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Self-Referential Observers in Quantum Dynamics: A Formal Theory of Internal Collapse and Cross-Observer Agreement

Chung Leung Danny Yeung
Independent Researcher, London, UK
Correspondence: SolitonSchooler@gmail.com
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Abstract

This paper develops a formal framework for "self-referential observers" within standard quantum theory. An observer is modeled as a process that records discrete measurement outcomes (its internal trace), selects future instruments adaptively as a measurable function of that trace, and updates the joint system–observer state via completely positive maps. Under mild measurability and continuity conditions, we prove the existence and uniqueness of the induced stochastic law over infinite outcome histories by constructing Born-rule transition kernels and invoking the Ionescu–Tulcea extension theorem; instruments are represented via Stinespring dilations. The model yields two internal phenomena without additional postulates: (i) delta-certainty over past outcomes (the observer's recorded events are fixed in its own filtration) and (ii) "latching" irreversibility, whereby adaptive policies make counterfactual branches diverge. We then formalize cross-observer agreement (AB-fixedness). Agreement holds when mapped effects commute, a frame transform consistently relates contexts and outcomes, and a record is accessible (e.g., stored locally or redundantly in the environment). Objectivity emerges in the presence of spectrum broadcast structures: redundant, fragmentwise encodings of pointer information drive high-probability consensus among many observers. Finally, we introduce a collapse-frame geometry: an indefinite "collapse interval" on the product of discrete tick-time and channel space, and the associated group of isometries ("Collapse-Lorentz" transformations) that preserve agreement/incompatibility relations. We present counterexamples demonstrating necessity of assumptions (measurability, commutation, redundancy, isometry) and outline equivalent formulations (operator-algebraic, process-tensor, categorical) to show the framework's robustness as a refereeable mathematical contribution rather than a new interpretation.

Keywords
quantum measurement; adaptive instruments; self-referential observers; stochastic processes on outcome histories; spectrum broadcast structure; objectivity; compatibility and commutation; process tensor; operator algebras; information geometry

Subject Areas
Quantum foundations; Mathematical physics; Quantum information theory

Ethics and Competing Interests
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Section 1. Introduction

1.1 Motivation and Scope
The standard quantum formalism combines unitary evolution for closed systems with stochastic state updates induced by measurement. The operational rules are clear, yet the ontological status of "collapse" remains unsettled across interpretations. This paper adopts a purely formal stance: we analyze observers as adaptive stochastic processes internal to quantum theory, without introducing extra postulates or metaphysical claims. An observer is modeled as a system that (i) records discrete outcomes ("trace"), (ii) selects future instruments as a measurable function of its recorded trace ("adaptive policy"), and (iii) updates the joint system–observer state via completely positive (CP) maps consistent with the Born rule. We study the internal certainty experienced by such observers, the conditions for cross-observer agreement, and a symmetry structure that preserves these relations.

1.2 Problem Statement
Two questions guide this work:
(1) Can self-referential, trace-conditioned measurement dynamics be defined as a mathematically well-posed stochastic process over infinite outcome histories, compatible with standard quantum mechanics?
(2) Under what conditions do distinct observers—possibly using different instruments and frames—assign delta-certainty to the same recorded event, and when does large-scale objectivity emerge?

1.3 Main Contributions
(A) Existence and Uniqueness of Adaptive Observer Processes. We construct Born-rule transition kernels from trace-preserving instruments selected by measurable policies and obtain a unique probability measure over infinite outcome sequences by the Ionescu–Tulcea extension theorem. Instruments are represented via Stinespring dilations to ensure compatibility with unitary embeddings.
(B) Internal Delta-Certainty and Latching. Conditioning on the observer's own filtration makes past outcomes delta-certain within that observer's frame. When policies depend on trace, counterfactual branches diverge irreversibly ("latching"), yielding the phenomenology of collapse without additional postulates.
(C) Cross-Observer Agreement (AB-Fixedness). We formalize agreement between observers through (i) a frame transform that maps contexts and outcomes, (ii) a compatibility condition (commuting effects or joint measurability), and (iii) accessibility of a record (local trace or redundant environment). Under these conditions, mapped outcomes are delta-certain for all parties.
(D) Objectivity via Redundancy. We show that spectrum broadcast structures—redundant encodings of pointer information across disjoint environment fragments—drive high-probability consensus among many observers, with disagreement rates that shrink under standard distinguishability bounds.
(E) Collapse-Frame Geometry and Invariance. We introduce an indefinite "collapse interval" on the product of discrete tick index and channel space, and define the group of isometries ("Collapse-Lorentz" transformations) that preserve agreement and incompatibility relations.
(F) Robustness and Necessity. We present counterexamples demonstrating that measurability, commutation/joint measurability, redundancy, and isometric frame mappings are each structurally necessary. We also recast the framework in operator-algebraic, process-tensor, and categorical terms to demonstrate equivalence across formalisms.

1.4 Conceptual Positioning
This is not a new interpretation of quantum mechanics. The theory treats "collapse" as the internal, conditionalized certainty that arises for an adaptive agent maintaining a trace of outcomes while interacting via standard instruments. Cross-observer agreement and objectivity

are derived from compatibility, record accessibility, and redundancy—features already available within the conventional formalism.

1.5 Assumptions and Modeling Choices
(i) Time is discretized by observer "ticks," each corresponding to an instrument application and outcome write. Between ticks, dynamics are latent (unitary or CPTP).
(ii) Outcome alphabets are finite or countable, and policy maps from finite histories to instrument labels are measurable with respect to the natural product σ-algebra.
(iii) Instruments are trace-preserving families of CP maps; updates follow the Born rule.
(iv) Frame transforms are maps between observers that consistently relabel contexts, effects, and outcomes; when required, they act as structure-preserving maps on the effect space.
(v) Redundancy is modeled via spectrum broadcast structures in the system–environment state; distinguishability assumptions are stated where needed.

1.6 Relation to Prior Work (brief orientation)
The technical backbone draws on established results and frameworks: Stinespring dilations for CP maps; Naimark dilation and joint measurability for compatibility; Ionescu–Tulcea extension for infinite-horizon stochastic processes; quantum Darwinism and spectrum broadcast structures for redundancy and objectivity; process tensors/quantum combs for causal modeling with memory; and operator-algebraic tools (von Neumann algebras, conditional expectations) for filtration and certainty. Our contribution integrates these ingredients into a single self-referential observer model with precise conditions for internal certainty, agreement, and invariance.

1.7 Organization of the Paper
Section 2 defines the mathematical framework: Hilbert spaces, instruments, outcome spaces, adaptive policies, and filtrations.
Section 3 proves existence and uniqueness of adaptive observer processes and establishes internal delta-certainty and latching.
Section 4 develops cross-observer agreement (AB-fixedness) under frame maps and compatibility, with precise conditions on records.
Section 5 analyzes objectivity via redundancy using spectrum broadcast structures.
Section 6 introduces the collapse-frame geometry and its isometries, clarifying invariants under frame changes.
Section 7 presents counterexamples that demonstrate the necessity of key assumptions.
Section 8 discusses implications, limitations, and extensions, and outlines practical avenues for simulation and control.
References and brief appendices (notation map; axioms-to-theorems chart; minimal simulation outline) complete the manuscript.

Section 2. Mathematical Framework

2.1 Spaces and States
Definition 2.1 (World, Observer, and Composite Space)
Let H_W be a separable Hilbert space for the "world" under observation. Let H_O be a separable Hilbert space for the observer's memory/ancilla. The total space is
$H := H_W \otimes H_O$.
Write D(H) for the set of density operators on H (positive, trace 1). Write B(H) for bounded operators on H.

Remark 2.1 (Picture)
We work in the Schrödinger picture for states and in the Heisenberg picture when talking about effects; both views are linked by the adjoint (dual) map.

2.2 Discrete Tick Index and Between-Tick Evolution
Definition 2.2 (Ticks)
Observation occurs at discrete "ticks" indexed by $k \in \{1, 2, \ldots\}$. We set the tick-time variable to

$\tau_k := k$ (so $\Delta\tau = 1$ by convention). Between ticks, the composite state evolves under a latent completely positive trace-preserving (CPTP) map:
$\rho_{k^-} := E_k(\rho_{k-1})$, where $E_k: D(H) \to D(H)$ is CPTP.
Measurement (collapse update) occurs only at ticks.

2.3 Outcome Alphabet and Product Sigma-Algebra

Definition 2.3 (Outcome Space)
Let Φ be a finite or countable outcome alphabet with the discrete sigma-algebra. For $k \geq 1$, write Φ^k for length-k strings and Φ^∞ for infinite sequences. Equip Φ^∞ with the product sigma-algebra generated by cylinder sets.

2.4 Instruments and Effects

Definition 2.4 (Quantum Instrument)
A measurement "context" (or "channel setting") is a label θ in a parameter space Θ. For each θ, a (trace-preserving) quantum instrument is a family of completely positive (CP), trace–nonincreasing maps
$M_{\theta,\varphi}: D(H) \to D(H)$, for $\varphi \in \Phi$,
such that the sum map $\sum_{\varphi\in\Phi} M_{\theta,\varphi}$ is CPTP. For any pre-measurement state $\sigma \in D(H)$,
$\Pr(\varphi \mid \theta, \sigma) = \mathrm{Tr}[\, M_{\theta,\varphi}(\sigma)\,]$ and $\sigma \mapsto M_{\theta,\varphi}(\sigma) / \mathrm{Tr}[M_{\theta,\varphi}(\sigma)]$.

Definition 2.5 (Associated Effects)
Let $M^*_{\theta,\varphi}: B(H) \to B(H)$ be the dual (Heisenberg) map. Define the effect operator
$E_{\theta,\varphi} := M^*_{\theta,\varphi}(I)$, with $0 \leq E_{\theta,\varphi} \leq I$ and $\sum_{\varphi} E_{\theta,\varphi} = I$.
Then $\Pr(\varphi \mid \theta, \sigma) = \mathrm{Tr}[\, E_{\theta,\varphi}\, \sigma\,]$. These effects will be used for compatibility/commutation in Section 4.

2.5 Channel (Context) Space and Regularity

Assumption A1 (Channel Space)
Θ is a standard Borel (e.g., Polish) space with its Borel sigma-algebra.
Assumption A2 (Continuity in θ)
For each φ, the map $\theta \mapsto M_{\theta,\varphi}$ is continuous in the strong (or diamond) norm topology sufficient to make $(\theta, \sigma) \mapsto \mathrm{Tr}[M_{\theta,\varphi}(\sigma)]$ jointly measurable. This ensures measurability of Born-rule kernels constructed below.

2.6 Adaptive Policies (Trace-Conditioned Context Selection)

Definition 2.6 (Adaptive Policy)
An adaptive policy is a sequence of measurable maps
$f_k: \Phi^{k-1} \to \Theta$, $k \geq 1$.
At tick k, the selected context is $\theta_k := f_k(\varphi_{1:k-1})$, where $\varphi_{1:k-1}$ is the realized outcome trace up to tick $k-1$.

2.7 Observer Trace and Filtration

Definition 2.7 (Trace and Filtration)
Let $\varphi_k \in \Phi$ be the realized outcome at tick k, and let
$\varphi_{1:k} := (\varphi_1, \ldots, \varphi_k) \in \Phi^k$.
The observer's information after tick k is captured by the sigma-algebra
$F_k := \sigma(\varphi_1, \ldots, \varphi_k)$,
the sigma-algebra generated by the observed prefix. The family $(F_k)_{k\geq 0}$ with F_0 trivial is an increasing filtration.

2.8 Outcome–State Dynamics at a Tick

Definition 2.8 (Tick Update Rules)
Given ρ_{k^-} (the pre-measurement state after between-tick evolution) and $\theta_k = f_k(\varphi_{1:k-1})$, the model specifies:
(i) Outcome sampling
$p_k(\varphi \mid \varphi_{1:k-1}) := \mathrm{Tr}[\, M_{\theta_k,\varphi}(\rho_{k^-})\,]$, for $\varphi \in \Phi$.
(ii) Post-measurement update

$\rho_k := M_{\theta_k,\varphi_k}(\rho_{k^-}) / \mathrm{Tr}[M_{\theta_k,\varphi_k}(\rho_{k^-})]$.
The process thus produces a sequence of outcome–state pairs (φ_k, ρ_k) for $k = 1, 2, \ldots$

Remark 2.2 (Kernels)
The functions $K_k(\varphi_{1:k-1}, \varphi) := p_k(\varphi \mid \varphi_{1:k-1})$ form measurable transition kernels on Φ (standard Borel because Φ is countable). Section 3 constructs from these kernels a unique probability measure on Φ^∞ via Ionescu–Tulcea.

2.9 Minimal Assumption Set (for Existence and Agreement)
We collect the standing assumptions referenced later:

A1. Θ is standard Borel; Φ is finite or countable.
A2. (Regularity) For each φ, $\theta \mapsto M_{\theta,\varphi}$ is continuous in a topology guaranteeing measurability of $(\varphi_{1:k-1} \mapsto \theta_k)$ composed with $(\theta, \sigma) \mapsto \mathrm{Tr}[M_{\theta,\varphi}(\sigma)]$.
A3. (Policy measurability) Each $f_k: \Phi^{k-1} \to \Theta$ is Borel-measurable.
A4. (Between-tick CPTP) Each E_k is CPTP; $\rho_{k^-} := E_k(\rho_{k-1})$.
A5. (Instrument normalization) For each θ, $\sum_{\varphi} M_{\theta,\varphi}$ is CPTP; associated effects satisfy $\sum_{\varphi} E_{\theta,\varphi} = I$.
A6. (Compatibility for agreement; used in Section 4) When two observers compare propositions, the relevant effects commute (or are jointly measurable) after any required frame mapping.
A7. (Redundancy for objectivity; used in Section 5) When modeling environment-mediated consensus, the joint system–environment state exhibits spectrum broadcast structure (or an explicitly stated distinguishability condition).

2.10 Notes on Frames and Maps (Preview)
While frame transforms are developed in Section 4, we record the structural requirement used there:

(i) A frame map T between observers acts on contexts and effects, producing $T_\Theta: \Theta^A \to \Theta^B$ and T_E on effect operators, such that probabilities are preserved when reading shared records.
(ii) When we reason about invariance (Section 6), we additionally require that T preserves a specified indefinite form on the product of tick index and channel space; this will define the “collapse-frame” isometries.

2.11 Summary of Section 2
We have specified: the composite Hilbert space; the discrete tick index with latent CPTP evolution; the countable outcome alphabet with product sigma-algebra; trace-preserving instruments and associated effects; measurable adaptive policies; the observer’s filtration; and the tickwise outcome–state update rules. With measurability and regularity in place, Section 3 constructs the global stochastic law on infinite traces, proves uniqueness, and formalizes internal delta-certainty and latching.

Section 3. Existence and Uniqueness of Adaptive Observer Processes

3.1 Problem Setup and Objectives
Given the framework of Section 2, we aim to construct a single, well-defined stochastic process over infinite outcome histories together with the induced post-measurement states. Concretely, from:
(a) an initial state $\rho_0 \in D(H)$,
(b) a family of trace-preserving instruments $\{ M_{\theta,\varphi} : \varphi \in \Phi \}$ indexed by $\theta \in \Theta$,
(c) a between-tick CPTP evolution $\{ E_k \}$, and
(d) a measurable adaptive policy sequence $f_k: \Phi^{k-1} \to \Theta$,
we will:

(i) build measurable transition kernels $K_k(\varphi_{1:k-1}, \varphi) = \mathrm{Tr}[M_{\theta_k,\varphi}(\rho_{k^-})]$ where $\theta_k := f_k(\varphi_{1:k-1})$ and $\rho_{k^-} := E_k(\rho_{k-1})$;
(ii) invoke the Ionescu–Tulcea extension theorem to obtain a unique probability measure P on

Φ^∞ with those kernels as conditionals;
(iii) define the state recursion ρ_k deterministically from the realized outcomes;
(iv) establish two internal properties: delta-certainty about past outcomes and "latching" irreversibility when the policy depends on history.

Throughout, we use the assumptions listed in 2.9 (A1–A5).

3.2 Regularity and Measurability of the Building Blocks

Lemma 3.1 (Measurability of composed kernels).
Fix $k \geq 1$. Under A1–A5, the map
$(\varphi_{1:k-1}, \varphi) \mapsto K_k(\varphi_{1:k-1}, \varphi) := \mathrm{Tr}[M_{f_k(\varphi_{1:k-1}), \varphi}(\rho_{k^-})]$
is measurable on $\Phi^{k-1} \times \Phi$ (with Φ countable and Φ^{k-1} endowed with the product sigma-algebra), where $\rho_{k^-} := E_k(\rho_{k-1})$ and ρ_{k-1} is itself a measurable function of $\varphi_{1:k-1}$.

Proof.
Step 1 (Policy measurability). By A3, f_k is Borel-measurable, so $\varphi_{1:k-1} \mapsto \theta_k$ is measurable.
Step 2 (Instrument regularity). By A2 and A5, for each fixed φ the map $\theta \mapsto M_{\theta,\varphi}$ is continuous in a topology strong enough to ensure $(\theta, \sigma) \mapsto \mathrm{Tr}[M_{\theta,\varphi}(\sigma)]$ is jointly measurable.
Step 3 (State dependence). By induction in k, the map $\varphi_{1:k-1} \mapsto \rho_{k-1}$ is measurable (details below), and E_k is CPTP (A4), hence continuous and measurable; thus $\varphi_{1:k-1} \mapsto \rho_{k^-}$ is measurable.
Step 4 (Composition). The composition
$\varphi_{1:k-1} \mapsto (\theta_k, \rho_{k^-}) \mapsto \mathrm{Tr}[M_{\theta_k,\varphi}(\rho_{k^-})]$
is measurable for each φ. Since Φ is countable, K_k is measurable as a function of $(\varphi_{1:k-1}, \varphi)$. ■

Lemma 3.2 (Measurability of post-measurement state recursion).
Define ρ_0 as given. For $k \geq 1$, let
$\rho_{k^-} := E_k(\rho_{k-1})$,
$\rho_k := M_{\theta_k,\varphi_k}(\rho_{k^-}) / \mathrm{Tr}[M_{\theta_k,\varphi_k}(\rho_{k^-})]$,
where $\theta_k := f_k(\varphi_{1:k-1})$. Then, as a function on the cylinder set $\{\varphi_{1:k}\}$, the map $\varphi_{1:k} \mapsto \rho_k$ is measurable.

Proof.
By induction. At k=1, $\varphi_1 \mapsto \theta_1 = f_1(\emptyset)$ is constant; $\varphi_1 \mapsto \mathrm{Tr}[M_{\theta_1,\varphi_1}(E_1(\rho_0))]$ is measurable in φ_1; the normalization divides by a strictly positive number on the event $\{\varphi_1$ realized$\}$ (events with zero probability are not realized). Hence $\varphi_1 \mapsto \rho_1$ is measurable.
Assume $\varphi_{1:k-1} \mapsto \rho_{k-1}$ measurable. Then $\varphi_{1:k-1} \mapsto \rho_{k^-} := E_k(\rho_{k-1})$ is measurable; with φ_k, the pair $(\theta_k, \varphi_k) \mapsto M_{\theta_k,\varphi_k}(\rho_{k^-})$ and its trace are measurable; normalization on the realized event gives $\varphi_{1:k} \mapsto \rho_k$ measurable. ■

Remark 3.1 (Non-zero denominators).
For any realized outcome φ_k, by construction the denominator $\mathrm{Tr}[M_{\theta_k,\varphi_k}(\rho_{k^-})]$ equals the kernel probability assigned to φ_k and is strictly positive on the event $\{\varphi_k$ realized$\}$. Thus the conditional state is well-defined almost surely.

3.3 Dilation of Instruments (Structural Embedding)

Proposition 3.3 (Stinespring–Naimark representation for instruments).
For each $\theta \in \Theta$, there exist a separable ancilla space K, an ancilla state $\xi \in D(K)$, a unitary U_θ on $H \otimes K$, and a projection-valued measure $\{ P_\varphi : \varphi \in \Phi \}$ on K such that for all $\sigma \in D(H)$ and $\varphi \in \Phi$:
$M_{\theta,\varphi}(\sigma) = \mathrm{Tr}_K[(I \otimes P_\varphi) U_\theta (\sigma \otimes \xi) U_\theta^\dagger (I \otimes P_\varphi)]$.
Consequently, the tick update can be realized as a unitary interaction with an ancilla followed by

a projective readout on K.

Sketch of justification.
This is standard: Stinespring dilation for each CP map component, and a Naimark dilation to represent the instrument's POVM components jointly as projections on an extended space. The normalization $\sum_\varphi \mathcal{M}_{\theta,\varphi}$ CPTP ensures the overall trace preservation after summing over φ. ■

Remark 3.2 (Use of dilation).
While the existence–uniqueness result below does not require constructing the dilation explicitly, it guarantees that our abstract tick updates admit a unitary–measurement realization on a larger Hilbert space.

3.4 Construction of the Global Law on Φ^∞
Theorem 3.4 (Existence and uniqueness of the adaptive observer process).
Assume A1–A5. Let $\rho_0 \in D(H)$ be fixed, and let $\{E_k\}$, $\{f_k\}$, and $\{M_{\theta,\varphi}\}$ be given as above. Then:

(a) There exists a unique probability measure P on (Φ^∞, product sigma-algebra) such that for every $k \geq 1$ and every cylinder event determined by $\varphi_{1:k-1}$, the conditional distribution of φ_k satisfies
$P(\varphi_k = \varphi \mid \varphi_{1:k-1}) = K_k(\varphi_{1:k-1}, \varphi) = \mathrm{Tr}[M_{f_k(\varphi_{1:k-1}), \varphi}(\rho_{k^-})]$
with $\rho_{k^-} := E_k(\rho_{k-1})$ and ρ_{k-1} defined recursively from $\varphi_{1:k-1}$.

(b) Given an outcome history $\omega = (\varphi_1, \varphi_2, \ldots)$ in the support of P, the post-measurement state sequence is uniquely determined by the recursion
$\rho_k(\omega) = M_{\theta_k(\omega), \varphi_k}(\rho_{k^-}(\omega)) / \mathrm{Tr}[M_{\theta_k(\omega), \varphi_k}(\rho_{k^-}(\omega))]$,
with $\theta_k(\omega) := f_k(\varphi_{1:k-1})$, $\rho_{k^-}(\omega) := E_k(\rho_{k-1}(\omega))$.

Proof.
Part (a): By Lemma 3.1, for each k the map $(\varphi_{1:k-1}, \cdot) \mapsto K_k(\varphi_{1:k-1}, \cdot)$ is a measurable probability kernel from Φ^{k-1} to Φ (standard Borel because Φ is countable). The Ionescu–Tulcea extension theorem then guarantees existence of a unique probability measure P on Φ^∞ whose finite-dimensional conditionals are exactly the K_k.
Part (b): By Lemma 3.2, for every finite prefix $\varphi_{1:k}$, the state ρ_k is a measurable function of that prefix; thus for any ω in the support of P the sequence $\{\rho_k(\omega)\}$ is uniquely defined. ■

Corollary 3.5 (Adaptedness and consistency with the filtration).
Let $F_k := \sigma(\varphi_1, \ldots, \varphi_k)$. Then:
(i) φ_k is F_k-measurable and (φ_k) is adapted to the filtration (F_k).
(ii) ρ_k is F_k-measurable and hence adapted.
(iii) For any bounded function g on D(H), the process $g(\rho_k)$ is adapted; if between-tick maps E_k are fixed, the pair (φ_k, ρ_k) is a well-defined adapted process.

Proof.
Immediate from the construction and measurability established above. ■

3.5 Internal Delta-Certainty (Fixedness of Past Outcomes)
Proposition 3.6 (Delta-certainty of recorded past).
Fix $k \geq 1$ and $j \leq k$. Let $x \in \Phi$. Then, almost surely,
$P(\varphi_j = x \mid F_k) = 1$ if x equals the realized φ_j, and 0 otherwise.
Equivalently, conditioning on F_k collapses the sigma-algebra to a point mass on each past coordinate.

Proof.
F_k is generated by the realized finite string $\varphi_{1:k}$. Conditioning on F_k fixes the values of all φ_j for $j \leq k$, so the conditional probability of any event $\{\varphi_j = x\}$ is either 1 (if x equals the realized value) or 0. ■

Remark 3.3 (Interpretation).
Within the observer's own filtration, past outcomes are "certain" in the strongest sense—delta measures. This is the formal content of internal collapse in the present framework.

3.6 Latching Irreversibility (Branch-Dependent Dynamics)

Definition 3.7 (Latching).
The process exhibits latching if there exist indices k and two feasible histories $h, h' \in \Phi^{k}$ that differ at some coordinate $\le k$ such that the subsequent context selections differ with positive probability (i.e., there exists $\ell \ge k+1$ and a set of nonzero probability under P for which $f_\ell(h \cdots) \neq f_\ell(h' \cdots)$). Intuitively, distinct recorded outcomes cause subsequent measurement paths to diverge.

Proposition 3.8 (Policy dependence implies latching).
Suppose there exists k and a Borel set $B \subset \Phi^{k}$ with $P(\varphi_{1:k} \in B) > 0$ such that the map f_{k+1} is non-constant on B (i.e., there exist $h, h' \in B$ with $f_{k+1}(h) \neq f_{k+1}(h')$). Then the process latches at time k: with positive probability, the future instrument at tick k+1 differs across the branches realizing h versus h'.

Proof.
On the event $\{\varphi_{1:k} = h\}$ the context at tick k+1 is $\theta_{k+1} = f_{k+1}(h)$; on $\{\varphi_{1:k} = h'\}$ it is $\theta'_{k+1} = f_{k+1}(h') \neq \theta_{k+1}$. Both events have positive probability because B has positive measure and Φ is countable; hence with positive probability the post-k dynamics diverge across these realized histories. ■

Remark 3.4 (Irreversibility).
The divergence is not undone by later conditioning: once a particular outcome is written, the policy follows the recorded history, making future distributions branch-specific. This formalizes the intuitive "latching" effect without introducing new postulates.

3.7 Structural Properties and Sanity Checks

Proposition 3.9 (Normalization and non-explosion).
For each k and each realized $\varphi_{1:k-1}$, we have $\sum_{\varphi \in \Phi} K_k(\varphi_{1:k-1}, \varphi) = 1$. Moreover, for realized φ_k, $\mathrm{Tr}[M_{\theta_k,\varphi_k}(\rho_{k^-})] > 0$ almost surely.

Proof.
By A5, $\sum_{\varphi} M_{\theta_k,\varphi}$ is CPTP; thus $\sum_{\varphi} \mathrm{Tr}[M_{\theta_k,\varphi}(\rho_{k^-})] = \mathrm{Tr}[\rho_{k^-}] = 1$. Since outcomes are sampled from K_k, any realized φ_k has strictly positive kernel probability almost surely. ■

Proposition 3.10 (History-dependent, but law-of-one-construction).
Although the process is generally non-Markovian in φ_k alone (because kernels depend on the entire prefix via f_k and ρ_{k^-}), the Ionescu–Tulcea construction yields a single, well-defined law P on Φ^∞. Different initial states ρ_0 or different policy sequences $\{f_k\}$ define different laws, but for fixed inputs the law is unique.

Proof.
Uniqueness follows directly from the extension theorem given the specified kernels. ■

3.8 Summary of Section 3

We have constructed the adaptive observer process rigorously:

- Kernels are measurable and normalized; states update measurably along realized histories.
- A unique probability measure on infinite outcome sequences exists by Ionescu–Tulcea.
- The observer's past is delta-certain within its own filtration (internal collapse).
- When policies depend on history, future dynamics latch to recorded outcomes (branch-dependent irreversibility).

These results require only the standing assumptions (A1–A5) and standard theorems

(Stinespring–Naimark dilation; Ionescu–Tulcea extension). Section 4 now turns to relations between distinct observers: frame maps, compatibility (commutation or joint measurability), records, and the resulting AB-fixedness (cross-observer delta-certainty).

Section 4. Cross-Observer Agreement (AB-Fixedness)

4.1 Joint Setting and Notation
We consider two observers, A and B, interacting (possibly at different ticks) with a common world subsystem and maintaining their own memories.

- World space: H_W (separable).
- Observer memories: H_O^A, H_O^B (separable).
- Composite spaces for local descriptions:

$H^A := H_W \otimes H_O^A$, $H^B := H_W \otimes H_O^B$.
For joint reasoning we may embed both into $H_{tot} := H_W \otimes H_O^A \otimes H_O^B \otimes H_E$, where H_E is an (optional) environment/register space used to carry "records."

Each observer uses a family of trace-preserving instruments indexed by a context $\theta \in \Theta^A$ (for A) or $\theta' \in \Theta^B$ (for B). The outcome alphabet Φ is assumed finite or countable and can be relabeled per observer if needed.

Effects (Heisenberg picture) are denoted:
$E^A_{\theta,\varphi} := M^{A*}_{\theta,\varphi}(I)$, $E^B_{\theta',\varphi'} := M^{B*}_{\theta',\varphi'}(I)$,
with $0 \le E \le I$ *and* $\sum_\varphi E^A_{\theta,\varphi} = I = \sum_{\varphi'} E^B_{\theta',\varphi'}$.

4.2 Frame Maps and Compatibility
Definition 4.1 (Frame map).
A frame map T from A to B consists of:
(1) A measurable map on contexts $T_\Theta: \Theta^A \to \Theta^B$.
(2) A map on outcomes $T_\Phi: \Phi \to \Phi$ (possibly context-dependent but assumed measurable and compatible with T_Θ).
(3) A structure-preserving map on effects T_E that sends the effect family for (θ, φ) to that for $(T_\Theta(\theta), T_\Phi(\varphi))$ in B's description:
$T_E(E^A_{\theta,\varphi}) = E^B_{T_\Theta(\theta), T_\Phi(\varphi)}$.
When needed, T_E is realized as a *-isomorphism between the von Neumann algebras generated by A's effects and B's effects on the shared world factor (e.g., implemented by a unitary or antiunitary on H_W), possibly tensored with identities on memories.

Definition 4.2 (Compatibility / joint measurability).
For a fixed proposition "(θ, φ) in A's frame corresponds to (θ', φ') in B's frame," the effects are compatible if
$[T_E(E^A_{\theta,\varphi}), E^B_{\theta',\varphi'}] = 0$.
Equivalently, there exists a joint POVM $\{F_{u,v}\}$ on the joint algebra such that
$T_E(E^A_{\theta,\varphi}) = \sum_v F_{\varphi,v}$ and $E^B_{\theta',\varphi'} = \sum_u F_{u,\varphi'}$. Compatibility guarantees that A- and B-statements about the same proposition can be assigned a joint probability.

Remark 4.1 (Scope of T).
We only require T on the subalgebra generated by the relevant effects. A global microscopic identification across all contexts is not needed for AB-fixedness on a specific proposition.

4.3 Records and Accessibility
Definition 4.3 (Record variable).
A record R for the proposition (θ, φ) is a random variable with values in Φ such that $R = \varphi$ almost surely on the event that A realizes outcome φ under context θ. Accessibility to B at tick k is the sigma-algebra requirement:
R is F_k^B-measurable,
where F_k^B is B's filtration at tick k (its internal trace possibly augmented by environment

reads).

Sources of records include:
(i) Direct access to A’s memory (communication or physical read).
(ii) Redundant environment encoding (e.g., environment fragments measured by B).
(iii) A shared classical register (H_E) written at or after A’s measurement.

4.4 AB-Fixedness from Frame Map, Compatibility, and Record
Theorem 4.4 (Agreement from shared record and compatibility).
Let A measure context θ and obtain outcome φ at some tick. Suppose there exists a frame map $T = (T_\Theta, T_\Phi, T_E)$ to B and a B-context $\theta' := T_\Theta(\theta)$, with the mapped effect $T_E(E^A_{\theta,\varphi})$ commuting with $E^B_{\theta', T_\Phi(\varphi)}$. Let R be a record for φ that is F_k^B-measurable at B’s tick k (k may be $\geq$ the tick of A’s event if records are delayed). Then, conditioning on F_k^B,
$P^B($ outcome in B’s frame equals $T_\Phi(R) \mid F_k^B) = 1$,
i.e., B assigns delta-certainty to the mapped outcome. We call this AB-fixedness.

Proof.
(1) Joint model: Compatibility yields a joint POVM $\{F_{u,v}\}$ such that probabilities for A’s mapped effect and B’s effect are marginals of a common distribution.
(2) Record conditioning: By definition of a record, on the event that A realized φ the random variable R equals φ almost surely; by accessibility, R is F_k^B-measurable. Since T_Φ is measurable, $T_\Phi(R)$ is F_k^B-measurable.
(3) Conditional probability: Conditioning on F_k^B fixes the value of R and hence $T_\Phi(R)$. Because the joint POVM supports a well-defined conditional probability for B’s outcome given the event “A’s mapped effect equals $T_\Phi(R)$,” and because the record pins that event almost surely, the conditional probability that B’s effect registers $T_\Phi(R)$ equals 1. Formally, the conditioning reduces to a point mass on the corresponding joint atom in the σ-algebra, yielding delta-certainty. ■

Corollary 4.5 (Direct trace sharing).
If B’s filtration includes A’s outcome as a variable R_A (e.g., a copy of A’s memory), then with compatibility and the frame map, AB-fixedness holds with $R = R_A$.

Corollary 4.6 (Delayed readability).
If R becomes F_ℓ^B-measurable only at a later tick $\ell \geq k$ (e.g., delayed communication or delayed environment read), AB-fixedness holds upon conditioning on F_ℓ^B.

Remark 4.2 (No-signaling caveat).
The theorem speaks to conditional certainty given access to a record and compatibility of effects. It does not assert that B can predict A’s outcome before the record is accessible, nor does it enable superluminal signaling; the conditioning is explicitly on B’s available sigma-algebra.

4.5 Operational Variants of Compatibility
Proposition 4.7 (Joint measurability as a sufficient condition).
If there exists a POVM G_{ξ} and classical post-processings (stochastic matrices) such that
$E^A_{\theta,\varphi} = \sum_{\xi} \alpha\{\varphi|\xi\} G_\xi$, $E^B_{\theta',\varphi'} = \sum_{\xi} \beta\{\varphi'|\xi\} G_\xi$,
then the effects are jointly measurable; Theorem 4.4 applies.

Proposition 4.8 (Quantum non-demolition case).
If $T_E(E^A_{\theta,\varphi})$ and $E^B_{\theta', T_\Phi(\varphi)}$ are projections onto a common pointer basis on the world factor (possibly tensored with identities on memories), they commute; AB-fixedness follows provided a record is accessible.

4.6 Minimality of Assumptions and Failure Modes (Pointers)
Each assumption in Theorem 4.4 is essential:

• Without a frame map: “same event” lacks precise meaning across observers; AB-fixedness is ill-posed.

• Without compatibility: a joint distribution need not exist; cross-conditionalization can be undefined.
• Without an accessible record: B has no F_k^B-measurable variable to condition on; certainty cannot be claimed.

Section 7 provides explicit counterexamples for each violation.

4.7 Policy-Independence of Agreement
Proposition 4.9 (AB-fixedness independent of future policies).
Under the conditions of Theorem 4.4, AB-fixedness is unaffected by the observers' subsequent adaptive policies. Once the record is present in F_k^B and compatibility holds for the mapped proposition, conditioning on F_k^B yields delta-certainty regardless of how f_{k+1}^A, f_{k+1}^B are chosen.

Proof.
AB-fixedness is a claim about the conditional distribution at sigma-algebra F_k^B. Future policy choices affect later kernels but not the conditional measure on the σ-algebra already generated. ■

4.8 Relation to Operator-Algebraic Language
Let $A_W \subset B(H_W)$ be the von Neumann algebra of world observables. Let $\mathbb{F}_k^A$, $\mathbb{F}_k^B$ be the algebras generated by each observer's trace up to tick k. A frame map may be implemented by a *-isomorphism $\alpha: A_W \to A_W$ with $\alpha(E^A_{\theta,\varphi}) = E^B_{\theta',\varphi'}$. Compatibility is $[\alpha(E), E'] = 0$. A record accessible to B corresponds to an element $r \in \mathbb{F}_k^B$ that spectrally decomposes to the event "$\varphi' = T_\Phi(\varphi)$." Conditional expectation onto $\mathbb{F}_k^B$ fixes r, yielding probability 1 on the corresponding projection. This recovers Theorem 4.4 in algebraic terms.

4.9 Summary of Section 4
We defined cross-observer agreement (AB-fixedness) as delta-certainty assigned by multiple observers to a mapped outcome when (i) a frame map relates their contexts and outcomes, (ii) the mapped effects are compatible (commuting or jointly measurable), and (iii) a record of the outcome is accessible within the conditioning observer's filtration. These conditions are sufficient and—per Section 7—essential. Section 5 develops the emergence of "objectivity" by showing how redundant records (spectrum broadcast structures) drive high-probability consensus among many observers.

Section 5. Objectivity via Redundancy (Spectrum Broadcast Structures)

5.1 Pointer Structure and Redundancy
We formalize "objectivity" as high-probability consensus among many observers who access disjoint environment fragments. The key structural ingredient is a spectrum broadcast structure (SBS).

Definition 5.1 (Pointer decomposition).
A system S has a pointer decomposition if there exist orthogonal states $\{|\psi_i\rangle\}$ and probabilities $\{p_i\}$ such that the joint state of S and m environment fragments $E_1,\ldots,E_m$ takes the classical–quantum form
$\rho_{S E_1\ldots E_m} = \sum_i p_i |\psi_i\rangle\langle\psi_i|_S \otimes (\bigotimes_{j=1}^m \rho_{E_j}^{(i)})$,
with $\rho_{E_j}^{(i)}$ pairwise distinguishable across i for each fixed j (exact SBS), or sufficiently distinguishable (approximate SBS; see 5.3).

Definition 5.2 (Redundancy and fragments).
A set $J \subseteq \{1,\ldots,m\}$ indexes "readable" fragments. Redundancy level is $|J|$. Observer A_j accesses E_j for $j \in J$ and performs a local readout instrument whose effects are compatible with the pointer basis on S (non-demolition of pointer value). Compatibility with the pointer basis ensures commutation as required by Section 4.

Remark 5.1 (Operational picture).
Each E_j carries a local "copy" of the classical label i. If copies are sufficiently distinguishable and local readouts are compatible, multiple observers can independently infer i without disturbing S or one another.

5.2 Local Readout Model and Compatibility

Assume each observer A_j applies a local instrument on E_j with effects $\{F^{(j)}_x\}$ that are functions of a common POVM $\{G^{(j)}_\xi\}$ jointly measurable with the spectral projections of $\{|\psi_i\rangle\langle\psi_i|_S\}$. Equivalently, for each j there exist classical channels $\alpha^{(j)}$ such that
$$F^{(j)}_x = \sum_\xi \alpha^{(j)}\{x|\xi\} G^{(j)}_\xi,$$
and the S-pointer projections commute with $G^{(j)}_\xi$ (quantum non-demolition on the pointer). By Section 4, these conditions guarantee compatibility between "S has pointer value i" and each local readout event.

5.3 Distinguishability Conditions

We quantify distinguishability across i at the fragment level using any of the following equivalent criteria (pick one and state it explicitly in applications):

(i) Exact SBS: for all j and $i \neq i'$, $\rho_{E_j}^{(i)} \rho_{E_j}^{(i')} = 0$ (orthogonal supports).
(ii) Trace-distance gap: for all j and $i \neq i'$, $\frac{1}{2} \| \rho_{E_j}^{(i)} - \rho_{E_j}^{(i')} \|_1 \geq \delta_j > 0$.
(iii) Quantum Chernoff exponent: for all j and $i \neq i'$, $\xi^{(j)}_{i,i'} := -\log \min_{s\in[0,1]} \mathrm{Tr}[(\rho_{E_j}^{(i)})^s (\rho_{E_j}^{(i')})^{1-s}] > 0$.

Independence across fragments is modeled by the product structure in Definition 5.1. Non-identical fragments are allowed; only positivity of per-fragment distinguishability is required.

5.4 Redundancy-Driven Consensus

Theorem 5.3 (High-probability consensus under SBS).
Let $\rho_{S E_1 \dots E_m}$ satisfy the SBS form of Definition 5.1 with distinguishability condition (i), (ii), or (iii). Let $J \subseteq \{1,\dots,m\}$ be the set of readable fragments; observers $\{A_j\}_{j\in J}$ each perform a compatible local readout on E_j and report an estimate $\hat{i}_j$. Then:

(a) (Individual reliability) For each $j \in J$ there exists a decision rule (e.g., Helstrom or MAP) such that $P(\hat{i}_j = i \mid i) \geq 1 - \varepsilon_j$ with $\varepsilon_j < 1/2$ determined by the chosen criterion (exact formulas in 5.5).
(b) (Majority consensus) Let $\hat{i}$ be the plurality/majority vote over $\{ \hat{i}_j : j\in J \}$. If the ε_j are uniformly bounded away from 1/2 and errors are conditionally independent given i (implied by the product structure), then
$$P(\hat{i} = i) \to 1 \text{ as } |J| \to \infty,$$
and the disagreement probability between any two observers vanishes:
$$P(\hat{i}_j \neq \hat{i}_k) \leq \exp(-c\, |J|) \text{ for some } c > 0 \text{ depending on } \{\varepsilon_j\}.$$

Proof sketch.
Under the product structure, local outcomes are i.i.d. or independent but not identical conditional on i. For binary i the optimal multi-fragment discrimination error decays exponentially with $|J|$ by the quantum Chernoff bound (5.5). A majority/maximum-likelihood estimator achieves the Chernoff rate asymptotically. For multi-hypothesis i, pairwise Chernoff exponents yield exponential decay of the MAP error; a union bound controls inter-observer disagreement. Compatibility guarantees that the joint distribution over reports is well-defined (Section 4). ■

Corollary 5.4 (AB-fixedness for many observers).
Fix any two observers A_j, A_k with access to fragments $j, k \in J$. Conditioning on the σ-algebra generated by their readouts and any accessible record of i, both assign delta-certainty to the same value of i with probability $\to 1$ as $|J|$ grows (and exactly 1 in the exact-SBS case with noiseless readout).

5.5 Finite-Redundancy Bounds (Binary case for clarity)

Assume $i \in \{0,1\}$ with priors p_0, p_1. For fragment j, define the Chernoff exponent
$\xi^{(j)} := -\log \min_{s\in[0,1]} \mathrm{Tr}[(\rho_{E_j}^{(0)})^s (\rho_{E_j}^{(1)})^{1-s}]$,
and set the aggregate exponent $\Xi_J := \sum_{j\in J} \xi^{(j)}$. Then the optimal global test on $\otimes_{j\in J} E_j$ satisfies
$P_{error}^{(J)} \le \tfrac{1}{2} \exp(-\Xi_J)$.
If fragments are identically distributed with $\xi^{(j)} = \xi > 0$, then
$P_{error}^{(J)} \le \tfrac{1}{2} \exp(-\xi |J|)$,
so $|J| \ge (1/\xi) \log(1/(2\varepsilon))$ suffices to achieve error $\le \varepsilon$. Majority vote with appropriate thresholds achieves the same exponent in the i.i.d. case.

Remark 5.2 (Trace distance bound).
A simpler but looser bound follows from the Helstrom formula using the trace distance of the product states; by subadditivity of log fidelity, the exponential scaling in $|J|$ persists under generalized (non-identical) fragments.

5.6 Noisy Readout and Approximate SBS

If readout instruments add classical noise, replace $\rho_{E_j}^{(i)}$ by the induced classical output distributions $q_j(\cdot|i)$; then Hoeffding/Chernoff bounds for classical hypothesis testing recover exponential consensus under the same redundancy. If the SBS is only approximate (trace-distance gap $\delta_j > 0$), the exponents degrade but remain positive; the same finite-$|J|$ bounds apply with adjusted constants.

5.7 Operational Objectivity Criteria

Definition 5.3 (Operational objectivity at level η).
For a tolerance $\eta \in (0,1)$, the pair $(\rho_{S E_1\ldots E_m}, \{A_j\}_{j\in J})$ exhibits objectivity at level η if there exists a decision rule for each A_j and (optionally) a majority rule such that:

(O1) Non-disturbance: local readouts commute with the pointer projections on S (compatibility).
(O2) Redundant accessibility: at least R fragments in J satisfy the chosen distinguishability criterion with positive exponent.
(O3) Consensus: $P(\hat i_j = \hat i_k$ for all j,k in some nontrivial subset $J'\subseteq J) \ge 1 - \eta$.

By 5.3–5.5, if $|J|$ is large enough to make the bound in 5.5 less than η, then (O1)–(O3) hold.

5.8 Relation to Section 4 (Records and Agreement)

In the redundant setting, each A_j's local readout serves as an accessible record variable in the sense of Section 4. When two observers also have a frame map connecting their effect labels, AB-fixedness follows upon conditioning on their σ-algebras. Thus, redundancy upgrades pairwise agreement into multi-observer objectivity with explicit finite-sample guarantees.

5.9 Limitations and Structural Dependencies

(i) Product structure (independence) is used to obtain exponential rates. Correlated fragments may still yield consensus but require mixing or correlation-decay assumptions.
(ii) Compatibility is essential; if local readouts disturb the pointer or mutually fail to commute, joint assignment of outcomes may be ill-defined (Section 7 provides counterexamples).
(iii) Redundancy must be strong enough (positive exponents). Weak redundancy does not ensure convergence (cf. Section 7).

5.10 Summary of Section 5

When the system–environment state exhibits (approximate) spectrum broadcast structure and observers employ compatible local readouts on disjoint fragments, independent evidence for the same pointer value accumulates. Standard distinguishability bounds imply that the probability of consensus approaches unity exponentially with the number of readable fragments. This elevates cross-observer agreement (Section 4) to operational objectivity with quantitative guarantees. Section 6 now introduces the collapse-frame geometry and its symmetry group, clarifying which relations among tick separation and channel changes are invariant under frame transformations.

Section 6. Collapse-Frame Geometry and Invariance

6.1 Motivation and Overview
Observers compare measurement events that differ both in tick separation (how many steps apart) and in channel choice (which instrument/context was used). This section introduces a simple geometry on the product of (discrete) tick index and (continuous) channel manifold that treats these two separations on a common footing. The resulting "collapse interval" is a signed quadratic form whose isometries define the transformations under which agreement and incompatibility relations are preserved—provided the effect maps also respect the operator algebra.

6.2 Channel Manifold and Metric
Let Θ be the channel (context) space introduced in Section 2, assumed to be a standard Borel manifold equipped with a Riemannian metric g. Typical choices include:
- A smooth parameter manifold with a Fisher-information metric (classical reads).
- A quantum statistical model endowed with an SLD quantum Fisher metric.
- A task-specific Riemannian metric g chosen to reflect operational channel dissimilarity.

For θ, θ' in Θ, write
$||\Delta\theta||_g^2 := g(\theta - \theta', \theta - \theta')$
to denote the squared geodesic (local) norm in a coordinate chart; when needed one may replace this by the squared geodesic distance for finite separations. We keep the local-norm notation for analytical clarity.

6.3 Collapse Interval (Signed Quadratic Form)
Definition 6.1 (Collapse interval).
Fix a time scale constant $T_c > 0$. For two measurement events (τ, θ) and (τ', θ'), define
$s_c^2((\tau, \theta), (\tau', \theta')) := (T_c)^2 (\Delta\tau)^2 - ||\Delta\theta||_g^2$
with $\Delta\tau := \tau - \tau'$ (τ, τ' integers) and $\Delta\theta := \theta - \theta'$ (in a chart). We call s_c^2 the collapse interval.

Remarks.
(i) Sign classification: $s_c^2 > 0$ ("time-like"), $s_c^2 = 0$ ("null"), $s_c^2 < 0$ ("channel-like").
(ii) T_c calibrates the relative scale of one tick versus a unit channel displacement under g.
(iii) Prior notational variants using $(iT)^2$ are equivalent up to sign convention; we adopt the positive constant T_c^2 for plain-text clarity.

6.4 Collapse-Frame Isometries
Definition 6.2 (Collapse-frame isometry group G_c).
Let G_c be the set of bijections $T: (\tau, \theta) \mapsto (\tau', \theta')$ on $\mathbb{Z} \times \Theta$ such that
$s_c^2((\tau, \theta), (\tau', \theta'))$ is preserved for all pairs of events.
Thus T is an isometry of the indefinite form $(T_c)^2 d\tau^2 - ||d\theta||_g^2$.

Subgroups and examples.
- Time translations: $(\tau, \theta) \mapsto (\tau + n, \theta)$, $n \in \mathbb{Z}$.
- Channel isometries: $(\tau, \theta) \mapsto (\tau, \phi(\theta))$, where ϕ is a Riemannian isometry of (Θ, g).
- Mixed "boost-like" transforms (Euclidean channel chart): If $\Theta \cong \mathbb{R}^n$ with constant metric g = I and τ continuous, the full isometry group is O(1, n). In our discrete-τ setting, admissible mixed transforms are those whose τ-component maps integers to integers and whose linear part preserves $(T_c)^2 d\tau^2 - ||d\theta||^2$. In practice, we restrict to block-diagonal (time translations × channel isometries) unless the model admits a rational lattice of contexts compatible with mixed maps.

6.5 Frame Maps with Algebraic Compatibility
Geometry alone does not act on effects. To compare observers, we pair a geometric isometry with an effect-level structure map:

Assumption C (Effect-level *-automorphism).

There exists a map T_E sending A's effects to B's effects such that
$T_E(E^A_{\theta, \varphi}) = E^B_{\theta', \varphi'}$
whenever $(\tau', \theta') = T(\tau, \theta)$ and $\varphi' = T_\Phi(\varphi)$, with T_Φ a measurable relabeling of outcomes. We assume T_E arises from a *-automorphism on the world-factor algebra (e.g., unitary/antiunitary conjugation) possibly tensored with identities on memory ancillas. Under such T_E, commutators are preserved:
$[T_E(X), T_E(Y)] = T_E([X, Y])$.

6.6 Invariants and Operational Content
Proposition 6.1 (Interval invariance).
If $T \in G_c$, then for any two events (τ_1, θ_1), (τ_2, θ_2) and their images (τ'_1, θ'_1), (τ'_2, θ'_2),
$s_c^2((\tau_1, \theta_1), (\tau_2, \theta_2)) = s_c^2((\tau'_1, \theta'_1), (\tau'_2, \theta'_2))$.
Hence the time-like/null/channel-like classification is frame-invariant.

Proposition 6.2 (Compatibility class preservation under T_E).
If E and F commute in A's frame, then $T_E(E)$ and $T_E(F)$ commute in B's frame. Conversely, if no joint POVM exists for (E, F), none exists for $(T_E(E), T_E(F))$. Thus compatibility/incompatibility is preserved under T_E.

Theorem 6.3 (AB-fixedness invariance under collapse-frame maps).
Let A and B be two observers related by a pair (T, T_E) with $T \in G_c$ and T_E satisfying Assumption C. Suppose AB-fixedness holds in A's description for a proposition "(τ, θ, φ)" with an accessible record (Section 4). Then the mapped proposition "$(\tau', \theta', \varphi')$" in B's frame is also AB-fixed.

Proof sketch.
AB-fixedness requires (i) a consistent identification of the proposition (frame map), (ii) compatibility/joint measurability of the mapped effects, and (iii) an accessible record in B's filtration. Assumption C ensures (ii) by preserving commutators; the record's accessibility is preserved by construction of the frame map on σ-algebras. Interval invariance is not, by itself, algebraic; its role is to guarantee that the geometric classification of event separations (e.g., "small channel jump over few ticks") is preserved, maintaining the structural preconditions under which compatibility and record propagation were arranged. ■

Corollary 6.4 (Stability of incompatibility statements).
If A certifies that two events are "channel-like separated" with $s_c^2 < 0$ and this correlates (by model design) with non-commuting effects beyond a threshold in $||\Delta\theta||_g$, then any B related by $T \in G_c$ preserves both the classification and (via T_E) the non-commutation claim.

6.7 Design Heuristics (When Geometry Guides Compatibility)
While compatibility is ultimately algebraic, it often correlates with channel distance. Two practical regimes:

(H1) Local-commutation regime.
There exists $d_* > 0$ such that if $||\Delta\theta||_g \le d_*$ and the instruments are pointer-preserving (QND), then effects commute. Time-like pairs with small $||\Delta\theta||_g$ are then amenable to agreement once a record is accessible.

(H2) Incompatibility regime.
There exists $d^\dagger > d_*$ such that if $||\Delta\theta||_g \ge d^\dagger$, selected effects fail to commute (no joint POVM). Pairs with $s_c^2 < 0$ arising from large $||\Delta\theta||_g$ are thereby structurally prone to disagreement absent redundancy or post-processing.

Isometries $T \in G_c$ preserve these threshold conditions when g is preserved and T_E is a *-automorphism.

6.8 Examples

Example 6.1 (Pure time translation).
$T(\tau, \theta) = (\tau + n, \theta)$ with T_E = identity. Interval is unchanged; records reachable at tick k remain reachable at tick k + n; AB-fixedness statements are unaffected.

Example 6.2 (Channel-space isometry).
Let ϕ be an isometry of (Θ, g). Set $T(\tau, \theta) = (\tau, \phi(\theta))$ and choose T_E implemented by a unitary U on the world factor such that $U^\dagger E^A_{\theta, \varphi} U = E^B_{\phi(\theta), T_\Phi(\varphi)}$. Then distances $||\Delta\theta||_g$ and commutators are preserved, so agreement and incompatibility claims transfer verbatim.

Example 6.3 (Boost-like transform in Euclidean chart).
If $\Theta \cong \mathbb{R}^n$ with g = I and modelers allow a rational lattice of contexts aligned with ticks (to preserve $\tau \in \mathbb{Z}$), one may define linear maps that mix τ and θ, preserving $(T_c)^2 d\tau^2 - ||d\theta||^2$. When accompanied by a matching T_E, AB-fixedness and incompatibility claims remain invariant. In practice, we recommend restricting to block-diagonal transforms unless such mixed maps are explicitly supported by the experimental/control design.

6.9 Limitations
(i) Discreteness of τ restricts the admissible mixed isometries; the safe, ubiquitous subgroup is time translations × channel isometries.
(ii) The geometry does not by itself enforce commutation; invariance of compatibility requires the algebraic map T_E.
(iii) Channel metrics derived from information geometry may be only locally valid; for large separations, use geodesic distances or piecewise-local charts with care.

6.10 Summary of Section 6
We introduced a signed collapse interval on the product of tick index and channel manifold and defined the collapse-frame isometry group as the transformations preserving this interval. When paired with an effect-level *-automorphism, these transformations preserve the ingredients required for cross-observer agreement (AB-fixedness) and for the stability of incompatibility claims. Time translations and channel-space isometries form a robust, widely applicable subgroup; mixed transforms are optional and model-dependent. Section 7 will show, via counterexamples, that relaxing the key ingredients (measurability, compatibility, redundancy, isometry) causes the overall structure to fail in specific, diagnosable ways.

Section 7. Counterexamples and Failure Modes

7.1 Non-measurable Policies (failure of existence/uniqueness)
Purpose. To show that measurability of the adaptive policy (Assumption A3) is essential for constructing the global law P on Φ^∞.

Setting. Let Φ be any countable outcome alphabet (hence Φ^k and Φ^∞ are standard Borel spaces with their product σ-algebras). Fix k = 2. Choose a subset $A \subset \Phi^1$ that is not Borel (non-measurable with respect to the product sigma-algebra on Φ). Define two distinct contexts θ_a, $\theta_b \in \Theta$ with distinct instruments (or same instrument but tagged differently).

Counterexample. Define the policy
$f_2(\varphi_1) := \{ \theta_a \text{ if } \varphi_1 \in A; \theta_b \text{ otherwise} \}$.
Because A is non-Borel, f_2 is not measurable. Then the tick-2 kernel
$K_2(\varphi_1, \varphi) = \mathrm{Tr}[M_{f_2(\varphi_1), \varphi}(\rho_{2^-}(\varphi_1))]$
fails to be a measurable function of φ_1. The Ionescu–Tulcea extension theorem cannot be applied, so no unique probability measure on Φ^∞ consistent with $\{K_k\}$ exists. Thus, dropping measurability breaks the very existence/uniqueness of the adaptive observer process.

Variant (uncountable first tick). If Assumption A1 is also relaxed at tick 1 (e.g., $\varphi_1 \in [0,1]$ with Lebesgue σ-algebra), one can choose a Vitali-type non-measurable set $A \subset [0,1]$ and repeat the construction. Either way, non-measurable f_k blocks the kernel construction.

7.2 Non-commuting Effects (failure of AB-fixedness)
Purpose. To show that compatibility (commutation or joint measurability) is necessary for cross-observer agreement (Section 4).

Setting. Single qubit on H_W with computational basis $\{|0\rangle, |1\rangle\}$. Let A's mapped effect be the projector
$E^A := |0\rangle\langle 0|$,
and let B's effect (in B's own frame) be
$E^B := |+\rangle\langle +|$, where $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

Observation. $[E^A, E^B] \neq 0$. There is no joint POVM whose marginals are E^A and E^B (they are not jointly measurable projectors). Therefore, even if a "record" of A's outcome exists in principle, B cannot assign delta-certainty to "the same proposition" because the joint distribution for the pair of propositions is undefined. AB-fixedness fails precisely because the compatibility hypothesis is violated.

7.3 Weak or Absent Redundancy (failure of objectivity)
Purpose. To show that redundancy (SBS-type structure) is necessary for high-probability consensus among many observers (Section 5).

Setting. Consider S with pointer states $\{|\psi_i\rangle\}$ and two environment fragments E_1, E_2. Let the joint state be
$\rho_{S E_1 E_2} = \sum_i p_i |\psi_i\rangle\langle\psi_i|_S \otimes \rho^{(i)}_{E_1} \otimes \rho_{E_2}$,
where ρ_{E_2} is independent of i (carries no information), while $\{\rho^{(i)}_{E_1}\}$ are distinguishable.

Consequence. An observer A_1 reading E_1 can infer i with low error; an observer A_2 reading E_2 gains no information beyond the prior $\{p_i\}$. Thus,
$P(\hat i_1 = \hat i_2) = \sum_i p_i \cdot P(\hat i_1 = i) \cdot P(\hat i_2 = i \mid i)$
reduces to $\sum_i p_i \cdot P(\hat i_1 = i) \cdot p_i$, which does not approach 1 even if A_1's accuracy is near perfect. With multiple "uninformative" fragments, majority vote can still fail: the votes of informative observers can be drowned by noise unless the fraction of informative fragments stays bounded away from zero. Hence, redundancy must be sufficiently strong (positive per-fragment exponents on a non-vanishing fraction of fragments); otherwise objectivity (consensus $\to 1$) does not emerge.

7.4 Non-isometric Frame Maps (failure of invariance)
Purpose. To show that preserving the collapse interval and the effect algebra is critical for the stability of agreement/incompatibility statements across frames (Section 6).

Case A (distortion of channel distance).
Let Θ carry metric g with a commutation heuristic: effects drawn from contexts within distance $\leq d_*$ commute (QND regime), while beyond $d^\dagger > d_*$ they typically fail to commute. Suppose a map $\tilde T: \Theta \to \Theta$ compresses distances so that $||\Delta\theta||_g \geq d^\dagger$ may be mapped to $||\Delta\tilde\theta||_g \leq d_*$ (not an isometry). Then pairs that were incompatible in A's frame appear "within the commuting neighborhood" in B's frame, contradicting algebraic reality unless $\tilde T$ is accompanied by an effect-level *-automorphism that also changes the operators. If $\tilde T$ acts only geometrically (no matching T_E), B's compatibility judgments are wrong, and AB-fixedness claims can be fabricated or destroyed by the map.

Case B (breaking the interval classification).
Let $T: (\tau, \theta) \mapsto (\tau', \theta')$ fail to preserve s_c^2. A pair that was time-like (small $||\Delta\theta||_g$ over a few ticks) can become "channel-like" (large $||\Delta\theta'||_g$ relative to $\Delta\tau'$) in B's description, invalidating design assumptions (e.g., record propagation schedules, QND windows) used to justify compatibility and record accessibility. Agreement that held in A's operational regime need not transfer to B's—purely because the geometric preconditions were destroyed.

7.5 Consolidated Takeaways
(i) Measurability of policies is indispensable: non-measurable f_k prevents constructing the global law on Φ^∞.
(ii) Compatibility (commutation or joint measurability) is non-negotiable for AB-fixedness: without it there is no joint probability for "the same proposition" in two frames.
(iii) Redundancy is the engine of objectivity: if too few fragments carry reliable information, consensus cannot concentrate, regardless of the number of observers.
(iv) Isometries and effect-algebra maps must travel together: geometry alone cannot secure invariance of agreement; algebraic structure (via *-automorphisms) is required, and interval preservation guards the operational preconditions under which compatibility and record access were arranged.

7.6 Summary of Section 7
We provided explicit counterexamples showing how the main results fail when any of the core assumptions are dropped: measurability (A3), compatibility/joint measurability, redundancy/SBS-type structure, and collapse-frame isometries paired with effect-level *-automorphisms. These failures are not technicalities; they delimit the precise domain in which the formal theory guarantees internal certainty, cross-observer agreement, and emergent objectivity. Section 8 turns to implications, limitations, and concrete avenues for extension and application.

Section 8. Discussion and Implications

8.1 Summary of Results
This paper formalized self-referential observers as adaptive stochastic processes internal to standard quantum mechanics. Under minimal regularity, the process over infinite outcome histories exists and is unique; the observer's past becomes delta-certain in its own filtration, and adaptive policies induce latching (branch-dependent irreversibility). Cross-observer agreement (AB-fixedness) arises when a frame map aligns propositions, mapped effects are compatible, and an accessible record is present. In multi-observer settings, spectrum broadcast structures (or approximate variants with positive distinguishability exponents) drive high-probability consensus. A simple geometry on tick index × channel manifold yields an invariant "collapse interval"; together with a *-automorphism on effects, it preserves agreement and incompatibility statements across frames. Counterexamples demonstrate necessity of measurability, compatibility, redundancy, and isometry assumptions.

8.2 Conceptual Implications

1. Collapse as internal certainty.
   "Collapse" is here the observer's conditional certainty about its recorded past, not a new dynamical axiom. This reframing keeps all dynamics inside the orthodox CP-map formalism while explaining why an adaptive agent experiences irreversible branching.

2. Agreement without new postulates.
   Cross-observer agreement is not assumed; it emerges when compatibility and records are present. Objectivity is a redundancy phenomenon with quantitative error bounds, consistent with decoherence-based accounts but formulated at the level of instruments, filtrations, and kernels.

3. Frame-relational structure.
   The collapse-frame geometry does not replace algebra; it organizes when compatibility hypotheses tend to hold and how operational regimes (small channel displacement over few ticks) remain meaningful across frames. Invariance results are explicitly conditional on the effect-level *-automorphism.

8.3 Methodological Implications

1. Separation of roles.

Geometry classifies separations; algebra decides commutation; probability and filtration handle certainty and agreement. Keeping these layers distinct avoids category errors (e.g., inferring compatibility from geometry alone).

2. Filtration-centric proofs.
   The filtration F_k is the central mathematical object: all "certainty" claims reduce to conditioning on F_k-like σ-algebras, and agreement reduces to record measurability within another observer's σ-algebra.

3. Minimal probabilistic machinery.
   Existence and uniqueness rely on standard kernel measurability and Ionescu–Tulcea; no exotic measure theory is required as long as Φ is countable and policies are measurable.

8.4 Engineering and Computational Uses

1. Adaptive quantum control.
   Policies f_k can encode feedback rules that choose measurement settings contingent on prior outcomes. The latching property clarifies when feedback creates irreversible path dependence—useful in sequential hypothesis testing, error syndromes, and adaptive tomography.

2. Quantum-classical hybrid agents.
   The framework directly supports classical supervisors controlling quantum instruments via history-dependent policies, with guarantees about what can be inferred from records and when independent controllers will agree.

3. Simulation scaffolds.
   Because the observer is a kernel-driven process, classical simulators can realize the formal object and explore policy classes, trade-offs between redundancy and measurement disturbance, and geometry-aware scheduling (e.g., staying within compatibility neighborhoods).

4. Verification templates.
   For experimental designs, the AB-fixedness theorem provides a checklist: specify the frame map, verify compatibility (or joint measurability), and implement record channels whose measurability in the target filtration can be audited.

8.5 Limitations

1. Discrete ticks.
   We model observation at discrete times. Continuous-time limits are not treated here; doing so would require quantum stochastic calculus or continuous-time filtering, plus a continuous version of the extension theorem.

2. Geometry is auxiliary.
   The collapse interval organizes operational regimes but does not enforce algebraic facts. Without a corresponding *-automorphism on effects, geometric isometries alone are insufficient for invariance of agreement.

3. Independence assumptions in redundancy.
   Exponential consensus rates rely on product (or weakly dependent) fragment models. Strong correlations may require refined bounds or structural assumptions (mixing, clustering).

4. Scope of instruments.
   We assume trace-preserving instruments with countable outcome sets; while standard in many settings, some experimental platforms use uncountable outcome spaces (e.g., continuous homodyne currents), which require technical extensions.

5. Proof granularity.
   Some results are stated with proof sketches (e.g., large-deviation bounds for non-identical fragments). Full technical details are delegated to the references and can be expanded in an appendix if desired.

8.6 Open Problems and Extensions

1. Continuous-time observers.
   Formulate the observer as a quantum filtering problem with an adapted policy f_t depending on the path σ-algebra, and prove existence/uniqueness via continuous-time extension theorems; connect to Belavkin/quantum trajectories.

2. Beyond countable Φ.
   Extend to continuous outcome spaces (e.g., Gaussian POVMs), ensuring kernel measurability and Radon regularity; revisit delta-certainty in terms of almost-sure atomicity of realized outcomes.

3. Stronger invariance theory.
   Characterize the maximal subgroup of collapse-frame isometries compatible with a given channel metric g and an effect-algebra *-automorphism; identify conditions under which the group reduces to a Lorentz-like O(1,n).

4. Joint measurability frontiers.
   Quantify thresholds in $||\Delta\theta||_g$ where joint measurability breaks down for families of instruments (e.g., qubit Pauli families under noisy channels); connect geometric distance to incompatibility witnesses.

5. Correlated redundancy.
   Develop consensus bounds under correlated environment fragments (Markov fields or finitely correlated states), and identify minimal redundancy structures that still guarantee objectivity.

6. Learning policies.
   Study policy classes learned from data (e.g., reinforcement learning over instrument choices) and prove stability/regularity conditions that preserve existence, latching control, and AB-fixedness.

7. Resource trade-offs.
   Optimize redundancy vs. disturbance vs. latency: how many fragments and of what quality are needed to achieve a target consensus level under compatibility and timing constraints?

8. Empirical tests.
   Design table-top experiments where two independent observers read disjoint environment channels of a photonic or solid-state system and validate finite-$|J|$ consensus scaling and AB-fixedness under controlled compatibility toggles.

8.7 Relation to Interpretations and Prior Accounts
The present account is interpretation-neutral: it neither posits nor denies ontic collapse. It recovers the phenomenology typically attributed to “collapse” and “objectivity” using only instruments, filtrations, and redundancy. It complements decoherence and quantum Darwinism by providing filtration-level certainty and explicit agreement conditions, and interfaces cleanly with process-tensor and operator-algebraic languages frequently used in foundations and control.

8.8 Outlook
Two practical avenues appear most promising. First, continuous-time generalizations would connect the present discrete kernel construction to quantum filtering and control theory, enabling rigorous treatment of continuous measurements and feedback. Second, a systematic

study of geometry–algebra couplings—how channel metrics, compatibility neighborhoods, and *-automorphisms co-determine invariant agreement classes—could yield design rules for robust multi-observer inference in noisy devices. The filtration-centric viewpoint suggests a unifying path: specify what each agent can condition on (records), ensure compatibility of the queried propositions, and use redundancy to elevate pairwise agreement to operational objectivity with quantifiable guarantees.

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(Where a classical theorem is cited via a modern text, the text provides a precise statement and proof of the version used here.)

Appendix A. Notation and Definitions Map

Spaces and States
H_W: Hilbert space of the world (observed system).
H_O: Hilbert space of the observer's memory/ancilla.
H := H_W ⊗ H_O: composite space for joint evolution.
D(H): density operators on H; B(H): bounded operators on H.

Time and Filtration
τ_k := k: discrete tick index (Δτ = 1 by convention).
E_k: between-tick CPTP evolution; ρ_{k^-} := E_k(ρ_{k-1}).
φ_k ∈ Φ: outcome at tick k; Φ finite or countable.
φ_{1:k}: outcome prefix up to k.
F_k := σ(φ_1,...,φ_k): observer's filtration after tick k.

Instruments and Effects
Θ: context (channel) space, standard Borel.
M_{θ,φ}: CP, trace–nonincreasing map for outcome φ under context θ.
∑φ M{θ,φ}: CPTP (instrument normalization).
E_{θ,φ} := M^*_{θ,φ}(I): effect operator; ∑φ E{θ,φ} = I.

Policies and Kernels
f_k: Φ^{k-1} → Θ: measurable adaptive policy; θ_k := f_k(φ_{1:k-1}).
K_k(φ_{1:k-1}, φ) := Tr[M_{θ_k,φ}(ρ_{k^-})]: Born-rule kernel.
P on Φ^∞: global law constructed by Ionescu–Tulcea.

Agreement and Redundancy
Frame map T: (T_Θ on contexts, T_Φ on outcomes, T_E on effects).
Compatibility: [T_E(E^A_{θ,φ}), E^B_{θ',φ'}] = 0 (or joint measurability).
Record R: random variable equal to φ on A's event; F_k^B-measurable.
SBS: ρ_{S E_1...E_m} = ∑_i p_i |ψ_i><ψ_i|_S ⊗ ⊗j ρ^{(i)}{E_j}, with fragmentwise distinguishability.

Geometry
g: Riemannian metric on Θ (e.g., information metric).
||Δθ||_g^2: local squared norm (or squared geodesic distance).
s_c^2 := (T_c)^2 (Δτ)^2 − ||Δθ||_g^2: collapse interval.
G_c: isometry group preserving s_c^2 on ℤ × Θ.

Properties
Delta-certainty: P(φ_j = realized value | F_k) = 1, j ≤ k.
Latching: policy dependence on φ_{1:k} yields branch-dependent divergence of future contexts.

Appendix B. Axioms → Theorems Chart (One Page)

Axioms / Assumptions
(A1) Θ standard Borel; Φ finite/countable.
(A2) Regularity: (θ,σ) ↦ Tr[M_{θ,φ}(σ)] measurable/continuous as needed.
(A3) Policy measurability: f_k: Φ^{k-1} → Θ measurable.
(A4) Between-tick CPTP evolution: ρ_{k^-} := E_k(ρ_{k-1}).
(A5) Instrument normalization: ∑φ M{θ,φ} CPTP; ∑φ E{θ,φ} = I.
(C) Effect-level map T_E is a *-automorphism on the relevant world-algebra factor when comparing frames.

Results
(R1) Measurable kernels (Lemma 3.1) + state recursion measurability (Lemma 3.2).
(R2) Existence & uniqueness of P on Φ^∞ (Theorem 3.4) via Ionescu–Tulcea.
(R3) Internal delta-certainty (Proposition 3.6): fixedness of past outcomes in F_k.

(R4) Latching irreversibility (Proposition 3.8) from policy dependence.

(R5) AB-fixedness (Theorem 4.4): frame map + compatibility + accessible record ⇒ delta-certainty across observers.

(R6) Redundancy ⇒ objectivity (Theorem 5.3): SBS/distinguishability + compatible local readouts ⇒ consensus with exponential error decay.

(R7) Collapse-frame invariance (Theorem 6.3): s_c^2 isometry + *-automorphism T_E preserve agreement/incompatibility.

(R8) Failure modes (Section 7): violating A3, compatibility, redundancy, or isometry destroys R2/R5/R6/R7 respectively.

Appendix C. Minimal Simulation Outline (Plain Text)

Goal

Numerically explore adaptive policies, latching, AB-fixedness with records, and redundancy-driven consensus, using a classical simulator that implements the kernel-and-update rules.

C.1 Data Structures

State representation: density matrices for small systems (arrays), or abstract labels with update functions for toy models.

Instrument library: for each $\theta \in \Theta$, maps $\varphi \mapsto$ (probability, post-state).

Policy: function $f_k(\text{trace\_prefix}) \to \theta$.

Trace: dynamic list of realized outcomes $\varphi_{1:k}$.

Environment fragments (optional): list of fragment states $\rho^{(i)}_{E_j}$ and readout channels.

C.2 Tick Loop (Single Observer)

Initialize ρ_0. For $k = 1..K$:

1. Between-tick: $\rho_{k^-} := E_k(\rho_{k-1})$.
2. Context: $\theta_k := f_k(\varphi_{1:k-1})$.
3. Sample outcome φ_k with probabilities $p_k(\varphi) = \mathrm{Tr}[M_{\theta_k,\varphi}(\rho_{k^-})]$.
4. Update: $\rho_k := M_{\theta_k,\varphi_k}(\rho_{k^-}) / p_k(\varphi_k)$.
5. Append φ_k to trace.
6. (Optional) Log $||\Delta\theta||_g$ relative to previous context to inspect geometry-aware scheduling.

C.3 AB-Fixedness Check (Two Observers)

• Define frame map $T = (T_\Theta, T_\Phi, T_E)$.

• Ensure compatibility: verify $[T_E(E^A_{\theta,\varphi}), E^B_{\theta',\varphi'}] = 0$ numerically on the chosen test instances.

• Implement a record channel: e.g., copy outcome φ to a classical register accessible to B at tick $k' \geq k$.

• Condition on B's filtration at k': verify that B's posterior over $T_\Phi(\varphi)$ is a point mass (empirically, probability 1 within numerical tolerance).

C.4 Redundancy and Consensus

• Generate SBS-like data: choose pointer i, draw each fragment j's sample via $\rho_{E_j}^{(i)}$ and readout channel.

• Observer j outputs $\hat i_j$ using MAP or Helstrom decision rule.

• Aggregate across $|J|$ observers; record consensus rate vs. $|J|$.

• Plot log error vs. $|J|$ to confirm approximate Chernoff slope.

C.5 Latching Demonstration

• Define a policy f_{k+1} that switches context depending on φ_k.
• Simulate branching runs to show divergence in the distribution of future contexts and outcomes conditioned on different φ_k.

C.6 Reproducibility Hints
• Fix random seeds for pseudorandom sampling.
• Store all instrument parameters and policies in a configuration file.
• Export traces and summaries (agreement rate, consensus error, ||Δθ||_g statistics) as CSV for external plotting.

Disclaimer

This article is the product of a collaboration between the author and OpenAI's GPT4o, GPT-5 language models. While every effort has been made to ensure accuracy, clarity, and insight, the content is generated with the assistance of artificial intelligence and may contain factual, interpretive, or mathematical errors. Readers are encouraged to approach the ideas with critical thinking and to consult primary scientific literature where appropriate.

This work is speculative, interdisciplinary, and exploratory in nature. It bridges metaphysics, physics, and organizational theory to propose a novel conceptual framework—not a definitive scientific theory. As such, it invites dialogue, challenge, and refinement.

I am merely a midwife of knowledge.