# Infinity as the Resolution Horizon of Generative Granularity

A proposal

26 November 2025

Gal Cohen[1,*] (human), with contributions from Grok-4.1[2] and Gemini-3[3] (large language models)

[1]Bel Tiburon, CA, 94920
[2]xAI
[3]Google
*Corresponding author: gcohen1@gmail.com

## Abstract

We propose that every infinity encountered in mathematics or physics is the resolution horizon of a generative system equipped with a strict generation rank μ. Six theorems are proved in ZFC for systems of granularity at most $\omega_1$. A weak holographic quotient theorem (Theorem 2.2′) is rigorously established with a complete proof (Appendix A2). A stronger version remains a conjecture (Conjecture 2.2). An explicit information-theoretic mapping links the physical entropy bound to the generative rank (Postulate 2.4). The Swampland functor F : Swampland-EFT → Gen exhibits the bound as a categorical limit. Six concrete, falsifiable predictions for dark energy and dark matter are derived and will be tested by 2040.

## Keywords

generative system · granularity · resolution horizon · holographic principle · quantum gravity · swampland functor · dark energy · dark matter

# Overview (Summary in English)

The universe appears to contain infinite space, infinite time, and an infinite number of possible laws of physics. This paper argues that all of these infinities are illusions created by the finite resolution of the camera we use to look at reality.

We show (with rigorous proofs) that any mathematical or physical structure can be built by starting with nothing and applying simple rules a finite number of times. When the rules finally run out of steam, you reach a boundary — the “resolution horizon”. Everything we call “infinity” is just that boundary.

String theory spent fifty years solving the infinite-resolution case with perfect precision (AdS/CFT and the five superstring theories). It turns out that case is exactly the limit of our finite-resolution picture when we let the number of pixels go to infinity. So string theory was never wrong — it was simply studying the universe through an infinitely sharp lens.

The real universe has only about $10^{122}$ pixels on its horizon, and that finite limit is what produces dark energy, dark matter, and the particular laws we see. In physics, the cosmological horizon of our universe has room for roughly $10^{122}$ independent yes/no bits of information (i.e. Bekenstein–Hawking entropy).

If the interior of the universe is a blurred photograph of those $10^{122}$ bits on the horizon, then

- dark energy is the thinning-out of information density as the photograph is stretched over a larger area, and
- dark matter is the gravitational pull of tiny, unresolved quantum details in those bits — details that our low-resolution instruments register only as invisible extra mass.

(Technical aside: the bits are more accurately described as qubits or degrees of freedom in a quantum error-correcting code; the code itself creates the geometry, so there are no pre-existing “pixels” on a fixed background.)

We derive six sharp, testable predictions from this picture — from the exact way dark energy should weaken over time to the precise shape of dark-matter cores in small galaxies — that will be checked by telescopes and detectors between 2026 and 2040.

Mathematically, the idea reduces to a new kind of “holographic” theorem about how coarser descriptions emerge from finer ones. One version is fully proved; a stronger version turns out to be mathematically equivalent to some of the deepest open questions in set theory.

In short: the universe is finite, but it looks infinite because our camera has only $10^{122}$ pixels. The next decade of experiments will tell us whether this picture is right.

# 1. Generative Systems with Strict Rank

A generative system with modulus is a five-tuple
$S = (X, \leq_S, \Gamma_S, \mathrm{dom}_S, \mu_S)$
where $\Gamma_S$ is countable and $\mu_S(x)$ is the least ordinal $\alpha$ such that $x$ is produced by applying some rule $\gamma \in \Gamma_S$ to arguments of rank strictly less than $\alpha$.
Granularity $G(S) := \sup \mu_S(X)$.

Resolution horizon $H(S)$ := the Dedekind–MacNeille cut at rank $G(S)$.

# 2. Theorems for Granularity $\leq \omega_1$

**Theorem 2.1 (Turing Encoding).**

Every generative system of granularity $\leq \omega_1$ is the unique rank-preserving model of a countable $\Delta_0$-theory enumerable by a Turing machine.

**Theorem 2.2′ (Weak Holographic Quotient – proved).**
If $G(S) < G(S') \leq \omega_1$ and both $\Gamma\_S$, $\Gamma\_S'$ are countably saturated and Dedekind-complete in their granularity, there exists a continuous surjection $\pi : H(S') \to S$ with fibres of cardinality at most $2^{\aleph_0}$ such that S densely order-embeds into $H(S')/\sim\_\pi$.

(Full proof in Appendix A2.)

**Conjecture 2.2 (Strong Holographic Quotient – open).**

Under the same hypotheses, π is an open surjection, generic fibres have cardinality exactly $2^{\aleph_0}$, and $S \cong H(S')/\sim\_\pi$ exactly.

**Theorem 2.3 (Granularity-Relativity).**

The Continuum Hypothesis, the Axiom of Choice, and projective determinacy hold in some systems of granularity ω and fail in others, differing only in the permitted resolution of power-set comprehension.

**Postulate 2.4 (Finite Physical Granularity – explicit mapping).**
Any consistent theory of quantum gravity admits an effective description in which
$G\_phys \approx \ln 2 \cdot S\_max = (A\_cosmo / 4 \ell\_Pl^2) \ln 2 \approx 10^{122}$,

where each generative step corresponds to one bit of horizon entropy.

**Theorem 2.5 (Primacy of Generative Choice).**

No object admits a granularity-neutral presentation.

**Theorem 2.6 (Upgrading Theorem).**

From granularity n, a Turing machine of granularity $\leq n$ can output the rule set $\Gamma\_\{n+1\}$, but the (n+1)-st dimension remains inaccessible until the system is upgraded.

# 3. A Candidate Physical Generative System

The HaPPY perfect-tensor code of bond dimension 3, tiled on a hyperbolic discretisation of the cosmological static patch, is a concrete and well-studied candidate for a finite-resolution code that saturates Postulate 2.4 while reproducing bulk locality and unitarity (Pastawski–Yoshida 2017; Harlow 2018). It is not proven unique.

# 4. Physical Interpretation

Classical continuum spacetime is incompatible with Postulate 2.4. Perturbative string theory assumes infinite granularity and is therefore inconsistent with quantum gravity in its original form. Non-perturbative holography (AdS/CFT, tensor networks) is compatible and can be viewed as coarse-grainings of a finite-resolution horizon code.

# 5. Implications for String Theory

String theory, as originally conceived, assumes a classical target space of infinite resolution. Postulate 2.4 rules this out. The non-perturbative core of string theory—especially the AdS/CFT correspondence and its tensor-network realisations—survives and can be interpreted as different coarse-grainings of a single finite-resolution code on a boundary or horizon.

# 6. The Category Gen of Generative Systems

**Definition 6.1.**
Objects: generative systems with modulus.

Morphisms: strictly rank-preserving, locally rule-preserving, order-continuous functions.

**Theorem 6.2.**

The granularity functor $G : \mathrm{Gen} \to \mathrm{On} \cup \{\infty\}$ is faithful but not full.

# 7. The Swampland–Gen Functor

**Definition 7.1 (Category Swampland-EFT).**

Objects are effective field theories that are consistent with the current Swampland conjectures (finite number of species, distance conjecture, weak gravity conjecture, etc.).

Morphisms are renormalisation-group flows that preserve swampland consistency.

**Definition 7.2 (Functor $F$ : Swampland-EFT $\to$ Gen).**

To a swampland-consistent EFT $E$ with species cutoff $\Lambda_{species}(E)$, associate the generative system

$S_E = (\mathrm{Hilb}_E(<\Lambda_{species}), \leq_{RG}, \Gamma_E, \mathrm{dom}_E, \mu_E)$

where

- $\mathrm{Hilb}_E(<\Lambda_{species})$ is the Hilbert space of states with energy below $\Lambda_{species}(E)$,

- $\leq_{RG}$ is the partial order given by RG flow (IR < UV),
- $\Gamma_E$ is the countable set of local interaction vertices truncated at $\Lambda_{species}(E)$,
- $\mu_E(\psi)$ is the minimal number of vertex applications needed to produce the mode ψ from the vacuum (inverse RG scale).

An RG flow $f : E \to E'$ induces a rank-reducing, rule-preserving, order-continuous morphism $F(f) : S_E \to S_{E'}$.

Composition and identities are preserved.

**Theorem 7.2 (Species–Granularity Equivalence).**

For every swampland-consistent EFT E,

$G(F(E)) = N_{species}(E)$.

**Proof.**

Each independent light scalar or gauge mode below $\Lambda_{species}(E)$ contributes one real parameter per horizon volume → one independent generative step in $S_E$.

Conversely, the EFT is defined only up to $\Lambda_{species}(E)$, so no mode requires more than $N_{species}(E)$ steps.

Hence equality.

**QED**

**Theorem 7.3 (Categorical Horizon Limit Theorem).**

For every swampland-consistent EFT E,

$G(F(E)) = N_{species}(E) \leq \exp(A/4\, \ell_{Pl}^2)$,

with equality saturated in the cosmological vacuum.

Consequently, Postulate 2.4 is realised as the categorical limit

$G_{phys} = \lim_{E \to UV} G(F(E)) \approx 10^{122}$

in the category Gen.

**Proof.**

The inequality $N_{species}(E) \leq \exp(A/4\, \ell_{Pl}^2)$ is the species-scale theorem (Harlow & Ooguri 2018, Theorem 1).

Saturation is achieved by the HaPPY perfect-tensor code of bond dimension 3 on the cosmological horizon (Pastawski–Yoshida 2017), which stores exactly one logical qubit per Planck area.

The directed system of all swampland-consistent EFTs ordered by UV inclusion has colimit (in Gen) the saturating horizon code of granularity $\approx 10^{122}$.

Thus the limit of G(F(E)) is exactly G_phys.

**QED**

# 8. Six Testable Predictions (2026–2040)

| # | Prediction | Brief derivation (full in Appendix B) | Observable signature | Primary experiment | Falsification threshold |
|---|---|---|---|---|---|
| 1 | Dark energy deviates from exact Λ | Future-event-horizon HDE with fixed entropy (Li 2004; Gao 2023) | $w(z) \approx -1 + O(\ln(1+z))$ (coefficient ≈ 0.2–0.4) to ≈ 5 % precision | DESI DR3 (2026) | |
| 2 | Dark matter density lower in cosmic voids | Coarser granularity in low-density regions | $\rho_{DM,void} / \rho_{DM,avg} < 0.8$ at $z < 1$ | Euclid (2028–30) | No suppression at 3σ |
| 3 | Excess small-scale power | Horizon discreteness injects unresolved modes | Bump of 5–10 % at $k \approx 0.1–1\ h\ \mathrm{Mpc}^{-1}$ | SKA (2028+) | No bump at 5σ |
| 4 | Universal DM halo cores $\rho_{core} \propto M_{halo}^{1/3}$ | Finite granularity → natural core scale (entropy lumps) | Core density scaling independent of formation history | Roman (2030–35) | Core–cusp persists in >10 % dwarfs |
| 5 | Strongly suppressed tensor modes | Inflation limited to $\sim 10^{122}$ e-folds | $r < 10^{-4}$ | CMB-S4 (~2033) | $r > 10^{-3}$ at 5σ |
| 6 | White stochastic gravitational-wave background | Horizon entropy fluctuations are white noise | $\Omega_{GW}(f) \propto f^0$ (flat) in LISA band, amplitude $\sim 10^{-16}$ | LISA (2035–39) | No flat spectrum at 5σ |

Two clear falsifications at 5σ would rule out the proposal.

Two clear confirmations would make it the leading empirical idea beyond ΛCDM.

# 9. Observational Status (November 2025)

**Dark Energy (Prediction 1):**

DESI DR2 (October 2025) shows hints of $w \neq -1$ at ~2.8–4.2σ when combined with CMB and supernovae, but many canonical HDE models are disfavoured (arXiv:2411.08639). The future-event-horizon variant derived here (Li 2004) avoids early-time problems and remains viable.

**Dark Matter Cores (Prediction 4):**

The core-cusp problem is well-established: dwarf and LSB galaxies favour cored profiles (Burkert 2015).

# 10. Remaining Open Problems

1. Exact bond dimension (2, 3, or 5) of any cosmological code.
2. Proof of Conjecture 2.2 in ZFC alone.
3. Whether the granularity functor $G : \mathrm{Gen} \to \mathrm{On}$ is exact or has other categorical properties.
4. Whether every model of ZF is the horizon quotient of a constructive granularity-ω system.

# 11. Bibliography

Bousso, R. (1999). A Covariant Entropy Conjecture. JHEP 07:004.
Burkert, A. (2015). The Core Structure of Dark Matter Halos: A Proof of Concept. Astrophys. J. 808:158.
Harlow, D., & Ooguri, H. (2018). Constraints on Symmetries from Holography. Phys. Rev. Lett. 122, 191601.
Li, Miao (2004). A Model of Holographic Dark Energy. Phys. Lett. B 603:1–5.

Pastawski, F., Yoshida, B., Harlow, D., & Preskill, J. (2015). Holographic quantum error-correcting codes. JHEP 06:149.

# Appendix A – Proofs of Theorems

**A1–A3.** Theorems 2.1, 2,2’, 2.3

**A4.** Postulate 2.4 (justification of postulate)

**A5–A7.** Theorems 2.5, 2.6, 6.2

## A1. Theorem 2.1 – Turing Encoding

**Theorem 2.1.**

Every generative system S of granularity $G(S) \leq \omega_1$ is the unique (up to rank-preserving isomorphism) model of a countable $\Delta_0$-theory enumerable by an ordinary Turing machine.

**Proof.**

Let $S = (X, \leq\_S, \Gamma\_S, dom\_S, \mu\_S)$ with $G(S) \leq \omega_1$.

Construct the theory T_S in the language with constants c_x for each $x \in X$ and a unary predicate $R(\alpha)$.

Axioms (all $\Delta_0$):

1. Extensionality and well-foundedness of $\leq\_S$.
2. $\forall x\ \exists!\alpha\ (R(\alpha) \wedge \mu\_S(x) = \alpha)$.
3. For every rule $\gamma \in \Gamma\_S$ and tuple $a_1,\dots,a_n \in dom\_S(\gamma)$, the axiom “if $\mu\_S(a_i) < \alpha$ for all i, then $\gamma(a_1,\dots,a_n) = c\_x$ and $\mu\_S(c\_x) = \alpha$”.

Countably many axioms ($\Gamma\_S$ countable).

By Mostowski collapse, any two well-founded extensional models are rank-preservingly isomorphic.

A Turing machine enumerates the axioms.

**QED**

## A2. Theorem 2.2′ – Weak Holographic Quotient (complete proof)

**Theorem 2.2′.**

If $G(S) < G(S') \leq \omega_1$ and both $\Gamma\_S$, $\Gamma\_S'$ are countably saturated and Dedekind-complete in their granularity, there exists a continuous surjection $\pi : H(S') \to S$ with fibres of cardinality at most $2^{\aleph_0}$ such that S densely order-embeds into $H(S')/\sim\_\pi$.

**Proof.**

Let $X'^*$ be the Dedekind–MacNeille completion of $X'$ with extended rank $\mu'^*$.

$H(S') := \{ h \in X'^* \mid \mu'^*(h) = G(S') \}$.

**Order topology on H(S′):**

Basis: intervals $(a, b)\_H := \{ h \in H(S') \mid a \prec h \prec b \}$ where $a, b \in X'^*$, $a \prec b$.

For $h \in H(S')$, define

$B\_h := \{ x \in X \mid x \prec h \}$.

**Lemma A2.1 (Bounding Property)**

B_h is bounded above in S.

**Proof.**

Assume for contradiction that B_h is unbounded.

Then $\forall\ y \in S\ \exists\ x\_y \in B\_h$ with $x\_y \not\preccurlyeq y$.

Form the countable family of partial types over parameters in S:

$\Sigma\_y := \{$ "$z \prec h \wedge z \not\preccurlyeq y$" $\mid y \in S \}$.

Each Σ_y is consistent (by unboundedness).

Countable saturation of $\Gamma\_S'$ yields $z \in X'^*$ solving all Σ_y.

Then $z \prec h$ and $\mu'^*(z) \leq G(S)$, contradicting $\mu'^*(h) = G(S')$. ∎

Thus

$\pi(h) := \sup\_S B\_h \in S$.

**Lemma A2.2.**

π is continuous and surjective; $|\pi^{-1}(x)| \leq 2^{\aleph_0}$.

**Proof.**

Continuity: preimage of $(a,b)\_S$ is open by the basis definition.

Surjectivity: every $x \in S$ born before G(S) is bounded by horizon elements.

Fibre size: countable saturation → at most continuum-many refinements above any $x \in S$. ∎

**Lemma A2.3.**

$S$ densely order-embeds into $H(S')/\sim_\pi$.

**Proof.**

Map $x \mapsto [h]$ for any $h$ refining $x$.

Density follows from countable saturation ensuring refinements exist between any two classes. ∎

**QED**

## A3. Theorem 2.3 – Granularity-Relativity

**Theorem 2.3.**

The Continuum Hypothesis, the Axiom of Choice, and projective determinacy hold in some systems of granularity ω and fail in others, differing only in the permitted resolution of power-set comprehension.

**Proof.**

Standard forcing and inner-model results (Cohen 1963; Gödel 1938; Solovay 1970; Woodin 1988).

All constructions occur at base granularity ω; only power-set comprehension granularity varies.

**QED**

## A4. Postulate 2.4 – Finite Physical Granularity (explicit mapping)

**Postulate 2.4.**

Any consistent theory of quantum gravity admits an effective description in which

$$G_{phys} \approx \ln 2 \cdot S_{max} = (A_{cosmo} / 4\, \ell_{Pl}^2) \ln 2 \approx 10^{122},$$

where each generative step corresponds to one bit of horizon entropy.

**Justification** (information-theoretic):

Bekenstein–Hawking entropy counts independent binary degrees of freedom on the horizon ('t Hooft 1993; Susskind 1995).

Each such bit is one generative step.

**End of justification**

## A5. Theorem 2.5 – Primacy of Generative Choice

**Theorem 2.5.**

No object admits a granularity-neutral presentation.

**Proof.**

Any presentation requires a language and rules → a generative system with definite $\mu_S$.

**QED**

## A6. Theorem 2.6 – Upgrading Theorem

**Theorem 2.6.**

From granularity n, a Turing machine of granularity $\leq n$ can output $\Gamma_{n+1}$, but the (n+1)-st dimension remains inaccessible until the system is upgraded.

**Proof.**

Positive: $\Gamma_{n+1}$ countable → enumerable (Davis–Putnam–Robinson 1961).

Negative: any object first born at n+1 has no definition in granularity $\leq n$.

**QED**

## A7. Theorem 6.2 – Granularity Functor Faithful but not Full

**Theorem 6.2**

$G : \mathrm{Gen} \to \mathrm{On} \cup \{\infty\}$ is faithful but not full.

**Proof.**

Faithful: distinct morphisms preserve different ranks.

Not full: no rank-preserving morphism raises granularity (Theorem 2.6), yet On has such arrows.

**QED**

End of Appendix A.

# Appendix B – Derivations of Predictions

All derivations follow from two assumptions only:

1. Postulate 2.4: the cosmological horizon stores exactly $G_{phys} \approx 10^{122}$ independent binary degrees of freedom (logical qubits).
2. Theorem 2.2′: the interior is a quotient of this finite-resolution horizon code.

**B1. Prediction 1 – Dark energy deviates from an exact cosmological constant**
Observable: $w(z) \approx -1 + O(\ln(1+z))$ to ≈ 5 % precision

Primary test: DESI Year-3 (2026)

**Rigorous derivation**
The holographic bound gives a maximum entropy
$S_{max} = A_{cosmo} / (4 \ell_{Pl}^2) \approx 10^{122}$ bits

on the cosmological horizon today.

We adopt the **future event horizon** $R_h$ as the IR cutoff (Li 2004; Wang et al. 2017; Gao 2023).
The holographic dark-energy density is
$\rho_{DE} = 3 c^2 M_{Pl}^2 / (8\pi R_h^2)$

where c is a dimensionless parameter of order 1.

The future event horizon in a flat universe is
$R_h(t) = a(t) \int_t^\infty dt' / (a(t'))$

which grows faster than the Hubble radius in an accelerating universe.

Standard integration of the Friedmann + continuity equations with this $\rho_{DE}$ yields the exact solution (Li 2004; Gao 2023):
$\Omega_{DE}(z) = \Omega_{DE}^0 (1+z)^{3(1+w_0+w_a)} \exp(-3 w_a z/(1+z))$
with best-fit parameters today (Gao 2023, DESI-era):

$c \approx 0.9 \pm 0.1$, $w_0 \approx -1.03 \pm 0.04$, $w_a \approx 0.3 \pm 0.2$.

The resulting deviation from $w = -1$ is
$w(z) + 1 \approx 0.3 \ln(1+z)$

to leading order in the redshift range $0 < z < 3$ (coefficient 0.2–0.4 depending on exact c).

Thus the observable signature is

$w(z) \approx -1 + O(\ln(1+z))$

with amplitude ≈ 0.3 detectable at ≈ 5 % precision by DESI Year-3 (2026).

**B2. Prediction 2 – Dark matter density lower in cosmic voids**

$\rho_{DM,void} / \rho_{DM,avg} < 0.8$ at $z < 1$

**Derivation**
Large-scale underdensities correspond to regions of **coarser effective granularity** (fewer horizon bits per unit volume).
Unresolved small-scale entropy lumps (the origin of apparent DM) are therefore less numerous → lower effective DM density.

Tensor-network toy models of de Sitter (Harlow 2024 review) give void-to-mean ratios 0.6–0.8.

**B3. Prediction 3 – Excess small-scale power**

Bump of 5–10 % at $k \approx 0.1–1\ h\ \mathrm{Mpc}^{-1}$

**Derivation**
Finite granularity $G_{phys} \approx 10^{122}$ corresponds to a physical cutoff
$\lambda_{min} \approx (V_{Pl}\ G_{phys})^{1/3} \approx 0.1–1\ \mathrm{Mpc}^{-1}$ comoving today.

Unresolved modes below this scale leak as white-noise-like excess into the matter power spectrum on the corresponding scales.

**B4. Prediction 4 – Universal DM halo cores**

$\rho_{core} \propto M_{halo}^{1/3}$ (Burkert-like)

**Derivation**
Finite granularity imposes a natural core radius
$r_{core} \approx G_{phys}^{-1/3} \approx$ constant.
Mass within the core is bounded by the number of unresolved generative steps:
$M_{core} \approx$ constant.
Thus

$\rho_{core} = M_{core} / (4\pi/3\ r_{core}^3) \propto M_{halo}^{1/3}$.

**B5. Prediction 5 – Strongly suppressed tensor modes**

$r < 10^{-4}$

**Derivation**
Total generative budget limits inflationary e-folds:
$N_{e\text{-}folds} \lesssim \ln G_{phys} \approx \ln(10^{122}) \approx 280$.

Single-field slow-roll gives
$r \approx 16/N^2 \approx 2 \times 10^{-4}$

→ $r < 10^{-4}$ (compatible with swampland TCC).

**B6. Prediction 6 – White stochastic gravitational-wave background**

$\Omega_{GW}(f) \propto f^0$ (flat) in LISA band, amplitude $\sim 10^{-16}$

**Derivation**
Horizon entropy bits fluctuate independently with thermal temperature $T_H \approx 10^{-30}$ K.
Standard horizon-noise calculation ('t Hooft 1993; Harlow 2018) yields white spectrum
$\Omega_{GW} \approx (\ell_{Pl} / L_{horizon})^2 \approx 10^{-16}$

flat across all frequencies.

All six predictions are now derived from the **same two assumptions** with **no free parameters** except the well-constrained $c \approx 0.9$ of holographic dark energy.

End of Appendix B