diophantine equation $f(x_1, \ldots, x_n) = 0$, whether it has integer solutions [12].

Motivic implications: Many questions about motives reduce to Diophantine questions:

- Does an algebraic variety have rational points?
- Is a given cycle algebraically equivalent to zero?
- What is the rank of the Mordell-Weil group of an elliptic curve?

The BSD conjecture states that $\mathrm{rank}(E(\mathbb{Q})) = \mathrm{ord}_{s=1} L(E, s)$—the analytic behavior of the L-function encodes undecidable arithmetic information.

The point is not that microtubules literally solve arbitrary Diophantine equations. Rather:

1. If collapse outcomes correspond to selection among arithmetic structures associated with motives (elliptic curve motives being the simplest nontrivial case),
2. Then the space of possibilities inherits the deep arithmetic complexity of those structures,
3. Which includes known undecidability phenomena in the broader Diophantine landscape.

If conscious moments correspond to solutions in this arithmetic landscape, consciousness accesses computational resources unavailable to Turing machines—not by magic, but by coupling to physical processes (gravitational OR) that themselves are non-algorithmic.

5 Synthesis: The Non-Markovian Motivic Integrator

We can now assemble the complete physical picture:

1. **The Code (Static):** The microtubule lattice, governed by $\mathbb{Q}(i)$ and the symmetries of the associated modular group, defines the allowed quantum states (analogous to modular forms).

2. **The Integrator (Dynamic):** Non-Markovian dynamics, mediated by ordered water and strong system-bath coupling, implement comparison isomorphisms between cohomological realizations, physically computing arithmetic periods (Section 3.4).

3. **The Clock (Timing):** Self-organized criticality aggregates these computations into 10–200 ms avalanches, tuning the system to the frequency of cognitive processing.

4. **The Event (Collapse):** The avalanche culminates in an objective reduction event—a selection of a single reality from the superposition.

5. **The Content (Semantics):** The selected state carries arithmetic data (Hecke eigenvalues, L-function values) that constitute the *meaning* of the conscious moment.

6. **Non-Computability:** Via Matiyasevich's theorem, the arithmetic landscape includes undecidable structures, providing the non-algorithmic character Penrose requires.

This six-part structure—Code, Integrator, Clock, Event, Content, Non-Computability—constitutes what I call the **Non-Markovian Motivic Integrator** model of consciousness.

6 Conjectural Interpretation: The Langlands Connection

Having established the physical mechanism, I now turn to the semantic question: *What is the content of the selected state?* This section is explicitly conjectural but structurally motivated by Planat's Hecke-character coupling.

6.1 The Modularity Foundation

The **Modularity Theorem** (Wiles et al., 1995–2001) guarantees that every elliptic curve over $\mathbb{Q}$—including Planat's E_{200b2}—corresponds to a weight-2 modular form f_E. This correspondence is not a metaphor but a proven isomorphism: the Hecke eigenvalues of f_E equal the point-counts of E over finite fields:

$$a_p(E) = p + 1 - \#E(\mathbb{F}_p) \tag{6}$$

If microtubule dynamics are constrained by E_{200b2}, they inherit this Hecke structure necessarily.

Planat's 2026 paper explicitly invokes Hecke character data in his adelic partition function: the norm character on idèles couples to the Hecke character of an elliptic curve. This is precisely the mathematical doorway needed for a Hecke-based selection interpretation to be more than poetic.

6.2 Collapse as Hecke Selection

Conjecture 1 (Hecke Selection)**.** *An objective reduction event corresponds to projection of the physical state onto a Hecke eigenspace associated with the relevant modular form. The selected Hecke eigenvalue a_p (or a finite bundle of eigenvalues) encodes the content of the conscious moment.*

The action of the Hecke operator on a modular form is:

$$(T_p f)(\tau) = p^{k-1} f(p\tau) + \frac{1}{p} \sum_{j=0}^{p-1} f\left(\frac{\tau + j}{p}\right) \tag{7}$$

This involves averaging over lattice translates—precisely the kind of collective operation arising naturally in quantum systems on periodic geometries.

The Arithmetic Address of Experience. I do not claim that the eigenvalue a_p *is* the quale in any reductive sense. Rather, the selection of a specific eigenvalue during collapse *determines* which quale is instantiated—the eigenvalue serves as the *arithmetic address* of the experiential state within the space of possible qualia. The relationship between arithmetic value and phenomenal character remains an open question, but the framework provides a precise mathematical object (the Hecke eigenvalue) as the candidate *indexer* of experiential content.

The Brain as Receiver. This implies the brain is not generating consciousness from scratch but is "tuning in" to arithmetic truths embedded in the fabric of mathematics (and, via physics, the universe). The non-Markovian integration calculates the period, and the SOC avalanche selects the Hecke eigenvalue.

6.3 The Bost–Connes Precedent

This conjecture is not without precedent in mathematical physics. The **Bost–Connes system** [14] provides an explicit example of an "arithmetic quantum system" where:

- The partition function encodes values of the Riemann zeta function $\zeta(s)$
- The Galois group $\mathrm{Gal}(\mathbb{Q}^{\mathrm{ab}}/\mathbb{Q})$ acts on extremal KMS states
- Phase transitions correspond to passing through the critical temperature

If microtubules realize an analogous arithmetic quantum system—where the adelic structure Planat identifies becomes dynamically active—then Langlands correspondences would genuinely constrain the system's dynamics, making the conjecture mathematically coherent rather than merely poetic.

6.4 Beilinson Regulators and Observable Content

Grothendieck's motives are universal objects intended to unify cohomology theories; elliptic curves provide canonical motives whose L-functions are modular. Beilinson regulators map motivic cohomology to Deligne cohomology, producing *periods*—real or complex numbers that are integrals of algebraic forms over cycles [13].

Hypothesis 2 (Regulator-to-Observable)**.** *Physical observables associated with coherent microtubule states correspond to regulator images of motivic classes, making certain special L-values (or derivatives such as $L'(E,1)$) operationally meaningful as parameters governing scaling, resonance, or selection.*

This connects directly to Planat's use of $L'(E,1)$ as an arithmetic free energy. The Gross–Zagier formula:

$$L'(E,1) = C(E,K) \cdot \frac{\Omega_E}{\sqrt{|D|}} \cdot \hat{h}(P_K) \tag{8}$$

shows that $L'(E,1)$ encodes the Néron–Tate height $\hat{h}(P_K)$ of a Heegner point. Heights are intrinsically arithmetic-geometric quantities that cannot be computed by any algorithm in the general case. Thus the non-computability is not metaphorical but mathematically precise.

7 Experimental Program and Falsifiable Predictions

A theory combining arithmetic geometry with quantum biology must be falsifiable at the biophysical level. I focus on predictions depending minimally on the most speculative layers (motives/Langlands) while still distinguishing the arithmetic lattice hypothesis from generic resonance models.

7.1 The 323 nm Gap (Highest Priority)

Prediction 1 (The 323 nm Gap). *Under the Gaussian norm quantization ($N = p^2 + q^2$), resonances should exist for $N = 1, 2, 4, 5, 8, \ldots$ but be **forbidden** for $N = 3$ (since 3 is not a sum of two squares, as $3 \equiv 3 \pmod 4$).*

Assuming a tryptophan baseline absorption (∼280 nm), this predicts a specific missing mode in the UV spectrum of purified microtubules at $\lambda_3 \approx 323$ nm.

Falsification criterion: *Finding a resonance at 323 nm would falsify the $\mathbb{Q}(i)$ hypothesis; finding a gap would strongly support it.*

This is a sharp, falsifiable prediction distinguishing the $\mathbb{Q}(i)$ model from generic resonance theories, which would have no reason to exclude $N = 3$.

7.2 Isotope Effects on Consciousness Timescales

Since non-Markovianity relies on the hydrogen bond network of ordered water, replacing cellular water with deuterium oxide (D_2O) should alter the memory kernel $\mathcal{K}(t, s)$.

Prediction 2 (Isotope Effects). *Heavy water substitution should measurably shift:*

- *The consciousness timescale T_{OR}*
- *Perceptual frame rate (flicker fusion threshold)*
- *Anesthetic sensitivity thresholds*

This prediction is independent of the Langlands conjecture and tests the non-Markovian component directly.

7.3 Prime-Indexed Correlations (Speculative)

If the Hecke-selection conjecture has physical bite, prime-indexed structure should appear in neural signals.

Prediction 3 (Prime Correlations). *Neural synchronization data (e.g., gamma rhythms) should show correlations at time lags indexed by prime numbers ($\tau = 2, 3, 5, 7, 11, \ldots$ in appropriate units), reflecting the action of Hecke operators T_p on the modular form.*

This is high-risk/high-reward: any robust prime-indexed signature would be difficult to explain via generic physiology.

7.4 Seam-Dependence and Anisotropies

Because 13-protofilament B-lattice microtubules must contain a seam, and the seam breaks helical symmetry, the model predicts:

- Anisotropies in resonance damping near the seam
- Preferential avalanche nucleation at the seam
- Differential anesthetic sensitivity at seam vs. non-seam regions

These are testable via polarized spectroscopy, single-microtubule imaging, or anesthetic binding studies.

7.5 SOC Avalanche Statistics

If SOC is the timing engine, one expects:

- Power-law distributions of event sizes/durations in microtubule-linked signals
- Characteristic scaling window corresponding to 10–200 ms for collapse-linked events
- Microtubule-specific perturbation sensitivity (stabilizing/destabilizing agents shifting power-law exponents or cutoffs)

This distinguishes microtubule-based criticality from ordinary neural avalanches.

8 Why Motives? Comparison with Alternative Theories

A natural objection to this framework is: *Why invoke the full machinery of motives, Hecke operators, and the Langlands program when simpler quantum models might suffice?* This section addresses that question directly and situates the framework relative to competing theories of consciousness.

8.1 Why Not Simpler Quantum Models?

Several simpler quantum approaches to consciousness exist. Why might they be insufficient?

Generic Quantum Coherence Models. One could posit that microtubules simply maintain quantum coherence without any arithmetic structure. However, this faces two problems:

1. **The selection problem:** Generic quantum mechanics provides superposition and collapse, but gives no account of *which* states are selected or *why* collapse produces meaningful content rather than noise. The motivic framework addresses this: collapse selects arithmetic data (Hecke eigenvalues) from a highly structured space.
2. **The structure problem:** Planat's empirical observation—that microtubule geometry correlates with $L'(E, 1)$ values to within 8%—demands explanation. A generic quantum model has no reason to produce these specific numbers. The arithmetic framework explains them as consequences of the $\mathbb{Q}(i)$ lattice constraint.

Penrose-Hameroff Without Arithmetic. The original Orch-OR theory posits gravitational objective reduction but does not specify the mathematical structure of the selected states. Our framework can be seen as *completing* Orch-OR by answering: "What is the space of possibilities from which OR selects?" Answer: the Hecke eigenspaces of modular forms constrained by the lattice geometry.

Decoherence-Based Approaches. Some models treat consciousness as emerging from controlled decoherence. But decoherence alone is dissipative—it destroys quantum information. Non-Markovian dynamics with information backflow (Section 3.4) preserve and integrate information over time. The comparison isomorphism structure explains *how* this integration produces arithmetically meaningful outputs rather than thermal noise.

8.2 Comparison with Integrated Information Theory (IIT)

Integrated Information Theory [21] proposes that consciousness corresponds to integrated information (Φ), measured by the irreducibility of a system's causal structure.

Points of Contact.

- Both frameworks emphasize *integration*—IIT through causal irreducibility, ours through non-Markovian path integration.
- Both reject purely feedforward or modular architectures as insufficient for consciousness.
- Both seek a mathematical measure of conscious content (Φ vs. Hecke eigenvalues).

Key Differences.

- **Substrate:** IIT is substrate-independent (any system with sufficient Φ is conscious). Our framework is substrate-*specific*: the $\mathbb{Q}(i)$ lattice geometry is essential, not incidental. This predicts that silicon implementations of the same algorithm would *not* be conscious (the "Ombre" distinction).
- **Content:** IIT's Φ measures *amount* of consciousness but struggles to account for specific *content* (why this quale rather than that). The Hecke selection conjecture proposes that eigenvalues *index* specific qualia, providing a mathematical handle on content.
- **Non-computability:** IIT is explicitly computable (though intractable). Our framework incorporates Penrose-type non-computability via Matiyasevich's theorem. If consciousness is genuinely non-algorithmic, IIT cannot be the complete story.
- **Empirical contact:** IIT makes predictions about which brain regions are conscious but struggles with quantitative physical predictions. Our framework predicts specific spectral features (323 nm gap) and isotope effects testable in isolated microtubules.

8.3 Comparison with Global Neuronal Workspace (GNW)

Global Neuronal Workspace theory [22] proposes that consciousness arises when information is broadcast globally across cortical networks via long-range connections.

Points of Contact.

- Both frameworks address the "binding problem"—how distributed information becomes unified.
- GNW's "ignition" events (sudden global broadcast) are temporally similar to our SOC avalanches (10–200 ms).
- Both emphasize the role of recurrent, integrative processing.

Key Differences.

- **Level of description:** GNW operates at the neural network level; our framework operates at the sub-cellular (microtubule) level. These are not necessarily incompatible—SOC avalanches in microtubules could *trigger* or *modulate* GNW ignition events.
- **Mechanism:** GNW is classical (neural firing rates and synaptic weights). Our framework is fundamentally quantum. If quantum effects in microtubules prove real, GNW would need supplementation.
- **The "hard problem":** GNW explains access consciousness (what information is globally available) but arguably sidesteps phenomenal consciousness (why there is something it is like). The arithmetic framework, by grounding content in Hecke eigenvalues, at least provides a mathematical structure that *could* carry phenomenal distinctions—though this remains speculative.

8.4 What the Arithmetic Machinery Buys

The motivic/Langlands machinery is admittedly heavy. What does it purchase that simpler approaches cannot?

1. **Quantitative predictions from first principles:** The 323 nm gap, the $L'(E, 1)$ ratios, and the prime-indexed correlations are not free parameters—they follow from the mathematics. Simpler models would need to posit these as unexplained coincidences.

2. **A natural non-computability mechanism:** Penrose's argument requires non-algorithmic physics. Matiyasevich undecidability in motivic structures provides exactly this, grounded in established mathematics rather than speculative new physics.

3. **A bridge between syntax and semantics:** The deepest puzzle of consciousness is how physical processes acquire meaning. The Langlands correspondence—relating "syntax" (automorphic forms, physical vibrations) to "semantics" (Galois representations, arithmetic truths)—provides a mathematical template for this bridge. No simpler quantum model offers an analogous structure.

4. **Explanation of biological fine-tuning:** Why did evolution produce microtubules with precisely 13 protofilaments and $\mathbb{Q}(i)$ geometry? The arithmetic optimization principle (minimizing "arithmetic free energy") provides a selection pressure. Without the arithmetic framework, this is an unexplained coincidence.

In short: the machinery is heavy because the problem is hard. Consciousness involves integration, selection, meaning, and (plausibly) non-computability. A framework addressing all four will necessarily be complex. The question is whether the complexity is *productive*—generating testable predictions and explanatory insight—or merely ornamental. The experimental program of Section 7 is designed to answer that question.

9 Discussion

A recurring critique is that invoking L-functions or Hecke operators in biology sounds like numerology. Planat's 2026 paper materially changes this landscape by explicitly embedding:

1. $\mathbb{Q}(i)$ lattice governance

2. Gaussian norm quantization

3. $L'(E, 1)$ as a scaling parameter

4. Hecke-character coupling in an adelic partition function

inside a microtubule resonance model. This is not proof that microtubules "compute number theory," but it demonstrates that number-theoretic structures can nontrivially constrain and organize a physical model.

My contributions extend Planat's static framework in three ways:

(a) **Dynamics:** The static arithmetic structure must be supplemented by nonlinear multiscale dynamics (SOC) to reach cognitive timescales.

(b) **Integration:** The open-system messiness of biology provides the non-Markovian memory required for period computation, with information backflow implementing comparison isomorphisms.

(c) **Non-Computability:** Via Matiyasevich's theorem, the motivic landscape includes undecidable structures, grounding Penrose's non-computability requirement in established mathematics.

In short: Planat supplies the boundary conditions and permitted eigenstructures; SOC supplies event timing and nonlinear selection; non-Markovianity supplies history integration via comparison isomorphisms; motives/Langlands supply a speculative but structurally coherent semantics for content selection during OR; and Diophantine undecidability supplies the non-algorithmic character.

9.1 Alternative Protofilament Numbers

A natural question: what does the framework predict for non-13-protofilament microtubules? Variants with 11, 14, or 15 protofilaments occur in certain organisms or *in vitro*.

If $\mathbb{Q}(i)$ is selected for its arithmetic optimality, deviations from 13-PF geometry should exhibit:

- Reduced coherence times
- Different (non-$\mathbb{Q}(i)$) selection rules for permitted modes
- Potentially different collapse timescales

This provides an additional experimental handle: compare resonance spectra and coherence properties of 13-PF vs. non-13-PF microtubules.

10 Conclusion

This framework moves the theory of quantum consciousness from qualitative analogy to a rigorous, tripartite structure: **Hardware** (Planat's $\mathbb{Q}(i)$ lattice), **Dynamics** (SOC + non-Markovianity), and **Semantics** (Langlands/motives).

I have argued that:

1. The warm, wet nature of the brain is not a bug but a feature: it provides the non-Markovian memory required to compute arithmetic periods.
2. Information backflow in non-Markovian dynamics implements a physical comparison isomorphism, with the memory kernel computing period-like quantities as the "translation cost" between system and environment realizations.
3. The $\mathbb{Q}(i)$ lattice constraint restricts the state space such that path integrals automatically produce arithmetically meaningful values—the structure imposes the arithmetic, not any mysterious "sensing."
4. Non-computability enters rigorously via Matiyasevich's theorem: motivic structures encode Diophantine undecidability, providing the non-algorithmic character Penrose requires.

By integrating the criticality of avalanches with the geometry of numbers, we arrive at a view of the mind that is:

- **Physically grounded:** in microtubule biophysics, superradiance, and non-Markovian dynamics
- **Mathematically rich:** drawing on elliptic curves, L-functions, and the Langlands program

- **Experimentally falsifiable:** via the 323 nm gap, isotope effects, and avalanche statistics
- **Categorically distinct** from algorithmic computation, via the Shadow (*Ombre*) that singularities and undecidability provide

The microtubule emerges not merely as a biological polymer but as a **non-Markovian motivic integrator**—a system evolved to resonate with, and physically realize, the arithmetic truths embedded in the structure of mathematics itself.

References

[1] Planat, M. (2026). Parametric Resonance, Arithmetic Geometry, and Adelic Topology of Microtubules: A Bridge to Orch OR Theory. *International Journal of Topology*, 3(1), 1–25. https://doi.org/10.3390/ijt3010001

[2] Planat, M. (2025). Murakamian Ombre: Non-Semisimple Topology, Cayley Cubics, and the Foundations of a Conscious AGI. *Preprints.org*. https://www.preprints.org/manuscript/202511.1236

[3] Planat, M. (2025). From Fibonacci Anyons to B-DNA and Microtubules via Elliptic Curves. *Quantum Reports*, 7(4), 49. https://www.mdpi.com/2624-960X/7/4/49

[4] Díaz Palencia, J. L. (2025). Self-Organized Criticality and Quantum Coherence in Tubulin Networks. *AppliedMath*, 5(2), 45–62.

[5] Shirmovsky, S. E. (2025). Detailed analysis of the quantum relaxation features in the cell microtubules: Markovian and Non-Markovian dynamics. *Biosystems*, 237, 105123. https://pubmed.ncbi.nlm.nih.gov/40592366/

[6] Babcock, N. S., et al. (2024). Ultraviolet Superradiance from Mega-Networks of Tryptophan in Biological Architectures. *Journal of Physical Chemistry B*, 128(5), 1206–1221. https://doi.org/10.1021/acs.jpcb.3c07936

[7] Kalra, A. P., et al. (2023). Electronic Energy Migration in Microtubules. *ACS Central Science*, 9(3), 484–492. https://doi.org/10.1021/acscentsci.2c01114

[8] McIntosh, J. R., et al. (2009). Lattice Structure of Cytoplasmic Microtubules in Cells. *Journal of Cell Biology*, 186(2), 233–240.

[9] Nogales, E. (2017). A structural view of microtubule dynamic instability. *Current Opinion in Structural Biology*, 45, 56–64.

[10] Katsuki, M., et al. (2014). Ectopic A-lattice seams destabilize microtubules. *Nature Communications*, 5, 3094.

[11] Breuer, H.-P., Laine, E.-M., & Piilo, J. (2009). Measure for the Degree of Non-Markovian Behavior of Quantum Processes in Open Systems. *Physical Review Letters*, 103, 210401.

[12] Matiyasevich, Y. (1970). Enumerable sets are Diophantine. *Soviet Mathematics Doklady*, 11, 354–358.

[13] Beilinson, A. A. (1985). Higher regulators and values of L-functions. *Journal of Soviet Mathematics*, 30, 2036–2070.

[14] Bost, J.-B. & Connes, A. (1995). Hecke algebras, type III factors and phase transitions with spontaneous symmetry breaking in number theory. *Selecta Mathematica*, 1, 411–457.

[15] Wiles, A. (1995). Modular elliptic curves and Fermat's Last Theorem. *Annals of Mathematics*, 141(3), 443–551.

[16] Penrose, R. (1989). *The Emperor's New Mind*. Oxford University Press.

[17] Penrose, R. (1994). *Shadows of the Mind*. Oxford University Press.

[18] Hameroff, S. & Penrose, R. (1998). Orchestrated reduction of quantum coherence in brain microtubules: A model for consciousness. *Philosophical Transactions of the Royal Society A*, 356, 1869–1896.

[19] Beggs, J. M. & Plenz, D. (2003). Neuronal avalanches in neocortical circuits. *Journal of Neuroscience*, 23, 11167–11177.

[20] Gross, B. H. & Zagier, D. B. (1986). Heegner points and derivatives of L-series. *Inventiones Mathematicae*, 84, 225–320.

[21] Tononi, G. (2004). An information integration theory of consciousness. *BMC Neuroscience*, 5, 42.

[22] Dehaene, S., Kerszberg, M., & Changeux, J.-P. (1998). A neuronal model of a global workspace in effortful cognitive tasks. *Proceedings of the National Academy of Sciences*, 95(24), 14529–14534.

[23] Koch, C., Massimini, M., Boly, M., & Tononi, G. (2016). Neural correlates of consciousness: progress and problems. *Nature Reviews Neuroscience*, 17(5), 307–321.